TABLES FOR SOLUTION OF THE HEAT-CONDUCTION EQUATION WITH A TIME-DEPENDENT HEATING RATE

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Department of Mechanical Engineering
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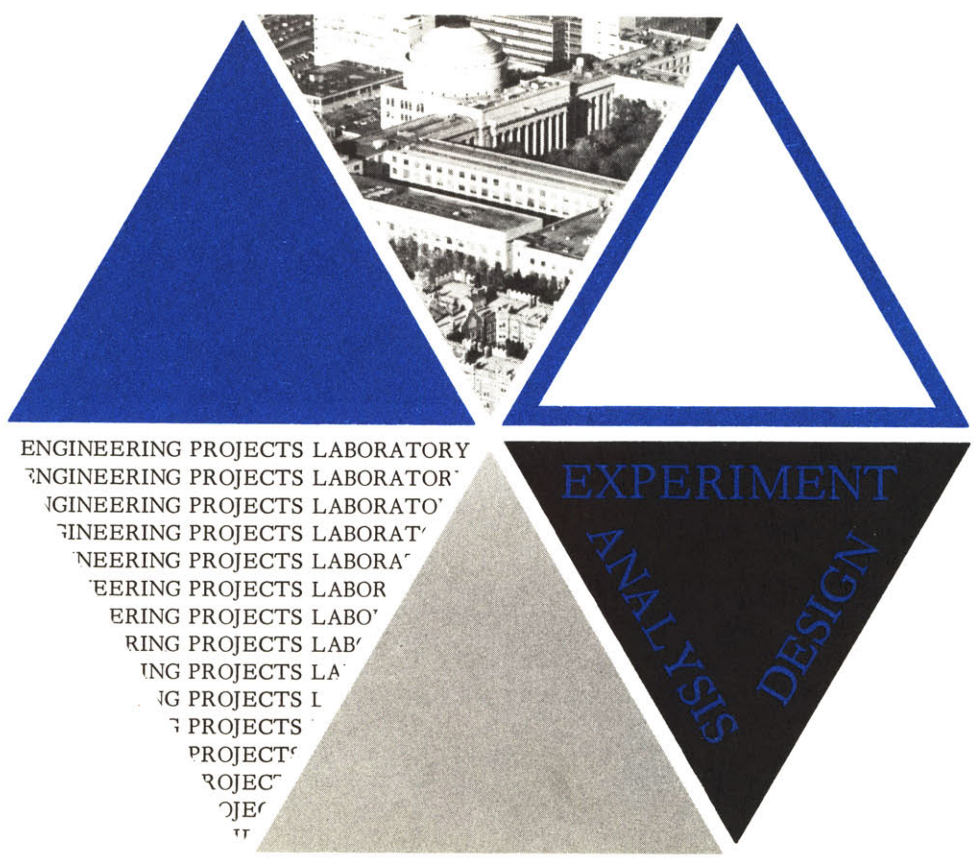

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TABLES FOR SOLUTION OF THE HEAT-CONDUCTION EQUATION WITH A TIME-DEPENDENT HEATING RATE

by

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ABSTRACT

Tables are presented for the solution of the transient one-dimensional heat flow in a solid body of constant material properties with the heating rate at one boundary dependent on time. These tables allow convenient and rapid estimation of the temperature distribution in the many practical cases where the mathematical model applies. Examples illustrating use of the tables are given.

INTRODUCTION

An important case of the heat-conduction equation concerns the large-slab geometry where one boundary is insulated and the heat input at the other boundary is an arbitrary function of time. This mathematical model approximates, for example, the aerodynamic heating of a flight vehicle re-entering the earth's atmosphere. Complex problems of this nature are normally handled by numerical integration and employ large-scale digital computers. However, the design engineer needs to determine rapidly the transient non-uniform temperature distribution in the skin of such a vehicle for any specified flight condition. In this respect the analytical solution offers considerable advantage over the numerical procedure as the entire heating history need not be considered to obtain the temperature distribution at any instant of time.

There has been considerable interest in this problem in recent years, and various solutions have been presented for the transient one-dimensional heat flow in a solid body of constant material properties with the heating rate at one boundary dependent on time.

This is a linear boundary-value problem which readily yields its solution to any of the elegant methods available for treating such problems. The solution for a constant flux into the solid is given in the standard work of Carslaw and Jaeger[1]*. A small-scale chart gives this infinite series solution over a range of parameters. Carslaw and Jaeger also give the solution for a time-dependent flux which can be expressed

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*Numbers in brackets denote References at end of paper.

as a single term. These single-term-flux solutions are, however, limited in their usefulness as most actual heating rates are more complex functions of time.

Sutton [2, 3] and Chen [4] presented the solution in integral form for an arbitrary heating rate. However, these integrals can be very difficult to evaluate depending on the form of the heating rate. This type of solution is only of academic interest to the engineer who seeks a convenient and rapid solution. Even the solutions given in reference [1] for certain time variations of the heat input are of limited practical usefulness as the computation labor involved in evaluating the infinite series is a major burden.

Solutions for a more general variation of the heat flux have been presented. Sutton [2] give a solution for a polynomial variation of the heat input with time. Certain terms in the solution were neglected so that the temperature distribution could be given as a polynomial in time and position. This reference includes an involved table of coefficients which can be used to calculate the desired temperatures. The solution is, however, accurate only for large times due to simplification of the solution.

A polynomial variation of the heat input was also considered by Bergles and Kaye [5] . The exact solution was given in terms of the infinite series of the repeated integrals of the error functions. The design charts given by Kaye and Yeh [6] can be used to rapidly estimate these infinite series. Such graphical solutions conserve on space; however, a limited range of parameters is considered, and their accuracy is limited to two significant figures.

The polynomial time variation of the heating rate is sufficiently general to be of considerable practical interest. In view of the fact that available graphical and tabular solutions are inaccurate and inconvenient to use, accurate tabular solutions were prepared based on the analysis of reference [5] .

ANALYSIS

Consider transient one-dimensional heat flow in a solid body of constant material properties with the heating rate at one boundary dependent on time for the slab shown in Fig. 1. The following solution is presented in quite general terms by consideration of heat generation and initial temperature distribution.

The general differential equation, assuming a heat-generation term of the form

$$P(N) = K + M N^2 \qquad (1)$$

is given by

$$\partial^2 \tau (N,X)/ \partial N^2 - \partial \tau (N,X)/ \partial X + K + MN^2 = 0 \qquad (2)$$

The initial temperature distribution is assumed to be represented by an even order polynomial of the form

$$\tau (N,0) = FN^2 + GN^4 \qquad (3)$$

The boundary conditions are as follows:

$$\partial \tau (0,X)/ \partial N = 0 \qquad (4)$$

$$\partial \tau (1,X)/ \partial N = Q(X) \qquad (5)$$

where the heat flux in Eq. (5) can be expressed as a polynomial of (s + 1) terms

$$Q(X) = \sum_{s=0}^{s} H_s X^s \qquad (6)$$

Equations (2) to (6) were solved by means of the Laplace transformation. The solution for the temperature distribution is given by

$$\begin{aligned} \tau(N,X) = & \sum_{s=0}^{s} H_s X^s Z_{2s+1} - (2F + 4G)Z_1 - 24GXZ_3 + \\ & (2F + K)X + (12G + M)X^2 + (M + 12G)XN^2 + FN^2 + GN^4 \end{aligned} \qquad (7)$$

where

$$\begin{aligned} Z_{2s+1} \equiv 2^{2s+1} s! X^{1/2} \sum_{r=0} \Big\{ & i^{2s+1}\mathrm{erfc} \left[(2r + 1 - N)/2X^{1/2} \right] + \\ & i^{2s+1}\mathrm{erfc} \left[(2r + 1 + N)/2X^{1/2} \right] \Big\} \end{aligned} \qquad (8)$$

It is seen that the solution to Eq. (2) can be simply represented as a polynomial in the functions of time and position defined by Eq. (8). The transient temperature distribution can be readily computed if the various temperature distribution functions are computed and arranged in tabular form.

COMPUTATION OF TEMPERATURE DISTRIBUTION FUNCTIONS

The infinite series of the repeated integrals of the error function, which comprises the temperature distribution function given by Eq. (8), was evaluated using the IBM 704 digital computer.

The repeated integrals of the error function were taken from the tables of Kaye [7]. The tables, together with appropriate differences, were stored in the machine. The Everett central-difference interpolation method was chosen as it is more accurate and converges more rapidly than the forward-difference methods such as those of Newton or Gauss.

Values of the index s were chosen from 0 to 5, and the position ratio N varied from 0 to 1.0 in steps of 0.2. The values of relative time X were chosen over a range of 0 - 40 so that linear interpolation could be used in the tables. Linear interpolation is valid throughout all but two per cent of the final tables which are presented in the Appendix.

APPLICATIONS

The problem of the slab with prescribed heat flux at its surface is of increasing importance in technical applications. The general requirement for employment of the present model is that the heat flux be independent of the temperature of the body. Heat can be supplied, for example, by a flat heater embedded in the solid; in this case there is no loss of heat at the surface, and the boundary condition is accurately satisfied if the thermal capacity of the heater is negligible. The boundary condition is also satisfied for a flight vehicle re-entering the earth's atmosphere where the allowable surface temperature is small compared with the gas temperature and can be neglected.

A special case of the present solution was derived in reference [6] for use in estimating the transient temperature distribution in a wedge-shaped wing flying at supersonic speeds. The flux variation was obtained from specified time variation of the surface coefficient of heat transfer and of the temperature difference between the adiabatic wall and the wall. The assumption of a time variation of the temperature difference is, however, equivalent to specifying the solution to the equation. The close agreement noted in that investigation between the analytical and numerical results is, therefore, to be expected since information from the numerical solution was used as a boundary condition for the analytical solution. An analytical solution for aerodynamic heating using a heat-transfer coefficient and adiabatic-wall temperature which are time dependent is given in reference [8]. The complexity of this solution is so great, however, that it is apparent that the numerical approach is more desirable for this type of aerodynamic heating problem.

The present solution is applicable to situations where the temperature gradient in all but one direction can be neglected. The temperature distribution for certain simple two- and three-dimensional geometries, such as the brick-shaped solid, can be treated using the tables and the standard product solution technique of Newman.

The tables have sufficiently fine intervals at low values of the relative time so that the very thick slabs or semi-infinite solids can be readily considered. Only the large values of the position ratio are used in this case.

The inverse problem arises when the surface heat flux versus time is sought from knowledge of an interior temperature versus time. Stolz [9] presented one of the few general treatments of the subject and developed a numerical inversion method. The present analytical method is applicable to all cases where the heat flux can be expressed as a polynomial and where the initial temperature is uniform.

The interior temperature at a location near the surface is monotored as a function of time. The values of N and X are readily calculated if the properties are known and the corresponding Z are obtained from the tables. The coefficients of the surface heat flux are then solved from the series of simultaneous equations derived from the data.

$$H_o Z_1 + H_1 Z_3 X + \ldots\ldots = t - t_b$$

$$H_o Z_1' + H_1 Z_3' X' + \ldots\ldots = t' - t_b$$

etc.

EXAMPLES

Satellite Re-entry

The temperature distribution in the skin of a flight vehicle can be readily estimated by the present method for an important case of aerodynamic heating. The solution given by Eq. (7) is valid for a situation where the heat input to a body can be determined independent of the surface temperature. This occurs, for example, when a vehicle re-enters the earth's atmosphere as its allowable surface temperature is small compared with the gas temperature and can be neglected [10]. For hypersonic re-entry velocities the heat input is determined primarily by the solution to the equation of motion.

Scala [11] presents the aerodynamic heating rate for the ballistic re-entry of a satellite from an initial orbit of 900,000 feet. The ballistic parameter $W/C_D A_D$ is chosen to be 200 lb/ft^2, and the re-entry velocity is approximately 24,000 ft/sec at a path angle of 92.5 degrees from the local vertical. The heating rate obtained in that study is presented in Fig. 2. Figure 3 includes one-dimensional transient heat-conduction calculations used in the heat sink section of Scala's work.

The temperature profiles were obtained by numerical analysis involving use of a large digital computer. The temperature distribution will now be approximated by means of the present method.

The heating rate to the satellite as given in Fig. 2 is first expressed as a polynomial in time. A simple four-point curve fit suffices to give a good representation of the temporal variation of the heating rate.

$$q/A = 1.76\text{x}10^{-4}\ \theta^3 - 9.887\text{x}10^{-3}\ \theta^2 + 0.5711\theta \qquad (9)$$

The thickness of the material is five inches. The product of ρc_p is 44 Btu/ft^3 ^{o}F, and the thermal conductivity is assumed to be 43.2 Btu/hr ft ^{o}F[12]. Using these properties the heating rate is expressed in the form of Eq. (6).

$$Q = 15.77\text{x}10^5\ X^3 - 13.91\text{x}10^4\ X^2 + 12.63\text{x}10^3\ X \qquad (10)$$

The solution as given by Eq. (7) for a uniform initial temperature of 170 F is

$$t - 170 = 15.77\text{x}10^5\ X^3\ Z_7 - 13.91\text{x}10^4\ X^2\ Z_5 + 12.63\text{x}10^3\ X\ Z_3 \qquad (11)$$

Temperature profiles were calculated for the same flight times as used in the computer solution. For a time of 100 sec, for example, the value of $X = \alpha\theta/l^2 = 0.1571$. At N = 1.0 the value of Z_7 using linear interpolation is 0.2044. The temperature profiles as calculated by the present method are shown to be in close agreement with the computer solution in Fig. 3. The small deviations in the profiles are due to consideration of gas-cap radiation and use of a temperature-dependent thermal conductivity in the computer solution. Approximately two hours of desk-calculator computations were necessary to obtain the four temperature profiles.

The rest of the temperature history can be calculated by fitting a curve to the remaining portion of the heat flux versus time plot and proceeding as above. The initial temperature distribution must, however, be included. An even-order-polynomial curve can be fitted to the temperature distribution at the time of application of the new heat input. Several such steps should serve in most cases to solve the problem if the entire heat-input history cannot be expressed accurately as a single polynomial.

Evaluation of Diffusivity

The present solution can be used to rapidly evaluate the thermal diffusivity of a material. Diffusivity experiments achieve one-dimensional heat flow by a. use of a large slab sample where only a small central portion is used for measurements, b. insulation of the sides so as to prevent heat losses, and c. use of a guard heater to prevent radial heat loss.

An experiment to determine the diffusivity of copper at 932 ^{o}F (500 ^{o}C) is devised using a large-slab sample. A heat source, such as an electric resistance element, is uniformly applied to the surface; whereas, the back face is insulated. Thermocouples are installed at two interior locations, $x = 1.0$ and 2.0 inches. The block is initially at a uniform temperature $t_b = 72$ ^{o}F. The heating, with a heat source producing a heat flux $q/A \sim 10^6$ Btu/hr ft^2, is commenced at time $\theta = 0$, and the temperatures at the interior locations are monotored. The time required for each of these points to reach 932 ^{o}F is then recorded.

$$x = 2.0 \text{ in.}, \quad N' = 0.8, \quad \Theta' = 31.3 \text{ sec.}$$

$$x = 1.0 \text{ in.}, \quad N'' = 0.4, \quad \Theta'' = 41.8 \text{ sec.}$$

The analytical solution as obtained from Eq. (7) is given as

$$\tau(N, X) = t - t_b = H_o Z_1 (N, X) = (q/A)(1/k) Z_1 \qquad (12)$$

It is readily seen that the temperature distribution functions must be equal for the two points. Thus

$$Z_1' (N', X') = Z_1'' (N'', X'') \qquad (13)$$

subject to the condition that

$$X'/X'' = \Theta'/\Theta'' = 31.3/41.8 \qquad (14)$$

The tables for Z_1, $N = 0.4$, and 0.8 give the desired solution

$$Z_1' = Z_1'' = 0.8684$$

$$X' = 0.7152, \quad X'' = 0.9551$$

from which the diffusivity is calculated.

$$\alpha = l^2 X/\Theta = (2.5/12)^2 \, 0.7152/(31.3/3600) = 3.57 \text{ ft}^2/\text{hr.}$$

It is noted that the evaluation of the diffusivity requires no measurement of the heat flux. If the density and specific heat are known from other simple experiments, this experimental determination also yields the thermal conductivity indirectly. The thermal conductivity can, however, be directly evaluated from Eq. (12) if the heat flux is measured.

The analytical solution to Eq. (7) for constant heat flux has been modified and used with success in diffusivity experiments by Butler and Inn [13] and Sheer, et al. [14] among others. These investigators made use of the linearity of the temperature distribution at values of the relative time $X > 0.5$. The present method is more general as the data can be taken for any value of the relative time.

ACKNOWLEDGMENT

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SYMBOLS

A	=	surface area
k	=	thermal conductivity
l	=	thickness of slab
N	=	position ratio = x/l
P	=	heat-generation term = q'/k
Q	=	heat-flux term = $(q/A)(l/k)$
q	=	rate of heat transfer
q'	=	heat generation per unit volume
r,s	=	integers, 0, 1, 2, ...
t	=	temperature
t_b	=	constant initial temperature
X	=	relative time = $\alpha\theta/l^2$
x	=	normal distance
Z	=	defined by Eq. (8)
F, G, H, K, M	=	constants
α	=	thermal diffusivity
τ	=	temperature difference = $(t - t_b)$
θ	=	time

Subscripts and Superscripts

w	=	wall position
s	=	integer

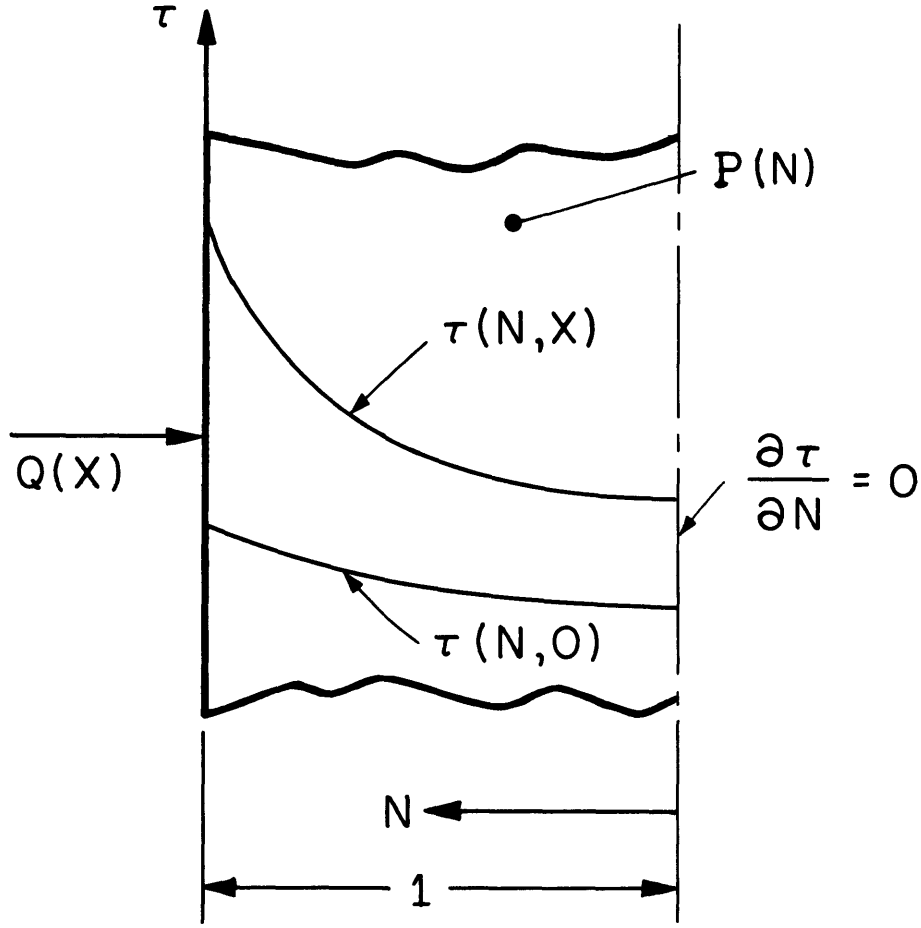

FIG. 1 ONE-DIMENSIONAL HEAT CONDUCTION IN A SLAB

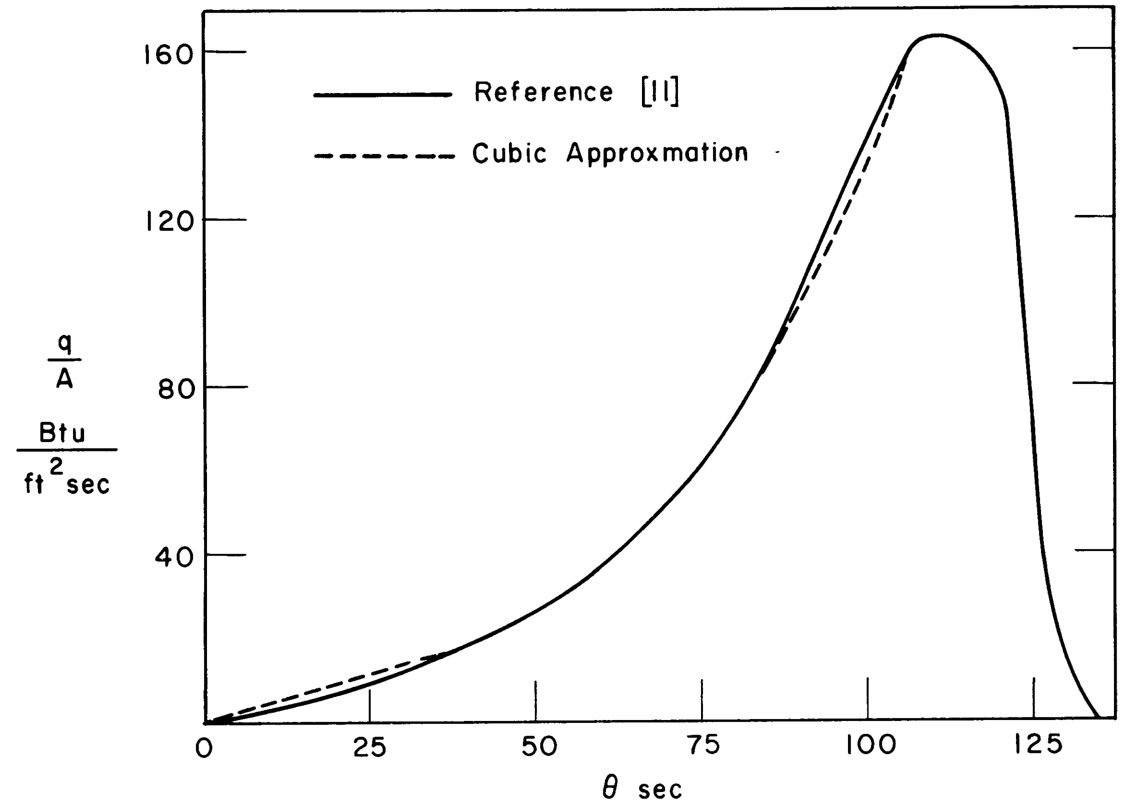

FIG. 2 HEATING RATE VERSUS TIME FOR RE-ENTRY SATELLITE

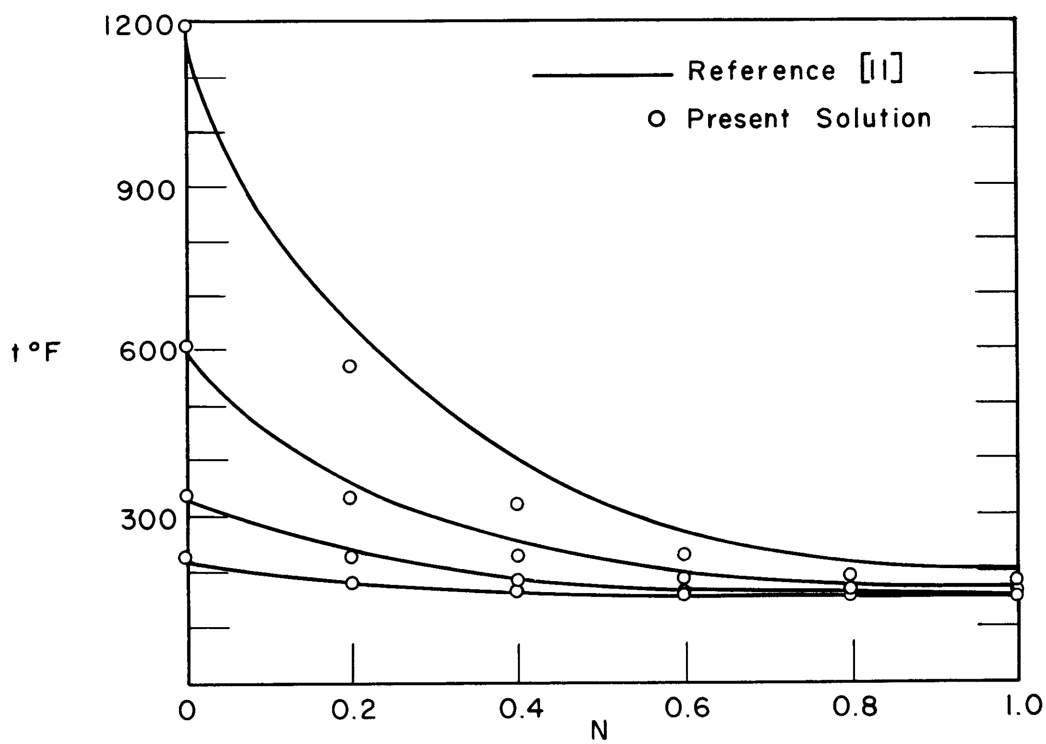

FIG. 3 COMPARISON OF TEMPERATURE PROFILES IN SATELLITE WALL

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_1

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
0.000	0.00000	0.00000	0.00000	0.0000	0.0000	0.0000
.002	.00000	.00000	.00000	.0000	.0000	.0505
.004	.00000	.00000	.00000	.0000	.0008	.0714
.006	.00000	.00000	.00000	.0000	.0029	.0874
.008	.00000	.00000	.00000	.0001	.0061	.1109
.010	.00000	.00000	.00000	.0002	.0101	.1128
.012	.00000	.00000	.00000	.0005	.0144	.1236
.014	.00000	.00000	.00001	.0009	.0190	.1335
.016	.00000	.00000	.00004	.0016	.0237	.1427
.018	.00000	.00000	.00008	.0024	.0285	.1514
.020	.00000	.00000	.00015	.0034	.0333	.1596
.022	.00000	.00001	.00026	.0046	.0382	.1674
.024	.00000	.00001	.00041	.0059	.0430	.1748
.026	.00000	.00003	.00060	.0073	.0478	.1820
.028	.00000	.00004	.00085	.0089	.0525	.1888
.030	.00000	.00007	.00115	.0105	.0572	.1954
.032	.00001	.00011	.00150	.0123	.0618	.2019
.034	.00002	.00016	.00190	.0141	.0664	.2081
.036	.00002	.00022	.00237	.0161	.0710	.2141
.038	.00004	.00030	.00288	.0181	.0754	.2200
.040	.00006	.00039	.00345	.0201	.0799	.2256
.042	.00008	.00051	.00407	.0222	.0842	.2312
.044	.00011	.00064	.00474	.0243	.0885	.2367
.046	.00016	.00079	.00546	.0265	.0928	.2420
.048	.00021	.00097	.00623	.0288	.0970	.2472
.050	.0003	.0012	.0070	.0310	.1012	.2523
.055	.0005	.0018	.0093	.0368	.1113	.2646
.060	.0008	.0025	.0117	.0426	.1212	.2764
.065	.0012	.0034	.0144	.0486	.1308	.2877
.070	.0017	.0045	.0173	.0546	.1402	.2985
.075	.0024	.0057	.0203	.0606	.1493	.3090
.080	.0032	.0071	.0235	.0667	.1582	.3192
.085	.0042	.0086	.0268	.0727	.1669	.3290
.090	.0052	.0103	.0303	.0788	.1754	.3385
.095	.0065	.0122	.0338	.0848	.1838	.3478
.100	.0079	.0141	.0375	.0908	.1919	.3568
.105	.0094	.0162	.0412	.0968	.1999	.3656
.110	.0111	.0185	.0450	.1027	.2078	.3742
.115	.0129	.0209	.0489	.1086	.2155	.3827
.120	.0149	.0234	.0529	.1145	.2230	.3909
.125	.0170	.0260	.0569	.1204	.2305	.3990
.130	.0192	.0287	.0609	.1262	.2378	.4069
.135	.0216	.0315	.0651	.1320	.2450	.4146
.140	.0240	.0344	.0692	.1378	.2521	.4222

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_1

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
.145	.0266	.0375	.0734	.1435	.2591	.4297
.150	.0293	.0406	.0777	.1492	.2660	.4371
.155	.0321	.0438	.0820	.1549	.2728	.4443
.160	.0350	.0471	.0863	.1605	.2795	.4515
.165	.0380	.0505	.0907	.1661	.2861	.4585
.170	.0411	.0539	.0951	.1717	.2927	.4654
.175	.0443	.0575	.0995	.1772	.2992	.4723
.180	.0476	.0611	.1040	.1828	.3056	.4790
.185	.0509	.0647	.1084	.1883	.3119	.4857
.190	.0544	.0685	.1130	.1938	.3182	.4922
.195	.0579	.0722	.1175	.1992	.3244	.4987
.200	.0615	.0761	.1220	.2046	.3306	.5052
.210	.0688	.0840	.1312	.2155	.3427	.5178
.220	.0764	.0920	.1405	.2262	.3546	.5302
.230	.0843	.1003	.1498	.2369	.3664	.5424
.240	.0923	.1087	.1592	.2475	.3780	.5544
.250	.1005	.1172	.1686	.2580	.3894	.5661
.260	.1089	.1259	.1781	.2685	.4007	.5778
.270	.1174	.1347	.1877	.2790	.4119	.5892
.280	.1261	.1437	.1973	.2894	.4230	.6006
.290	.1349	.1527	.2069	.2998	.4340	.6118
.300	.1438	.1618	.2166	.3101	.4448	.6228
.310	.1528	.1710	.2263	.3204	.4556	.6338
.320	.1619	.1803	.2360	.3307	.4664	.6447
.330	.1711	.1896	.2457	.3409	.4770	.6555
.340	.1804	.1991	.2555	.3511	.4876	.6663
.350	.1897	.2085	.2653	.3613	.4982	.6769
.360	.1991	.2180	.2751	.3715	.5086	.6875
.370	.2086	.2276	.2850	.3817	.5191	.6981
.380	.2181	.2372	.2948	.3919	.5295	.7086
.390	.2276	.2468	.3047	.4020	.5398	.7190
.400	.2372	.2565	.3145	.4121	.5502	.7294
.420	.2566	.2770	.3343	.4323	.5707	.7501
.440	.2760	.2955	.3542	.4525	.5912	.7707
.460	.2955	.3151	.3740	.4727	.6116	.7911
.480	.3151	.3348	.3939	.4928	.6319	.8115
.500	.3348	.3545	.4138	.5129	.6522	.8319
.520	.3545	.3743	.4337	.5330	.6724	.8521
.540	.3743	.3941	.4536	.5530	.6925	.8723
.560	.3941	.4140	.4736	.5731	.7127	.8925
.580	.4140	.4339	.4935	.5931	.7328	.9127

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_1

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
.600	.4339	.4538	.5135	.6132	.7529	.9328
.650	.4837	.5036	.5634	.6632	.8031	.9830
.700	.5335	.5535	.6134	.7133	.8532	1.0331
.750	.5835	.6034	.6634	.7633	.9032	1.0832
.800	.6334	.6534	.7134	.8133	.9533	1.1333
.850	.6834	.7034	.7633	.8633	1.0033	1.1833
.900	.7334	.7534	.8133	.9133	1.0533	1.2333
.950	.7834	.8033	.8633	.9633	1.1033	1.2833
1.0	.833	.853	.913	1.013	1.153	1.333
2.0	1.833	1.853	1.913	2.013	2.153	2.333
3.0	2.833	2.853	2.913	3.013	3.153	3.333
4.0	3.833	3.853	3.913	4.013	4.153	4.333
5.0	4.833	4.853	4.913	5.013	5.153	5.333
6.0	5.833	5.853	5.913	6.013	6.153	6.333
7.0	6.833	6.853	6.913	7.013	7.153	7.333
8.0	7.833	7.853	7.913	8.013	8.153	8.333
9.0	8.833	8.853	8.913	9.013	9.153	9.333
10.0	9.833	9.853	9.913	10.013	10.153	10.333
11.0	10.833	10.853	10.913	11.013	11.153	11.333
12.0	11.833	11.853	11.913	12.013	12.153	12.333
13.0	12.833	12.853	12.913	13.013	13.153	13.333
14.0	13.833	13.853	13.913	14.013	14.153	14.333
15.0	14.833	14.853	14.913	15.013	15.153	15.333
16.0	15.833	15.853	15.913	16.013	16.153	16.333
17.0	16.833	16.853	16.913	17.013	17.153	17.333
18.0	17.833	17.853	17.913	18.013	18.153	18.333
19.0	18.833	18.853	18.913	19.013	19.153	19.333
20.0	19.833	19.853	19.913	20.013	20.153	20.333
21.0	20.833	20.853	20.913	21.013	21.153	21.333
22.0	21.833	21.853	21.913	22.013	22.153	22.333
23.0	22.833	22.853	22.913	23.013	23.153	23.333
24.0	23.833	23.853	23.913	24.013	24.153	24.333
25.0	24.833	24.853	24.913	25.013	25.153	25.333
26.0	25.833	25.853	25.913	26.013	26.153	26.333
27.0	26.833	26.853	26.913	27.013	27.153	27.333
28.0	27.833	27.853	27.913	28.013	28.153	28.333
29.0	28.833	28.853	28.913	29.013	29.153	29.333
30.0	29.833	29.853	29.913	30.013	30.153	30.333
31.0	30.833	30.853	30.913	31.013	31.153	31.333
32.0	31.833	31.853	31.913	32.013	32.153	32.333
33.0	32.833	32.853	32.913	33.013	33.153	33.333
34.0	33.833	33.853	33.913	34.013	34.153	34.333
35.0	34.833	34.853	34.913	35.013	35.153	35.333
36.0	35.833	35.853	35.913	36.013	36.153	36.333
37.0	36.833	36.853	36.913	37.013	37.153	37.333
38.0	37.833	37.853	37.913	38.013	38.153	38.333
39.0	38.833	38.853	38.913	39.013	39.153	39.333
40.0	39.833	39.853	39.913	40.013	40.153	40.333

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_3

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
0.000	0.00000	0.00000	0.00000	0.00000	0.0000	0.0000
.002	.00000	.00000	.00000	.00000	.0000	.0336
.004	.00000	.00000	.00000	.00000	.0002	.0476
.006	.00000	.00000	.00000	.00000	.0007	.0583
.008	.00000	.00000	.00000	.00001	.0016	.0673
.010	.00000	.00000	.00000	.00003	.0029	.0752
.012	.00000	.00000	.00000	.00008	.0045	.0824
.014	.00000	.00000	.00000	.00017	.0062	.0890
.016	.00000	.00000	.00000	.00030	.0081	.0952
.018	.00000	.00000	.00001	.00049	.0101	.1009
.020	.00000	.00000	.00002	.00073	.0122	.1064
.022	.00000	.00000	.00004	.00102	.0143	.1116
.024	.00000	.00000	.00006	.00137	.0165	.1165
.026	.00000	.00000	.00010	.00177	.0187	.1213
.028	.00000	.00001	.00014	.00222	.0210	.1259
.030	.00000	.00001	.00020	.00272	.0232	.1303
.032	.00000	.00001	.00027	.00326	.0255	.1346
.034	.00000	.00002	.00035	.00384	.0278	.1387
.036	.00000	.00003	.00045	.00447	.0300	.1427
.038	.00000	.00004	.00056	.00513	.0323	.1466
.040	.00001	.00006	.00069	.00583	.0346	.1505
.042	.00001	.00008	.00084	.00656	.0368	.1542
.044	.00001	.00010	.00100	.00732	.0391	.1578
.046	.00002	.00012	.00118	.00811	.0413	.1613
.048	.00003	.00016	.00137	.00892	.0436	.1648
.050	.00003	.00019	.0016	.0098	.0458	.1682
.055	.00006	.00031	.0022	.0120	.0513	.1764
.060	.00011	.00046	.0029	.0143	.0567	.1843
.065	.00018	.00065	.0037	.0167	.0620	.1918
.070	.00027	.00089	.0045	.0192	.0673	.1990
.075	.00039	.00117	.0055	.0217	.0725	.2060
.080	.00054	.00149	.0065	.0243	.0775	.2128
.085	.00072	.00187	.0076	.0270	.0825	.2193
.090	.00094	.00229	.0088	.0297	.0875	.2257
.095	.00120	.00276	.0100	.0325	.0923	.2319
.100	.00150	.00328	.0113	.0352	.0971	.2379
.105	.00184	.00384	.0126	.0380	.1018	.2438
.110	.00222	.00446	.0140	.0408	.1064	.2495
.115	.00265	.00512	.0154	.0436	.1110	.2551
.120	.00312	.00583	.0169	.0465	.1155	.2606
.125	.00363	.00658	.0184	.0493	.1200	.2660
.130	.00418	.00738	.0200	.0522	.1244	.2712
.135	.00478	.00822	.0216	.0550	.1287	.2764
.140	.00543	.00910	.0232	.0579	.1330	.2815

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_3

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
.145	.00611	.01003	.0249	.0607	.1372	.2865
.150	.00684	.01099	.0265	.0636	.1414	.2914
.155	.00761	.01200	.0283	.0664	.1455	.2962
.160	.00842	.01305	.0300	.0693	.1496	.3009
.165	.00927	.01413	.0318	.0721	.1536	.3056
.170	.01016	.01525	.0336	.0750	.1576	.3102
.175	.01109	.01640	.0354	.0778	.1616	.3147
.180	.01206	.01759	.0372	.0807	.1655	.3192
.185	.01306	.01882	.0391	.0835	.1694	.3236
.190	.01411	.02008	.0410	.0863	.1732	.3279
.195	.01518	.02137	.0429	.0891	.1770	.3322
.200	.0163	.0227	.0448	.0920	.1808	.3365
.210	.0186	.0254	.0487	.0976	.1882	.3448
.220	.0211	.0283	.0527	.1032	.1955	.3530
.230	.0236	.0312	.0567	.1088	.2027	.3609
.240	.0263	.0343	.0608	.1143	.2097	.3687
.250	.0291	.0374	.0649	.1199	.2167	.3764
.260	.0321	.0406	.0691	.1254	.2235	.3839
.270	.0351	.0440	.0733	.1309	.2303	.3913
.280	.0382	.0474	.0775	.1363	.2370	.3986
.290	.0413	.0508	.0818	.1418	.2436	.4058
.300	.0446	.0544	.0862	.1472	.2501	.4128
.310	.0479	.0580	.0905	.1527	.2566	.4198
.320	.0514	.0617	.0949	.1581	.2630	.4266
.330	.0549	.0654	.0993	.1634	.2693	.4334
.340	.0584	.0692	.1038	.1688	.2756	.4401
.350	.0620	.0730	.1083	.1742	.2818	.4457
.360	.0657	.0769	.1128	.1795	.2879	.4532
.370	.0694	.0809	.1173	.1848	.2940	.4597
.380	.0732	.0849	.1218	.1902	.3001	.4661
.390	.0771	.0889	.1264	.1955	.3061	.4725
.400	.0810	.0930	.1310	.2007	.3121	.4788
.420	.0889	.1012	.1402	.2113	.3239	.4912
.440	.0970	.1096	.1495	.2218	.3356	.5034
.460	.1052	.1181	.1588	.2323	.3471	.5155
.480	.1135	.1267	.1682	.2427	.3586	.5274
.500	.1219	.1354	.1776	.2531	.3699	.5398
.520	.1305	.1443	.1871	.2635	.3812	.5508
.540	.1392	.1532	.1966	.2738	.3923	.5623
.560	.1479	.1622	.2061	.2842	.4034	.5738
.580	.1567	.1712	.2157	.2945	.4144	.5851

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_3

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
.600	.1656	.1802	.2253	.3048	.4254	.5964
.650	.1882	.2032	.2494	.3304	.4525	.6242
.700	.2111	.2264	.2736	.3560	.4793	.6516
.750	.2342	.2499	.2979	.3815	.5059	.6787
.800	.2576	.2735	.3223	.4069	.5323	.7056
.850	.2812	.2974	.3468	.4323	.5585	.7322
.900	.3049	.3213	.3713	.4576	.5846	.7586
.950	.3288	.3454	.3959	.4829	.6106	.7849
1.0	.353	.370	.421	.508	.637	.811
2.0	.843	.861	.917	1.011	1.145	1.322
3.0	1.340	1.359	1.416	1.512	1.648	1.826
4.0	1.838	1.857	1.915	2.012	2.149	2.328
5.0	2.337	2.357	2.415	2.512	2.650	2.829
6.0	2.837	2.856	2.915	3.012	3.151	3.330
7.0	3.336	3.356	3.414	3.513	3.651	3.830
8.0	3.836	3.855	3.914	4.013	4.151	4.331
9.0	4.335	4.355	4.414	4.513	4.651	4.831
10.0	4.835	4.855	4.914	5.013	5.152	5.331
11.0	5.335	5.355	5.414	5.513	5.652	5.831
12.0	5.835	5.855	5.914	6.013	6.152	6.331
13.0	6.335	6.355	6.414	6.513	6.652	6.832
14.0	6.835	6.854	6.914	7.013	7.152	7.332
15.0	7.335	7.354	7.414	7.513	7.652	7.832
16.0	7.835	7.854	7.914	8.013	8.152	8.332
17.0	8.334	8.354	8.414	8.513	8.652	8.832
18.0	8.834	8.854	8.914	9.013	9.152	9.332
19.0	9.334	9.354	9.414	9.513	9.652	9.832
20.0	9.834	9.854	9.914	10.013	10.152	10.332
21.0	10.334	10.354	10.414	10.513	10.652	10.832
22.0	10.834	10.854	10.914	11.013	11.153	11.332
23.0	11.334	11.354	11.414	11.513	11.653	11.832
24.0	11.834	11.854	11.914	12.013	12.153	12.332
25.0	12.334	12.354	12.414	12.513	12.653	12.832
26.0	12.834	12.854	12.914	13.013	13.153	13.332
27.0	13.334	13.354	13.414	13.513	13.653	13.832
28.0	13.834	13.854	13.914	14.013	14.153	14.332
29.0	14.334	14.354	14.414	14.513	14.653	14.832
30.0	14.834	14.854	14.914	15.013	15.153	15.333
31.0	15.334	15.354	15.414	15.513	15.653	15.833
32.0	15.834	15.854	15.914	16.013	16.153	16.333
33.0	16.334	16.354	16.414	16.513	16.653	16.833
34.0	16.834	16.854	16.914	17.013	17.153	17.333
35.0	17.334	17.354	17.414	17.513	17.653	17.833
36.0	17.834	17.854	17.914	18.013	18.153	18.333
37.0	18.334	18.354	18.414	18.513	18.653	18.833
38.0	18.834	18.854	18.914	19.013	19.153	19.333
39.0	19.334	19.354	19.414	19.513	19.653	19.833
40.0	19.834	19.854	19.914	20.013	20.153	20.333

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_5

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
0.000	0.00000	0.00000	0.00000	0.00000	0.0000	0.0000
.002	.00000	.00000	.00000	.00000	.0000	.0269
.004	.00000	.00000	.00000	.00000	.0000	.0381
.006	.00000	.00000	.00000	.00000	.0002	.0466
.008	.00000	.00000	.00000	.00000	.0006	.0538
.010	.00000	.00000	.00000	.00001	.0012	.0602
.012	.00000	.00000	.00000	.00002	.0020	.0659
.014	.00000	.00000	.00000	.00005	.0029	.0712
.016	.00000	.00000	.00000	.00009	.0039	.0761
.018	.00000	.00000	.00000	.00015	.0050	.0807
.020	.00000	.00000	.00000	.00024	.0061	.0851
.022	.00000	.00000	.00001	.00035	.0074	.0893
.024	.00000	.00000	.00002	.00048	.0087	.0932
.026	.00000	.00000	.00002	.00064	.0100	.0970
.028	.00000	.00000	.00004	.00083	.0113	.1007
.030	.00000	.00000	.00005	.00104	.0127	.1042
.032	.00000	.00000	.00008	.00128	.0141	.1077
.034	.00000	.00000	.00010	.00154	.0156	.1110
.036	.00000	.00001	.00013	.00182	.0170	.1142
.038	.00000	.00001	.00017	.00212	.0185	.1173
.040	.00000	.00001	.00022	.00245	.0199	.1204
.042	.00000	.00002	.00027	.00280	.0214	.1233
.044	.00000	.00002	.00033	.00317	.0229	.1262
.046	.00000	.00003	.00039	.00355	.0243	.1291
.048	.00001	.00004	.00046	.00396	.0258	.1318
.050	.00001	.00005	.0005	.0044	.0273	.1346
.055	.00001	.00009	.0008	.0055	.0310	.1411
.060	.00003	.00013	.0011	.0067	.0347	.1474
.065	.00004	.00019	.0014	.0080	.0383	.1534
.070	.00007	.00027	.0018	.0094	.0420	.1592
.075	.00010	.00037	.0022	.0108	.0456	.1648
.080	.00014	.00049	.0026	.0123	.0491	.1702
.085	.00020	.00062	.0031	.0138	.0527	.1755
.090	.00027	.00078	.0037	.0154	.0562	.1805
.095	.00035	.00096	.0043	.0170	.0596	.1855
.100	.00045	.00116	.0049	.0186	.0630	.1903
.105	.00056	.00138	.0055	.0203	.0664	.1950
.110	.00069	.00163	.0062	.0220	.0698	.1996
.115	.00084	.00190	.0069	.0237	.0731	.2041
.120	.00101	.00219	.0077	.0255	.0764	.2085
.125	.00119	.00250	.0085	.0272	.0796	.2128
.130	.00140	.00284	.0093	.0290	.0828	.2170
.135	.00162	.00320	.0101	.0308	.0860	.2211
.140	.00186	.00358	.0110	.0326	.0892	.2252

THE TEMPERATURE DISTRIBTION FUNCTION - Z_5

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
.145	.00213	.00399	.0119	.0344	.0923	.2292
.150	.00241	.00442	.0128	.0362	.0954	.2331
.155	.00272	.00487	.0137	.0380	.0984	.2369
.160	.00304	.00534	.0147	.0399	.1014	.2407
.165	.00339	.00583	.0156	.0417	.1044	.2445
.170	.00376	.00634	.0166	.0436	.1074	.2481
.175	.00414	.00688	.0176	.0454	.1103	.2518
.180	.00455	.00743	.0186	.0473	.1133	.2553
.185	.00498	.00801	.0197	.0491	.1161	.2589
.190	.00543	.00860	.0207	.0510	.1190	.2623
.195	.00589	.00921	.0218	.0528	.1218	.2658
.200	.0064	.0098	.0229	.0547	.1247	.2691
.210	.0074	.0112	.0251	.0584	.1302	.2758
.220	.0085	.0126	.0274	.0622	.1357	.2823
.230	.0097	.0140	.0297	.0659	.1411	.2887
.240	.0109	.0155	.0321	.0696	.1464	.2949
.250	.0123	.0171	.0345	.0733	.1516	.3010
.260	.0136	.0188	.0369	.0771	.1568	.3069
.270	.0151	.0205	.0394	.0808	.1619	.3128
.280	.0166	.0223	.0420	.0845	.1670	.3186
.290	.0182	.0241	.0445	.0882	.1719	.3242
.300	.0198	.0260	.0471	.0919	.1768	.3298
.310	.0215	.0279	.0497	.0956	.1817	.3353
.320	.0232	.0298	.0524	.0992	.1865	.3407
.330	.0250	.0319	.0550	.1029	.1913	.3460
.340	.0268	.0339	.0577	.1066	.1960	.3513
.350	.0287	.0360	.0605	.1102	.2006	.3565
.360	.0306	.0381	.0632	.1139	.2052	.3616
.370	.0326	.0403	.0660	.1175	.0298	.3667
.380	.0346	.0425	.0688	.1212	.2143	.3717
.390	.0367	.0448	.0715	.1248	.2188	.3766
.400	.0388	.0471	.0744	.1284	.2233	.3815
.420	.0431	.0517	.0801	.1356	.2321	.3911
.440	.0475	.0565	.0858	.1428	.2408	.4005
.460	.0521	.0614	.0936	.1500	.2493	.4098
.480	.0567	.0628	.0975	.1571	.2578	.4189
.500	.0615	.0714	.1034	.1643	.2661	.4279
.520	.0664	.0766	.1094	.1713	.2744	.4367
.540	.0714	.0819	.1154	.1784	.2826	.4455
.560	.0764	.0872	.1214	.1855	.2907	.4541
.580	.0816	.0926	.1275	.1925	.2987	.4626

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_5

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
.600	.0868	.0980	.1336	.1996	.3066	.4710
.650	.1001	.1119	.1489	.2170	.3262	.4916
.700	.1139	.1261	.1645	.2344	.3455	.5118
.750	.1279	.1405	.1801	.2518	.3645	.5316
.800	.1422	.1552	.1959	.2690	.3832	.5511
.850	.1567	.1701	.2117	.2862	.4018	.5702
.900	.1715	.1852	.2276	.3034	.4201	.5892
.950	.1864	.2004	.2436	.3205	.4383	.6079
1.0	.201	.216	.260	.338	.456	.626
2.0	.518	.535	.587	.675	.804	.979
3.0	.846	.864	.918	1.010	1.142	1.319
4.0	1.176	1.195	1.250	1.344	1.478	1.656
5.0	1.508	1.526	1.583	1.678	1.813	1.991
6.0	1.840	1.859	1.916	2.012	2.148	2.326
7.0	2.172	2.191	2.249	2.345	2.482	2.660
8.0	2.505	2.524	2.582	2.679	2.816	2.994
9.0	2.838	2.857	2.915	3.012	3.150	3.328
10.0	3.171	3.190	3.248	3.346	3.483	3.662
11.0	3.503	3.523	3.581	3.679	3.817	3.996
12.0	3.837	3.856	3.914	4.012	4.151	4.330
13.0	4.170	4.189	4.248	4.346	4.484	4.663
14.0	4.503	4.522	4.581	4.679	4.818	4.997
15.0	4.836	4.855	4.914	5.013	5.151	5.330
16.0	5.169	5.189	5.248	5.346	5.485	5.664
17.0	5.502	5.522	5.581	5.679	5.818	5.997
18.0	5.835	5.855	5.914	6.013	6.151	6.331
19.0	6.169	6.188	6.247	6.346	6.485	6.664
20.0	6.502	6.522	6.581	6.679	6.818	6.998
21.0	6.835	6.855	6.914	7.013	7.152	7.331
22.0	7.168	7.188	7.247	7.346	7.485	7.665
23.0	7.502	7.521	7.581	7.680	7.818	7.998
24.0	7.835	7.855	7.914	8.013	8.152	8.331
25.0	8.168	8.188	8.247	8.346	8.485	8.665
26.0	8.501	8.521	8.580	8.680	8.819	8.998
27.0	8.835	8.854	8.914	9.013	9.152	9.332
28.0	9.168	9.188	9.247	9.346	9.485	9.665
29.0	9.501	9.521	9.580	9.680	9.819	9.998
30.0	9.835	9.854	9.914	10.013	10.152	10.332
31.0	10.168	10.188	10.247	10.346	10.485	10.665
32.0	10.501	10.521	10.580	10.680	10.819	10.998
33.0	10.835	10.854	10.914	11.013	11.152	11.332
34.0	11.168	11.188	11.247	11.346	11.485	11.665
35.0	11.501	11.521	11.580	11.679	11.819	11.998
36.0	11.835	11.854	11.914	12.013	12.152	12.332
37.0	12.168	12.188	12.247	12.346	12.485	12.665
38.0	12.501	12.521	12.580	12.679	12.819	12.998
39.0	12.835	12.854	12.914	13.013	13.152	13.332
40.0	13.168	13.188	13.247	13.346	13.485	13.665

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_7

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
0.000	0.00000	0.00000	0.00000	0.00000	0.0000	0.0000
.002	.00000	.00000	.00000	.00000	.0000	.0231
.004	.00000	.00000	.00000	.00000	.0000	.0326
.006	.00000	.00000	.00000	.00000	.0001	.0400
.008	.00000	.00000	.00000	.00000	.0003	.0461
.010	.00000	.00000	.00000	.00000	.0006	.0516
.012	.00000	.00000	.00000	.00001	.0010	.0565
.014	.00000	.00000	.00000	.00002	.0016	.0610
.016	.00000	.00000	.00000	.00003	.0022	.0652
.018	.00000	.00000	.00000	.00006	.0028	.0692
.020	.00000	.00000	.00000	.00010	.0036	.0729
.022	.00000	.00000	.00000	.00014	.0044	.0765
.024	.00000	.00000	.00000	.00021	.0052	.0799
.026	.00000	.00000	.00001	.00028	.0061	.0832
.028	.00000	.00000	.00001	.00037	.0070	.0863
.030	.00000	.00000	.00002	.00048	.0080	.0893
.032	.00000	.00000	.00003	.00060	.0089	.0923
.034	.00000	.00000	.00004	.00073	.0099	.0951
.036	.00000	.00000	.00005	.00088	.0109	.0979
.038	.00000	.00000	.00007	.00104	.0119	.1006
.040	.00000	.00000	.00008	.00122	.0130	.1032
.042	.00000	.00001	.00011	.00141	.0140	.1057
.044	.00000	.00001	.00013	.00162	.0151	.1082
.046	.00000	.00001	.00016	.00184	.0161	.1106
.048	.00000	.00001	.00019	.00207	.0172	.1130
.050	.00000	.00002	.00023	.00231	.0183	.1153
.055	.00000	.00003	.00033	.00297	.0210	.1210
.060	.00001	.00005	.00047	.00369	.0237	.1264
.065	.00001	.00007	.00063	.00448	.0265	.1315
.070	.00002	.00010	.00081	.00532	.0292	.1365
.075	.00003	.00014	.00103	.00622	.0319	.1413
.080	.00005	.00019	.00127	.00716	.0347	.1459
.085	.00007	.00025	.00154	.00814	.0374	.1504
.090	.00010	.00032	.00183	.00916	.0401	.1547
.095	.00013	.00041	.00215	.01022	.0427	.1590
.100	.00017	.00050	.00250	.01130	.0454	.1631
.105	.00021	.00060	.00287	.01242	.0480	.1671
.110	.00027	.00072	.00326	.01356	.0507	.1711
.115	.00033	.00085	.00368	.01472	.0533	.1749
.120	.00040	.00099	.00412	.01591	.0558	.1787
.125	.00048	.00115	.00457	.01711	.0584	.1824
.130	.00057	.00132	.00505	.01834	.0609	.1860
.135	.00067	.00150	.00555	.01958	.0634	.1895
.140	.00078	.00170	.00607	.02083	.0659	.1930

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_7

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
.145	.00090	.00190	.00660	.02210	.0684	.1964
.150	.00104	.00213	.00716	.02337	.0709	.1998
.155	.00118	.00236	.00773	.02466	.0733	.2031
.160	.00133	.00261	.00831	.02597	.0757	.2063
.165	.00150	.00287	.00892	.02727	.0781	.2095
.170	.00168	.00315	.00953	.02859	.0805	.2127
.175	.00187	.00344	.01016	.02992	.0829	.2158
.180	.00207	.00374	.01081	.03125	.0852	.2189
.185	.00228	.00405	.01147	.03259	.0876	.2219
.190	.00251	.00438	.01214	.03393	.0899	.2248
.195	.00274	.00472	.01283	.03528	.0921	.2278
.200	.0030	.0051	.0135	.0366	.0944	.2307
.210	.0035	.0058	.0150	.0393	.0989	.2364
.220	.0041	.0066	.0164	.0421	.1034	.2420
.230	.0047	.0074	.0179	.0448	.1077	.2474
.240	.0054	.0083	.0195	.0476	.1121	.2527
.250	.0061	.0092	.0211	.0503	.1163	.2579
.260	.0069	.0102	.0227	.0531	.1206	.2630
.270	.0077	.0112	.0244	.0559	.1247	.2681
.280	.0085	.0123	.0261	.0586	.1288	.2730
.290	.0094	.0134	.0278	.0614	.1329	.2778
.300	.0103	.0145	.0295	.0642	.1369	.2826
.310	.0113	.0157	.0313	.0669	.1409	.2873
.320	.0123	.0169	.0331	.0697	.1448	.2919
.330	.0133	.0181	.0349	.0725	.1487	.2964
.340	.0144	.0194	.0368	.0752	.1525	.3009
.350	.0155	.0207	.0386	.0780	.1563	.3053
.360	.0167	.0220	.0405	.0808	.1601	.3097
.370	.0179	.0233	.0424	.0835	.1639	.3140
.380	.0191	.0247	.0443	.0863	.1676	.3182
.390	.0203	.0262	.0463	.0890	.1713	.3224
.400	.0216	.0276	.0482	.0918	.1749	.3266
.420	.0242	.0306	.0522	.0973	.1821	.3347
.440	.0270	.0336	.0562	.1027	.1892	.3427
.460	.0298	.0368	.0603	.1082	.1962	.3505
.480	.0328	.0401	.0644	.1137	.2031	.3582
.500	.0358	.0434	.0685	.1191	.2099	.3657
.520	.0389	.0468	.0717	.1245	.2166	.3731
.540	.0422	.0503	.0770	.1299	.2224	.3804
.560	.0454	.0538	.0813	.1353	.2298	.3876
.580	.0488	.0574	.0856	.1407	.2362	.3946

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_7

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
.600	.0522	.0611	.0899	.1460	.2427	.4016
.650	.0610	.0705	.1009	.1594	.2585	.4187
.700	.0702	.0802	.1121	.1726	.2740	.4353
.750	.0797	.0901	.1234	.1858	.2892	.4515
.800	.0895	.1003	.1348	.1990	.3041	.4674
.850	.0995	.1107	.1463	.2121	.3188	.4829
.900	.1097	.1213	.1579	.2252	.3334	.4982
.950	.1201	.1320	.1695	.2382	.3478	.5132
1.0	.131	.143	.181	.251	.362	.528
2.0	.360	.375	.423	.506	.631	.803
3.0	.601	.618	.670	.759	.888	1.062
4.0	.847	.865	.918	1.010	1.141	1.317
5.0	1.094	1.113	1.167	1.260	1.394	1.570
6.0	1.343	1.361	1.417	1.511	1.645	1.823
7.0	1.591	1.610	1.666	1.761	1.896	2.074
8.0	1.840	1.859	1.916	2.011	2.147	2.325
9.0	2.090	2.109	2.166	2.262	2.398	2.576
10.0	2.339	2.358	2.415	2.512	2.648	2.827
11.0	2.589	2.608	2.665	2.762	2.899	3.077
12.0	2.838	2.857	2.915	3.012	3.149	3.328
13.0	3.088	3.107	3.165	3.262	3.399	3.578
14.0	3.337	3.357	3.415	3.512	3.650	3.829
15.0	3.587	3.607	3.665	3.762	3.900	4.079
16.0	3.837	3.856	3.915	4.012	4.150	4.329
17.0	4.087	4.106	4.165	4.262	4.400	4.579
18.0	4.337	4.356	4.414	4.512	4.651	4.830
19.0	4.586	4.606	4.664	4.762	4.901	5.080
20.0	4.836	4.856	4.914	5.013	5.151	5.330
21.0	5.086	5.106	5.164	5.263	5.401	5.580
22.0	5.336	5.355	5.414	5.513	5.651	5.830
23.0	5.586	5.605	5.664	5.763	5.901	6.080
24.0	5.836	5.855	5.914	6.013	6.151	6.331
25.0	6.086	6.105	6.164	6.263	6.401	6.581
26.0	6.336	6.355	6.414	6.513	6.651	6.831
27.0	6.585	6.605	6.664	6.763	6.901	7.081
28.0	6.835	6.855	6.914	7.013	7.152	7.331
29.0	7.085	7.105	7.164	7.263	7.402	7.581
30.0	7.335	7.355	7.414	7.513	7.652	7.831
31.0	7.585	7.605	7.664	7.763	7.902	8.081
32.0	7.835	7.855	7.914	8.013	8.152	8.331
33.0	8.085	8.105	8.164	8.263	8.402	8.581
34.0	8.335	8.355	8.414	8.513	8.652	8.831
35.0	8.585	8.605	8.664	8.763	8.902	9.081
36.0	8.835	8.855	8.914	9.013	9.152	9.331
37.0	9.085	9.105	9.164	9.263	9.402	9.581
38.0	9.335	9.355	9.414	9.513	9.652	9.831
39.0	9.585	9.605	9.664	9.763	9.902	10.081
40.0	9.835	9.855	9.914	10.013	10.152	10.331

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_9

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
0.000	0.00000	0.00000	0.00000	0.00000	0.0000	0.0000
.002	.00000	.00000	.00000	.00000	.0000	.0205
.004	.00000	.00000	.00000	.00000	.0000	.0290
.006	.00000	.00000	.00000	.00000	.0000	.0355
.008	.00000	.00000	.00000	.00000	.0001	.0410
.010	.00000	.00000	.00000	.00000	.0002	.0459
.012	.00000	.00000	.00000	.00000	.0003	.0502
.014	.00000	.00000	.00000	.00000	.0006	.0543
.016	.00000	.00000	.00000	.00001	.0009	.0580
.018	.00000	.00000	.00000	.00001	.0013	.0615
.020	.00000	.00000	.00000	.00003	.0018	.0648
.022	.00000	.00000	.00000	.00004	.0023	.0680
.024	.00000	.00000	.00000	.00007	.0028	.0710
.026	.00000	.00000	.00000	.00010	.0034	.0739
.028	.00000	.00000	.00000	.00014	.0040	.0767
.030	.00000	.00000	.00000	.00019	.0047	.0794
.032	.00000	.00000	.00001	.00025	.0054	.0820
.034	.00000	.00000	.00001	.00031	.0061	.0845
.036	.00000	.00000	.00002	.00039	.0068	.0870
.038	.00000	.00000	.00002	.00047	.0075	.0894
.040	.00000	.00000	.00003	.00057	.0083	.0917
.042	.00000	.00000	.00004	.00067	.0091	.0940
.044	.00000	.00000	.00005	.00079	.0099	.0962
.046	.00000	.00000	.00006	.00091	.0107	.0983
.048	.00000	.00000	.00007	.00105	.0115	.1005
			.00009	.00119	.0123	
.050	.00000	.00001	.00011	.00134	.0131	.1025
.055	.00000	.00001	.00016	.00175	.0152	.1075
.060	.00000	.00002	.00023	.00222	.0173	.1123
.065	.00001	.00003	.00032	.00273	.0195	.1169
.070	.00001	.00005	.00042	.00329	.0216	.1213
.075	.00001	.00006	.00054	.00389	.0238	.1256
.080	.00002	.00009	.00068	.00453	.0260	.1297
.085	.00003	.00012	.00084	.00520	.0281	.1337
.090	.00004	.00015	.00101	.00591	.0303	.1376
.095	.00005	.00020	.00120	.00665	.0325	.1413
.100	.00007	.00024	.00141	.00741	.0346	.1450
.105	.00009	.00030	.00164	.00820	.0368	.1486
.110	.00012	.00036	.00188	.00902	.0389	.1521
.115	.00014	.00043	.00214	.00986	.0410	.1555
.120	.00018	.00050	.00242	.01071	.0432	.1588
.125	.00022	.00059	.00271	.01159	.0453	.1621
.130	.00026	.00068	.00301	.01248	.0474	.1653
.135	.00031	.00078	.00333	.01339	.0494	.1685
.140	.00037	.00089	.00367	.01431	.0515	.1716

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_9

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
.145	.00043	.00101	.00402	.01525	.0536	.1746
.150	.00050	.00114	.00438	.01620	.0556	.1776
.155	.00058	.00128	.00476	.01716	.0576	.1805
.160	.00066	.00142	.00515	.01814	.0597	.1834
.165	.00074	.00157	.00555	.01912	.0617	.1862
.170	.00084	.00174	.00596	.02011	.0636	.1891
.175	.00094	.00191	.00639	.02111	.0656	.1918
.180	.00105	.00209	.00683	.02212	.0676	.1945
.185	.00116	.00227	.00727	.02314	.0695	.1972
.190	.00129	.00247	.00773	.02417	.0715	.1999
.195	.00142	.00268	.00820	.02520	.0734	.2025
.200	.00156	.00289	.0087	.02624	.0753	.2051
.210	.00186	.00335	.0097	.02832	.0791	.2101
.220	.00219	.00383	.0107	.03043	.0829	.2151
.230	.00255	.00435	.0118	.03256	.0866	.2199
.240	.00294	.00490	.0128	.03470	.0902	.2246
.250	.00337	.00549	.0140	.03685	.0939	.2293
.260	.00383	.00610	.0151	.03901	.0974	.2338
.270	.00431	.00675	.0163	.04119	.1010	.2383
.280	.00483	.00742	.0175	.04336	.1045	.2426
.290	.00537	.00813	.0187	.04555	.1079	.2469
.300	.00594	.00887	.0200	.04774	.1114	.2512
.310	.00654	.00963	.0213	.04993	.1147	.2553
.320	.00717	.01042	.0226	.05213	.1181	.2594
.330	.00783	.01124	.0239	.05433	.1214	.2634
.340	.00851	.01208	.0253	.05653	.1247	.2674
.350	.00922	.01295	.0266	.05873	.1280	.2713
.360	.00995	.01384	.0280	.06094	.1312	.2752
.370	.01071	.01476	.0294	.06314	.1344	.2790
.380	.01149	.01570	.0308	.06535	.1376	.2828
.390	.01230	.01666	.0322	.06755	.1407	.2865
.400	.0131	.0177	.0337	.06975	.1438	.2901
.420	.0149	.0197	.0366	.07416	.1500	.2973
.440	.0167	.0218	.0396	.07856	.1560	.3043
.460	.0186	.0240	.0427	.08295	.1620	.3112
.480	.0206	.0263	.0457	.08734	.1679	.3180
.500	.0227	.0286	.0489	.09172	.1738	.3246
.520	.0248	.0310	.0520	.09609	.1795	.3311
.540	.0270	.0334	.0552	.10045	.1852	.3375
.560	.0293	.0360	.0585	.10481	.1908	.3438
.580	.0316	.0385	.0617	.10916	.1963	.3501

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_9

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
.600	.0340	.0412	.0650	.1135	.2018	.3562
.650	.0402	.0479	.0734	.1243	.2152	.3711
.700	.0468	.0550	.0820	.1351	.2284	.3855
.750	.0536	.0623	.0906	.1458	.2413	.3995
.800	.0607	.0698	.0994	.1564	.2539	.4132
.850	.0680	.0775	.1083	.1671	.2664	.4266
.900	.0755	.0854	.1173	.1776	.2786	.4397
.950	.0832	.0935	.1263	.1882	.2907	.4526
1.0	.091	.102	.135	.199	.303	.465
2.0	.267	.281	.326	.405	.524	.695
3.0	.457	.473	.522	.607	.733	.906
4.0	.651	.668	.720	.809	.938	1.113
5.0	.848	.865	.919	1.009	1.141	1.317
6.0	1.046	1.064	1.118	1.210	1.343	1.519
7.0	1.244	1.262	1.317	1.411	1.544	1.721
8.0	1.443	1.461	1.517	1.611	1.745	1.923
9.0	1.642	1.660	1.716	1.811	1.946	2.124
10.0	1.841	1.860	1.916	2.011	2.147	2.325
11.0	2.040	2.059	2.116	2.211	2.347	2.525
12.0	2.240	2.259	2.316	2.412	2.548	2.726
13.0	2.439	2.458	2.515	2.612	2.748	2.927
14.0	2.639	2.658	2.715	2.812	2.949	3.127
15.0	2.838	2.857	2.915	3.012	3.149	3.327
16.0	3.038	3.057	3.115	3.212	3.349	3.528
17.0	3.238	3.257	3.315	3.412	3.549	3.728
18.0	3.438	3.457	3.515	3.612	3.750	3.928
19.0	3.637	3.657	3.715	3.812	3.950	4.129
20.0	3.837	3.856	3.915	4.012	4.150	4.329
21.0	4.037	4.056	4.115	4.212	4.350	4.529
22.0	4.237	4.256	4.315	4.412	4.550	4.729
23.0	4.437	4.456	4.514	4.612	4.750	4.929
24.0	4.636	4.656	4.714	4.812	4.950	5.130
25.0	4.836	4.856	4.914	5.012	5.151	5.330
26.0	5.036	5.056	5.114	5.212	5.351	5.530
27.0	5.236	5.256	5.314	5.412	5.551	5.730
28.0	5.436	5.456	5.514	5.612	5.751	5.930
29.0	5.636	5.655	5.714	5.812	5.951	6.130
30.0	5.836	5.855	5.914	6.012	6.151	6.330
31.0	6.036	6.055	6.114	6.212	6.351	6.530
32.0	6.236	6.255	6.314	6.412	6.551	6.730
33.0	6.436	6.455	6.514	6.612	6.751	6.930
34.0	6.636	6.655	6.714	6.812	6.951	7.131
35.0	6.835	6.855	6.914	7.013	7.151	7.331
36.0	7.035	7.055	7.114	7.213	7.351	7.531
37.0	7.235	7.255	7.314	7.413	7.551	7.731
38.0	7.435	7.455	7.514	7.613	7.751	7.931
39.0	7.635	7.655	7.714	7.813	7.951	8.131
40.0	7.835	7.855	7.914	8.013	8.151	8.331

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_{11}

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
0.000	0.00000	0.00000	0.00000	0.00000	0.00000	0.0000
.002	.00000	.00000	.00000	.00000	.00000	.0186
.004	.00000	.00000	.00000	.00000	.00000	.0264
.006	.00000	.00000	.00000	.00000	.00003	.0323
.008	.00000	.00000	.00000	.00000	.00009	.0373
.010	.00000	.00000	.00000	.00000	.00020	.0417
.012	.00000	.00000	.00000	.00000	.00036	.0457
.014	.00000	.00000	.00000	.00000	.00058	.0493
.016	.00000	.00000	.00000	.00001	.00085	.0527
.018	.00000	.00000	.00000	.00001	.00116	.0559
.020	.00000	.00000	.00000	.00002	.00152	.0589
.022	.00000	.00000	.00000	.00004	.00191	.0618
.024	.00000	.00000	.00000	.00005	.00234	.0646
.026	.00000	.00000	.00000	.00007	.00279	.0672
.028	.00000	.00000	.00000	.00010	.00328	.0697
.030	.00000	.00000	.00000	.00014	.00379	.0722
.032	.00000	.00000	.00001	.00018	.00432	.0746
.034	.00000	.00000	.00001	.00022	.00487	.0769
.036	.00000	.00000	.00001	.00027	.00545	.0791
.038	.00000	.00000	.00001	.00033	.00603	.0813
.040	.00000	.00000	.00002	.00040	.00664	.0834
.042	.00000	.00000	.00002	.00047	.00725	.0854
.044	.00000	.00000	.00003	.00055	.00788	.0874
.046	.00000	.00000	.00004	.00064	.00851	.0894
.048	.00000	.00000	.00005	.00073	.00916	.0913
.050	.00000	.00000	.00006	.00083	.0098	.0932
.055	.00000	.00000	.00008	.00110	.0115	.0978
.060	.00000	.00001	.00012	.00142	.0132	.1021
.065	.00000	.00002	.00017	.00177	.0149	.1063
.070	.00000	.00002	.00024	.00216	.0167	.1103
.075	.00000	.00003	.00031	.00258	.0185	.1142
.080	.00000	.00004	.00039	.00303	.0202	.1179
.085	.00001	.00006	.00049	.00352	.0220	.1215
.090	.00002	.00008	.00060	.00403	.0238	.1251
.095	.00003	.00010	.00072	.00457	.0256	.1285
.100	.00004	.00012	.00085	.00512	.0274	.1318
.105	.00005	.00015	.00100	.00571	.0292	.1351
.110	.00006	.00019	.00116	.00631	.0310	.1382
.115	.00008	.00023	.00133	.00693	.0328	.1414
.120	.00009	.00027	.00151	.00758	.0346	.1444
.125	.00011	.00033	.00170	.00824	.0364	.1474
.130	.00013	.00039	.00191	.00892	.0382	.1503
.135	.00016	.00045	.00213	.00961	.0399	.1532
.140	.00019	.00051	.00236	.01032	.0417	.1560

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_{11}

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
.145	.00022	.00058	.00260	.01104	.0434	.1587
.150	.00026	.00066	.00284	.01177	.0452	.1614
.155	.00030	.00075	.00310	.01252	.0469	.1641
.160	.00035	.00084	.00337	.01327	.0486	.1667
.165	.00039	.00093	.00366	.01404	.0504	.1693
.170	.00045	.00103	.00395	.01481	.0521	.1719
.175	.00051	.00114	.00425	.01559	.0538	.1744
.180	.00057	.00125	.00456	.01639	.0555	.1768
.185	.00065	.00137	.00488	.01719	.0571	.1793
.190	.00072	.00150	.00521	.01799	.0588	.1817
.195	.00080	.00163	.00555	.01881	.0605	.1841
.200	.00089	.00177	.00590	.0196	.0621	.1864
.210	.00107	.00206	.00661	.0213	.0654	.1910
.220	.00127	.00237	.00736	.0230	.0687	.1955
.230	.00149	.00271	.00813	.0247	.0719	.1999
.240	.00173	.00308	.00893	.0264	.0751	.2042
.250	.00199	.00348	.00976	.0281	.0782	.2084
.260	.00228	.00389	.01061	.0299	.0813	.2125
.270	.00259	.00433	.01149	.0316	.0844	.2166
.280	.00292	.00479	.01239	.0334	.0875	.2206
.290	.00328	.00527	.01332	.0352	.0905	.2245
.300	.00365	.00578	.01426	.0370	.0935	.2283
.310	.00404	.00631	.01523	.0388	.0964	.2321
.320	.00445	.00685	.01622	.0406	.0994	.2358
.330	.00488	.00742	.01722	.0424	.1023	.2395
.340	.00533	.00801	.01824	.0442	.1052	.2431
.350	.00580	.00862	.01929	.0460	.1080	.2466
.360	.00630	.00925	.02034	.0478	.1109	.2501
.370	.00682	.00990	.02142	.0497	.1137	.2536
.380	.00736	.01057	.02251	.0515	.1165	.2570
.390	.00792	.01125	.02361	.0533	.1192	.2604
.400	.0085	.0120	.0247	.0551	.1220	.2637
.420	.0097	.0134	.0270	.0588	.1274	.2702
.440	.0110	.0149	.0293	.0624	.1327	.2766
.460	.0123	.0165	.0317	.0661	.1380	.2828
.480	.0137	.0182	.0341	.0697	.1432	.2890
.500	.0152	.0199	.0365	.0734	.1483	.2950
.520	.0168	.0217	.0390	.0771	.1534	.3008
.540	.0183	.0235	.0415	.0808	.1584	.3066
.560	.0199	.0254	.0441	.0844	.1633	.3123
.580	.0216	.0273	.0467	.0881	.1682	.3179

THE TEMPERATURE DISTRIBUTION FUNCTION - Z_{11}

X	N = 0	N = .2	N = .4	N = .6	N = .8	N = 1.0
.600	.0234	.0292	.0493	.0917	.1730	.3234
.650	.0279	.0343	.0559	.1008	.1848	.3368
.700	.0328	.0397	.0627	.1098	.1964	.3498
.750	.0379	.0453	.0697	.1188	.2077	.3624
.800	.0433	.0510	.0767	.1278	.2188	.3746
.850	.0488	.0570	.0838	.1367	.2297	.3865
.900	.0546	.0631	.0911	.1456	.2404	.3982
.950	.0605	.0694	.0984	.1545	.2509	.4096
1.0	.067	.076	.106	.163	.261	.421
2.0	.206	.220	.262	.336	.452	.620
3.0	.362	.377	.424	.506	.629	.800
4.0	.522	.538	.588	.674	.801	.975
5.0	.685	.702	.753	.842	.971	1.146
6.0	.848	.866	.919	1.009	1.140	1.316
7.0	1.013	1.031	1.085	1.177	1.309	1.485
8.0	1.178	1.196	1.251	1.344	1.477	1.653
9.0	1.344	1.362	1.417	1.511	1.644	1.822
10.0	1.509	1.528	1.583	1.678	1.812	1.989
11.0	1.675	1.694	1.750	1.844	1.979	2.157
12.0	1.841	1.860	1.916	2.011	2.146	2.324
13.0	2.007	2.026	2.083	2.178	2.314	2.492
14.0	2.173	2.192	2.249	2.345	2.481	2.659
15.0	2.340	2.359	2.416	2.512	2.648	2.826
16.0	2.506	2.525	2.582	2.679	2.815	2.993
17.0	2.672	2.691	2.749	2.845	2.982	3.160
18.0	2.838	2.858	2.915	3.012	3.149	3.327
19.0	3.005	3.024	3.082	3.179	3.316	3.494
20.0	3.171	3.191	3.248	3.345	3.483	3.661
21.0	3.338	3.357	3.415	3.512	3.649	3.828
22.0	3.504	3.524	3.582	3.679	3.816	3.995
23.0	3.671	3.690	3.748	3.846	3.983	4.162
24.0	3.837	3.857	3.915	4.012	4.150	4.329
25.0	4.004	4.023	4.081	4.179	4.317	4.496
26.0	4.171	4.190	4.248	4.346	4.483	4.662
27.0	4.337	4.356	4.415	4.512	4.650	4.829
28.0	4.503	4.523	4.581	4.679	4.817	4.996
29.0	4.670	4.689	4.748	4.846	4.984	5.163
30.0	4.836	4.856	4.915	5.013	5.151	5.330
31.0	5.003	5.023	5.081	5.179	5.317	5.496
32.0	5.170	5.189	5.248	5.346	5.484	5.663
33.0	5.336	5.356	5.414	5.513	5.651	5.830
34.0	5.503	5.522	5.581	5.679	5.818	5.997
35.0	5.669	5.689	5.748	5.846	5.984	6.163
36.0	5.836	5.855	5.914	6.013	6.151	6.330
37.0	6.002	6.022	6.081	6.179	6.318	6.497
38.0	6.169	6.189	6.247	6.346	6.485	6.664
39.0	6.336	6.355	6.414	6.513	6.651	6.831
40.0	6.502	6.522	6.581	6.679	6.818	6.998