

Happy birthday, ultra-cold neutron!*

Hartmut Abele^{1,a}, Tobias Jenke², and Hartmut Lemmel^{1,2}

¹ Technische Universität Wien, Atominstytut, Stadionallee 2, 1020 Wien, Austria

² Institut Laue-Langevin, 71 avenue des Martyrs, CS 20156, 38042 Grenoble Cedex 9, France

Abstract. What is driving the accelerated expansion of the universe and do we have an alternative for Einstein's cosmological constant? What is dark matter made of? Do extra dimensions of space and time exist? Is there a preferred frame in the universe? To which extent is left-handedness a preferred symmetry in nature? What's the origin of the baryon asymmetry in the universe? These fundamental and open questions are addressed by precision experiments using ultra-cold neutrons. This year, we celebrate the 50th anniversary of their first production, followed by first pioneering experiments. Actually, ultra-cold neutrons were discovered twice in the same year – once in the eastern and once in the western world [1,2]. For five decades now research projects with ultra-cold neutrons have contributed to the determination of the force constants of nature's fundamental interactions, and several technological breakthroughs in precision allow to address the open questions by putting them to experimental test. To mark the event and tribute to this fabulous object, we present a birthday song for ultra-cold neutrons with acoustic resonant transitions [3], which are based solely on properties of ultra-cold neutrons, the inertial and gravitational mass of the neutron m , Planck's constant h , and the local gravity g . We make use of a musical intonation system that bears no relation to basic notation and basic musical theory as applied and used elsewhere [4] but addresses two fundamental problems of music theory, the problem of reference for the concert pitch and the problem of intonation.

1. A tribute to a fabulous object

What is an ultra-cold neutron? Following a pragmatic definition [5], such a neutron is able to become reflected from a given surface under any angle of incidence. This feature allows us to store neutrons in a box or in a so-called neutron bottle. As a consequence, an observation time of up to few times its lifetime is manageable, thus enabling highly sensitive experiments. The ultra-cold neutron became thus a tool and an object for precision experiments. The search for a permanent electric dipole moment of the neutron investigates a high-energy scale in particle physics that cannot be reached by accelerators on Earth. The present experimental limit on this quantity is $|d_n| < 3.0 \times 10^{-26}$ e cm (90% C.L.) [6]. At the current level of sensitivity, energy changes down to 10^{-22} eV can be detected. A common technique uses the Ramsey resonance method of separate oscillating fields [7], where polarized neutrons precess in a magnetic field. The precession frequency will change in the presence of an electric field if the neutron has an electric dipole moment. The measurements are made with ultra-cold neutrons stored in a cell permeated by uniform electric and magnetic fields [6,8–16]. This search investigates CP

violating mechanisms beyond the Standard Model. These are necessary to explain the matter-antimatter asymmetry in our universe. A byproduct is a search for axionlike dark matter through nuclear spin precession. “This null result sets the first laboratory constraints on the coupling of axion dark matter to gluons, which improve on astrophysical limits by up to 3 orders of magnitude” [17].

This is just an example how neutrons contribute to key issues that have emerged in particle physics and cosmology and are expected to be decisive for unanswered fundamental questions. Addressed by physics with neutrons as a tool [18,19] are basic questions like “What is driving the accelerated expansion of the universe and do we have an alternative for Einstein's cosmological constant?”, “What is dark matter made of?”, “Do extra dimensions of space and time exist?”, “Is there a preferred frame in the universe?”, “To what extent is left-handedness a preferred symmetry in nature?”, “What's the origin of the baryon asymmetry in the universe?”, and so on.

With neutrons serving as an object basic properties have been studied. Examples include the neutron lifetime [20–27] and other decay parameters like β -decay correlation coefficients [28–30], measurements of its magnetic moment, quantum mechanical [31,32] or neutron optical [33,34] properties, and searches for a charge of the neutron [35]. The β -decay measurements are complemented by experiments with cold neutrons for the lifetime [36–39], see also the review [40], and measurements of correlation coefficients [41–55]. Other searches include a conversion of a neutron into an anti-neutron [56] or mirror-neutron [57,58] or a decay into a hypothetical dark matter particle [59].

* This article covers the music part of a lecture given by H.A. at the International Workshop on Particle Physics at Neutron Sources, PPNS, in Grenoble/France in May 2018 with a composition on the occasion of the 50th anniversary of the first production of ultra-cold neutrons. [1] H. Abele, H. Lemmel, T. Jenke, Nature **572**, 178 (2019).

^a e-mail: abele@ati.ac.at



Figure 1. This photograph taken at the PPNS workshop in May 2018 shows Yuri N. Pokotilovskii, member of the Dubna team (right) and Albert Steyerl (left), two of the pioneers of research with ultra-cold neutrons.

2. Ultra-cold neutrons

Neutrons are abundantly produced in a spallation source or a research reactor. At production, these neutrons are very hot with an energy of several MeV. On the other side of the energy scale, some ultra-cold neutron experiments use particles with peV energies in the direction of gravity, about 18 orders of magnitude less. This tremendous reduction is achieved in several steps. In a first step, spallation or fission neutrons thermalize, e.g., in a heavy water tank at a temperature of 300 K. The thermal fluxes are distributed in energy close to Maxwellian law. Cold neutrons with energies in the meV range are obtained in a second moderator stage. At the Institut Laue-Langevin this is achieved by a 24 K liquid deuterium cold moderator near the uranium core of the 58.3 MW reactor. As for propagation of light in matter, one can assign a neutron refractive index to materials but it is often less than unity. Thus, one considers the surface of matter as constituting a potential step of height V , see Table 1. Neutrons with transversal energy $E_{\perp} < V$ will be totally reflected. Following our pragmatic definition, ultra-cold neutrons are, in contrast to faster neutrons, capable of undergoing total external reflection even at normal incidence in a given material or magnetic device [5]. When the surface roughness of the mirror is small enough, the ultra-cold neutron reflection is specular. This feature makes it possible to build simple and efficient retroreflectors. Such a neutron mirror makes use of the strong interaction between ultra-cold neutrons and wall nuclei, resulting in an effective spatially extended potential step of the order of 10^{-15} m and 100 neV. We will need a neutron mirror together with gravity for our birthday song in Sect. 3.

Table 1. Ultra-cold neutrons can be trapped by the optical potential of matter, by local gravity g , or by magnetic fields.

Optical Potential	~ 100 neV	Material dependencies
Gravity Potential	100 neV/m	$V = m \times g \times z$
Magnetic Field	60 neV/T	Zeeman Splitting

First attempts to produce ultra-cold neutrons started by V.I. Lushchikov, Y.N. Pokotilovskii, A.V. Strelkov, and F.L. Shapiro at Joint Institute for Nuclear Research in Dubna [1], and by A. Steyerl at Technische Hochschule München [2]. Their efforts made first ultra-cold neutrons available exactly 50 years ago. The photo in Fig. 1 shows two of the pioneers during a coffee break at the PPNS workshop. Original design drawings together with short descriptions of the experimental arrangements are shown in Fig. 2. In the following years, ultra-cold neutron densities have been drastically increased using more powerful reactor and cold neutron sources at the Petersburg Nuclear Physics Institute (PNPI) and at ILL. Ultra-cold neutrons are taken from the low energy tail of the continuous cold spectrum. At ILL, they are transported vertically upwards by a curved guide, which transmits neutrons below a threshold energy, acting as a low-velocity filter. Neutrons with a velocity of up to 50 m/s arrive at a rotating turbine with blades consisting of high-quality Ni mirrors. Colliding with the blades moving in the same direction with half their velocity, the neutrons lose almost all their energy and leave the turbine as ultra-cold neutrons with typical velocities of a few meters per second. They are then guided to several experimental areas. The total output per beam port today is close

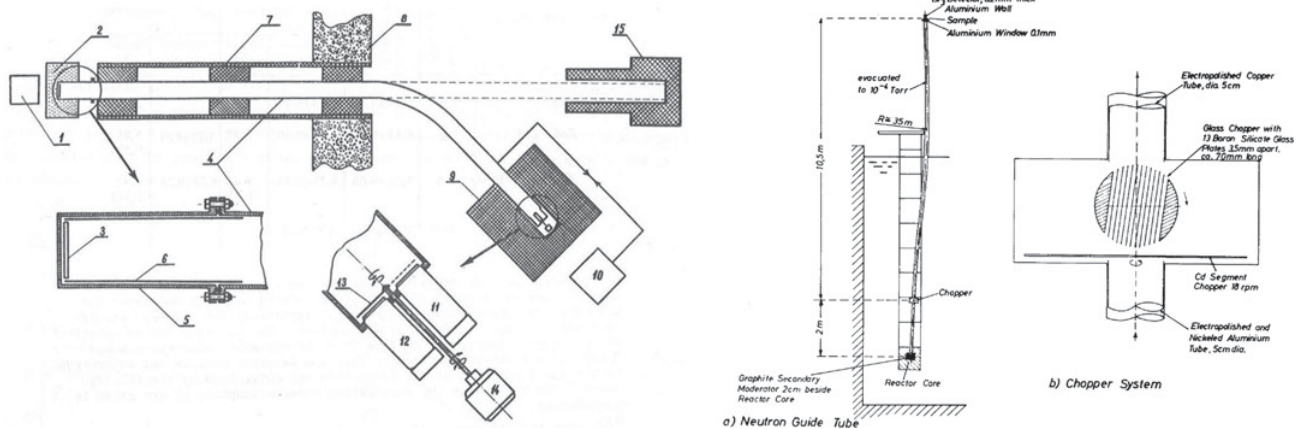


Figure 2. The first production of ultra-cold neutrons as an episode during the era of the cold war. *Left:* experiment of the east, the Dubna experiment by V. I. Lushchikov et al. [1]. The neutron source was the IBR pulsed reactor operating at an average power of 6 kW and with a flash repetition frequency of one every 5 sec. The only neutrons that could reach the detector in the bent tube in practice were those emitted from moderator labeled 3 with velocities lower than 5.7 m/s for copper [1]. *Right:* experiment of the west, the Munich experiment by A. Steyerl [2]. Very cold neutrons were selected from the spectrum of a graphite secondary moderator near the core of the FRM reactor through a vertical neutron guide tube and energy-analysed with the time-of-flight method. Total cross sections for gold and aluminium at 300 K were measured for neutron velocities down to 5 m/s.

to 10^6 neutrons per second. This instrument, nowadays called PF2, was designed by A. Steyerl and P. Ageron and colleagues and constructed in 1985 [60], replacing the former PN5 on level C of the ILL reactor. To date, most experiments using ultra-cold neutrons are limited by counting statistics. Therefore, major efforts are undertaken in order to improve existing ultra-cold neutron sources or develop new concepts. From the experimental point of view, ideas exist for techniques that could further increase count rates due to a higher ultra-cold neutron phase space density. One way employs down-scattering of cold neutrons in superfluid ^4He below 1 K [61–63]. An alternative approach is the use of solid deuterium at about 5 K [64–66]. Ultra-cold neutron sources are in operation or under construction at many sites, including facilities at the ILL (France), Paul-Scherrer-Institute (Switzerland), University of Mainz (Germany), Los Alamos National Lab (United States), Forschungsneutronenquelle Heinz Maier-Leibnitz (Germany), TRIUMF (Canada), PNPI (Russia), KEK (Japan), North Carolina State University (United States). A comparison of operating ultra-cold neutron sources for fundamental physics measurements can be found in [67].

3. Birthday song for ultra-cold neutrons

In the following, we present a birthday song in a novel manner for the 50th anniversary of the first production of ultra-cold neutrons. We make use of a musical intonation system that bears no relation to basic notation and basic music theory as applied and used elsewhere. Instead it is based on natural constants and local gravity and addresses two fundamental problems of music theory.

3.1. The problem of reference for the concert pitch

The first problem is the practical adjustment of absolute pitch of musical instruments. The concert pitch is the reference to which a group of musical instruments is tuned

for a performance. Today 440 Hertz is the frequency of the standard tuning tone, called A4 according to Fig. 3. Between 1700 and 1820 the concert pitch was pretty constant [4]: Bach’s tuning fork vibrated at 415.5 Hz and Händel used 422.5 Hz, Berlin was at 422 Hz in 1752 and Mozart’s tuning fork gave 421.6 Hz, the pitch in Paris was 423 Hz. Striving for high brilliance in orchestral sound, a rise in frequency started. In 1858 the following pitch frequencies were used: Paris 449 Hz, Milano 451 Hz, Berlin 452 Hz, London 453 Hz. In 1880 Steinway tuned the pianos to 458 Hz. In an attempt to halt such a pitch inflation the Wien Conference of 1885 recommended 435 Hz as the standard pitch, but in the following decades the pitch rose again to about 443 Hz on average. In 1939, the London Conference of International Federation of the National Standardizing Associations (ISA) recommended that the A be tuned to 440 Hz, now known as concert pitch, confirmed by the International Organization for Standardization as ISO 16 in 1975. Such case that a standard reference was lacking a deeper justification or was subject to fluctuations was also observed in physical science and metrology.

As an example, we point to the definition of the kilogram as for long established by International System of Units (SI). The standard was based on a platinum alloy cylinder, the International Prototype Kilogram (IPK), manufactured in 1889, and saved in Saint-Cloud. IPK has diverged from its replicas by approximately 50 μg since their production, see Fig. 4. For this reason it has been desirable to replace the kilogram artifact with a definition based directly on physical constants. Such a definition was approved by the General Conference on Weights and Measures (CGPM) on 16 November 2018. It is based on constants of nature, in particular the Planck constant, which is now defined, thereby fixing the value of the kilogram in terms of the second and the metre, and thus eliminating the need for the IPK.

For problems of drifting standards a solution similar to the definition of physical units can be adopted and transferred to musicology. We suggest as a standard for the concert pitch the resonance frequency of a $|5\rangle \leftrightarrow |8\rangle$ level

85	a ^{'''}	A ₇	3520.000
84	g ^{'''} /a ^{''}	G ₇ /A ₇	3322.438
83	g ^{''}	G ₇	3135.963
82	f ^{'''} /g ^{''}	F ₇ /G ₇	2959.955
81	f ^{''}	F ₇	2793.826
80	e ^{'''}	E ₇	2637.020
79	d ^{'''} /e ^{''}	D ₇ /E ₇	2489.016
78	d ^{''}	D ₇	2349.318
77	c ^{'''} /d ^{''}	C ₇ /D ₇	2217.461
76	c ^{''} 4-line octave	C ₇ Double high C	2093.005
75	b ^{'''}	B ₆	1975.533
74	a ^{'''} /b ^{''}	A ₆ /B ₆	1864.655
73	a ^{''}	A ₆	1760.000
72	g ^{'''} /a ^{''}	G ₆ /A ₆	1661.219
71	g ^{''}	G ₆	1567.982
70	f ^{'''} /g ^{''}	F ₆ /G ₆	1479.978
69	f ^{''}	F ₆	1396.913
68	e ^{'''}	E ₆	1318.510
67	d ^{'''} /e ^{''}	D ₆ /E ₆	1244.508
66	d ^{''}	D ₆	1174.659
65	c ^{'''} /d ^{''}	C ₆ /D ₆	1108.731
64	c ^{''} 3-line octave	C ₆ Soprano C (High C)	1046.502
63	b ^{'''}	B ₅	987.7666
62	a ^{'''} /b ^{''}	A ₅ /B ₅	932.3275
61	a ^{''}	A ₅	880.0000
60	g ^{'''} /a ^{''}	G ₅ /A ₅	830.6094
59	g ^{''}	G ₅	783.9909
58	f ^{'''} /g ^{''}	F ₅ /G ₅	739.9888
57	f ^{''}	F ₅	698.4565
56	e ^{'''}	E ₅	659.2551
55	d ^{'''} /e ^{''}	D ₅ /E ₅	622.2540
54	d ^{''}	D ₅	587.3295
53	c ^{'''} /d ^{''}	C ₅ /D ₅	554.3653
52	c ^{''} 2-line octave	C ₅ Tenor C	523.2511
51	b ^{'''}	B ₄	493.8833
50	a ^{'''} /b ^{''}	A ₄ /B ₄	466.1638
49	a ^{''}	A ₄ 4440	440.0000
48	g ^{'''} /a ^{''}	G ₄ /A ₄	415.3047
47	g ^{''}	G ₄	391.9954
46	f ^{'''} /g ^{''}	F ₄ /G ₄	369.9944
45	f ^{''}	F ₄	349.2282
44	e ^{'''}	E ₄	329.6276
43	d ^{'''} /e ^{''}	D ₄ /E ₄	311.1270
42	d ^{''}	D ₄	293.6648
41	c ^{'''} /d ^{''}	C ₄ /D ₄	277.1826
40	c ^{''} 1-line octave	C ₄ Middle C	261.6256
39	b ^{'''}	B ₃	246.9417
38	a ^{'''} /b ^{''}	A ₃ /B ₃	233.0819
37	a ^{''}	A ₃	220.0000
36	g ^{'''} /a ^{''}	G ₃ /A ₃	207.6523
35	g ^{''}	G ₃	195.9977
34	f ^{'''} /g ^{''}	F ₃ /G ₃	184.9972
33	f ^{''}	F ₃	174.6141
32	e ^{'''}	E ₃	164.8138
31	d ^{'''} /e ^{''}	D ₃ /E ₃	155.5635
30	d ^{''}	D ₃	146.8324
29	c ^{'''} /d ^{''}	C ₃ /D ₃	138.5913
28	c ^{''} small octave	C ₃	130.8128
27	B	B ₂	123.4708
26	A ^{'''} /B ^{''}	A ₂ /B ₂	116.5409
25	A	A ₂	110.0000

Figure 3. Relationship between musical notations and physical frequencies for five octaves between A2 and A7. 1st column: key number on a keyboard, 2nd column: Helmholtz's notation. 3rd column: English notation. *Right:* corresponding frequencies in equal-tempered intonation. (Figure source: Wikipedia.)

transition of ultra-cold neutrons in the gravitational field as explained in Sect. 3.3. This frequency is based on natural constants as well as the local gravity g .

3.2. The problem of intonation

The second problem is related to the choice of intervals between notes. While the basic grid of notes, as represented by the modern piano scale, has been quite uniform over different times and cultures, the fine

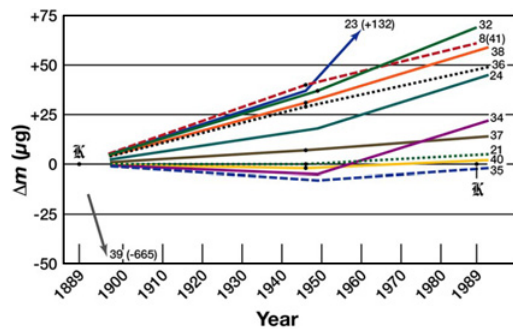


Figure 4. Kilogram definition and mass drift over time of national prototypes K21-K40, plus two of the IPK's sister copies: K32 and K8(41) (Wikipedia).

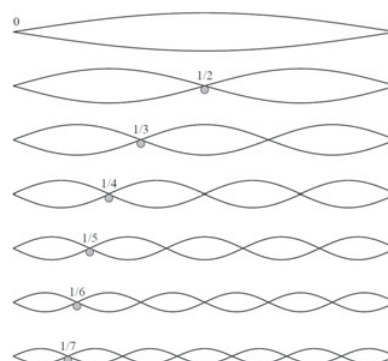


Figure 5. Harmonics of a string, based on sine and cosine functions (Wikipedia).

tuning of those notes has always been a subject of discussion, and has led to the creation of various musical tuning systems. Extreme systems are the modern "equal-tempered" intonation, offering a maximum flexibility for switching between tonalities, as opposed to "just" or "pure" intonation, where the intervals are based on whole-number ratios of frequencies, offering a maximum of clarity within a single tonality. An example is the Pythagorean intonation system, using ratios of 2 and 3 as well as their powers. A property of Pythagorean tuning is an excess of 12 perfect fifths over 7 octaves, which is $(3/2)^{12} : (2/1)^7 = 531441/524288$. The discrepancy is 23.45% of a semi-tone, or nearly a quarter of a semi tone, and the consequences are a misfit in frequency between enharmonic equal notes like A_b and G_{\sharp} . Medieval compositions avoid such mistunes by a restriction to nine notes, for example B_b -F-C-G-D-A-E-B-F $_{\sharp}$, see for details [4].

The number ratios in just intonation are given by the harmonic overtone spectrum of pitched musical instruments, which are often based on an acoustic resonator such as a string or a column of air. The eigenmodes of such resonators form standing waves, see Fig. 5, which correspond to integral frequency ratios. Numerous such modes oscillate simultaneously.

The musical pitch of a note is usually perceived as the lowest, fundamental frequency, which may be the one created by vibration over the full length of the string. The musical timbre of the tone is strongly affected by the relative strength of each harmonic. It is interesting to note that the American composer Ben Johnston is experimenting with pure intonation and has proposed the

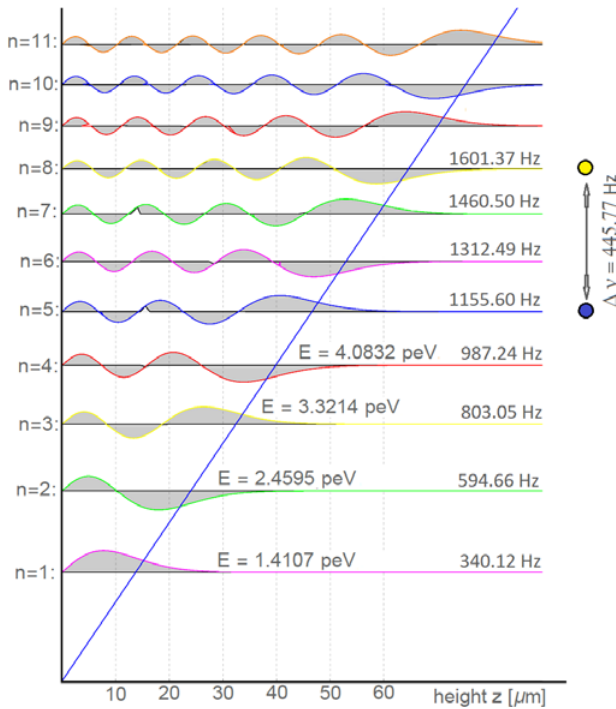


Figure 6. Energy eigenvalues and eigenfunctions of a neutron bound in the gravity potential of the Earth and corresponding frequency equivalents. By oscillating the mirror with a frequency, which correspond to the energy difference between quantum states, transitions are introduced. The oscillation frequencies are in the acoustic frequency range.

term “extended just intonation” for composition involving ratios that contain prime numbers beyond five (7, 11, 13 etc.), as mentioned at Wikipedia.

In the equal-tempered intonation system, the modes deviate slightly from the Pythagorean frequency ratios. It parts the octave in 12 equal semitones separated by factors $2^{(1/12)}$. Five octaves of music notes are shown in Fig. 3 in the notation of Helmholtz, in English notation, together with corresponding frequencies in equal-tempered intonation. We introduce an intonation system, which is neither based on the Pythagorean harmonic system nor on the 12th square root of 2.

3.3. Neutron’s intonation and the quantum bouncing ball

We approach both mentioned problems of music theory by resorting to the natural frequency spectrum given by an ultra-cold neutron prepared as quantum bouncing ball [68]. The neutron is bound between a flat lying mirror and the raising potential of gravity above, $V = mgz$, where m denotes the neutron mass, g the local gravity, and z the distance above the mirror. Every bound system in quantum mechanics has discrete energy levels. In our case the lowest energy eigenvalues E_n ($n = 1, 2, 3, 4, 5$) are 1.4107 peV, 2.4595 peV, 3.3214 peV, 4.0832 peV, and 4.77906 peV. In Fig. 6 they are shown together with the corresponding neutron wavefunctions, the well-known Airy functions. Similarly, in the neutron whispering gallery quantum states occur in a binding well formed by the centrifugal potential and a circular boundary [69].

The interesting point is that one can drive transitions between these eigenstates by vibrating the mirror. Within

Table 2. A selection of cities with corresponding local acceleration g . Each city has its own character. To take this fact into account it is suggested to establish a local concert pitch for music performances based on local g and on ultra-cold neutrons.

Location	Local acceleration g [m/s ²]	Concert pitch [Hz] based on ultra-cold neutrons
Amsterdam	9.813	446.01
Athens	9.800	445.61
Bangkok	9.783	445.10
Cape Town	9.796	445.49
Chicago	9.803	445.70
Grenoble	9.805	445.77
Helsinki	9.819	446.19
Havana	9.788	445.25
Istanbul	9.808	445.85
Jakarta	9.781	445.04
London	9.796	445.49
Mexico City	9.779	444.97
Paris	9.809	445.88
San Francisco	9.800	445.61
Sydney	9.797	445.52
Vienna	9.808	445.85
Zurich	9.807	445.82

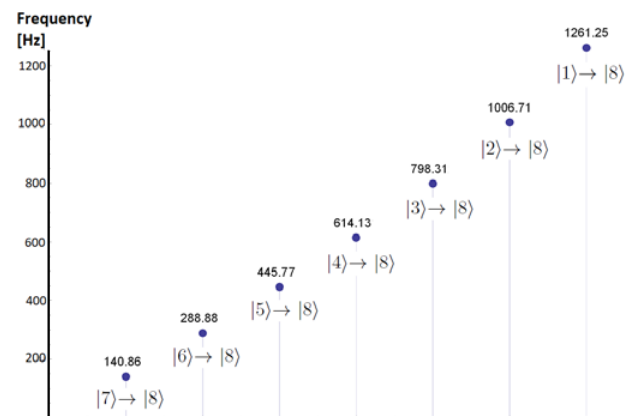


Figure 7. Planck’s constant h , the mass of the neutron m and local gravity g define a series of frequency, here exemplarily demonstrated for acoustic transitions between $|7\rangle \rightarrow |8\rangle$ at 140.86 Hz and $|1\rangle \rightarrow |8\rangle$ at 1261.25 Hz. In this paper the frequencies ν are rounded to two decimal places. Such a frequency resolution is reached during a time span of 10 s taking the classical uncertainty relationship $\Delta\omega \times \Delta t = \frac{1}{2}$ into consideration.

the qBounce experiment [3, 70–73], several resonance spectroscopy measurements with different geometric parameters have been performed, resulting in different resonance frequencies and widths. In general, the oscillator frequency at resonance for a transition between states with energies E_p and E_q is

$$\nu_{p,q} = \frac{E_q - E_p}{h} = \nu_q - \nu_p. \quad (1)$$

Table 3. Neutron’s intonation system based on acoustic resonant transitions [3], here in a 30×14 matrix M . Shown is the frequency range for transitions between $|14\rangle \leftrightarrow |15\rangle$ at 112.45 Hz and $|1\rangle \leftrightarrow |30\rangle$ at 3585.57 Hz spanning the piano key notes from A2 to A7 in Fig. 3. We can consider this matrix as an instrument, which is able to perform pieces of music with these frequencies, and perform “Happy Birthday to you” for the ultra-cold neutron. The frequencies have been calculated for the local gravity $g = 9.80507 \text{ m/s}^2$ at our experimental setup PF2 in Grenoble.

	1)	2)	3)	4)	5)	6)	7)	8)	9)	10)	11)	12)	13)	14)
1)	0.	254.542	462.939	647.12	815.486	972.374	1120.39	1261.25	1396.17	1526.04	1651.53	1773.19	1891.44	2006.64
2)	254.542	0.	208.396	392.578	560.944	717.831	865.846	1006.71	1141.63	1271.49	1396.99	1518.65	1636.9	1752.1
3)	462.939	208.396	0.	184.182	352.548	509.435	657.45	798.312	933.231	1063.1	1188.59	1310.25	1428.5	1543.7
4)	647.12	392.578	184.182	0.	168.366	325.254	473.269	614.131	749.049	878.915	1004.41	1126.07	1244.32	1359.52
5)	815.486	560.944	352.548	168.366	0.	156.888	304.903	445.765	580.683	710.549	836.045	957.704	1075.96	1191.16
6)	972.374	717.831	509.435	325.254	156.888	0.	148.015	288.877	423.795	553.662	679.157	800.816	919.069	1034.27
7)	1120.39	865.846	657.45	473.269	304.903	148.015	0.	140.862	275.78	405.647	531.142	652.801	771.054	886.253
8)	1261.25	1006.71	798.312	614.131	445.765	288.877	140.862	0.	134.918	264.785	390.28	511.939	630.192	745.391
9)	1396.17	1141.63	933.231	749.049	580.683	423.795	275.78	134.918	0.	129.866	255.362	377.021	495.274	610.473
10)	1526.04	1271.49	1063.1	878.915	710.549	553.662	405.647	264.785	129.866	0.	125.495	247.154	365.407	480.606
11)	1651.53	1396.99	1188.59	1004.41	836.045	679.157	531.142	390.28	255.362	125.495	0.	121.659	239.912	355.111
12)	1773.19	1518.65	1310.25	1126.07	957.704	800.816	652.801	511.939	377.021	247.154	121.659	0.	118.253	233.452
13)	1891.44	1636.9	1428.5	1244.32	1075.96	919.069	771.054	630.192	495.274	365.407	239.912	118.253	0.	115.199
14)	2006.64	1752.1	1543.7	1359.52	1191.16	1034.27	886.253	745.391	610.473	480.606	355.111	233.452	115.199	0.
15)	2119.08	1864.54	1656.14	1471.96	1303.59	1146.71	998.691	857.829	722.91	593.044	467.549	345.89	227.637	112.438
16)	2229.	1974.46	1766.06	1581.88	1413.52	1256.63	1108.61	967.753	832.834	702.968	577.472	455.813	337.56	222.361
17)	2336.62	2082.08	1873.69	1689.5	1521.14	1364.25	1216.24	1075.37	940.455	810.588	685.093	563.434	445.181	329.982
18)	2442.12	2187.58	1979.18	1795.	1626.64	1469.75	1321.73	1180.87	1045.95	916.087	790.592	668.933	550.68	435.481
19)	2545.66	2291.12	2082.72	1898.54	1730.17	1573.28	1425.27	1284.41	1149.49	1019.62	894.127	772.468	654.215	539.016
20)	2647.37	2392.83	2184.43	2000.25	1831.88	1674.99	1526.98	1386.12	1251.2	1121.33	995.837	874.178	755.925	640.726
21)	2747.38	2492.83	2284.44	2100.26	1931.89	1775.	1626.99	1486.12	1351.21	1221.34	1095.84	974.186	855.933	740.734
22)	2845.79	2591.25	2382.85	2198.67	2030.3	1873.42	1725.4	1584.54	1449.62	1319.75	1194.26	1072.6	954.346	839.147
23)	2942.7	2688.16	2479.77	2295.58	2127.22	1970.33	1822.32	1681.45	1546.54	1416.67	1291.17	1169.51	1051.26	936.063
24)	3038.21	2783.67	2575.27	2391.09	2222.72	2065.84	1917.82	1776.96	1642.04	1512.18	1386.68	1265.02	1146.77	1031.57
25)	3132.39	2877.84	2669.45	2485.27	2316.9	2160.01	2012.	1871.13	1736.22	1606.35	1480.85	1359.2	1240.94	1125.74
26)	3225.3	2970.76	2762.36	2578.18	2409.81	2252.93	2104.91	1964.05	1829.13	1699.26	1573.77	1452.11	1333.86	1218.66
27)	3317.02	3062.48	2854.08	2669.9	2501.53	2344.65	2196.63	2055.77	1920.85	1790.98	1665.49	1543.83	1425.58	1310.38
28)	3407.6	3153.06	2944.66	2760.48	2592.12	2435.23	2287.21	2146.35	2011.43	1881.57	1756.07	1634.41	1516.16	1400.96
29)	3497.1	3242.56	3034.17	2849.98	2681.62	2524.73	2376.71	2235.85	2100.93	1971.07	1845.57	1723.91	1605.66	1490.46
30)	3585.57	3331.03	3122.63	2938.45	2770.09	2613.2	2465.18	2324.32	2189.4	2059.54	1934.04	1812.38	1694.13	1578.93

The transfer from state $|p\rangle$ to $|q\rangle$ is referred to as Rabi transition, which can be induced by applying the right frequency in the acoustic range. For example, the $|1\rangle \leftrightarrow |2\rangle$ transition corresponds to the frequency $\nu_{1,2} = 254.542 \text{ Hz}$, the transition $|1\rangle \leftrightarrow |3\rangle$ has a frequency of $\nu_{1,3} = 462.94 \text{ Hz}$, and $\nu_{5,8} = 445.77 \text{ Hz}$ is close to concert pitch of 444 Hz, which is used by many orchestras although the official value is 440 Hz.

We therefore suggest to use the acoustic $|5\rangle \leftrightarrow |8\rangle$ transition as a definition for the concert pitch, satisfying our requirement for natural constants, the mass of the neutron m and Planck’s constant h , together with the local gravity g .

$$\nu_{5,8} = \frac{E_8 - E_5}{h} = \nu_8 - \nu_5 = 445.77 \text{ Hz.} \quad (2)$$

We would like to note that, as a particularity of a quantum system involving gravity as a binding force, this definition is still open to display regional diversity. This, however, is not due to an arbitrary choice, as for the pitch in a concert hall, but defined by nature. A local gravity $g = 9.797 \text{ m/s}^2$ at Sydney results in a concert pitch of 445.52 Hz, whereas the opera house in Helsinki with $g = 9.819$ would use a

pitch of 446.19 Hz. Concert pitches at other locations can be found in Table 2. For our calculations we use the local gravity $g = 9.80507 \text{ m/s}^2$ at our experimental setup PF2 in Grenoble.

The transitions between $|7\rangle \rightarrow |8\rangle$ at 140.835 Hz and $|1\rangle \rightarrow |8\rangle$ at 1263.25 Hz define a series of frequency, see Fig. 7. Taking all transitions $|p\rangle \rightarrow |q\rangle$ into account one obtains neutron’s intonation system, here shown in Table 3 in a matrix of frequencies up to level 30. Many frequencies are close to notes of the modern piano scale, cf. Fig. 3, and we therefore suggest to consider the neutron transition frequencies a musical instrument. Selected frequencies can be used to tune the piano scale, addressing the second problem mentioned above. However, the complete spectrum is much more diverse than a standard musical instrument. Each pair of levels has a unique transition frequency, as a consequence of the Airy functions. In principle the spectrum offers vast musical possibilities but for now we restrict ourselves to selected frequencies suitable for the traditional song “Happy Birthday”. We play the song in B♭ major starting with an f. With 440 Hz tuning and equal temperament the f corresponds to 349.2282 Hz. In the neutron

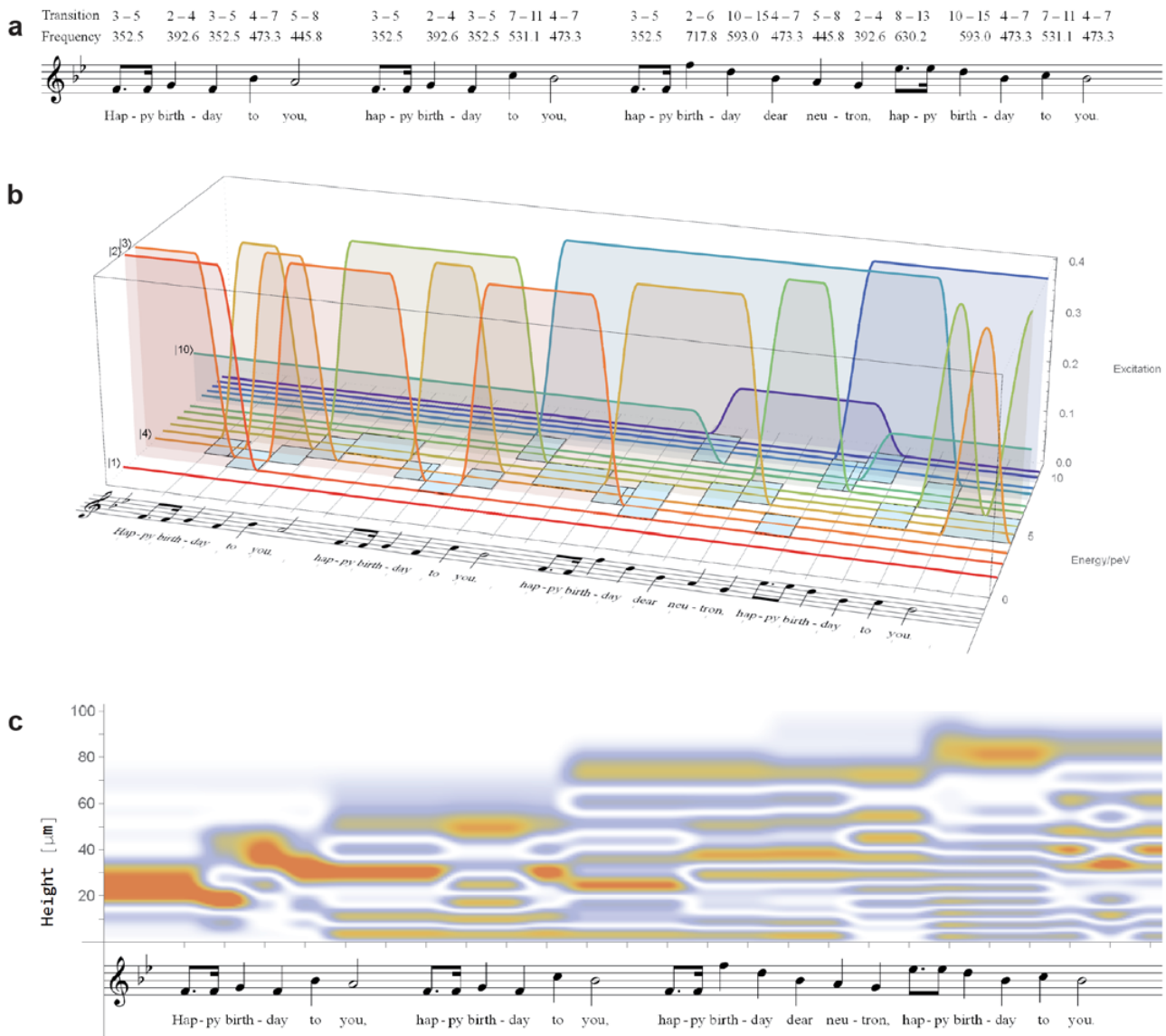


Figure 8. A birthday song for ultra-cold neutrons. (a) Level couplings and frequencies used for the birthday song. (b) Each song note couples two levels, indicated by the light blue rectangles. As long as the note is sounding the level amplitudes make a Rabi oscillation. The tone duration of a quarter note creates a π flip. (c) Neutron wave function $|\psi|^2$ of the dancing neutron.

tuning we use the $|3\rangle \rightarrow |5\rangle$ transition corresponding to 352.548 Hz and continue with other transitions as shown in Fig. 8a. The audio file can be downloaded from our web site.

As it's their 50 year's birthday, it's not us but the ultra-cold neutrons to listen to the song. It's us to play it and watch their reaction. We send the signal to the neutron mirror and let it vibrate. Each note starts a Rabi transition between the corresponding two levels. The rhythm (tone duration) determines the coupling time and therefore the amount of amplitude change. We adjust the vibration strength such that a quarter note induces an exact π flip, swapping the complete amplitude between two levels. As initial state we prepare the superposition $\sqrt{0.4}|2\rangle + \sqrt{0.4}|3\rangle + \sqrt{0.2}|10\rangle$. Figure 8b shows the couplings and the excitation amplitudes of the levels during the song. Figure 8c shows the resulting neutron probability density as a function of time and vertical position over the mirror. The neutron is dancing to its birthday song:

Happy Birthday Ultra-Cold Neutron!

H.A. would like to thank G. Horvath, who presented the neutron's time evolution with respect to the first 30 s of Wolfgang Amadeus Mozart's Opera *Così fan tutte*, Aria *Fiordiligi, Come scolgì*.

References

- [1] V.I. Lushchikov, Y.N. Pokotilovskii, A.V. Strelkov, F.L. Shapiro, *JETP Lett.* (U.S.S.R., Engl. Transl.) **9**, 40 (1969)
- [2] A. Steyerl, *Phys. Lett. B* **29**, 33 (1969)
- [3] G. Cronenberg, P. Brax, H. Filter, P. Geltenbort, T. Jenke, G. Pignol, M. Pitschmann, M. Thalhammer, H. Abele, *Nat. Phys.* **14**, 1022 (2018)
- [4] Riemann, *Musiklexikon* (F.A. Brockhaus, B. Schotts Söhne, Wiesbaden, Mainz, 1978)
- [5] A. Steyerl, *Very Low Energy Neutrons* (Springer Berlin Heidelberg, Berlin, Heidelberg, 1977), pp. 57–130, ISBN 978-3-540-37543-2, <https://doi.org/10.1007/BFb0041487>

- [6] J.M. Pendlebury, S. Afach, N.J. Ayres, C.A. Baker, G. Ban, G. Bison, K. Bodek, M. Burghoff, P. Geltenbort, K. Green et al., *Phys. Rev. D* **92**, 092003 (2015)
- [7] N.F. Ramsey, *Phys. Rev.* **76**, 996 (1949)
- [8] I.S. Altarev et al., *JETP Lett.* **29**, 730 (1979)
- [9] I.S. Altarev, Y.V. Borisov, N.V. Borovikova, A.B. Brandin, A.I. Egorov, V.F. Ezhov, S.N. Ivanov, V.M. Lobashev, V.A. Nazarenko, V.L. Ryabov et al., *Phys. Lett. B* **102**, 13 (1981)
- [10] J.M. Pendlebury, K.F. Smith, R. Golub, J. Byrne, T.J.L. McComb, T.J. Sumner, S.M. Burnett, A.R. Taylor, B. Heckel, N.F. Ramsey et al., *Phys. Lett. B* **136**, 327 (1984)
- [11] I. Altarev, et al., *JETP Lett. (U.S.S.R., Engl. Transl.)* **44**, 460 (1986)
- [12] K.F. Smith, N. Crampin, J.M. Pendlebury, D.J. Richardson, D. Shiers, K. Green, A.I. Kilvington, J. Moir, H.B. Prosper, D. Thompson et al., *Phys. Lett. B* **234**, 191 (1990)
- [13] I.S. Altarev, Y.V. Borisov, N.V. Borovikova, S.N. Ivanov, E.A. Kolomensky, M.S. Lasakov, V.M. Lobashev, V.A. Nazarenko, A.N. Pirozhkov, A.P. Serebrov et al., *Phys. Lett. B* **276**, 242 (1992)
- [14] I.S. Altarev, Y.V. Borisov, N.V. Borovikova, A.I. Egorov, S.N. Ivanov, E.A. Kolomensky, M.S. Lasakov, V.M. Lobashev, V.A. Nazarenko, A.N. Pirozhkov et al., *Phys. Atom. Nuclei* **59**, 1152 (1996)
- [15] P.G. Harris, C.A. Baker, K. Green, P. Iaydjiev, S. Ivanov, D.J.R. May, J.M. Pendlebury, D. Shiers, K.F. Smith, M. van der Grinten et al., *Phys. Rev. Lett.* **82**, 904 (1999)
- [16] C.A. Baker, D.D. Doyle, P. Geltenbort, K. Green, M.G.D. van der Grinten, P.G. Harris, P. Iaydjiev, S.N. Ivanov, D.J.R. May, J.M. Pendlebury et al., *Phys. Rev. Lett.* **97**, 131801 (2006)
- [17] C. Abel, N.J. Ayres, G. Ban, G. Bison, K. Bodek, V. Bondar, M. Daum, M. Fairbairn, V.V. Flambaum, P. Geltenbort et al., *Phys. Rev. X* **7**, 041034 (2017)
- [18] D. Dubbers, M.G. Schmidt, *Rev. Mod. Phys.* **83**, 1111 (2011)
- [19] H. Abele, *Prog. Part. Nucl. Phys.* **60**, 1 (2008)
- [20] W. Mampe, P. Ageron, C. Bates, J.M. Pendlebury, A. Steyerl, *Phys. Rev. Lett.* **63**, 593 (1989)
- [21] S. Arzumanov, L. Bondarenko, S. Chernyavsky, W. Drexel, A. Fomin, P. Geltenbort, V. Morozov, Y. Panin, J. Pendlebury, K. Schreckenbach, *Phys. Lett. B* **483**, 15 (2000)
- [22] A.P. Serebrov, V.E. Varlamov, A.G. Kharitonov, A.K. Fomin, Y.N. Pokotilovski, P. Geltenbort, I.A. Krasnoschekova, M.S. Lasakov, R.R. Taldaev, A.V. Vassiljev et al., *Phys. Rev. C* **78**, 035505 (2008)
- [23] V. Ezhov, A. Andreev, G. Ban, B. Bazarov, P. Geltenbort, F. Hartman, A. Glushkov, M. Groshev, V. Knyazkov, N. Kovrizhnykh et al., *Nucl. Instrum. Meth. Phys. Res. A* **611**, 167 (2009)
- [24] A. Pichlmaier, V. Varlamov, K. Schreckenbach, P. Geltenbort, *Phys. Lett. B* **693**, 221 (2010)
- [25] A.P. Serebrov, E.A. Kolomensky, A.K. Fomin, I.A. Krasnoschekova, A.V. Vassiljev, D.M. Prudnikov, I.V. Shoka, A.V. Chechkin, M.E. Chaikovskiy, V.E. Varlamov et al., *Phys. Rev. C* **97**, 055503 (2018)
- [26] R.W. Pattie, N.B. Callahan, C. Cude-Woods, E.R. Adamek, L.J. Broussard, S.M. Clayton, S.A. Currie, E.B. Dees, X. Ding, E.M. Engel et al., *Science* **360**, 627 (2018)
- [27] V.F. Ezhov, A.Z. Andreev, G. Ban, B.A. Bazarov, P. Geltenbort, A.G. Glushkov, V.A. Knyazkov, N.A. Kovrizhnykh, G.B. Krygin, O. Naviliat-Cuncic et al., *JETP Lett.* **107**, 671 (2018)
- [28] R.W. Pattie, J. Anaya, H.O. Back, J.G. Boissevain, T.J. Bowles, L.J. Broussard, R. Carr, D.J. Clark, S. Currie, S. Du et al., *Phys. Rev. Lett.* **102**, 012301 (2009)
- [29] M.P. Mendenhall, R.W. Pattie, Y. Bagdasarova, D.B. Berguno, L.J. Broussard, R. Carr, S. Currie, X. Ding, B.W. Filippone, A. García et al. (UCNA Collaboration), *Phys. Rev. C* **87**, 032501 (2013)
- [30] M.A.P. Brown, E.B. Dees, E. Adamek, B. Allgeier, M. Blatnik, T.J. Bowles, L.J. Broussard, R. Carr, S. Clayton, C. Cude-Woods et al. (UCNA Collaboration), *Phys. Rev. C* **97**, 035505 (2018)
- [31] V.I. Luschikov, A.I. Frank, *JETP Lett.* **28**, 559 (1978)
- [32] H. Rauch, S.A. Werner, *Neutron Interferometry: Lessons in Experimental Quantum Mechanics, Wave-Particle Duality, and Entanglement* (Oxford University Press, 2015), ISBN 978-0-19-871251-0
- [33] A.I. Frank, P. Geltenbort, G.V. Kulin, D.V. Kustov, V.G. Nosov, A.N. Strepetov, *JETP Lett.* **84**, 363 (2006)
- [34] A.I. Frank, P. Geltenbort, M. Jentschel, D.V. Kustov, G.V. Kulin, A.N. Strepetov, *JETP Lett.* **93**, 361 (2011)
- [35] K. Durstberger-Rennhofer, T. Jenke, H. Abele, *Phys. Rev. D* **84**, 036004 (2011)
- [36] J. Byrne, P.G. Dawber, C.G. Habeck, S.J. Smidt, J.A. Spain, A.P. Williams, *Europhys. Lett. (EPL)* **33**, 187 (1996)
- [37] M.S. Dewey, D.M. Gilliam, J.S. Nico, F.E. Wietfeldt, X. Fei, W.M. Snow, G.L. Greene, J. Pauwels, R. Eykens, A. Lamberty et al., *Phys. Rev. Lett.* **91**, 152302 (2003)
- [38] J.S. Nico, M.S. Dewey, D.M. Gilliam, F.E. Wietfeldt, X. Fei, W.M. Snow, G.L. Greene, J. Pauwels, R. Eykens, A. Lamberty et al., *Phys. Rev. C* **71**, 055502 (2005)
- [39] A.T. Yue, M.S. Dewey, D.M. Gilliam, G.L. Greene, A.B. Laptev, J.S. Nico, W.M. Snow, F.E. Wietfeldt, *Phys. Rev. Lett.* **111**, 222501 (2013)
- [40] F.E. Wietfeldt, G.L. Greene, *Rev. Mod. Phys.* **83**, 1173 (2011)
- [41] P. Bopp, D. Dubbers, L. Hornig, E. Klemt, J. Last, H. Schütze, S.J. Freedman, O. Schärpf, *Phys. Rev. Lett.* **56**, 919 (1986)
- [42] K. Schreckenbach, P. Liaud, R. Kossakowski, H. Nastoll, A. Bussiere, J.P. Guillaud, *Phys. Lett. B* **349**, 427 (1995)
- [43] H. Abele, S. Baeßler, D. Dubbers, J. Last, U. Mayerhofer, C. Metz, T.M. Müller, V. Nesvizhevsky, C. Raven, O. Schärpf et al., *Phys. Lett. B* **407**, 212 (1997)
- [44] B. Yerozolimsky, I. Kuznetsov, Y. Mostovoy, I. Stepanenko, *Phys. Lett. B* **412**, 240 (1997)
- [45] A.P. Serebrov, I.A. Kuznetsov, I.V. Stepanenko, A.V. Aldushchenkov, M.S. Lasakov, Y.A. Mostovoi,

- B.G. Erozolinskii, M.S. Dewey, F.E. Wietfeldt, O. Zimmer et al., JETP **86**, 1074 (1998)
- [46] H. Abele, M. Astruc Hoffmann, S. Baeßler, D. Dubbers, F. Glück, U. Müller, V. Nesvizhevsky, J. Reich, O. Zimmer, Phys. Rev. Lett. **88**, 211801 (2002)
- [47] T. Soldner, L. Beck, C. Plonka, K. Schreckenbach, O. Zimmer, Phys. Lett. B **581**, 49 (2004)
- [48] M. Kreuz, T. Soldner, S. Baeßler, B. Brand, F. Glück, U. Mayer, D. Mund, V. Nesvizhevsky, A. Petoukhov, C. Plonka et al., Phys. Lett. B **619**, 263 (2005)
- [49] M. Schumann, T. Soldner, M. Deissenroth, F. Glück, J. Krempel, M. Kreuz, B. Märkisch, D. Mund, A. Petoukhov, H. Abele, Phys. Rev. Lett. **99**, 191803 (2007)
- [50] M. Schumann, M. Kreuz, M. Deissenroth, F. Glück, J. Krempel, B. Märkisch, D. Mund, A. Petoukhov, T. Soldner, H. Abele, Phys. Rev. Lett. **100**, 151801 (2008)
- [51] T.E. Chupp, R.L. Cooper, K.P. Coulter, S.J. Freedman, B.K. Fujikawa, A. García, G.L. Jones, H.P. Mumm, J.S. Nico, A.K. Thompson et al., Phys. Rev. C **86**, 035505 (2012)
- [52] A. Kozela, G. Ban, A. Białek, K. Bodek, P. Gorel, K. Kirch, S. Kistryn, O. Naviliat-Cuncic, N. Severijns, E. Stephan et al., Phys. Rev. C **85**, 045501 (2012)
- [53] D. Mund, B. Märkisch, M. Deissenroth, J. Krempel, M. Schumann, H. Abele, A. Petoukhov, T. Soldner, Phys. Rev. Lett. **110**, 172502 (2013)
- [54] G. Darius, W.A. Byron, C.R. DeAngelis, M.T. Hassan, F.E. Wietfeldt, B. Collett, G.L. Jones, M.S. Dewey, M.P. Mendenhall, J.S. Nico et al., Phys. Rev. Lett. **119**, 042502 (2017)
- [55] B. Märkisch, H. Mest, H. Saul, X. Wang, H. Abele, D. Dubbers, M. Klopff, A. Petoukhov, C. Roick, T. Soldner et al., Phys. Rev. Lett. **122**, 242501 (2019)
- [56] M. Baldo-Ceolin, P. Benetti, T. Bitter, F. Bobisut, E. Calligarich, R. Dolfini, D. Dubbers, P. El-Muzeini, M. Genoni, D. Gibin et al., Zeitschrift für Physik C Particles and Fields **63**, 409 (1994)
- [57] A.P. Serebrov, E.B. Aleksandrov, N.A. Dovator, S.P. Dmitriev, A.K. Fomin, P. Geltenbort, A.G. Kharitonov, I.A. Krasnoschekova, M.S. Lasakov, A.N. Murashkin et al., Phys. Lett. B **663**, 181 (2008)
- [58] I. Altarev, C.A. Baker, G. Ban, K. Bodek, M. Daum, P. Fierlinger, P. Geltenbort, K. Green, M.G.D. van der Grinten, E. Gutmiedl et al., Phys. Rev. D **80**, 032003 (2009)
- [59] X. Sun, E. Adamek, B. Allgeier, M. Blatnik, T.J. Bowles, L.J. Broussard, M.A.P. Brown, R. Carr, S. Clayton, C. Cude-Woods et al., Phys. Rev. C **97**, 052501 (2018)
- [60] A. Steyerl, H. Nagel, F.X. Schreiber, K.A. Steinhauser, R. Gähler, W. Gläser, P. Ageron, J. Astruc, W. Drexel, G. Gervais et al., Phys. Lett. A **116**, 347 (1986)
- [61] R. Golub, J.M. Pendlebury, Phys. Lett. A **62**, 337 (1977)
- [62] O. Zimmer, K. Baumann, M. Fertl, B. Franke, S. Mironov, C. Plonka, D. Rich, P. Schmidt-Wellenburg, H.F. Wirth, B. van den Brandt, Phys. Rev. Lett. **99**, 104801 (2007)
- [63] F.M. Piegsa, M. Fertl, S.N. Ivanov, M. Kreuz, K.K.H. Leung, P. Schmidt-Wellenburg, T. Soldner, O. Zimmer, Phys. Rev. C **90**, 015501 (2014)
- [64] A.P. Serebrov, et al., Pis'ma Zh. Eksp. Teor. Fiz. **10**, 764 (1995)
- [65] R. Pattie, LANL-UCN Team Team, *Commissioning of the Upgraded Ultracold Neutron Source at Los Alamos Neutron Science Center*, in *APS Division of Nuclear Physics Meeting* (2016), p. DJ.005
- [66] A. Anghel, F. Atchison, B. Blau, B. van den Brandt, M. Daum, R. Doelling, M. Dubs, P.A. Duperrex, A. Fuchs, D. George et al., Nucl. Instrum. Meth. Phys. Res. A **611**, 272 (2009)
- [67] G. Bison, M. Daum, K. Kirch, B. Lauss, D. Ries, P. Schmidt-Wellenburg, G. Zsigmond, T. Brenner, P. Geltenbort, T. Jenke et al., Phys. Rev. C **95**, 045503 (2017)
- [68] H. Abele, H. Leeb, New J. Phys. **14**, 055010 (2012)
- [69] V.V. Nesvizhevsky, A.Y. Voronin, R. Cubitt, K.V. Protasov, Nat. Phys. **6**, 114 (2010)
- [70] H. Abele, T. Jenke, H. Leeb, J. Schmiedmayer, Phys. Rev. D **81**, 065019 (2010)
- [71] T. Jenke, P. Geltenbort, H. Lemmel, H. Abele, Nat. Phys. **7**, 468 (2011)
- [72] T. Jenke, G. Cronenberg, J. Burgdörfer, L.A. Chizhova, P. Geltenbort, A.N. Ivanov, T. Lauer, T. Lins, S. Rotter, H. Saul et al., Phys. Rev. Lett. **112**, 151105 (2014)
- [73] T. Jenke, et al., EPJ Web of Conf. **219**, 05003 (2019)