On Decision Making in Tandem Networks

by

Manal Dia

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Abstract

We study the convergence of Bayesian learning in a tandem social network. Each agent receives a noisy signal about the underlying state of the world, and observes her predecessor's action before choosing her own. We characterize the conditions under which, as the network grows larger, agents' beliefs converge to the true state of the world. The literature has predominantly focused on the case where the number of possible actions is equal to that of alternative states. We examine the case where agents pick three-valued actions to learn one of two possible states of the world. We focus on myopic strategies, and distinguish between learning in probability and learning almost surely. We show that ternary actions are not sufficient to achieve learning (almost sure or in probability) when the likelihood ratios of the private signals are bounded. When the private signals can be arbitrarily informative (unbounded likelihood ratios), we show that there is learning, in probability. Finally, we report an experimental test of how individuals learn from the behavior of others. We explore sequential decision making in a game of three players, where each decision maker observes her immediate predecessor's binary or ternary action. Our experimental design uses Amazon Mechanical Turk, and is based on a setup with continuous signals, discrete actions and a cutoff elicitation technique introduced in [ÇK05]. We replicate the findings of the experimental economics literature on observational learning in the binary action case and use them as a benchmark. We find that herds are less frequent when subjects use three actions instead of two. In addition, our results suggest that with ternary actions, behavior in the laboratory is less consistent with the predictions of Bayesian behavior than with binary actions.

Thesis Supervisor: John N. Tsitsiklis
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Mais malheur à l'auteur qui veut toujours instruire!
Le secret d'ennuyer est celui de tout dire.
– Voltaire, *Discours En Vers Sur L'homme.*

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Chapter 1

Introduction

1.1 Motivation

Social network models have emerged as important frameworks to study the propagation of information, and the formation of opinions and behaviors; they arise in a variety of settings, from bank runs and technology adoption, to mass hysteria, medical treatment choices and political campaigns. In these networks, individuals observe each other's behavior, and obtain private information and ideas directly from their environment, for instance through the media.

Consider a sequence of agents, each with a noisy signal about some hidden state of the world, where each agent needs to make a decision based on her best estimate of what that true state is: for instance, which political candidate should she vote for? What employment or education opportunities should she pursue? What stocks should she invest in? The behavior of such a decision maker obviously depends on her private information. Additionally, when her behavior is observed by other agents who are faced with the same decision problem, the other agents will in turn want to reassess their own beliefs. Rather than blind imitation or passive updating, this framework requires a complicated interactive reasoning that is adapted to the corresponding network setting. This in turns raises questions about what decision strategies will lead to learning the true underlying state of the world, i.e., cause the network to converge to the best possible use of the information available to its constituents. Our focus is on understanding whether sequential social networks lead a society to aggregate information – under what conditions do individuals learn the true state of the world?

The case of perfect information where each decision maker observes all previous individuals' actions has been widely studied [BHW92, Ban92]. It is more realistic though to investigate the case where individuals have access to only a subset of their predecessors' actions. Other work, e.g., [ADLO08] allows for constrained access to previous actions, but only looks at cases where decisions are binary. In view of this, it is of interest to understand how beliefs propagate in tandem networks, where each individual can observe a ternary message decision transmitted from her predecessor. Allowing for a ternary message helps capture situations in which a decision maker may abstain from making a decision, declaring a less costly 'I don't know' decision.

1.2 Problem Statement

The model we analyze consists of the following:

- An unknown state of the world, which is transmitted through noisy signals to each decision maker. We distinguish between bounded and unbounded beliefs, which is a distinction used by Cover [Cov69] and formalized by Smith and Sorensen [SS00], and pertains to whether the likelihood ratio implied by these private signals is finite and bounded away from 0 and from infinity. Unbounded beliefs refer to the case where the signals, can be arbitrarily informative, by having arbitrarily large or small likelihood ratios, whereas bounded beliefs refer to the case where the amount of information in any one signal is capped and cannot provide arbitrarily strong evidence on the state of the world.

- A cost function that is conditional on the underlying unknown state, and is identical for all agents.

- Sequential myopic decisions based on private signals and observation of ternary messages transmitted from a predecessor.

Our problem consists of studying conditions under which learning occurs in the above model, and contrasting them with already established findings for tandem networks communicating with binary messages. We argue that allowing for the number of actions to be higher than that of possible states is a powerful and useful extension; it not only provides a more realistic representation of information propagation conditions, but also introduces a richer approach to information herding and cascades.

1.3 Thesis Contributions

We establish learning conditions for tandem networks communicating with three-valued messages: we show that learning (in probability or almost surely) does not occur under bounded private beliefs, while learning in probability occurs under unbounded private beliefs. We explore an experimental test of social learning where each individual receives a private but noisy signal regarding the underlying state of nature in an exogenously determined order. After receiving their signal each subject communicates his or her belief about the state of the world in a sequential manner. Our experimental results suggest that herd behavior is less frequent with ternary actions than with binary actions. A heuristic where agents follow their own signal seems to be a better predictor of behavior than Bayesian learning theory – more so in the ternary case than in the binary case.

1.4 Thesis Outline

Chapter 2 gives an overview of the relevant literature, organized around two central themes: hypothesis testing and observational learning. We lay out our learning model in detail in Chapter 3, as well as definitions of learning and relevant network properties in Chapter 4. Chapter 5 presents our main theoretical findings, for both the bounded and unbounded likelihood ratio cases. We describe our experimental test in Chapter 6. In Chapter 7, we give our conclusions and discuss future work.

Chapter 2

Background and Prior Work

The literature on this subject draws from and applies to various areas of research. We focus on two main themes: decentralized hypothesis testing and social and observational learning.

2.1 Decentralized Hypothesis Testing

In the field of Engineering, there is a vast literature that studies decentralized hypothesis testing, motivated by distributed detection with decentralized sensors. The standard framework builds on the classic two-hypothesis testing problem $H_0 : \mathcal{P} = \mathcal{P}_0$ vs. $H_1 : \mathcal{P} = \mathcal{P}_1$. Instead of communicating with unconstrained sufficient statistics however, the model requires that data be reduced after each observation X_n to a finite statistic $T_n \in \{1, 2, \cdots, m\}$, where T_n is updated according to a time-varying algorithm of the form $T_{n+1} = f_n(T_n, X_{n+1})$. Cover [Cov69] proves the existence of a four-valued statistic (two-valued when beliefs are unbounded) that achieves a zero-limiting probability of error under either hypothesis. Koplowitz [Kop75] improves on Cover's algorithm with a scheme that requires a three-valued statistic only; he proves that given a sequence of iid Bernoulli observations, $m + 1$ states are necessary and sufficient to resolve the m-hypothesis test with a zero-limiting probability of error; for the network to learn, the number of messages therefore needs to be one higher than the number of alternative hypotheses.

Cover and Hellman [HC70] focus on time-invariant algorithms of the form $T_{n+1} = f(T_n, X_{n+1})$. They characterize the asymptotic probability of error and show that it is always strictly positive regardless of m ("No hope exists in the time invariant problem."). Cover and Hellman model the two-hypotheses problem using a finite Markov chain, with two states corresponding to the two distinct hypotheses, and time-invariant transition probabilities. They show that in general, no optimal rules f exist; however, they prove the existence of an ϵ-optimal class of rules that are determined by the likelihood ratio $d\mathcal{P}_0/d\mathcal{P}_1$ and require randomization. They conclude on a compelling note; "It would be of interest to see whether human beings, in problems to which they have allotted finite memory (such as "like," "indifference," and "dislike") demonstrate an optimal randomized learning procedure similar to that suggested by this paper."

Papastavrou and Athans [PA92] consolidate the previous literature by examining an infinite tandem architecture communicating with M-valued messages to perform M-ary hypothesis testing. They distinguish between "optimal" networks, where the objective is to minimize the probability of error for the last decision maker in the tandem, and "suboptimal" networks, where each decision maker tries selfishly to maximize the performance of her own decision. They show that in the former case, the necessary and sufficient condition for the probability of error to asymptotically go to zero is that the log-likelihood ratio of the observation of each decision maker be unbounded , either from above or below. For suboptimal networks, they show that the necessary and sufficient condition is that the log-likelihood ratio be unbounded from both above and below. Papastavrou and Athans point out that "since under the suboptimal decision scheme each decision maker tries to minimize his personal probability of error, it does not make sense to assign to each decision maker a number of messages different from the number of the alternative hypotheses." We argue that allowing for a number of messages that is different from that of alternative hypotheses is an adequate representation for certain models of learning where it might be more beneficial for an agent to declare a cheaper "I don't know" message, than to make an uncertain costly guess of what hypothesis is true.

Tay, Tsitsiklis and Win [TTW08] show that the rate of error probability decay in Bayesian decentralized binary hypothesis testing is sub-exponential. They also motivate the study of tandem networks as a tool for the study of more complicated network topologies, such as trees, or as an adequate representation of a single agent receiving private signals at multiple time periods, while remembering her latest decision.

2.2 Social and Observational Learning

In Economics, there is a similarly extensive literature on herd behavior, information cascades and social or observational learning. A large majority of work in this area has considered belief propagation when all previous actions are observed. Banerjee [Ban92] and Bikhchandani, Hirshleifer and Welch [BHW92] show that when each decision maker observes all previous actions and has bounded beliefs, there will not be asymptotic learning. Conversely, Smith and Sorenson [SS00] prove that when private beliefs are unbounded, again under the perfect information setup where agents observe all previous actions, there will be asymptotic learning.

Acemoglu et al. [ADLO08] consider a model where rational agents make a once in a lifetime binary decision based on a private signal, and observations of previous actions taken by a stochastically generated neighborhood of agents. Their main theorem states that if agents have unbounded beliefs, and in the case of *expanding observations*, i.e., in the absence of agents who are excessively influential, the network is guaranteed convergence to the correct decision.[1] Conversely, they show that the existence of an excessively influential group hinders social learning. In addition, the paper generalizes findings discussed above, proving that bounded beliefs usually imply no asymptotic learning, except in some special stochastic network topologies in which the flow of new information supersedes private signals, causing the pertinent information to aggregate, and the network to learn.

2.3 Herding and Information Cascades: An Experimental Approach

Empirical approaches to test for cascade behavior are mostly focused on laboratory experiments using human subjects. Anderson and Holt [AH97] were the first to develop an experiment based on Bikhchandani's model. In their setup, at the beginning of each round, subjects are randomly ordered, and one of two possible states of the world is randomly selected with equal probability. The states are represented by two urns, labeled A and B. Urn A contains 1/3 white balls and 2/3 black balls. Balls in urn B are 2/3 white and 1/3 black. The initial draw is not revealed until the end of the round, so that agents do not know whether balls are chosen from A or B. Each agent, however, receives a private signal about the true state of the world in the form of one sample (with replacement) from the urn. Agents then have to make sequential public binary decisions. At the end of the game, each agent who has guessed the true state of the world gets a fixed reward, while agents who guessed the wrong state get nothing. Anderson and Holt find that information cascades develop frequently in the lab.

[1] Note that a tandem network has expanding observations.

Kübler and Weizsäcker [KW05] build on [AH97] by examining the robustness of information cascades in laboratory experiments when private signals are no longer free: they look at the case where decision makers decided whether or not to receive a private signal at a small but positive cost. Though the equilibrium of their game dictates that only the first agent buys a signal and the remaining agents herd on the decision of the first, the laboratory results show that too many signals are bought compared to what the equilibrium predicts. They attribute this discrepancy to high levels of noise of the subjects' inferences.

Çelen and Kariv [ÇK04a] set out to experimentally distinguish information cascades from herd behavior.[2] Informational cascades occur when, after some finite time, all decision makers completely ignore their private signal when choosing an action, while herd behavior occurs when, after some finite time, all decision makers choose the same action, not necessarily ignoring their private information.[3] Çelen and Kariv introduce a technique called "cutoff elicitation", which consists of asking subjects about their strategy before seeing their private information; instead of choosing an action directly, each agent needs to provide a cutoff such that, if the signal is higher than the cutoff, action A is chosen, and if the signal is lower than the cutoff, action B is chosen. They observe that in the lab, both herd behavior and cascades occur frequently, and attribute the frequency of cascades to a noisy deviation from Bayes rationality. The same authors [ÇK04b] analyze observational learning under perfect and imperfect information. In the former, each decision maker is able to observe actions of all preceding agents, while, in the later, each agent only observes his immediate predecessor's action. They show that under imperfect information, there is no convergence of actions, though over time private information is ignored and increasingly longer periods of uniform behaviors, or imitation patterns, emerge. They thus establish that the imperfect information state introduces occasional, sharp and increasingly rare shifts of actions that punctuate periods of long uniform herd behavior.

2.4 Summary of Previous Work for Tandem Networks

This section summarizes previous work and motivates the work of this thesis. We categorize the findings established in the literature for tandem networks using 3 variables:

[2][BHW92] introduced the terminology of a *cascade* to describe an infinite train of individuals acting irrespective of the content of their signals.[SS00] were the first to decouple the notions of *cascades* and *herds*.

[3]An informational cascade implies herd behavior but herd behavior is not necessarily the result of an informational cascade.

1. The number D of distinct messages that each decision maker can send: Binary vs. Ternary. Whereas the literature has mostly focused on binary messages ($D = 2$), this thesis looks at the case where agents can pick between three communication messages ($D = 3$).

2. The decision objective: Designed vs. Myopic. The Engineering literature looks at the problem of designing the decision rules of each agent in the network so as to minimize the error probability of the last agent. A typical example is to specify the decision rules of a team of sensors connected in a line, where only the decision of the last agent is taken into account. In contrast, the Economics literature looks at the problem of minimizing the probability of error at each decision maker, claiming that the latter is a better approximation to real-world social networks. In this myopic case, each agent picks her strategy so as to maximize her own expected payoff, given the strategies of all other agents.[4]

3. The likelihood ratio implied by the the private signal structure: Bounded vs. Unbounded. When the likelihood ratio is bounded away from zero and infinity, there is a maximum amount of information in any individual signal: the private beliefs are said to be bounded. Alternatively, when the likelihood ratio can be arbitrarily high or low, private beliefs are said to be unbounded.[5]

As mentioned in Sections 2.1–2.3 above, previous work has mostly focused on models where both the number of distinct messages between agents and the underlying state of the world are binary. Cover established that no learning occurs in the Binary×Designed×Bounded case. It follows trivially that no learning occurs in the Binary×Myopic×Bounded case (see first column in Table 2.1). Papastavrou and Athans showed that there is learning in the Binary×Myopic×Unbounded case. It follows that there is learning in both the Binary×Designed ×Unbounded and the Ternary×Designed×UnBounded cases. Finally, Koplowitz established that there is learning in the Ternary×Designed×Bounded case.

This thesis fills the gaps in the literature by proving results for myopic networks communicating with ternary messages. We establish that learning occurs in the the Ternary×Myopic× Unbounded case, but not in Ternary×Myopic×Bounded case (see first row in Table 2.2). Ternary messages capture ample situations in social and economic networks where agents have a third option to declare a "no trade" or "don't know" message that is less costly than a wrong guess.

[4]See Section 3.2 for a precise definition.
[5]See Section 4.2 for a precise definition.

	Bounded	Unbounded
Myopic	N	Y [PA92]
Designed	N [Cov69]	Y

Table 2.1: Binary Messages: Y and N refer to learning, and no learning in probability, respectively. In the binary messages case, the network only converges to the true state when beliefs are unbounded.

	Bounded	Unbounded
Myopic	N [Our Results]	Y [Our Results]
Designed	Y [Kop75]	Y

Table 2.2: Ternary Messages: no learning in the myopic case with bounded beliefs, and learning with unbounded beliefs.

In addition, this thesis addresses another shortcoming of the literature corresponding to the definition of learning. The literature defines learning as convergence in probability. The only instance where learning is established in the almost sure sense is found in [Cov69] for the Binary$\times$Designed $\times$Unbounded, where almost sure convergence is used to establish convergence in probability. In this work, we draw a clear distinction between learning in probability and learning almost surely, and investigate these as two separate types of learning.

Chapter 3

The Model

3.1 The Network: Agents and Interaction

The underlying state of the world, H, is unknown, and assumed to be binary, i.e., $H = H_0$ or H_1, with prior probabilities $\mathbf{P}(H_0)$ and $\mathbf{P}(H_1)$, respectively. We will assume $\mathbf{P}(H_0) = \mathbf{P}(H_1) = 1/2$. The network consists of a countably infinite number n of decision makers, who each make their decisions sequentially as illustrated in Figure 3-1, and in the following manner:

Each agent t forms her belief about the state of the world based on:

1. a private signal X_t about the realization of the state of the world, independently generated according to a probability measure determined by H, i.e., $\mathcal{P}_{H_0}$ or $\mathcal{P}_{H_1}$;

2. for $t > 1$, the message received from her predecessor M_{t-1}, which is ternary, and can take one of three values $\{0, 1, *\}$.

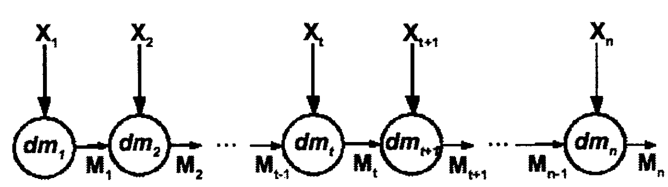


Figure 3-1: Tandem Configuration: A number n of decision makers are configured in a tandem network. Each decision maker t receives an independent message X_t and observes her predecessor's message M_{t-1} before transmitting her own message M_t to her successor.

3.2 The Decision Problem

Let us use the convention $M_0 = 0$. Given her information set $I_t = \{X_t, M_{t-1}\}$, each agent t's strategy is a mapping from the space of private signals and predecessor messages back into the set of messages, according to some decision rule $\gamma_t : M_t = \gamma_t(X_t, M_{t-1})$. Her decision problem consists of minimizing her local cost function J_t, where J_t is identical for all agents and is described as follows,

$$J_t = \{0, 1, *\} \times \{H_0, H_1\} \longrightarrow \mathbb{R}, \qquad \forall t \in \{1, \cdots, n\}.$$

Throughout this paper, we will assume the following cost structure for all agents t,

$$J_t(M_t, H) = \begin{cases} c, & \text{if } M_t = *; \\ 0, & \text{if } M_t = H; \\ 1, & \text{otherwise.} \end{cases}$$

where $c \in (0, .5)$ is a given constant. Under this cost function, a 0 or 1 message can be thought of as a best guess of the state H, whereas a $*$ corresponds to a "don't know".

Definition 1. Given all possible decision rules Γ available to an agent t and her cost function J_t, we say that agent t acts *myopically* when

$$\gamma_t = \arg\min_{\gamma \in \Gamma} \mathbf{E}[J_t(\gamma(X_t, M_{t-1}), H)].$$

A network therefore operates myopically if each agent tries to minimize the probability of error of her own decision: that is, each agent tries to optimize the performance of her own decision as opposed to the global team decision, ignoring all of her successors in the team.

Chapter 4

Learning: Definition and Characterization

In this chapter, we introduce three important concepts in the study of information aggregation in networks. First, we distinguish between two definitions of learning. Second, we carefully define the likelihood ratio characteristics of private signals. Finally, we specify a class of strategies that turns out to be optimal for the decision problem we have presented.

4.1 Almost Sure Learning vs. Learning in Probability

We distinguish between two different definitions of learning: *in probability* and *almost surely.* A network learns in probability if the probability of error asymptotically goes to zero. A stronger definition of learning, almost surely, requires convergence with probability 1.

Definition 2. The network learns *almost surely (A.S.)* if $\lim_{t\to\infty} M_t = M^*$ almost surely, where $M^* = j$ under H_j, for $j = 0, 1$.

Definition 3. The network learns *in probability (I.P.)* if and only if $\lim_{t\to\infty} \mathbf{P}(M_t = H_j | H_j) = 1$, for $j = 0, 1$.

4.2 Bounded vs. Unbounded Private Signals

The likelihood ratio of the densities that generate private signals plays a key role in the study of social learning. Whereas previous work distinguishes between bounded and unbounded likelihood ratios, it generally glosses over the distinction between one-sided and two-sided unboundedness.[1]

Definition 4. Let $X_1, X_2, \cdots$ be a sequence of independent identically distributed random variables generated according to a probability measure $\mathcal{P}$. We consider the two hypothesis testing problem $H_0 : \mathcal{P} = \mathcal{P}_0$ vs. $H_1 : \mathcal{P} = \mathcal{P}_1$, and define the likelihood ratio $L(X_t)$ as the Radon-Nikodym derivative $d\mathcal{P}_1/d\mathcal{P}_0$, where $\mathcal{P}_0$ and $\mathcal{P}_1$ are assumed to be absolutely continuous with respect to each other.

(a) The signal structure $(\mathcal{P}_0, \mathcal{P}_1)$ has *bounded beliefs* if there exist some β_1 and β_2 such that

$$0 < \beta_1 < L(X_t) < \beta_2 < \infty,$$

with probability one (wp1), under each hypothesis.

(b) The signal structure $(\mathcal{P}_0, \mathcal{P}_1)$ is:

(i) *unbounded from the left* if there is no $\beta_1 > 0$ such that $\beta_1 < L(X_t)$ with probability one (wp1). This is the definition of unboundedness used in [Cov69].

(ii) *unbounded from the right* if there is no β_2 such that $L(X_t) < \beta_2$ wp1.

(ii) *unbounded* if it is both unbounded from the left and the right. This is the definition of unboundedness used in [ADLO08, SS00].

In the bounded beliefs case, an agent can only derive a bounded amount of evidence about the underlying state of the world. In the unbounded beliefs case, she can receive an arbitrarily strong signal about the underlying state of the world.

4.3 Threshold Strategies

Threshold Strategies are crucial to defining optimal decisions rules for the agents.

Definition 5. Let $L(X_t)$ be the likelihood ratio associated with the observation X_t made by decision maker t. A strategy for an agent t is a *threshold strategy* if it can be expressed as follows:

[1] In one exception, Papastravou and Athans [PA92] show that one sided unboundedness is necessary and sufficient for learning with unbounded binary messages in the myopic network, and two-sided is necessary and sufficient in the designed case.

(i) for the first agent in the tandem $t = 1$, there exists a pair of thresholds (α, β), such that,

$$\gamma_t(X_1) = \begin{cases} 0, & \text{if } L(X_1) < \alpha \\ *, & \text{if } L(X_1) \in [\alpha, \beta) \\ 1, & \text{if } L(X_1) \geq \beta). \end{cases}$$

(ii) for each agent $t \geq 2$, there exist 3 pairs of thresholds $(\alpha^{M_{t-1}}, \beta^{M_{t-1}})$, $M_{t-1} = 0, *, 1$, such that,

$$\gamma_t(X_t, M_{t-1}) = \begin{cases} 0, & \text{if } L(X_t) < \alpha^{M_{t-1}}; \\ *, & \text{if } L(X_t) \in [\alpha_t^{M_{t-1}}, \beta^{M_{t-1}}); \\ 1, & \text{if } L(X_t) \geq \beta^{M_{t-1}}). \end{cases}$$

The next lemma has been widely established in the decentralized hypothesis literature, and follows easily from the existence results in [Tsi93b] and Section 4 in [Tsi93a].

Lemma 1. *There exists an optimal strategy for a myopic agent t that is a threshold strategy.*

Chapter 5

Learning With Three-valued Messages

Our main theoretical results are presented in this chapter. They are organized in two sections, covering the bounded and unbounded likelihood ratio cases, respectively. To simplify notation, we assume both values of the state of the world are equally likely, as mentioned in Chapter 4. An easy modification would extend our results to the case of unequal priors.

5.1 Learning: The Bounded Likelihood Ratio Case

This section establishes no learning in the bounded beliefs case. This result is to be contrasted with the results from designed systems, where three-valued messages are enough to guarantee almost sure learning [Kop75].

5.1.1 Almost Sure Learning

Theorem 1. *In the bounded likelihood ratio case, a tandem network operating myopically with three-valued messages does not learn almost surely.*

Proof. We shall prove this theorem by contradiction. Suppose that the myopic sequence of messages converges a.s. Consider the corresponding finite state Markov chains $(A_t^{H_0})$ and $(A_t^{H_1})$, under hypotheses H_0 and H_1, respectively. Each Markov chain consists of 3 states, corresponding to every possible value of message M_t at stage t. The transition probabilities are $\mathbf{P}(M_t = k|M_{t-1} = l) = p_{kl}^t$ for $l, k \in \{0, *, 1\}$ under H_0, and $\mathbf{P}(M_t = k|M_{t-1} = l) = q_{kl}^t$ under H_1 (Figure 5-1). Note that we have used the shorthand $p_{kl}^t = p_{kl}(\gamma_t)$ and $q_{kl}^t = q_{kl}(\gamma_t)$ since these transition probabilities are a function of the decision rule γ_t of agent t. From our assumption, $(A_t^{H_0})$

converges a.s. to state "0", whereas $(A_t^{H_1})$ converges a.s. to state "1". It follows that $M_t \to 0$ wp1 under H_0. If the probability of ever getting to states "1" or "*" is zero, then for all $t > 0$,

$$p_{01}^t = 0,\ p_{0*}^t = 0, \text{ and } M_t = 0.$$

Otherwise, it follows from the Borel-Cantelli Lemma that

$$\begin{aligned} &\sum_{t=0}^{\infty} p_{01}^t < \infty \\ &\sum_{t=0}^{\infty} p_{0*}^t < \infty \\ &\sum_{t=0}^{\infty} (p_{10}^t + p_{1*}^t) = \infty \end{aligned} \tag{5.1}$$

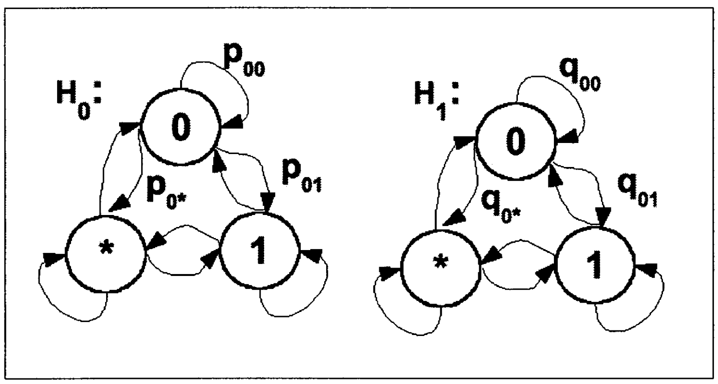


Figure 5-1: Markov Chains $(A_t^{H_0})$ and $(A_t^{H_1})$. Each Markov chain consists of 3 states, corresponding to every possible value of message M_t at stage t.

Since the likelihood ratios are bounded,[1] (5.1) implies that

$$\sum_{t=0}^{\infty} (q_{10}^t + q_{1*}^t) = \infty$$

Then, the Borel-Cantelli lemma implies that the second Markov chain $(A_t^{H_1})$ does not converge to state "1" with probability 1. $\square$

[1]This means that there exist positive quantities β_1 and β_2 such that $0 < \beta_1 < q_{01}^t/p_{01}^t < \beta_2 < \infty$ and $0 < \beta_1 < q_{0*}^t/p_{0*}^t < \beta_2 < \infty$.

5.1.2 Learning in Probability

We now strengthen our previous result by studying the convergence in probability of the tandem network.

Theorem 2. *In the bounded likelihood ratio case, a tandem network operating myopically with three-valued messages does not learn in probability.*

Proof. From the total probability theorem, the probability of error e_T for some agent $T \geq 1$ is given by

$$\begin{aligned} e_T &= \mathbf{P}(M_T = *) + \mathbf{P}(M_T = 1, H_0) + \mathbf{P}(M_T = 0, H_1) \\ &= \mathbf{P}(M_T = *) + \frac{1}{2}\big[\mathbf{P}(M_T = 1|H_0) + \mathbf{P}(M_T = 0|H_1)\big]. \end{aligned}$$

Given some error bound $\epsilon > 0$, one of two cases are possible:

Case 1 For all T, there exists some $t > T$ such that $e_t \geq \epsilon$; i.e, some subsequent decision maker has a probability of error greater than ϵ, so the network, by definition, does not learn in probability.

Case 2 There exists some T such that for all $t \geq T$, the error probability $e_t < \epsilon$. Let us fix such a T. At that particular time, we have

$$\begin{aligned} \mathbf{P}(M_T = 1|H_0) &< 2\epsilon \\ \mathbf{P}(M_T = *|H_1) &< 2\epsilon \\ \mathbf{P}(M_T = 0|H_1) &< 2\epsilon. \end{aligned}$$

Using Bayes' Rule and the assumption that states H_0, and H_1 are equally likely, this implies

$$\begin{aligned} \mathbf{P}(H_0|M_T = 1) &= \frac{\mathbf{P}(M_T = 1|H_0)}{\mathbf{P}(M_T = 1|H_0) + \mathbf{P}(M_T = 1|H_1)} \\ &= \left[1 + \frac{\mathbf{P}(M_T = 1|H_1)}{\mathbf{P}(M_T = 1|H_0)}\right]^{-1} \\ &< [1 + \frac{1}{2\epsilon}]^{-1} \\ &< 2\epsilon. \end{aligned}$$

For all private signals x observed by agent $T+1$, another Bayes' Rule application yields

$$\begin{aligned}
\mathbf{P}(H_0|M_T=1, X_{T+1}=x) &= \frac{\mathbf{P}(M_T=1, X_{T+1}=x|H_0)}{\mathbf{P}(M_T=1, X_{T+1}=x|H_0)+\mathbf{P}(M_T=1, X_{T+1}=x|H_1)} \\
&= \frac{\mathbf{P}(M_T=1|H_0)}{\mathbf{P}(M_T=1|H_0)+\mathbf{P}(M_T=1|H_1)L(x)}
\end{aligned} \tag{5.2}$$

where $L(x) = d\mathcal{P}_1/d\mathcal{P}_0(x)$ is the likelihood ratio from Definition 4, and we have used the fact that conditional on state H_0, the private signals and the observed decisions are independent, i.e., $\mathbf{P}(M_T = 1, X_{T+1} = x|H_0) = \mathbf{P}(M_T = 1|H_0)\mathbf{P}(X_{T+1} = x|H_0)$. We can then show that

$$\begin{aligned}
\mathbf{P}(M_T=1|H_1) &> 1-4\epsilon \\
\mathbf{P}(M_T=1|H_1)L(x) &> \beta_1(1-4\epsilon).
\end{aligned} \tag{5.3}$$

where the first line follows from $\mathbf{P}(M_T = 1|H_1) + \mathbf{P}(M_T = 0|H_1) + \mathbf{P}(M_T = *|H_1) = 1$, and the second line uses the fact that the likelihood ratio $L(x)$ is bounded, i.e., that there exists $\beta_1 > 0$ such that $L(x) > \beta_1 > 0$ wp1 for all x.

It then follows from equations (5.2) and (5.3) that

$$\mathbf{P}(H_0|M_T=1, X_{T+1}=x) < \frac{4\epsilon}{\beta_1(1-4\epsilon)} \triangleq h(\epsilon).$$

For ϵ small enough, $\mathbf{P}(H_0|M_T = 1, X_{T+1} = x)$ is small enough,[2] and agent $T+1$ will always choose $M_{T+1} = 1$, i.e., copy her predecessor, so

$$\mathbf{P}(M_{T+1=1}|M_T=1, X_{T+1}=x, H_0) = 1.$$

By integrating or summing over the range of the private signal X_{T+1}, it follows that

$$\mathbf{P}(H_0, M_{T+1}=1) \geq \mathbf{P}(H_0, M_T=1).$$

A parallel argument can show that $\mathbf{P}(H_1, M_{T+1} = 0) \geq \mathbf{P}(H_1, M_T = 0)$. By adding both

[2]A series expansion of $h(\epsilon)$ around $\epsilon = 0$ gives $h(\epsilon) = 1/\beta_1(2\epsilon + 4\epsilon^2 + 8\epsilon^3 + 16\epsilon^4 + 32\epsilon^5 + O(\epsilon^6))$.

probabilities, we have

$$
\begin{aligned}
e_{T+1} &\geq \mathbf{P}(H_1, M_{T+1} = 0) + \mathbf{P}(H_0, M_{T+1} = 1) \\
&\geq \mathbf{P}(H_1, M_T = 0) + \mathbf{P}(H_0, M_T = 1) \\
&> 0.
\end{aligned}
$$

Therefore we have a lower bound on the probability of error which is positive and monotonically nondecreasing for all $t > T$. Thus, e_t does not converge to zero, and the network does not learn in probability.

□

5.2 Learning: The Unbounded Likelihood Ratio Case

As we discussed in the Literature Review, learning occurs in the unbounded belief case for two-valued messages. Intuitively, this result should generalize to the case of three-valued messages where we expect the tandem network to do at least as well as with two-valued messages. Proving this result, however, requires a careful analysis.

5.2.1 Learning in Probability

Lemma 2. *The expected cost $\zeta_t = \mathbf{E}[J(\gamma_t(X_t, M_{t-1}), H)]$ is monotonically non-increasing: i.e., for all $t, \zeta_{t+1} \leq \zeta_t$.*

Proof. Myopic agent $t+1$ picks her optimal strategy from all available strategies Γ as given in Definition 1, i.e.,

$$\gamma_{t+1} = \arg\min_{\gamma \in \Gamma} \mathbf{E}[J(\gamma(X_{t+1}, M_t), H)].$$

Since Γ contains the copying strategy, $\hat{\gamma}_{t+1}$, it follows that agent $t+1$ never expects to do worse than agent t. Because agent $t+1$ always has the option to copy agent t's decision, thereby ensuring that her expected cost is as big as her predecessor's ($\zeta_{t+1} = \zeta_t$), she will only deviate when she expects to do better ($\zeta_{t+1} < \zeta_t$). More formally,

$$\begin{aligned}
\zeta_{t+1} &= \min_{\gamma \in \Gamma} \mathbf{E}[J(\gamma(X_{t+1}, M_t), H)] \\
&\leq \mathbf{E}[J(\hat{\gamma}_{t+1}(X_{t+1}, M_t), H)] \\
&= \mathbf{E}[J(\gamma_t(X_t, M_{t-1}), H)] \\
&= \zeta_t.
\end{aligned}$$

□

Therefore, $\zeta_{t+1} \leq \zeta_t$. The question remains whether $\{\zeta_t\}$ converges or not to some limit, and if so, whether it converges to zero or to a strictly positive number.

Theorem 3. *In the unbounded likelihood ratio case, a tandem network operating myopically with three-valued messages learns in probability.*

Proof. Consider a sequence of t agents, where the first $t-1$ agents have each made their decision according to the optimal myopic rule given in Definition 1. Now suppose that agent t, instead of acting myopically like her $t-1$ predecessors, decides to apply the following strategy $\tilde{\gamma}_t$: she will copy her neighbor's decision M_{t-1}, unless she receives an extremely high private signal, in which case she will switch to 1. More formally, for some ρ big enough,[3]

$$\tilde{M}_t = \tilde{\gamma}_t(X_t, M_{t-1}) = \begin{cases} M_{t-1}, & \text{if } L(X_t) < \rho; \\ 1, & \text{if } L(X_t) \geq \rho. \end{cases}$$

Suppose there is no learning, i.e., there exists a strictly positive quantity δ, such that for all $t > 0$, the expected cost ζ_t is bounded above δ.

We shall proceed with a proof by contradiction, first by proving that there is strict improvement (I), and second by showing that this strict improvement is by a positive factor of δ (II).

For $i = 0, 1$, define the conditional probability of passing for agent $t-1$ as $q_i = \mathbf{P}(M_{t-1} = *|H_i)$. Also define the conditional probability of an incorrect guess for agent $t-1$ under H_0 and H_1 as $p_0 = \mathbf{P}(M_{t-1} = 1|H_0)$ and $p_1 = \mathbf{P}(M_{t-1} = 0|H_1)$, respectively. Finally, define the conditional probability of getting an extremely high private signal as $r_i = \mathbf{P}(L(X_t) \geq \rho|H_i)$.

[3] We will define what "big enough" means at the end of the proof.

Lemma 3. *We have $r_1 \geq \rho r_0$.*

Proof. For simplicity, we consider the case where private signals are discrete. The same reasoning applies to the continuous time case, replacing probability mass functions with probability densities and summation with integration.

Consider a realization of a private signal $X_t = x$ such that $L(x) = \mathbf{P}(X_t = x|H_1)/\mathbf{P}(X_t = x|H_0) \geq \rho$. Note that $\mathbf{P}(X_t = x|H_1) \geq \rho \mathbf{P}(X = x|H_0)$ is true for all x such that $L(x) \geq \rho$. Therefore,

$$r_1 = \sum_{x:L(x)\geq\rho} \mathbf{P}(X_t = x|H_1) \geq \rho \sum_{x:L(x)\geq\rho} \mathbf{P}(X_t = x|H_0) = \rho r_0.$$

□

I. Strict Improvement. Because we use equal priors $\mathbf{P}(H_0) = \mathbf{P}(H_1) = 1/2$, the expected cost of agent $t-1$, assumed be bounded below by δ, is given by

$$\zeta_{t-1} = \frac{1}{2}\big[p_0 + cq_0 + p_1 + cq_1\big] \geq \delta.$$

Consequently, either $p_0 + cq_0 \geq \delta$ or $p_1 + cq_1 \geq \delta$. Without loss of generality, assume $p_1 + cq_1 \geq \delta$. (We can argue for the case of $p_0 + cq_0 \geq \delta$ by symmetry.)

We need to argue that correct switching (i.e. switching to 1 under H_1) is much more likely than incorrect switching (i.e. switching to 1 under H_0), or equivalently that $r_1 \gg r_0$. The expected cost of agent t, had she acted myopically, is less than her expected cost with her suboptimal strategy $\tilde{\gamma}_t$. Therefore,

$$\begin{aligned}
\zeta_t &= \min_{\gamma\in\Gamma} \mathbf{E}[J(\gamma(X_t, M_t), H)] \\
&\leq \mathbf{E}[J(\tilde{\gamma}_t(X_t, M_t), H)] \\
&= \frac{1}{2}\big[(1 - p_0 - q_0)r_0 + cq_0(1 - r_0) + q_0 r_0 + p_1(1 - r_1) + cq_1(1 - r_1)\big] \\
&\leq \frac{1}{2}\big[p_0 + cq_0 + p_1 + cq_1 + r_0 - (p_1 + cq_1)r_1\big] \\
&= \zeta_{t-1} + \frac{1}{2}\big[r_0 - (p_1 + cq_1)r_1\big] \\
&\leq \zeta_{t-1} + \frac{1}{2}\big[r_0 - \delta r_1\big],
\end{aligned}$$

where we have used Figure 5-2 to calculate $\mathbf{E}[J(\tilde{\gamma}_t(X_t, M_t), H)]$.

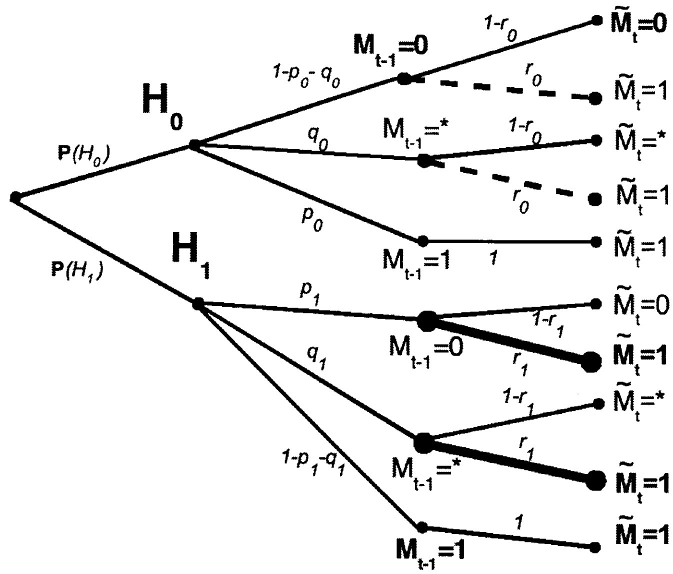


Figure 5-2: Universe of possibilities for agent t who applies a suboptimal strategy $\tilde{\gamma}_t$. Each branch is associated with a conditional probability (in italics). Messages in bold correspond to the correct decisions. The dashed branch correspond to an incorrect switching, while the bolded branches correspond to a correct switching. We argue that correct switching is much more likely than incorrect switching.

To prove strict improvement, we need to argue that $r_0 - \delta r_1$ is negative. Using Lemma 3, we get

$$\zeta_t \leq \zeta_{t-1} + \frac{1}{2}\big[\frac{r_1}{\rho} - \delta r_1\big].$$

II. By a positive factor of δ. To complete our proof, we would like to establish a good bound for the strict improvement described in I. For this, we pick a threshold ρ that is big enough, i.e., $r_1/\rho \leq \delta r_1$ or $\rho \geq 1/\delta$. For example, we pick $\rho = 2/\delta$. In this case,

$$\zeta_t \leq \zeta_{t-1} - \frac{1}{4}\delta r_1.$$

Since this inequality would have to be true for every t, and since $\zeta_t \geq 0$, we obtain a contradiction, thus completing our proof. □

We now extend Theorem 3 to the case where agents can communicate with D-valued messages, $D \geq 2$. When $D \geq 3$, we consider the following cost structure

$$J_t(M_t, H) = \begin{cases} 0, & \text{if } H = H_0 \text{ and } M_t = 0 \text{ or } H = H_1 \text{ and } M_t = D-1; \\ c_i, & \text{if } H = H_0 \text{ and } M_t = 1, \cdots, D-2; \\ d_i, & \text{if } H = H_1 \text{ and } M_t = 1, \cdots, D-2; \\ 1, & \text{otherwise,} \end{cases}$$

where c_i and d_i are in $(0, 0.5)$.

Theorem 4. *In the unbounded likelihood ratio case, a tandem network operating myopically with D-valued messages, $D \geq 2$ learns in probability.*

Proof. A proof for the case $D = 2$ can be found in [PA92, Proposition 3]. Theorem 3 proved the case $D = 3$. The proof for $D \geq 4$ proceeds as for Theorem 3, by coalescing messages 1 through $D-2$ into one "don't know" or * message. $\square$

5.2.2 Almost Sure Learning

As mentioned in Chapter 2, the decentralized detection literature finds that tandem networks can learn almost surely with two messages and unbounded private beliefs (e.g., [Cov69, Theorem 1]). The standard herding models however are limited to learning in probability (e.g., [BHW92, SS00, ADLO08]. In this section we discuss learning almost surely when the private beliefs are unbounded. We conjecture that almost sure learning can be achieved with three-valued messages, with certain private signal structures.

Rates of Convergence for Decentralized Hypothesis Testing with Binary Messages

Tay, Tsitsiklis and Win [TTW08] study the decay rate of the probability of error over a tandem network with two-valued messages. They consider a Gaussian hypothesis testing problem; private signals are distributed according to a zero-mean normal distribution with variance σ_0^2 under H_0 and σ_1^2 under H_1 $(0 < \sigma_0^2 < \frac{1}{2} < \sigma_1^2)$.[4] They prove that a selfish strategy[5] where an

[4]Note that this is a one-sided unbounded private signal structure (unbounded from the left).

[5]Formally, the strategy γ_t is defined for $t \geq 1$ as,

$$\gamma_t(M_{t-1}, X_t) = \begin{cases} 0, & \text{if } X_t^2 \leq \log t \text{ and } M_{t-1} = 0; \\ 1, & \text{otherwise.} \end{cases}$$

where $M_0 = 0$.

agent t decides $M_t = 1$ if and only if $X_i^2 > \log t$ for some $i \leq t$ achieves a probability of error $O(t^{1-\frac{1}{2\sigma_0^2}})$. Building on this example, a particular Gaussian hypothesis testing problem where $0 < \sigma_0^2 < 1/4$ achieves almost sure learning with binary messages.

Rates of Convergence for Social Learning with Binary Messages

Lobel et al. [LADO09] show that convergence to the correct decision is faster than a polynomial rate when private beliefs have polynomial shape.[6] They consider unbounded private beliefs with polynomial distributions, defined as $F_0(x) = x - x^2$ under H_0 and $F_1(x) = x^2$ under H_1, x in $[0,1]$. They show that an optimal strategy achieves a probability of error that decays as $\Theta(t^{-1})$ [LADO09, Section IV]. This suggests that the sum of private errors diverges, leaving open the possibility that the network does not converge almost surely in this case. Indeed, we conjecture that it does not.

Learning Almost Surely with Three Messages

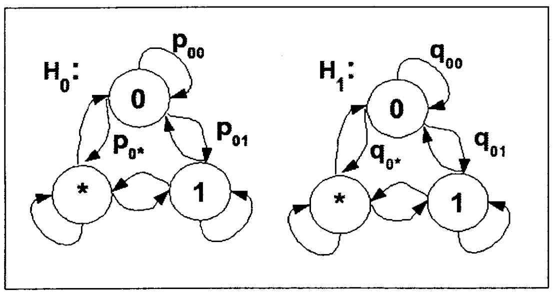
Given the examples in the previous two sections, we conjecture that in the unbounded likelihood ratio case, a tandem network operating myopically with three-valued messages does not always learn almost surely, but that it does for certain special cases. The following theorem might prove useful for the cases in which the network does not converge a.s. in the binary case.

Theorem 5. *In the unbounded likelihood ratio case, a tandem network that does not learn almost surely with two-valued messages, will not learn almost surely with three-valued messages.*

We proceed with a proof by contradiction, that starts exactly like the proof of Theorem 1. We suppose that the myopic sequence of messages converges a.s. and consider the corresponding finite state Markov chains $(A_t^{H_0})$ and $(A_t^{H_1})$, under hypotheses H_0 and H_1, respectively.

We construct two new Markov chains $(B_t^{H_0})$ and $(B_t^{H_1})$ from $(A_t^{H_0})$ and $(A_t^{H_1})$ as follows: if $(A_t^{H_0})$ transitions from state "0" into state " $*$ " , $(B_t^{H_0})$ transitions from "0" to "1"; if $(A_t^{H_0})$ transitions from state "1" into state " $*$ ", $(B_t^{H_0})$ switches from "1" to "0". For $(B_t^{H_1})$, the same procedure is used with the roles of states "0" and "1" interchanged. The transition probabilities are shown in Figure 5-3.

[6] Their model is similar to the one we use, except agents communicate with binary messages. They refer to the tandem topology as "immediate neighbor sampling".



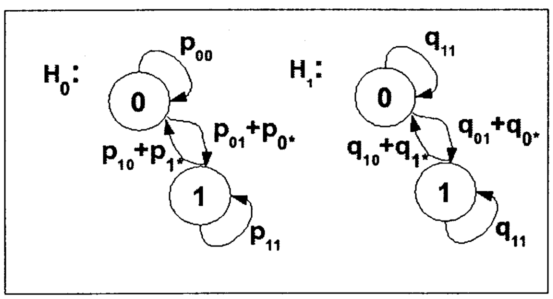


Figure 5-3: Markov Chains $(A_t^{H_0})$ and $(A_t^{H_1})$ (above), $(B_t^{H_0})$ and $(B_t^{H_1})$ (below). If $(A_t^{H_0})$ and $(A_t^{H_1})$ converge a.s., then so do $(B_t^{H_0})$ and $(B_t^{H_1})$, yielding a contradiction.

It follows from (5.1-5.3) and the Borel-Cantelli Lemma that that $(B_t^{H_0})$ converges to state "0" wp1. A parallel argument can be used to prove that $(B_t^{H_1})$ converges to state "1" wp1. This is a contradiction because if $(B_t^{H_i})$ converges to state "i" wp1, for $i = 0, 1$, we have specified a possible sequence of strategies $\gamma_1, \gamma_2, \cdots$ that learns almost surely with two-valued messages and unbounded beliefs, contradicting our conjectured result for the two-valued messages case.

Chapter 6

An Experimental Test of Social Learning

We take an experimental look at a variant of the social learning model discussed previously, and investigate empirical evidence for the findings. We report an experimental test of how individuals learn by observing the behavior of their immediate predecessors. Our experimental design is based on the ball-and-urn observational learning experiments paradigm laid out by Anderson and Holt [AH97] and extended by Çelen and Kariv [ÇK05]. In particular, we adopt the continuous signal-space and cutoff elicitation method in [ÇK04a]. Our contribution to the experimental learning literature is two-fold. First, our experiment looks at the case of ternary actions, whereas the information cascades literature has been surprisingly limited to binary-signal-binary-action. Second, we introduce the use of Amazon Mechanical Turk as a powerful framework to run learning experiments. We replicate results in the experimental economics literature, and extend them to the three-valued actions case. The experimental economics literature has so far mostly focused on the use of student subjects in a laboratory setting. We find that herd behavior is less frequent with ternary actions than binary actions. A heuristic where agents follow their own signal seems to be better predictor of behavior than Bayesian rationality – more so than in the binary case.

6.1 Motivation

Information cascades arise when individuals rationally choose identical actions despite having different private information. In economic environments where decision makers have imperfect information about the true state of the world, it can be rational to ignore one's own private information and make decisions based on what are believed to be more informative public signals. In particular, if decisions are made sequentially and some or all of the earlier decisions become publicly known, information cascades may emerge. They are observed in a variety of settings, from bank runs and technology adoption, to mass hysteria, medical treatment choices, and political campaigns. They also play an important role in financial markets, most notably in the context of bubbles and crashes.

It is difficult to test information cascades empirically, because the effects of reputation, bounded rationality, and payoff externalities are often entangled in empirical data. This problem can be overcome in an experimental design in which we can isolate control variables – in particular, private information that agents have when making decisions, as well as information aggregation mechanisms. In our experiment, three decision makers receive incomplete private information about an unknown state of the world, and observe one or all past actions. Given both their private signal and their observations, they choose, sequentially and according to a predetermined order, whether they want to guess which of two states of the world is true, or whether they will abstain from guessing, at a cost. Such a binary-signal-ternary-action setup has not been examined in the literature, and raises key questions about the effect of a richer yet still limited communication structure on the information pipeline.

6.2 Hypotheses

We set out to investigate a setting where the number of actions is different from that of alternative hypotheses. We argue the latter is a particularly adequate representation for learning in settings where an agent has the option to declare a cheaper "don't know" action, rather than to make an uncertain costly guess of what hypothesis is true.

Questions that motivate this design are:

- Do subjects, in problems to which they have allotted finite memory (such as "yes," "indifferent" and "no") demonstrate learning procedures similar to the ones suggested in the binary case?

- How do the convergence results (herding/information cascades/learning) compare? Is the Bayesian model (or a some mix of a Bayesian and irrational decision maker) capable of explaining the results?

- How often do we observe imitation and switches along the tandem when the option of passing is allowed at a cost?

The hypotheses that we propose to test are:

(1) Decision makers update information according to Bayes' Rule and recognize the cascade behavior of others across information and payoff structures, specifically:

  (1a) Subjects tend to rely more on the information revealed by the predecessor's action rather their private signal as their position increases.

  (1b) In line with the theoretical results, early moving players choose clear-cut signaling guesses and later players are able to decipher them and act accordingly.

(1′) Alternatively, decision makers are Bayes rational, but do not take into account the rationality or cascade behavior of others.

(2) The frequency of herding is lower, and that of action switches is higher when a third "Passing" or "No Trading" option is allowed.

6.3 Experimental Design

As a step towards investigating the above hypotheses, we propose a simple design based on the the number of alternative decisions (binary or ternary) available to agents. We adopt the same local payoff structure described in Chapter 4, where agents each attempt to maximize their local payoff. We also use the imperfect information setup defined in Chapter 3, where an agent only knows the decision of her immediate predecessor. As opposed to previous experimental designs [AH97, ÇK04a], we examine a network of $n = 3$ agents.[1] In the first, baseline, treatment,

[1] $n = 3$ is big enough to allow the observation of learning patterns while still being theoretically tractable. How these patterns extend to longer tandems could be explored in a further experiment, but is outside the scope of this design.

subjects choose one of two actions. In the second treatment, subjects choose one of three actions.

A round consists of 3 decision problems, or rounds where subjects are randomly assigned a decision turn. Common to both treatments are the following features. In each round, every subject receives a private signal in the form of a number chosen uniformly at random in the set of real numbers $[-10, 10]$ (rounded to 2 decimal points). Subjects then have to choose between alternative actions 1 or 0, and possibly a third option of passing, or no guessing, denoted $*$. Action 1 yields 2.5 cents only if the sum of all private signals is positive (H_1 is true), and otherwise yields zero. Action 0 yields 2.5 cents only if the sum of all private signals is negative (H_0 is true), and otherwise yields zero. Action $*$ yields an unconditional lower profit regardless of private signals.[2]

6.4 Experimental Procedures

We conducted the above experiment[3] using the Amazon Mechanical Turk service. Mechanical Turk (MTurk) is an Internet service provided by Amazon that gives developers the ability to pay people small rewards to do human computation tasks. It is sometimes referred to as a crowdsourcing[4] or micro-task marketplace, and an artificial artificial intelligence system, in the sense that it uses human powered artificial intelligence.[5] Developers use the Amazon web services API to submit tasks (or Human Intelligence Tasks, HITs) to the MTurk web site, review completed tasks, and integrate the results into their applications.

We use TurKit, a toolkit for programming iterative processes on MTurk developed by the MIT User Interface Design group. Little et al. [LCMG09] present a number of examples of iterative tasks, involving iterative text improvement and sorting, where they ask turkers to label images, brainstorm, vote on each other's output or act as comparison functions in a sort algorithm. Following guidelines presented in [KCS08, SF08], as well as the "cutoff elicitation"

[2] As an important technical point, this setup differs from the model presented in Chapter 3 in that the private signals are conditionally dependent given the state of the world.

[3] MIT institutional review board approval, for use of human subjects in research, was obtained for this study under COUHES Protocol# 0905003265.

[4] According to Wikipedia, crowdsourcing is "a neologism for the act of taking a task traditionally performed by an employee or contractor, and outsourcing it to an undefined, generally large group of people or community in the form of an open call."

[5] In 1769, Wolfgang von Kempelen built a chess automaton that toured the United States and Europe for years, defeating such famous opponents as Benjamin Franklin, Napolean Bonaparte, and Edgar Allen Poe [Sta02]. The secret to the automaton was in fact a human chess master hidden inside of the wooden mannequin.

method introduced in [ÇK04a], we design our tasks to elicit specific information regarding strategies and we attempt to reduce invalid answers. Instructions are attached as Appendix B.

6.5 Theory

The Network: Agents and Interaction As in Chapter 4, the underlying state of the world, H is unknown, and assumed to be binary, i.e., $H \in \{H_0, H_1\}$, with equal prior probabilities. The economy consists of a finite number n of Bayesian rational agents indexed by $t = 1, 2, \cdots, n$. In this design, we consider $n = 3$ agents. Each agent t makes a once-in-a-lifetime decision: to choose action 1 profitable under H_0, to choose action 0 profitable under H_1, or to Pass – and say "I don't know". These decisions correspond to $M_t = 1$, 0 and $*$ respectively. Agents make their decisions sequentially according to a predetermined random order.

Each agent t forms her belief about the state of the world based on:

1. a private signal X_t about the realization of the state of the world, independently generated according to a uniform distribution over $[-1, 1]$. [6] Private signals are thus bounded and their support is continuous and symmetric around zero.

2. unless $t = 1$, the action of her predecessor M_{t-1}, which can take one of three values $\{1, 0, *\}$.

The Payoff: Selfish Structure We adopt the same local or selfish setup as in Chapter 4, where each agent's payoff is represented by the payoff function $\Pi(M_t)$, normalized for simplicity of notation, and defined as follows

$$\Pi(M_t) = \begin{cases} 1 & \text{if } M_t = 1 \text{ and } H = H_1 : \sum_{k=1}^{n} X_k \geq 0 \text{ or if } M_t = 0 \text{ and } H = H_0 : \sum_{k=1}^{n} X_t < 0; \\ c = 3/5 & \text{if } M_t = *; \\ 0 & \text{otherwise.} \end{cases}$$

In other words, agents who pick action 1 earn +1 if the sum of all private signals is positive (H_1 is true), and nothing otherwise. On the other hand, agents who pick action 0 earn +1 if the sum of all private signals is negative (H_0 is true), and nothing otherwise. Agents who pick action

[6] We scale the distribution support from $[-10, 10]$ to $[-1, 1]$ for purposes of the analysis.

$*$ earn $c = 3/5$ regardless of private signals.[7] Also, we consider the binary case (equivalent to $c = 0$).

Information Structure: Perfect vs. Imperfect Information We use the imperfect information economy setup, where decision makers are not informed of the full history of actions taken, but only of their predecessor's decision, that is, for any agent $t > 1$,

$$I_t = \{X_t, M_{t-1}\}.$$

The information structure is assumed to be common knowledge; therefore every agent is assumed to know whose actions other agents observe.

The Decision Problem Given her information set I_t, each agent t's strategy is a mapping from the space of private signals and predecessor's action back into the set of actions, according to some decision rule $\gamma_t : M_t = \gamma_t(I_t)$. Her decision problem consists of maximizing her expected payoff. More formally, given all possible decision rules Γ available to an agent t and her payoff function Π, agent t picks a strategy γ_t such that

$$\gamma_t = \arg\max_{\gamma \in \Gamma} \mathbf{E}[\Pi_t(\gamma(I_{t-1})].$$

Proposition 1. *In the binary action case, there exists an optimal strategy for an agent t that is a threshold strategy.*

(i) For the first agent $t = 1$, the optimal decision rule γ_1 consists of following the sign of the private signal,

$$\gamma_1(X_1) = \begin{cases} 0, & \text{if } X_1 < 0; \\ 1, & \text{if } X_1 \geq 0. \end{cases}$$

(ii) For each agent $t = 2$ and 3, there exists a threshold $\alpha_t > 0$ such that t will always follow her predecessor, unless her private signal is higher than α_t or lower than $-\alpha_t$, in which

[7] The payoff c of action $*$ needs to be at least 0.5, otherwise it is never optimal to choose it.

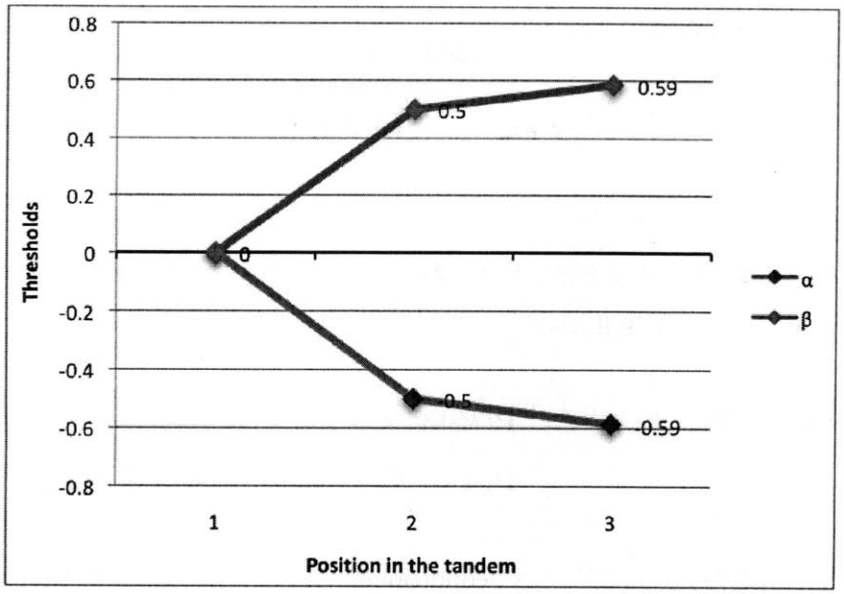


Figure 6-1: Binary Actions – Theoretical thresholds as a function of player's position in the tandem.

case, switching is optimal.

$$\gamma_t(X_t, M_{t-1}) = \begin{cases} 0, & \textit{if } X_t < -\alpha_t \textit{ and } M_{t-1} = 1 \textit{ or } X_t < \alpha_t \textit{ and } M_{t-1} = 0; \\ 1, & \textit{if } X_t \geq \alpha_t \textit{ and } M_{t-1} = 0 \textit{ or } X_t \geq -\alpha_t \textit{ and } M_{t-1} = 1. \end{cases}$$

We calculate $\alpha_2 = 1/2$ *and* $\alpha_3 = \sqrt{2} - 2$.

Proof. Each agent t wishes to maximize her expected payoff given her information set, i.e.,

$$\max_{\gamma} \mathbf{E}[\Pi_t(\gamma(I_{t-1})] = \max_{\gamma} \big[\mathbf{P}(H_0, I_t, M_t = 0) + \mathbf{P}(H_1, I_t, M_t = 1)\big].$$

Knowing only her private signal X_1, Player 1 picks action 0 when $\mathbf{P}(H_0|X_1) > 1/2$. Equivalently, she picks action 1 when $\mathbf{P}(H_1|X_1) \leq 1/2$. To show that such a strategy is indeed a threshold strategy, we can express $\mathbf{P}(H_0|X_1)$ as a function of X_1

$$\mathbf{P}(H_0|X_1) = \mathbf{P}(X_1 + X_2 + X_3 < 0|X_1) = \mathbf{P}(X_2 + X_3 < -X_1|X_1).$$

Using the cumulative distribution function of $X_2 + X_3$,[8] we show that an optimal strategy for

[8]See Appendix A for an explicit CDF.

Player 1 consists of a threshold rule because

$$\mathbf{P}(H_0|X_1) > 1/2 \Leftrightarrow F_{X_2+X_3}(-X_1) > 1/2 \Leftrightarrow X_1 < 0.$$

Player 2 observes Player 1's decision in addition to her private signal X_2. Again, she maximizes her payoff by picking action 0 when $\mathbf{P}(H_0|M_1, X_2) > 1/2$. We rewrite this probability as a function of X_2 when $M_1 = 0$ as follows,

$$\begin{aligned} \mathbf{P}(H_0|M_1 = 0, X_2) &= \mathbf{P}(H_0|X_1 < 0, X_2) \\ &= \mathbf{P}(X_1 + X_3 < -X_2|X_1 < 0, X_2). \end{aligned}$$

Using the cumulative distribution of $X_1 + X_3$ conditioned on $X_1 < 0$,[9] we show that an optimal strategy for Player 2 consists of a threshold rule because

$$\begin{aligned} \mathbf{P}(H_0|M_1 = 0, X_2) > 1/2 \quad &\Leftrightarrow F_{X_1+X_3|M_1=0}(-X_2) > 1/2 \quad \Leftrightarrow X_2 < 1/2 \\ \mathbf{P}(H_0|M_1 = 1, X_2) > 1/2 \quad &\Leftrightarrow F_{X_1+X_3|M_1=1}(-X_2) > 1/2 \quad \Leftrightarrow X_2 < -1/2, \end{aligned}$$

where the first line results from solving for X_2 in the inequality $F_{X_1+X_3|M_1=0}(-X_2) > 1/2$, and the second line follows from symmetry.

Similarly, Player 3 observes Player 2's decision in addition to her private signal X_3. Again, she maximizes her payoff by picking action 0 when $\mathbf{P}(H_0|M_2, X_3) > 1/2$. Suppose Player 3 observes Player 2's decision $M_2 = 0$. Player 2 might have copied Player 1 (if $X_1 < 0$ and $X_2 < 1/2$) or she might have switched given a low enough private signal (if $X_1 \geq 0$ and $X_2 < -1/2$). By a simple Bayes' Rule application, Player 3 assigns probability 3/4 to the

[9] *Ibid.*

former case ($M_1 = 0|M_2 = 0$), and 1/4 to the latter ($M_1 = 1|M_2 = 0$), so that

$$\begin{aligned}
\mathbf{P}(H_0|M_2 = 0, X_3) &= \frac{3}{4}\mathbf{P}(H_0|M_1 = 0, M_2 = 0, X_3) + \frac{1}{4}\mathbf{P}(H_0|M_1 = 1, M_2 = 0, X_3) \\
&= \frac{3}{4}\mathbf{P}(H_0|X_1 < 0, X_2 < \frac{1}{2}, X_3) + \\
&\quad \frac{1}{4}\mathbf{P}(H_0|X_1 \geq 0, X_2 < -\frac{1}{2}, X_3) \\
&= \frac{3}{4}\mathbf{P}(X_1 + X_2 < -X_3|X_1 < 0, X_2 < \frac{1}{2}, X_3) + \\
&\quad \frac{1}{4}\mathbf{P}(X_1 + X_2 < -X_3|X_1 \geq 0, X_2 < -\frac{1}{2}, X_3).
\end{aligned}$$

Using the cumulative distribution of $X_1 + X_2$ conditioned on $M_2 = 0$,[10] we show that an optimal strategy for Player 3 consists of a threshold rule because

$$\begin{aligned}
\mathbf{P}(H_0|M_2 = 0, X_3) > 1/2 \quad &\Leftrightarrow F_{X_1+X_2|M_2=0}(-X_3) > 1/2 \quad \Leftrightarrow X_3 < \sqrt{2} - 2 \\
\mathbf{P}(H_0|M_2 = 1, X_3) > 1/2 \quad &\Leftrightarrow F_{X_1+X_2|M_2=1}(-X_3) > 1/2 \quad \Leftrightarrow X_3 < 2 - \sqrt{2}.
\end{aligned}$$

We note that this last threshold is different from Çelen and Kariv's (5/8), though their experimental payoff structure is identical to ours [ÇK05]. While their theoretical framework has a different payoff structure than the one used in their experimental tests,[11] we believe our calculation makes sense given our underlying assumptions. □

Proposition 2. *In the ternary action case, there exists an optimal strategy for an agent t that is a threshold strategy.*

(i) For the first agent $t = 1$, there exists a pair of thresholds (α_1, β_1), such that,

$$\gamma_1(X_1) = \begin{cases} 0, & \text{if } X_1 < \alpha_1; \\ *, & \text{if } \alpha_1 \leq X_1 < \beta_1; \\ 1, & \text{if } X_1 \geq \beta_1. \end{cases}$$

For our game, we calculate thresholds $\alpha_1 = -4\sqrt{5}/5 + 2$ and $\beta_1 = 4\sqrt{5}/5 - 2$.

(ii) For each agent $t = 2, 3$, there exist 3 pairs of thresholds $(\alpha_t^{M_{t-1}}, \beta_t^{M_{t-1}})$, $M_{t-1} = 1, *, 0$

[10] *Ibid.*

[11] Their theoretical framework sets the payoff to be equal to the sum of private signals [ÇK04b], while their experiments reward correct decisions with a fixed \$2.

such that,

$$\gamma_t(X_t, M_{t-1}) = \begin{cases} 0, & \text{if } X_t < \alpha_t^{M_{t-1}}; \\ *, & \text{if } \alpha_t^{M_{t-1}} \le X_t < \beta_t^{M_{t-1}}; \\ 1, & \text{if } X_t \ge \beta_t^{M_{t-1}}. \end{cases}$$

For our game, we calculate the following thresholds

Agent 2	$\alpha_2^0 = (13 - 4\sqrt{5})/10$ $\alpha_2^* = -1/5$ $\alpha_2^1 = (4\sqrt{5} - 17)/10$	$\beta_2^0 = (17 - 4\sqrt{5})/10$ $\beta_2^* = 1/5$ $\beta_2^1 = (4\sqrt{5} - 13)/10$
Agent 3	$\alpha_3^0 = 2 - 4\sqrt{3}/5$ $\alpha_3^* = 2(2\sqrt{5} + \sqrt{35 - 12\sqrt{5}})/5 - 3$ $\alpha_3^1 = 4\sqrt{2}/5 - 2$	$\beta_3^0 = 2 - 4\sqrt{2}/5$ $\beta_3^* = 3 - 2(2\sqrt{5} + \sqrt{35 - 12\sqrt{5}})/5$ $\beta_3^1 = 4\sqrt{3}/5 - 2$

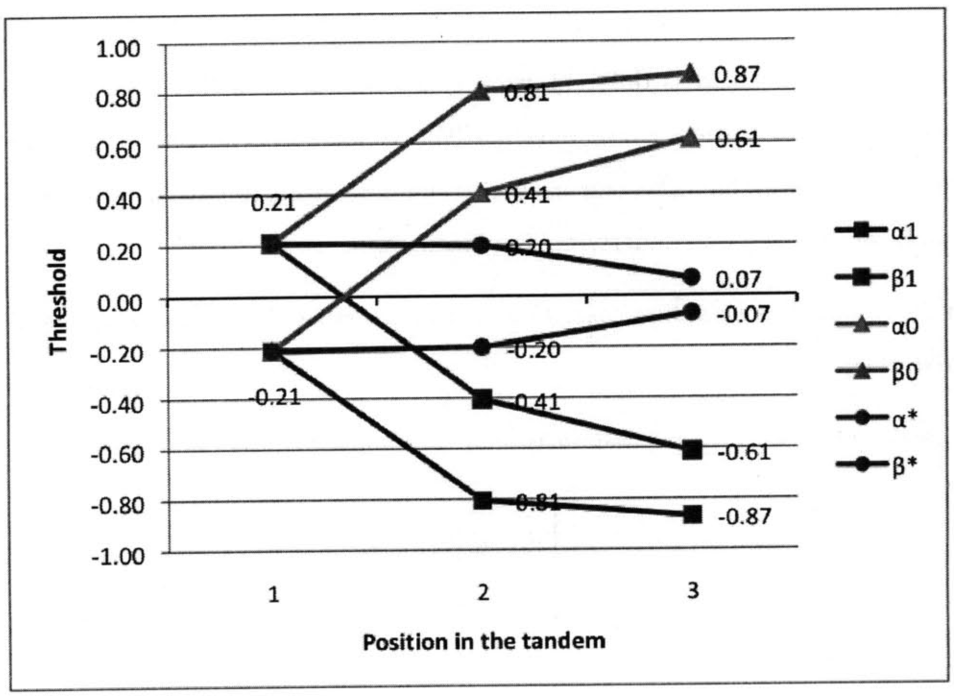


Figure 6-2: Ternary Actions – Theoretical thresholds as a function of player's position in the tandem. $\alpha^{M_{t-1}}$ and $\beta^{M_{t-1}}$ represent the optimal lower and higher cutoffs respectively, given the predecessor's action $M_{t-1} = 0, 1, *$.

Proof. Wishing to maximize his payoff and knowing only his private signal X_1, Player 1 picks action 0 when $\mathbf{P}(H_0|X_1) > c$, where c is the payoff for passing. Equivalently, he picks action 1 when $\mathbf{P}(H_1|X_1) > c$. To show that such a strategy is indeed a threshold strategy, we can express $\mathbf{P}(H_0|X_1)$ as a function of X_1 as follows,

$$\mathbf{P}(H_0|X_1) = \mathbf{P}(X_1 + X_2 + X_3 < 0|X_1) = \mathbf{P}(X_2 + X_3 < -X_1).$$

Using the cumulative distribution of $X_2 + X_3$,[12] we show that an optimal strategy for Player 1 consists of a threshold rule because

$$\begin{aligned}
\mathbf{P}(H_0|X_1) > c \quad &\Leftrightarrow F_{X_2+X_3}(-X_1) > c \quad &&\Leftrightarrow X_1 < 2(\sqrt{2(1-c)} - 1) \\
\mathbf{P}(H_1|X_1) > c \quad &\Leftrightarrow F_{X_2+X_3}(-X_1) \leq 1 - c \quad &&\Leftrightarrow X_1 \geq -2(\sqrt{2(1-c)} - 1).
\end{aligned}$$

Since our model assumes $c = 3/5$, we calculate threshold $\alpha_1 = -4\sqrt{5}/5 + 2$ and $\beta_1 = -\alpha_1$.

Player 2 observes Player 1's decision in addition to her private signal X_2. Again, he maximizes his payoff by picking action 0 when $\mathbf{P}(H_0|M_1, X_2) > c$. Using the cumulative distribution of $X_1 + X_3$ conditioned on $M_1 = 0$, i.e., $X_1 < \alpha_1$,[13] we show that an optimal strategy for Player 2 consists of a threshold rule because

$$\begin{aligned}
\mathbf{P}(H_0|M_1 = 0, X_2) > c \quad &\Leftrightarrow F_{X_1+X_3|M_1=0}(-X_2) > c \quad &&\Leftrightarrow X_2 < \alpha_2^0 \\
\mathbf{P}(H_1|M_1 = 0, X_2) > c \quad &\Leftrightarrow F_{X_1+X_3|M_1=0}(-X_2) \leq 1 - c \quad &&\Leftrightarrow X_2 \geq \beta_2^0,
\end{aligned}$$

where we calculate $\alpha_2^0 = (13 - 4\sqrt{5})/10$ and $\beta_2^0 = (17 - 4\sqrt{5})/10$ from the CDF for $c = 3/5$. Repeating the same process for $M_1 = 1$ and $M_1 = *$, we calculate the remaining 4 thresholds for Agent 2 shown in Proposition 2(ii) above.

Player 3 observes Player 2's decision in addition to his private signal X_3. Again, he maximizes his payoff by picking action 0 when $\mathbf{P}(H_0|M_2, X_3) > c$. Suppose Player 3 observes that Player 2 picked $M_2 = 0$. Player 3 knows this happened in one of three ways:

- Player 2 copied Player 1 (if $X_1 < \alpha_1$ and $X_2 < \alpha_2^0$). Using Bayes' Rule, this happens with probability $p_2^0 = (\alpha_2^1 + 1)(1 + \alpha_1)/(\alpha_2^1 + 1)(1 + \alpha_1) + (\alpha_2^0 + 1)(1 - \beta_1) + (a_2^* + 1)(\beta_1 - \alpha_1)$.
- Player 2 overturned Player 1's decision from 1 to 0 given a low enough private signal (if

[12]See Appendix A for an explicit CDF.
[13]*Ibid.*

$X_1 \geq \beta_1$ and $X_2 < \alpha_2^1$). This happens with probability $p_2^1 = (\alpha_2^0+1)(1-\beta_1)/(\alpha_2^1+1)(1+\alpha_1)+(\alpha_2^0+1)(1-\beta_1)+(a_2^*+1)(\beta_1-\alpha_1)$.

- Player 2 overturned Player 1's decision from $*$ to 1 given a low enough private signal (if $\alpha_1 \leq X_1 < \beta_1$ and $X_2 < \alpha_2^*$). his happens with probability $p_2^* = (a_2^*+1)(\beta_1-\alpha_1)/(\alpha_2^1+1)(1+\alpha_1)+(\alpha_2^0+1)(1-\beta_1)+(\alpha_2^*+1)(\beta_1-\alpha_1)$.

Therefore,

$$
\begin{aligned}
\mathbf{P}(H_0|M_2=0,X_3) &= p_2^0\,\mathbf{P}(H_0|M_1=0,M_2=0,X_3)+p_2^1\,\mathbf{P}(H_0|M_1=1,M_2=0,X_3)\\
&\quad +p_2^*\,\mathbf{P}(H_0|M_1=*,M_2=0,X_3)\\
&= p_2^0\,\mathbf{P}(H_0|X_1<\alpha_1,X_2<\alpha_2^0,X_3)\\
&\quad +p_2^1\,\mathbf{P}(H_0|X_1\geq\beta_1,X_2<\alpha_2^1,X_3)\\
&\quad +p_2^*\,\mathbf{P}(H_0|\alpha_1\leq X_1<\beta_1,X_2<\alpha_2^*,X_3)\\
&= p_2^0\,\mathbf{P}(X_1+X_2<-X_3|X_1<\alpha_1,X_2<\alpha_2^0,X_3)\\
&\quad +p_2^1\,\mathbf{P}(X_1+X_2<-X_3|X_1\geq\beta_1,X_2<\alpha_2^1,X_3)\\
&\quad +p_2^*\,\mathbf{P}(X_1+X_2<-X_3|\alpha_1\leq X_1<\beta_1,X_2<\alpha_2^*,X_3).
\end{aligned}
$$

Using the cumulative distribution of X_1+X_2 conditioned on $M_2=0$,[14] we show that an optimal strategy for Player 3 consists of a threshold rule because

$$
\begin{aligned}
\mathbf{P}(H_0|M_1=0,X_3)>c &\Leftrightarrow F_{X_1+X_2|M_2=0}(-X_3)>c &&\Leftrightarrow X_3<\alpha_3^0\\
\mathbf{P}(H_1|M_1=0,X_3)>c &\Leftrightarrow F_{X_1+X_2|M_2=0}(-X_3)<1-c &&\Leftrightarrow X_3\geq\beta_3^0.
\end{aligned}
$$

□

6.6 Experimental Results

6.6.1 Descriptive Statistics - Group behavior

We use the distinction between herding and cascade behavior established in [ÇK05]; A subject engages in cascade behavior when she disregards her private signal, and reports a cutoff of -10

[14] *Ibid.*

Table 6.1: Summary of Group Behavior Results.

Group Behavior	Tenary Action	Binary Action
#Rounds	51	51
Earnings per decision(cents)	1.54	1.5
%Correct Final Decision	23	29
%Pass Final Decision	19	N/A
%Incorrect Final Decision	9	22
Herds*	6	12
%of Herds**	11.8%	23.5%
Cascades	1	1
%of Cascades**	2%	2%
Overturns***	64	48
% of Overturns***	62.8%	47.1%
* of all 3 subjects.		
** Out of all 51 rounds.		
*** Out of all decision points excluding the first decision turn.		

or 10. Alternatively, a subject engages in herding behavior when she reports a cutoff different than -10 or 10, but still ends up picking the same action as her predecessor. A cascade is said to occur when at least the last 2 subjects follow cascade behavior. A herd is said to occur when all three subjects follow herd behavior.

Table 6.1 summarizes our experimental results at the group level. With ternary actions, herds were observed in 6 out of the 51 sessions(11.8%). All herds were correct (i.e., consistent with optimal behavior) except one. With binary actions, herds were observed in 12 out of the 51 sessions(23.5%), 9 of which were correct. Herds were less frequent and overturns more frequent with ternary actions,. Finally, no significant difference was found in the earnings between two treatments, or the frequency of cascades. The ternary action group involves a significantly lower percentage of incorrect final decisions (defined as the decision of the third member), though this can be attributed to the fact a substantial number of subjects chose to pass.

6.6.2 Descriptive Statistics - Individual behavior

Table 6.2 summarizes our experimental results at the individual level. A lower correct and incorrect decision rate is achieved in the ternary case compared to the binary case – this can be attributed to the fact approximately a third of players choose to pass. Cascade behavior is less frequent in the ternary case, and its frequency matches that reported in classroom exper-

Table 6.2: Summary of Individual Behavior Results

Individual Behavior	Tenary Action	Binary Action
%Correct Decision	39.2%	54.9%
%Pass Decision	37.9%	N/A
%Incorrect Decision	22.9%	45.1%
Information Cascade Behavior	3	8
%Information Cascade Behavior	5.6%	7.8%
Unconditional Passing	8	N/A
%Unconditional Passing	5.2%	N/A
Concurring Decisions	24.5%	42.2%
Contrary Decisions	72.6%	54.9%
Neutral Decisions	3%	3%

iments in the binary case. A somewhat surprising finding is that very few subjects choose to unconditionally pass in the ternary case, i.e., to set their lower and upper cutoffs to -10 and 10 respectively.

We again borrow the terminology used in [ÇK05] to give more color to the decisions reported: in particular, we distinguish between *concurring, contrary,* and *neutral* decisions. A subject makes a concurring decision if she picks a positive(negative) cutoff when she observes that her predecessor picked action 1(0). She makes a neutral decision if she picks zero cutoffs. She makes a contrary decision otherwise. Partitioning the data into these three categories gives more insight into the decision mechanisms at work.

Overall, subjects tended to weigh their predecessor's action far less than the theory predicts. In the ternary action group, over all decisions excluding first turns, only 24.5% of decisions were concurring, and 72.5% were contrary. In the binary case, we get a less severe breakdown, with 42.2% concurring decisions, and 54.9% contrary.

If we condition on contrary decisions, the intensity of disagreement is severe – when subjects disagree with their predecessor's action, they do so in an extreme way. Similar to [ÇK05], we measure the intensity of disagreement in two different ways: Disagreement 1 measures the absolute value of the distance between the cutoff actually chosen and the theoretical cutoff rule. Disagreement 2 measures the absolute value of the distance of the chosen cutoff from zero. Figure 6-3 presents the results in the binary action group. Our results seem to reproduce the severity of the disagreement reported in the classroom setting [ÇK05]. If we condition on concurring decisions, we find interesting conformity of behavior with the theory prediction in

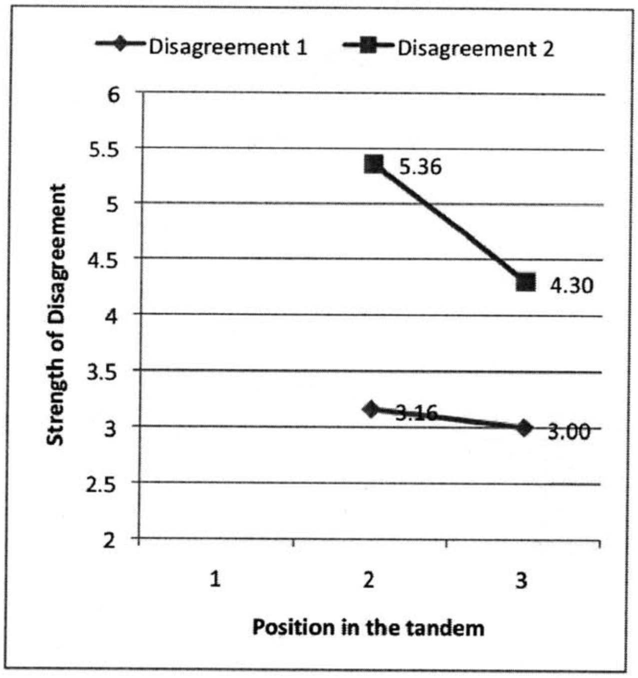


Figure 6-3: Binary Action Group – Strength of disagreement of contrary decisions, based on [ÇK05]. Disagreement 1 measures distance from the theoretical cutoffs. Disagreement 2 measures distance from zero.

the binary case. Figure 6-4 presents the theoretical cutoffs and the mean cutoff (in magnitude) of concurring decisions turn by turn. Bayesian learning does seem to capture the magnitude of the cutoffs adopted by the subjects in this instance. Figure 6-5 shows that in the ternary action group, subjects tend to follow their private information much more than in the binary case. Figure 6-6 shows that the compositional difference that was obtained by conditioning the data on concurrent and contrary decisions in the binary case does not seem to apply in the ternary case.

When the results are not conditioned on concurring or contrary data, a significant difference appears to exist between experimental findings and theoretical predictions. We find, as did [ÇK05, Wei08], that the heuristic of following one's private signal is a better predictor of behavior than Bayesian learning. In the binary action group, we are able to replicate the experimental literature finding that the difference from the prediction of the theory is a compositional difference over concurring and contrary decisions and not a reflection of how persuasive predecessors actions are once they are followed. We were not able to replicate this finding in the ternary action group. The heuristic of following one's private signal seems a better predictor of behavior

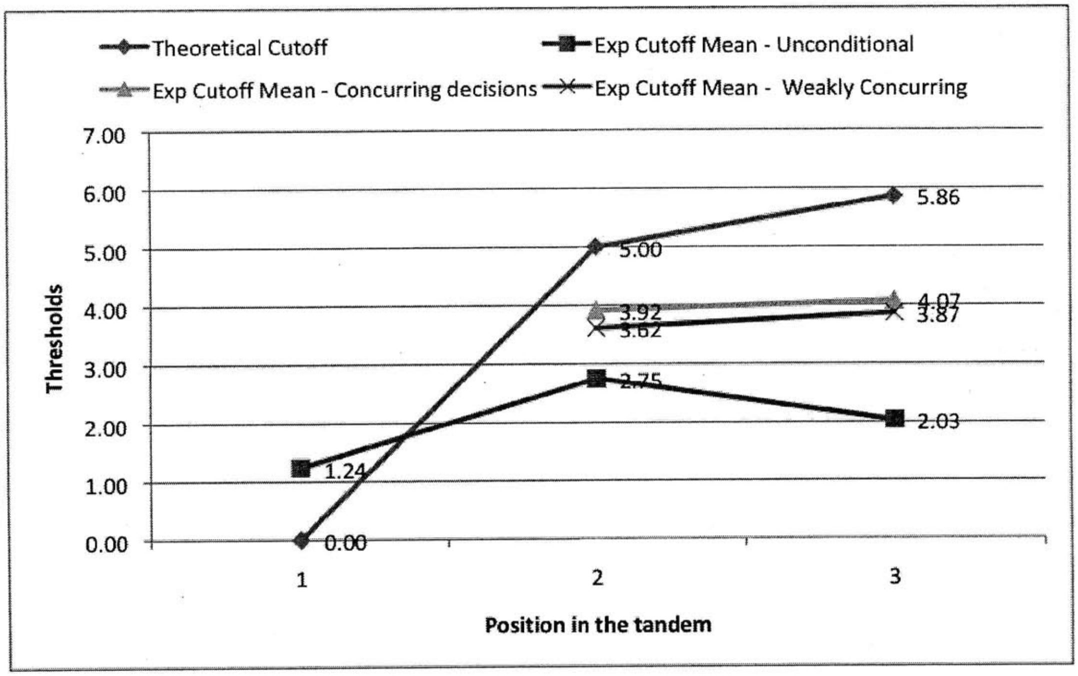


Figure 6-4: Binary Action Group – Conditional and Unconditional cutoff means by position. Exp Cutoff Mean refer to the experimental cutoff mean calculated from the binary action data. Weakly concurring decisions include both concurring and neutral decisions.

than Bayesian learning in the ternary case, whether or not results are conditioned on concurring or contrary data.

6.7 Discussion

MTurk. A number of limitations should be emphasized concerning our use of MTurk, some of which are specific to observational learning experiments, while others are common to online experimentation in general. Despite the limitations of this particular medium however, we were able to closely reproduce the findings of the experimental literature in the binary action group.

- The unknown nature of MTurk's user base is a double edged sword: because of its diversity, it can potentially generalize to a wider population, increasing the experiment's external validity. In contrast, the limited experimenter contact, the unknown expertise of the user base and the possibility of self-selection bias might be problematic.

- The complex nature of the experiment, and the need to synchronize across different players raise important challenges to guaranteeing quality assurance, in particular (a) making sure

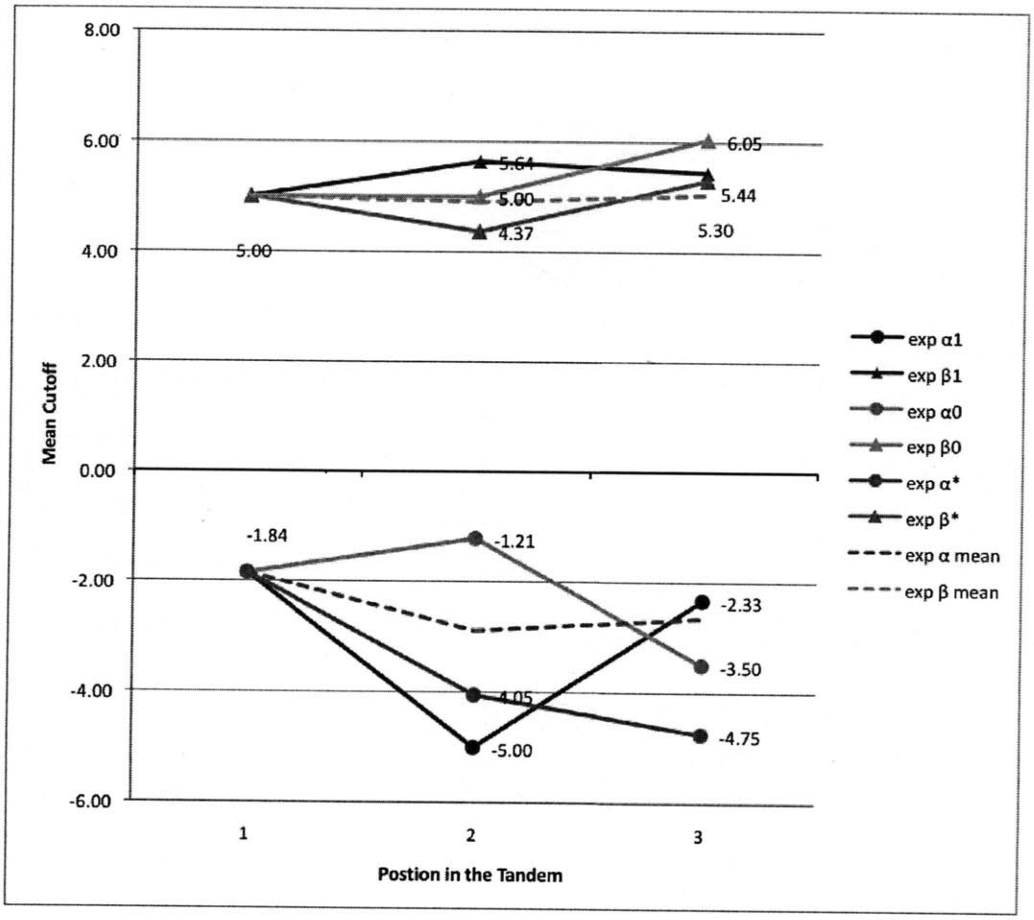


Figure 6-5: Ternary Action Group – Conditional and Unconditional experimental mean cutoffs by position t. $\alpha^{M_{t-1}}$ and $\beta^{M_{t-1}}$ refer to the lower and upper cutoffs respectively given the predecessor picked action $M_{t-1} = 0, 1, *$. The dashed line shows the unconditional experimental mean. No compositional difference emerges when conditioning the data on concurrent and contrary decisions.

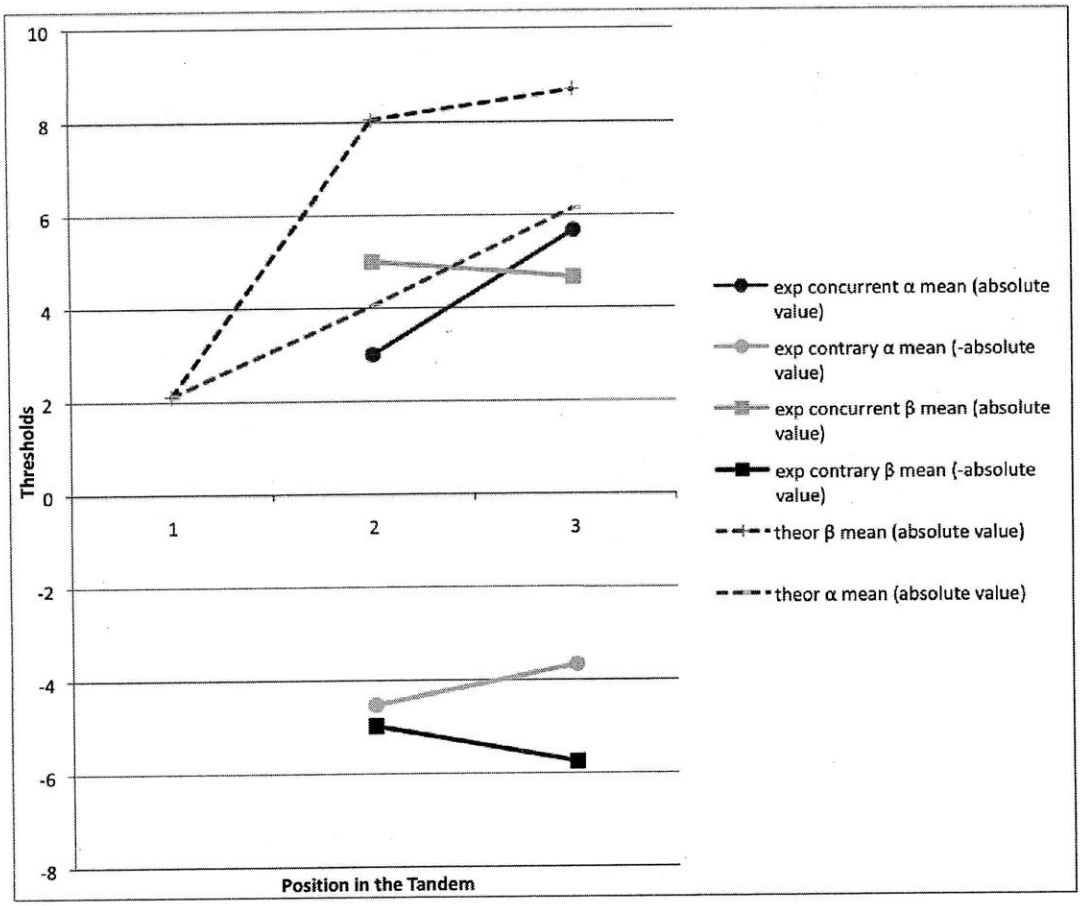


Figure 6-6: Ternary Action Group – Conditional cutoff means by position, excluding subjects who picked $*$. Exp α and β means refer to the experimental lower and upper cutoff mean calculated from the ternary action data. Theor cutoffs refer to the theoretical cutoffs calculated in Proposition 2.

Table 6.3: Illustrations of Heuristics I - Comments taken from questionnaire.

Bayesian heuristics:
– Player 2 said pos because he has a pos number or high neg. and #1 said pos; or he had a very high postive and #1 said neg. More likely #1 said pos then so #2 has pos or high neg. So I will guess pos if I have high neg or better.
– With a moderate amount of information and a negative sum predicted I would need to see a fairly positive number to guess positive.
– Since the player before me picked negative sum I would assume that either: 1. He has a large negative number (-8 -9 or -10) 2. He is near zero but the player before him guessed negative sum I would assume that the aggregate up to me was negative and unless my number was quite large (7 or higher) I would guess negative sum.
– I assume that since the guy before me was 1st and picked positive he had a positive number. Since there is one positive already I am going to lean to there being a slightly higher chance of finishing completely positive. If he was thinking logically he only would guess positive if he has a STRONG (> 5) positive number as the normal average would be to tend towards 0.

Influence of position:
– I was fairly influenced by his/her guess. Since we are fairly early I assume that the guesses so far are more accurate than the later ones in the game.
– Anchoring and adjusting heuristic (previous player guessed a negative sum therefore I adjusted down to reach a positive sum) Given my position in the order I would say the number my neighbor picked would weigh highly if they understood the parameters of the game.

that players understand the requested task and try to perform it well; (b) recovering from errors; and (c) detecting and preventing cheating.

- MTurk works best when tasks admit a bona fide response. Because our experiment consists of collecting user beliefs, it is harder to spot turkers who provide completely random answers to minimize the amount of time spent on each task.
- The lack of full control of the experimental setting raises important questions concerning the ecological validity of the findings. Differences in concentration levels or browsing experiences may significantly impact findings.

Tables 6.3, 6.4 and 6.5 present illustrative quotes from the questionnaire collected from subjects at the end of each decision round. The first two discuss examples of heuristics used, whereas the third highlights limitations we encountered in our experiment.

Table 6.4: Illustrations of Heuristics II - Comments taken from questionnaire.

Non Bayesian Heuristics:

- I chose 6 and negative 6 because it makes just more than half in each direction.

- I realized if I picked 0 and never pass I'd be in a better position to get a bonus. I didn't weigh the decision of my neighbor at all.

- Looking for the largest bonus forces me to try to keep my pass zone width small. It cannot be a width of 0 without eliminating the zone entirely or biasing it to the negative (-1 and 0) or to the positive (0 and 1). I haven't used the decision from my neighbor since they're the 2nd to play the game. I've centered my guess around 0.

- Since previous guess was pass I assume it was near zero so unless there is a strong negative or positive the average will tend to zero. So limits were picked to snag only strong in either direction. I weighed the previous guess the same as my number.

Risk Aversion:

- The players guesses have no impact on the actual sum of the numbers fixed once they have been chosen. Neighbors guesses are irrelevant to choice I make. The expected sum is zero. If you choose either positive or negative you should expect to win half of the time. The expected payout is therefore half of 2.5 cents or 1.25 cents. By passing your payout is 1.5 cents. Setting the lower cutoff to -10 and the upper cutoff to 10 will force a pass.

- guaranteed pass (better payoff than 2.5/2 odds).

- Since you have little data to use for statistical accuracy I set the range to the near extreme since I rather take a near 100% chance at 1.5 cents than a 50% chance at 2.5. 2. My neighbors guess had no influence on my decision because I have little clue as to what he put as his range.

Table 6.5: Illustrations of loss of experimental control – Comments taken from questionnaire.

Failure to recognize that own reward depended on cutoffs and guesses of others – not a game:

– *I guessed and that's about it. Zero is the hero!*
– *Age=51=5+1=6 Game no=45=5+4=9 9-6=3.*
– *You offered a bonus of six cents so naturally the number in front of my eye is 6.*
– *Sum of zipcode is 7.*
– *Favorite/lucky number.*

Tendency to guess positive:

– *People tend to guess on the positive side-human nature.*

Failure to understand how rewards are determined.

– *I believe extremes are best.*
– *I like to guess high.*
– *Based solely on intuition.*
– *Wild guess.*

Failure to understand probabilistic concepts:

– *If it is truly random the probability should be high that the sum equals zero.*
– *Assuming that all the numbers assigned are completely random it is safe to guess that the sum of the numbers would be above -5 but below 10.*

General lack of attention:

– *I don't really understand how this game works.*
– *I was a bit lost.*

6.8 Notes

Pilot. Appendix C presents the instructions of an in-class experiment that we conducted earlier in the term as part of an MIT class recitation for 6.986 - Fundamentals of Network Science and Engineering : this "pilot" effort[15] occurred in a one hour recitation with 14 student subjects who were divided into 2 control groups, with binary vs. ternary decisions. Instead of guessing whether the sum of private signals is positive or negative as proposed in the above design, students were told that one of two possible states of the world, H_0 or H_1, is true with equal prior probabilities. The attached instructions correspond to the +T(ternary actions) treatment. Results showed the emergence of learning regularities in the both treatments, with shorter periods of uniform behavior (shorter cascades) in the ternary than in the binary condition, and more frequent switches. Logistically, the pilot experiment suggested it would be beneficial to move away from Gaussian formulations, which were not always well understood. Additionally, the exercise showed that it would be beneficial to use computer terminals to facilitate the flow of information as opposed to paper slips.

Context. We considered wording the instructions in a more concrete way, by attributing meaning to private signals and more generally to the type of information being shared. Possible concretizing routes involve framing decisions in the context of:

- A disease test: Private information would be presented as the result of a disease test, and the decision would be whether a patient has the disease or not.

- Investment opportunities: Private information would correspond to the fundamental value of one of two risky assets A or B, and the decision would be investing in A , B, or Not Trading.

- A job interview: Private information would be presented as the result of a performance on an interview of a given candidate, and the decision would be whether to hire the candidate or not.

- "Bandwagon effects": Private information would correspond to input from the media about certain consumer products, and the decision would incorporate consumer purchase choices.

[15] This experiment is reported here in the spirit of following the "methodological exhortation" outlined in "Keeping the Con Out of Experimental Econ." [Rot94]

While these interpretations might increase the understanding of the experimental setup, we abstained from including them in order to avoid introducing biases (especially related to risk aversion).

Private Signal Structure. We initially considered discrete private signals (like those used in Anderson and Holt) but realized a move to a continuous signal space would allow for a richer exploration of cascades, specifically for the ternary actions and perfect information group.

6.9 Conclusion

We presented a design to test an imperfect-information observational learning model using Amazon Mechanical Turk. Applying procedures from [ÇK05] such as continuous signals, discrete actions and a cutoff elicitation technique, we test how well Bayesian learning approximates the actual behavior observed in the laboratory.

Our findings can be summarized as follows. First, imitation and herd behavior are much less frequent with ternary actions than with binary actions. Because herds are rarer, overturns and contrary decisions are more frequent with ternary actions. We also find strong evidence that behavior is less consistent with the predictions of Bayes' rationality in the ternary case. Whereas conditioning on concurrent decisions shows some uniformity with Bayesian learning in the binary case, no such consistency emerges with ternary actions. A somewhat surprising finding is that very few subjects choose to unconditionally pass. This leaves open the question of how the size of payoffs affects outcomes in economics experiments. [16]

[16] We would expect a higher number to unconditionally pass if the reward for passing was higher.

Chapter 7

Conclusion

7.1 Research Summary

We examined learning in a setting where agents receive noisy signals about the underlying state of the world, and communicate using three-valued messages. We extended the literature results that were applicable to binary actions, distinguishing between learning in probability and almost surely. Our main results proved that tandem networks do not learn (almost surely or in probability) with three-valued messages when the signal structure of the private messages is bounded. We also prove that tandem networks learn in probability when private messages are unbounded.

In addition, we examined experimental tests of a learning model with imperfect information and ternary actions. We used crowdsourcing (Amazon Mechanical Turk) to run observational learning experiments. We tested the robustness of literature results based on experiments in the classroom, and were able to replicate the findings in the binary action case and use them as a benchmark. We extended the experimental setup to games with three actions, and found that herd behavior is less frequent with ternary actions than binary actions. We also found that with three actions, behavior in the laboratory is much less consistent with Bayesian rationality than the theory predicts. A heuristic where agents follow their own signal seems to be better predictor of behavior – more so than in the binary case.

While real-world social and economic networks are more complex than the sequential topology investigated here, our results do offer insights into sequential decision making, and the way opinions propagate in a setting with imperfect information.

7.2 Areas for Future Research

There are many directions in which this research can be extended meaningfully: It is natural to ask about more general network topologies, and to generalize to any number of hypotheses and messages. Further extensions of the framework could explore topologies where the same decision maker makes more than one decision, and remembers a subset of the signals she has received in the past. On the experimental side, future work might consider more elaborate factorial designs, for instance ones that, in addition to the the number of alternative decisions, would incorporate (i) the payoff structure(global/collaborative vs. local/selfish), (ii) the structure of information (perfect information where agents know all previous decisions, vs. imperfect information like in our design, where an agent only knows the decision of her immediate predecessor), (iii) length of network, and (iv) situating decisions, e.g., gambling games, workplace or voting decisions.

Appendix A

Thresholds Calculation

We list the cumulative distribution functions used for calculating thresholds in Chapter 6, Section 6.5.

A.1 Distribution Functions - Binary Messages

The CDF of $X_2 + X_3$ is given by

$$F_{X_2+X_3}(y) = \begin{cases} 0, & \text{for } y < -2. \\ (2+y)^2/8, & \text{if } -2 \leq y < 0 \\ (4+4y-y^2)/8, & \text{if } 0 \leq y < 2 \\ 1, & \text{for } y \geq 2. \end{cases}$$

The CDF of $X_1 + X_3$ conditioned on Player 1 picking action 0 is given by

$$F_{X_1+X_3|M_1=0}(y) = \begin{cases} 0, & \text{for } y < -2. \\ (2+y)^2/4, & \text{if } -2 \leq y < -1 \\ (3+2y)/4, & \text{if } -1 \leq y < 0 \\ (3+2y-y^2)/4, & \text{if } 0 \leq y < 1 \\ 1, & \text{for } y \geq 1. \end{cases}$$

The CDF of $X_1 + X_2$ conditioned on Player 2 picking action 0 is given by

$$F_{X_1+X_2|M_2=0}(y) = \begin{cases} 0, & \text{for } y < -2. \\ (2+y)^2/8, & \text{if } -2 \leq y < 0 \\ (7+4y-2y^2)/8, & \text{if } 0 \leq y < 2 \\ 7+4y-4y^2)/8, & \text{if } 2 \leq y \\ 1, & \text{for } y \geq \frac{1}{2}. \end{cases}$$

A.2 Distribution Functions - Ternary Messages

The CDF of $X_2 + X_3$ is given by

$$F_{X_2+X_3}(y) = \begin{cases} 0, & \text{for } y < -2. \\ (2+y)^2/8, & \text{if } -2 \leq y < 0 \\ (4+4y-y^2)/8, & \text{if } 0 \leq y < 2 \\ 1, & \text{for } y \geq 2. \end{cases}$$

The CDF of $X_1 + X_3$ conditioned on Player 1 picking action 0 is given by

$$F_{X_1+X_3|M_1=0}(y) = \begin{cases} 0, & \text{for } y < 1-4/\sqrt{5}. \\ (-5+4\sqrt{5}+5y)^2/(20(-5+4\sqrt{5})), & \text{if } 1-4/\sqrt{5} \leq y < 0 \\ -1/4+1/\sqrt{5}+y/24, & \text{if } 0 \leq y < 3-4/\sqrt{5} \\ (40-16\sqrt{5}+5(-4+y)y)/(20-16\sqrt{5}), & \text{if } 3-4/\sqrt{5} \leq y < 2 \\ 1, & \text{for } y \geq 2. \end{cases}$$

The CDF of $X_1 + X_2$ conditioned on Player 2 picking action 0 is given by

$$F_{X_1+X_2|M_2=0}(y) = \begin{cases} 0, & \text{for } y < -2. \\ 5(2+y), & \text{if } -2 \le y < 3/10 - 2/\sqrt{5} \\ (3(37+8\sqrt{5}) - 10y(-26+8\sqrt{5}+5y))/160, & \text{if } 3/10 - 2/\sqrt{5} \le y < -11/5 + 4/\sqrt{5} \\ -291/160 + 5\sqrt{5}/4 + (2-5y)y/8, & \text{if } -11/5 + 4/\sqrt{5} \le y < 0 \\ (-291 + 200\sqrt{5} + 10(4-15y)y)/160, & \text{if } 0 \le y < 9/5 - 4/\sqrt{5} \\ (31 + 56\sqrt{5} - 20y(7 - 4\sqrt{5} + 5y))/160, & \text{if } 9/5 - 4/\sqrt{5} \le y < (4\sqrt{5} - 7)/10 \\ 1, & \text{for } y \ge 2/\sqrt{5} - 7/10. \end{cases}$$

Appendix B

Amazon Turk Instructions

B.1 Instructions for the Ternary Action Group

Please read these instructions very carefully to earn up to 8 cents of extra bonus! Feel free to accept other similar HITs, though you will not be allowed to participate more than once in the same round.

At the beginning of this game, the computer randomly picks 3 secret numbers from all decimal numbers between -10 and 10. 3 players are each assigned one of the secret numbers, and try to guess (one after the other) whether the sum of the 3 numbers is Positive or Negative.

- Unless you are the first player, you can see the guess of the player who played right before you.
- Instead of letting you see the secret number assigned to you, we ask for your *strategy* (given the guess of the previous player).
  - We ask for your LOWER CUTOFF - the secret number below which you would guess the sum of all 3 numbers is Negative.
  - We ask for your UPPER CUTOFF - the secret number above which you would guess the sum of all 3 numbers is Positive.
- If the secret number assigned to you is below your LOWER CUTOFF, the next player will be told that you guessed that the sum of the numbers is Negative.
  - You get a **BONUS of 2.5 cents** if the sum is indeed negative, nothing if the sum is positive.

- If the secret number assigned to you was above your UPPER CUTOFF, the next player will be told that you guessed the sum of the numbers is Positive.
  - You get a **BONUS of 2.5 cents** if the sum is indeed positive, nothing if the sum is negative.
- If the secret number is between your LOWER CUTOFF and UPPER CUTOFF, the next player will be told your decision was a Pass.
  - You get a **BONUS of 1.5 cents**. Note that you can pick UPPER CUTOFF=LOWER CUTOFF and never Pass.

1. Pick your LOWER CUTOFF - the secret number below which you would guess the sum of all 3 numbers is Negative
2. Pick your UPPER CUTOFF - the secret number above which you would guess the sum of all 3 numbers is Positive
3. Optional Questionnaire for extra bonus! We will award a bonus of 1 to 6 cents depending on how precisely you formulate your statements. For example,
   (a) What decision rule or heuristic did you use in your guess?
   (b) How strong have you weighed your secret number compared to the decision of your neighbor?
   (c) Are there any comments or ideas you would like to share about playing this game?

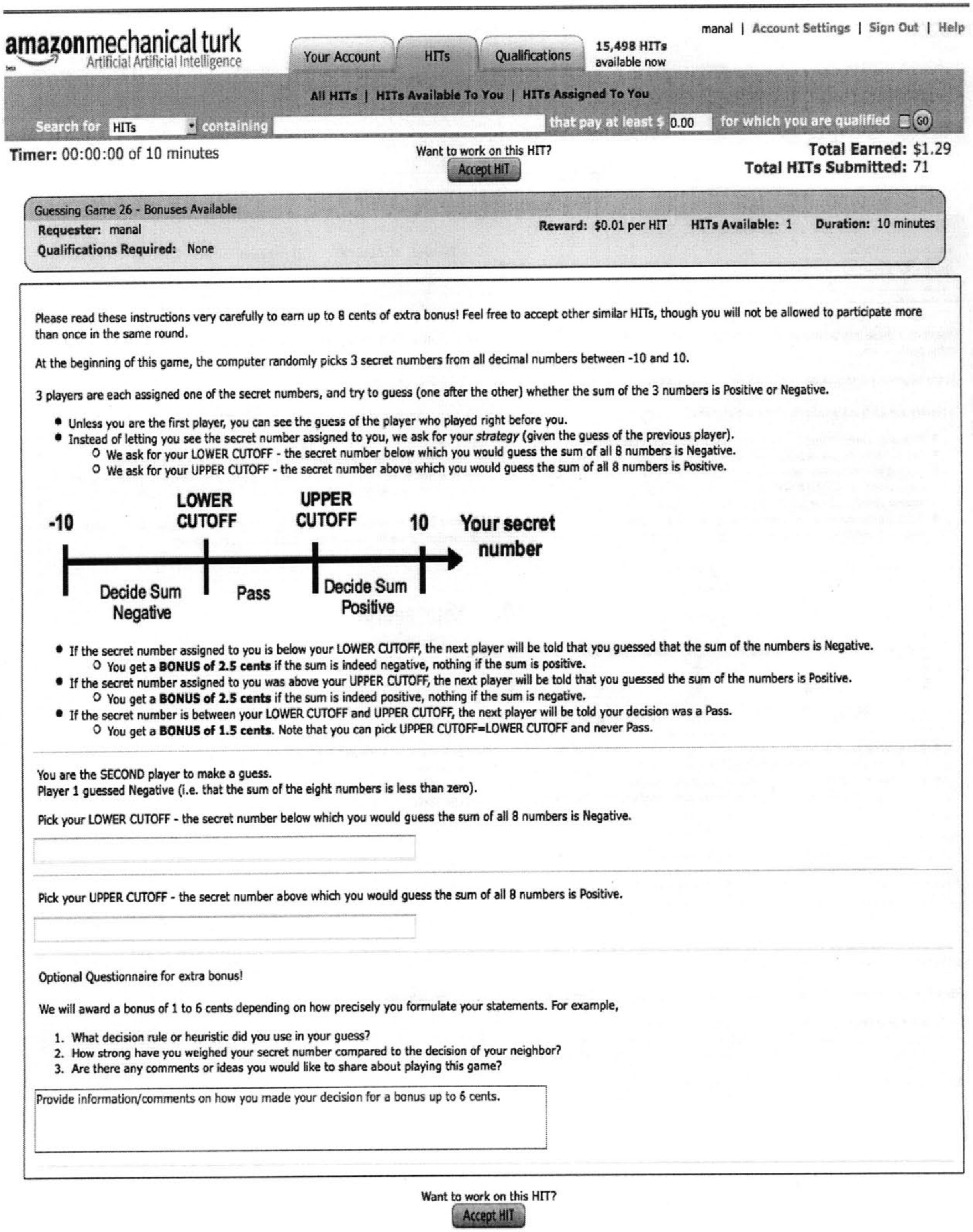


Figure B-1: MTurk Screenshot: the HIT(human intelligence task) used in the Ternary Action Group.

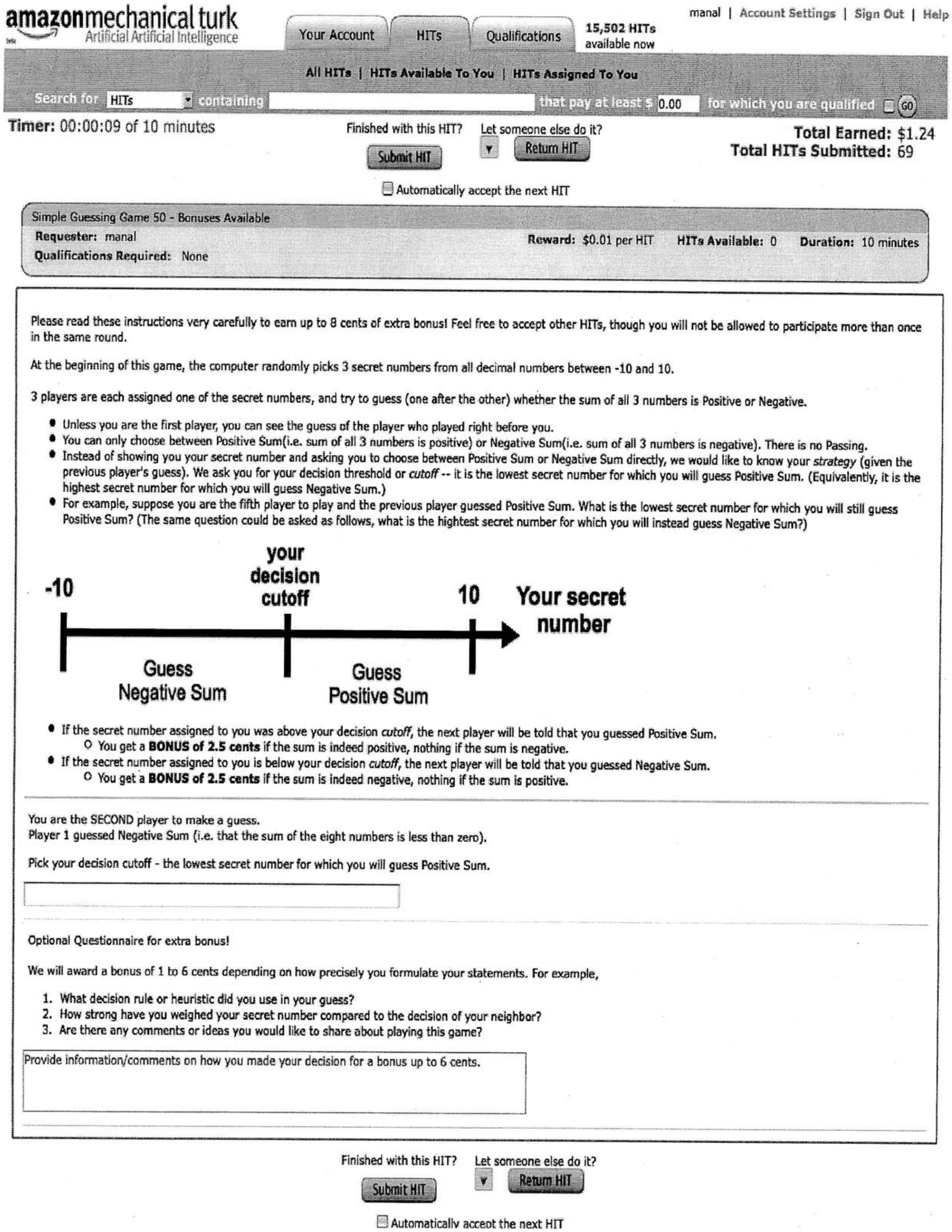


Figure B-2: MTurk Screenshot: the HIT(human intelligence task) used in the Binary Action Group.

Appendix C

Pilot In-Class Experiment

Introduction[1] The following is an experiment in the dynamics of information propagation and decision making. You will participate in 10 independent and identical decision problems, or rounds. The class will be broken into 2 groups. You will be rewarded with one homework point for your participation. In addition, 5, 4 and 3 homework points will be awarded to the top three performers in each group, as measured by the total score on all rounds.

In this experiment, you will be asked to guess which of two known probability distributions generated the data, and record your guess on your Decision Grid (See Table C.1).

In each round, each participant will make one decision. Participants will make their decision one after the other, in a line, or tandem, structure. At first, your position in the line will be based on the alphabetical order of your last name in the group. After all participants have made a decision in the first round, you will be assigned a new position in the second round based on your position in the initial (alphabetical) order. For example, assuming there are 3 people in your group, the position number in the first round is assigned alphabetically, i.e. Ben Bitdiddle, Alicia Hacker and Louis Reasoner line up in that order,"1 2 3". Next, you may be given "2 1 3" (on the board), which will determine the new line order for the second round (i.e. Ben and Alicia swap).

Decision Problem We will begin each round by privately tossing a coin. If the coin flip is Heads, we will draw from a $\mathcal{N}(0, 100)$ distribution. If it is Tails, we will draw from a $\mathcal{N}(10, 100)$ distribution. Therefore it is equally likely that either hypothesis is true.

[1]was conducted as part of an MIT in-class exercise with advanced EECS students who have fluency in probabilistic concepts. Please see Remark 1 from Section 7 for more information.

Once the distribution is picked, we will come around to each of you and deliver a folded paper slip with a number drawn from the distribution picked above. The result of this draw will be your private signal and should not be shared with other participants. Do not unfold the paper at first. Private signals are independent of each other, and of the numbers used on any other round.

In each round, you will be asked to choose one of 3 actions: L(low), H(high) or Pass. Action Pass will always yield +3(think of it as an I-don't-know, or I'm-not-sure statement). Action L will yield +5 if the distribution used to generate the private information has mean 0 (i.e. if the coin was heads), otherwise it will yield 0. Action H yields +5 if distribution used to generate the private information has mean 10 (i.e. if the coin was tails), otherwise it will yield 0 (refer to Table C-2b).

Unless you are the first participant to make a decision, you will get to observe the decision of the person right before you in the line. For example, if you are the 3rd participant, you will be told the action of the 2nd participant before having to make your own decision. After observing your predecessor's action and before being told your private signal, you will make your decision as follows:

Instead of choosing L, H or Pass directly, you will have to state the two thresholds[2] for which you will choose each action, and write them down on your Decision Grid.

For example, suppose you are the 3rd participant. The 2nd participant just passed you his decision, a L. Next you will have to state two threshold numbers t_L and t_H for your private signal x_i, for which you will choose L, H or Pass. For example, you may decide you will pick L if the private signal $x_i <= 3$, Pass if $3 < x_i <= 8$, and H if $x_i > 8$. You may want to adjust your thresholds as your position changes in the tandem.

After all participants have made a decision, participants are informed what the distribution that generated the private signals was in their group. If the distribution used to generate the data was $\mathcal{N}(0, 100)$, then everyone who chose L wins +5, those who picked Pass win +3, and those who decided H get nothing.

Rules We ask that you not talk to anyone during the experiment. This experiment is aimed to test the propagation of information in tandem, and as such, the tandem structure needs to be strictly enforced.

[2]potentially between - infinity and infinity

Recap At the beginning of each round, you are asked to stand in line according to last name alphabetical order. Next, you are given a folded sheet of paper with your private signal on it. Please keep this closed at first. The person at the end of the line first decides what thresholds she will use to make her decision, then writes them down on her Decision Grid. She then looks at her private number, and writes down what action(L,H or Pass) her chosen thresholds result in on a new paper slip. She finally passes off her decision(L, H or Pass) to the person behind her. The former repeats the process, first writing down the thresholds, then looking at his private signal, then writing down what action his thresholds generated, and finally communicating this action to the third person in line.

This game will be repeated until all 10 rounds are completed. Your final score will be the sum of the payoffs (+5, +3 or +0) on all 10 rounds, as calculated from your Decision Grid.

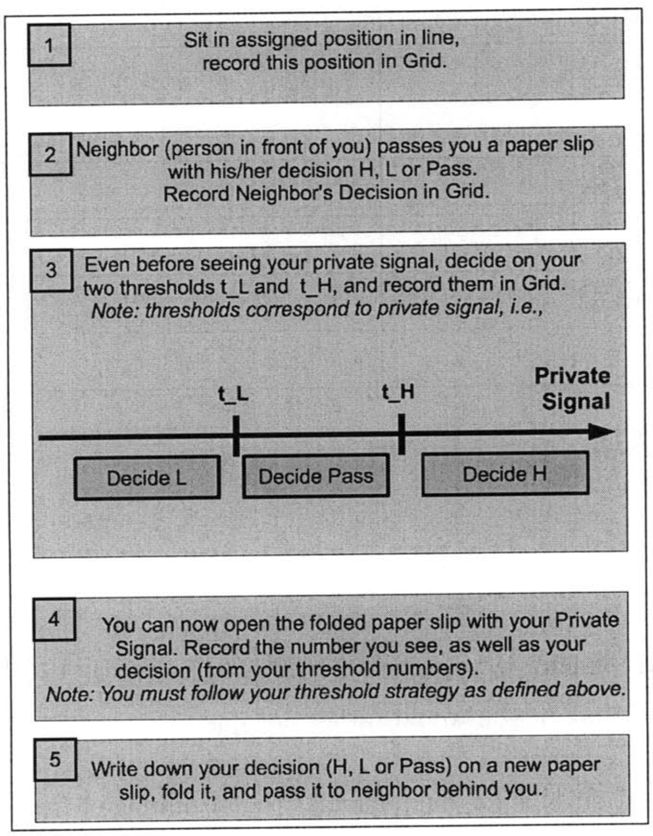


Figure C-1: Summary of Instructions

Decision Grid **Name:** ____________________

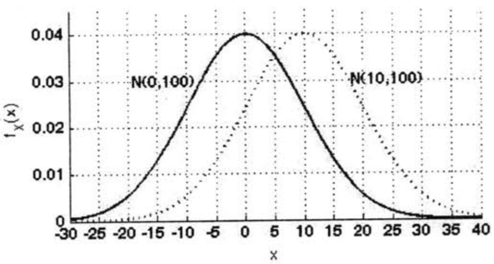
Round	Position	Neighbor Decision	Your Decision Thresholds	Your Private Signal	Your Decision
Example 1	3	H	2 and 12	-1	L
Example 2	8	P	-8 and 5	13	H
1	(Alphabetical)				
2	(See board)				
3	(See board)				
4	(See board)				
5	(See board)				
6	(See board)				
7	(See board)				
8	(See board)				
9	(See board)				
10	(See board)				

Table C.1: Decision Grid

Notes.

- **Decision:** either H ("high" if you guess $\mathcal{N}(\mathbf{10}, 100)$ is used), L ("low" if you guess $\mathcal{N}(\mathbf{0}, 100)$ is used) or Pass (if you would rather not guess).
- **Position:** First (Position 1) is for the person at the beginning of the line.
- **Neighbor Decision:** H(high), L(low) or Pass.
- **Private Signal Thresholds:** 2 numbers that fully specify your strategy. You pick L if your private signal falls below the first threshold, and pick H if it falls above the second threshold number. You pick Pass if private signal falls between the two threshold numbers.

- **Private Signal:** Record the number you obtain in the folded paper.
- **Your Decision:** Directly flows from thresholds and Seeing Private Signals. You are not allowed to modify your thresholds.



(a) At the beginning of every round, we flip a fair coin. If we get "heads", we generate private signals from distribution $\mathcal{N}(0,100)$. If we get "tails", we generate private signals from distribution $\mathcal{N}(10,100)$.

Payoff Per Round		
+5	CORRECT DECISION	H when N(10,100) or L when N(0,100)
+3	PASS	PASS
0	INCORRECT DECISION	H when N(0,100) or L when N(10,100)

(b) Payoff Table: +5 if your guess is wrong, +3 if Pass, 0 if wrong.

Bibliography

[ADLO08] Daron Acemoglu, Munther A. Dahleh, Ilan Lobel, and Asuman Ozdaglar, *Bayesian learning in social networks*, Working Paper 14040, National Bureau of Economic Research, May 2008.

[AH97] Lisa R Anderson and Charles A Holt, *Information cascades in the laboratory*, American Economic Review **87** (1997), no. 5, 847–62.

[Ban92] Abhijit V. Banerjee, *A simple model of herd behavior*, The Quarterly Journal of Economics **107** (1992), no. 3, 797–817.

[BHW92] Sushil Bikhchandani, David Hirshleifer, and Ivo Welch, *A theory of fads, fashion, custom, and cultural change in informational cascades*, Journal of Political Economy **100** (1992), no. 5, 992–1026.

[ÇK04a] Boğaçhan Çelen and Shachar Kariv, *Distinguishing informational cascades from herd behavior in the laboratory*, American Economic Review **94** (2004), no. 3, 484–498.

[ÇK04b] ______, *Observational learning under imperfect information*, Games and Economic Behavior **47** (2004), no. 1, 72–86.

[ÇK05] ______, *An experimental test of observational learning under imperfect information*, Econom. Theory **26** (2005), no. 3, 677–699. MR MR2214180

[Cov69] Thomas M. Cover, *Hypothesis testing with finite statistics*, Ann. Math. Statist. **40** (1969), 828–835. MR MR0240906 (39 #2251)

[HC70] Martin E. Hellman and Thomas M. Cover, *Learning with finite memory*, Ann. Math. Statist **41** (1970), 765–782. MR MR0272100 (42 #6981)

[KCS08] Aniket Kittur, Ed H. Chi, and Bongwon Suh, *Crowdsourcing user studies with mechanical turk*, CHI '08: Proceeding of the twenty-sixth annual SIGCHI conference on Human factors in computing systems (New York, NY, USA), ACM, 2008, pp. 453–456.

[Kop75] Jack Koplowitz, *Necessary and sufficient memory size for m-hypothesis testing*, IEEE Trans. Information Theory **IT-21** (1975), 44–46. MR MR0368982 (51 #5220)

[KW05] Dorothea Kübler and Georg Weizsäcker, *Are longer cascades more stable?*, Journal of the European Economic Association **3** (2005), no. 2-3, 330–339.

[LADO09] Ilan Lobel, Daron Acemoglu, Munther A. Dahleh, and Asuman Ozdaglar, *Rate of convergence of learning in social networks*, Proceedings of Conference on Decision and Control (CDC), 2009.

[LCMG09] Greg Little, Lydia B. Chilton, Rob Miller, and Max Goldman, *TurKit: Tools for Iterative Tasks on Mechanical Turk*, 2009.

[PA92] J. D. Papastavrou and M. Athans, *Distributed detection by a large team of sensors in tandem*, IEEE Transactions on Aerospace Electronic Systems **28** (1992), 639–653.

[Rot94] Alvin E Roth, *Lets keep the con out of experimental econ.: A methodological note*, Empirical Economics **19** (1994), no. 2, 279–89.

[SF08] A. Sorokin and D. Forsyth, *Utility data annotation with amazon mechanical turk*, Computer Vision and Pattern Recognition Workshops (2008).

[SS00] Lones Smith and Peter Sørensen, *Pathological outcomes of observational learning*, Econometrica **68** (2000), no. 2, 371–398. MR MR1748010 (2001c:91047)

[Sta02] T. Standage, *The Turk: The Life and Times of the Famous Eighteenth-Century Chess-Playing Machine*, Walker and Company, 2002.

[Tsi93a] John N. Tsitsiklis, *Decentralized detection*, In Advances in Statistical Signal Processing, JAI Press, 1993, pp. 297–344.

[Tsi93b] ———, *Extremal properties of likelihood-ratio quantizers*, IEEE Trans. Commun. **41** (1993), 550–558.

[TTW08] Wee Peng Tay, John Tsitsiklis, and Moe Win, *On the sub-exponential decay of detection error probabilities in long tandems*, IEEE Transactions on Information Theory **54** (2008), no. 10, 4767–4771.

[Wei08] Georg Weizsäcker, *Do we follow others when we should? a simple test of rational expectations*, IZA Discussion Papers 3616, Institute for the Study of Labor (IZA), July 2008.