

A Multi-period Multi-objective Location-routing Model for Relief Chain Management under Uncertainty

Mohsen Saffarian^{a,*}, Farnaz Barzinpour^b and Seyed Mahmood Kazemi^a

^a Faculty of Industrial Engineering, Birjand University of Technology, Birjand, Southern Khorasan, Iran

^b Department of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

Abstract

Natural disasters, accidents, and crises that cause widespread destruction and inflict heavy casualties accentuate the importance of careful planning to deal with the aftermath and mitigate the impacts responsively. Thus, the logistics of disaster relief is one of the main activities in disaster management. In this paper, the response phase of the disaster management cycle is considered and a multi-objective model for location and routing of vehicles is presented. Uncertainties of transfer time, demands of regional warehouses in the damaged areas and inventories at supply centers in different periods are taken into account. Three objectives are set in this model. Two objectives consist of minimizing the total time required to reach the damaged areas and maximizing satisfaction of the damaged areas. The third objective, which is of secondary importance, is to minimize total costs, including startup, transfer, and shortage costs. In order to convert the proposed multi-objective formulation to a single-objective one, Global Criterion approach is applied. Afterwards, the obtained single objective model is solved using an efficient genetic algorithm and simulated annealing. Finally, a case study in Southern Khorasan is conducted and the applicability of the proposed model is examined.

Keywords: Relief logistics; Location-routing problem; Cumulative vehicle routing; Multi-objective optimization; Uncertainty.

1. Introduction

With respect to the increasing number of disasters, millions of people are affected by natural or man-made disasters every year and the number of victimized people in recent decades has been increased conspicuously. In addition, critical nature of these crises entails keeping response times as short as possible (Thomas and Kopczak, 2005). Therefore, an appropriate planning can play an important role in reducing the impacts of such catastrophes. A number of studies estimate that logistics and supply chain management activities comprise more than 80% of total relief operations (Van Wassenhove, 2006). As literature presents, operational research models are able to successfully support different types of humanitarian operations (Van Wassenhove and Pedarza Martinez, 2010). Logistics can lead to greater coordination in transporting commodities between regional warehouses and damaged areas. Nonetheless, there are conflicting objectives in relief logistics planning, such as minimizing total unsatisfied demands, distribution costs and delay time besides maximizing the level of satisfaction and fairness in distribution of commodities.

Due to the contradictory natures of these objectives, an appropriate integrated planning is necessary to effectively fulfill the requirements of the affected areas. In this regard, determining active regional warehouses and allocating appropriate commodities so that most people obtain their needed goods in the shortest time can yield promising results.

To address these issues, a multi-objective formulation is proposed in this paper and is applied for solving a real-world. these issues, a multi-objective formulation is proposed in this paper and is applied for solving a real-world problem.

The rest of the paper is organized in seven sections as follows. In Section 2, a brief review of research works on relief fuzzy response chain is provided. The proposed mathematical model is formulated in Section 3. Efficient genetic algorithm (GA) and simulated annealing (SA) approaches are developed in Sections 4 and 5, respectively. The investigated case study along with the numerical results are provided in Section 6. Finally, in Section 7 concluding remarks are drawn and a number of future research opportunities are outlined.

2. Literature review

One of the first studies in the field of transportation in relief logistics was performed by Knott (1988). In the mentioned work, a linear programming model was presented to determine the optimal food transportation schedule. Oh and Haghani (1996) provided a model for transporting different supplies such as food, clothing, medicine and rescue teams with some types of vehicles for relief operations. Barbarosoglu et al. (2000) developed a mathematical model to solve the problem of operational and tactical timing for helicopter activity. Barbarosoglu and Arda (2004), presented a model that raised the issue of uncertainty in relief chains and provided a two-step stochastic programming framework for transportation planning in the response phase of the disaster relief chain. Ozdamar et al. (2004) studied emergency logistics planning for delivering commodities to distribution centers in the damaged areas. Nolz et al. (2011) presented a multi-objective model for transportation of commodities in the response phase; their model had three objective functions, which consisted of reducing risk of accountability, cover interval of each vehicle and total travel time. Lin et al. (2011) provided a multi-period, multi-product and multi-vehicle model for logistic planning of major and prioritized commodities in the disaster response phase; the model had two objective functions that one of them was to minimize unsatisfied demand and the other was to minimize total travel time. Moreover, Berkoune et al. (2012) offered a mathematical model for planning the transportation of commodities in the response phase and minimized the time spent for traveling of vehicles. Also, Eshghi and Najafi (2013) presented a bi-objective model in order to reduce the unmet demands along with the number of victims not arrived at hospitals.

Another aspect of routing problems, which is addressed in this study, is periodic routing problems where customer services must be done periodically during a planning horizon. The aim of periodic routing is to determine the motion paths from service centers to customers at each period so that total routing costs incurred throughout the planning horizon is minimized. Periodic routing problem was first proposed in 1974 (Beltrami and Bodin, 1974) while the first mathematical model of the problem was then presented in 1984 (Christofides and Beasley, 1984).

Rath and Gutjahr (2014) presented a three-objective optimization model with a medium-term economic sector, a short-term economic sector, and an accident objective function. To solve the problem, a meta-heuristic scheme based on genetic algorithm was also provided. Ngueveu et al. (2010) introduced a transportation routing model with stacked capacity where the aim was to minimize the total time of vehicle to get applicants. Ke and Feng (2013) presented an innovative two-step method for solving a routing problem with stacked capacity. Ahmadi et al. (2015) developed a multi-depot location-routing model considering network failure, multiple uses of vehicles, and standard relief time. The model was then extended to a two-step stochastic program with random travel time to determine the locations of distribution centers.

Wang et al. (2014) proposed a nonlinear integer open location-routing model for relief distribution problem considering travel time, the total cost, and reliability with split delivery. The non-dominated sorting genetic algorithm and non-dominated sorting differential evolution algorithm were employed to solve the proposed model. Tofighia et al. (2016) considered a two-echelon humanitarian logistics network design problem involving multiple central warehouses (CWs) and local distribution centers (LDCs) and developed a novel two-step scenario-based possibilistic-stochastic programming (SBPSP) approach to cope with the problem. Barzinpour et al. (2014) proposed a multi-objective model for distribution centers which are located and allocated periodically to the damaged areas in order to distribute the offered relief commodities.

USLU et al. (2017) considered a multi-depot vehicle routing problem with stochastic demands and developed a chance constrained mathematical model to cope with the problem. They also conducted a case study for Ankara city in Turkey. JHA et al. (2017) developed a multi-objective model for a humanitarian relief supply chain that included supply of relief goods and an evacuation chain in case of a natural disaster. The objective considered included demand satisfaction in relief chain, demand satisfaction in evacuation chain and overall logistic costs. Golabi et al. (2017) investigated a stochastic facility location problem for a possible earthquake in Tehran where unmanned aerial vehicles (UAV) are utilized. Jabbour et al. (2017) provided a state-of-the-art literature review on humanitarian logistics and supply chain

management. John (2018) conducted a review on empirical studies in the context of humanitarian supply chains based on methodologies considered and drew directions for future researches.

The main focus of this paper is on providing a decision making model in the context of relief logistics where the cost and time to reach the damaged areas must be minimized and the level of service that leads to the satisfaction of affected people is to be maximized. Moreover, the theory of fuzzy sets is used to tackle the prevailing uncertain environment. The reasons for applying fuzzy sets theory is twofold. Firstly, there is no historical data for uncertain parameters in many real problems hence obtaining proper distribution functions is not applicable. Also, using scenario based stochastic programming can lead to computationally challenging problems when a large number of scenarios is taken into consideration (Pishvae and Torabi, 2010). Saffarian et al. (2015) proposed a bi-objective model for relief chain logistics in an uncertain environment while considering uncertainty in both traveling times and demands of the damaged areas.

Thus, the contributions of this work can be summarized as follows:

- A three-echelon multi-period multi-item location-routing model in the context of relief logistics is developed.
- Strategic and tactical decisions regarding establishing warehouses and routing decisions are made simultaneously taking into account inherent uncertainties of important parameters.
- A new multi-objective optimization framework to address three opposing objectives is developed; minimizing delivery times, minimizing total costs and maximizing fairness in distribution of commodities to the affected regions.
- A case study is conducted in Southern Khorasan and the applicability of the proposed model is investigated.

3. The proposed mathematical model

The model description is as follows. A number of candidate points are given as potential sites for establishing temporary warehouses that are to be utilized as suppliers of commodities to the damaged areas. Demand points, on the other hand, are the regional warehouses located in the damaged areas. Due to the inherent uncertainties of the demand values, triangular fuzzy numbers are employed. A finite multi-period planning horizon is considered where the length of each period can be either one day or one week based on the motives and preferences of the user. In each period, a vehicle starts its journey from a central temporary warehouse and returns to the same warehouse after visiting all of the allocated regional warehouses. It must be noted that a vehicle can have several travels per period, provided that travel time restrictions be respected.

Assumptions

The following assumptions and features are included in the developed formulation:

- ✓ A vehicle can have several travels in each period.
- ✓ Every travel starts from central warehouse and returns again to the central warehouse after delivery of commodities to a series of regional warehouses.
- ✓ Several types of vehicles are available.
- ✓ A time limit is put on using each vehicle in each period.
- ✓ Multiple commodities are considered.
- ✓ A limit is imposed on the maximum number of temporary warehouses that can be active in each period.

Indices

i, j : Index of the damaged areas and central temporary warehouses, respectively

l_1 : Index of supplier facilities

t : Index of period

m : Index of vehicles

c : Index of commodities

v : Index of routes

Parameters

\tilde{d}_{ict} : Demand of i^{th} damaged area for commodity c in period of t

$Wcap_{wmt}$: Weight capacity of vehicle m from warehouse w in period t

$Vcap_{wmt}$: Volume capacity of vehicle m from warehouse w in period t

\tilde{W}_{wmijt} : The time interval between two nodes i and j by m^{th} vehicle related to central warehouse w in period t

\tilde{C}_{wmijt} : The cost of transfer between two nodes i and j by m^{th} vehicle related to central warehouse w in period t

\tilde{C}_{i_wct} : The cost of allocating each unit of commodity c from supply center i_i to temporary warehouse w in period t

\tilde{inv}_{ict} : Inventory of commodity c in supply center i_i in period t

LT_{wmt} : Loading time of vehicle m in warehouse w in period t

α_c : Duration of unloading each unit of commodity c

β_{wmct} : If the vehicle m in warehouse w at period t carries commodity c it will be 1, otherwise 0.

nW_t : The maximum number of open warehouses in period t

q_w : The cost of establishing warehouses w

p_c : The cost of each unit of commodity c

h_{wmt} : The allowed time for work of vehicle m from warehouse w in period t

$I = \{k+1, k+2, \dots, k+n\}$: Set of damaged areas

$M_{wt} = \{1, 2, \dots, m_{wt}\}$: Set of vehicles in warehouse w in period t

m_{wt} : Number of vehicles in warehouse w in period t

$C = \{1, 2, \dots, c\}$: Set of commodities

$I_1 = \{1, 2, \dots, i_1\}$: Set of all supply centers

$T = \{1, 2, \dots, t\}$: Set of periods in the planning horizon

$I' = \{1, 2, \dots, k, k+1, \dots, k+n, k+n+1, k+n+2, \dots, k+n+k\}$: Set of all damaged areas and central warehouses

$W = \{1, 2, \dots, k\}$: Set of central warehouses

$Mbig$: Very large number

Decision variables

x_{wimvt} : If the vehicle m that is started from warehouse w , moves from node i to node j in travel v in period t it will be 1, otherwise 0

x'_{i_wct} : The amount of commodity c allocated from central supply i_1 to temporary warehouse w in period t

$time_{wmivt}$: The time for vehicle m to get to damaged area i in travel v in period t from warehouse w .

d'_{wicmvt} : The net demand of damaged area i from commodity c in travel v in period t from warehouse w .

inv'_{ict} : Inventory of commodity surplus c from period t to period $t+1$ in warehouse i

y_{wt} : If the central warehouse w is active in period t it will be 1, otherwise 0

de_{ict} : Unmet demand that is transferred from period t to period $t+1$.

Mathematical formulation

$$\min f_1 = \sum_{t \in T} \sum_{i_1 \in I_1} \sum_{w \in W} \sum_{c \in C} c'_{i_1 w c t} x'_{i_1 w c t} + \tag{1}$$

$$\sum_{t \in T} \sum_{w \in W} \sum_{m \in M_{wt}} \sum_{j \in I' \setminus \{1, 2, \dots, k\}} \sum_{i \in I' \setminus \{n+k+1, \dots, n+2k\}} \sum_{v \in TR} \tilde{c}_{w m j t} x_{w m j v t} +$$

$$\sum_{w \in W} \sum_{t \in T} \max(0, y_{wt} - \sum_{l=1}^{t-1} y_{wl}) q_w + \sum_{t \in T} \sum_{c \in C} \sum_{i \in I} (de_{ict}) \times p_c$$

$$\min f_2 = \sum_{w \in W} \sum_{t \in T} \sum_{v \in TR} \sum_{i \in I} \sum_{m \in M_{wt}} time_{w m i t} \tag{2}$$

$$\max f_3 = \sum_{t \in T} \sum_{c \in C} \min_{i \in I} \left[\frac{\sum_{w \in W} \sum_{m \in M_{wt}} \sum_{v \in TR} d'_{w i c m v t}}{d_{i c t}} \right] \tag{3}$$

$$\sum_{i \in I} x_{w i v m i t} - \sum_{i \in I} x_{w i (k+n+w) m v t} = 0 \quad \forall m \in M_{wt}, t \in T, v \in TR, w \in W \tag{4}$$

$$\sum_{j \in I' \setminus \{i\}} x_{w j m i t} - \sum_{j \in I' \setminus \{i\}} x_{w j m v t} = 0 \quad \forall i \in I, w \in W, m \in M_{wt}, v \in TR, t \in T \tag{5}$$

$$\sum_{i \in I' \setminus \{n+k+1, \dots, n+2k\}} \sum_{j \in I' \setminus \{1, \dots, k\}} x_{w j m v t} \leq \left(\sum_{i \in I} x_{w i v m i t} \right) \times Mbig \quad \forall w \in W, m \in M_{wt}, v \in TR, t \in T \tag{6}$$

$$\sum_{c \in C} \sum_{i \in I} x'_{i_1 w c t} + \sum_{i \in I' \setminus \{n+k+1, \dots, n+2k\}} \sum_{j \in I' \setminus \{1, 2, \dots, k\}} \sum_{m \in M_{wt}} \sum_{v \in TR} x_{w j m v t} \leq Mbig y_{wt} \quad \forall w \in W, t \in T \tag{7}$$

$$\sum_{i \in I' \setminus \{n+k+1, \dots, n+2k\}} \sum_{j \in I' \setminus \{i\}} \sum_{c \in C} w h_c x_{w j m v t} d'_{w j c m v t} \leq w cap_{w m t} \quad \forall w \in W, m \in M_{wt}, t \in T, v \in TR \tag{8}$$

$$\sum_{i \in I' \setminus \{n+k+1, \dots, n+2k\}} \sum_{j \in I' \setminus \{i\}} \sum_{c \in C} v_c x_{w j m v t} d'_{w j c m v t} \leq v cap_{w m t} \quad \forall w \in W, m \in M_{wt}, t \in T, v \in TR \tag{9}$$

$$time_{w m (n+k+w)(v-1)t} + LT_{w m t} + \tilde{w}_{w m v j t} + \sum_{c \in C} \alpha_c d'_{w j c m v t} - (1 - x_{w i v j m t}) Mbig \leq time_{w m j v t} \tag{10}$$

$$\forall w \in W, j \in I' \setminus \{1, \dots, k\}, m \in M_{wt}, v \in TR, t \in T$$

$$time_{w m i t} + \tilde{w}_{w m j i t} + \sum_{c \in C} \alpha_c d'_{w j c m v t} - (1 - x_{w i v j m t}) Mbig \leq time_{w m j v t} \tag{11}$$

$$\forall w \in W, i \in I' \setminus \{n+k+1, \dots, n+2k\}, j \in I' \setminus \{1, \dots, k\}, m \in M_{wt}, v \in TR, t \in T$$

$$time_{j-n-k m j v t} \leq h_{(j-n-k) m t} \quad v \in TR, t \in T, j \in \{n+k+1, \dots, n+2k\}, m \in M_{(j-n-k)t} \tag{12}$$

$$inv'_{w c t} = \sum_{i_1 \in I_1} x'_{i_1 w c t} + inv'_{w c (t-1)} y_{w(t-1)} - \sum_{i \in I} \sum_{m \in M_{wt}} \sum_{v \in TR} d'_{w i c m v t} \quad \forall c \in C, t \in T, w \in W \tag{13}$$

$$de_{i c t} = \tilde{d}_{i c t} + de_{i c t-1} - \sum_{w \in W} \sum_{m \in M_{wt}} \sum_{v \in TR} d'_{w i c m v t} \quad \forall i \in I, c \in C, t \in T \tag{14}$$

$$d'_{w j c m v t} \leq \tilde{d}_{j c t} \sum_{i \in I' \setminus \{n+k+1, \dots, n+2k\}} x_{w j m i t} \quad \forall w \in W, j \in I, c \in C, m \in M_{wt}, v \in TR, t \in T \tag{15}$$

$$\sum_{w \in W} x'_{i_1 w c t} \leq \tilde{inv}_{i c t} \quad \forall t \in T, c \in C, i_1 \in I_1 \tag{16}$$

$$\sum_{j \in I} x_{w j m i t} \leq 1 \quad \forall m \in M_{wt}, w \in W, v \in TR, t \in T \tag{17}$$

$$Time_{vmnvt} \geq Time_{vm(n+k+w)(v-1)t} \quad \forall m \in M_{vt}, w \in W, v \in TR, t \in T \quad (18)$$

$$Time_{vmnvt} \leq Time_{vm(n+k+w)t} \quad \forall m \in M_{vt}, w \in W, v \in TR, t \in T \quad (19)$$

$$\sum_{t \in T} \sum_{v \in TR} \sum_{w \in W} \sum_{m \in M_{vt}} \sum_{i \in I} \sum_{j \in I} x_{wijmvt} = 0 \quad (20)$$

$$\sum_{t \in T} \sum_{v \in TR} \sum_{w \in W} \sum_{m \in M_{vt}} \sum_{i \in I} \sum_{j \in \{n+k+1, \dots, n+2k\}, j \neq n+k+w} x_{wijmvt} = 0 \quad (21)$$

$$\sum_{w \in W} y_{wt} \leq nwt \quad \forall t \in T \quad (22)$$

$$x_{wijmvt} \in \{0, 1\} \quad \forall w \in W, i \in I', j \in I', m \in M_{vt}, v \in TR, t \in T \quad (23)$$

$$\sum_{i \in I} \sum_{v \in TR} d'_{vicmvt} \leq Mbig \times \beta_{vmct} \quad \forall v \in W, c \in C, m \in M_{vt}, t \in T \quad (24)$$

$$in'_{jt} \geq 0 \ \&integer \quad \forall j \in \{1, \dots, k\}, c \in C, t \in T \quad (25)$$

$$de_{ia} \geq 0 \ \&integer \quad \forall i \in I, c \in C, t \in T \quad (26)$$

$$d'_{vicmvt} \geq 0 \ \&integer \quad \forall w \in W, i \in I, c \in C, m \in M_{vt}, v \in TR, t \in T \quad (27)$$

$$y_{wt} \in \{0, 1\} \quad \forall w \in W, t \in T \quad (28)$$

$$time_{vmnvt} \geq 0 \quad \forall w \in W, j \in I, m \in M_{vt}, v \in TR, t \in T \quad (29)$$

$$x'_{i_1wct} \geq 0 \quad \forall v \in W, t \in T, c \in C, i_1 \in I_1 \quad (30)$$

The proposed mathematical model includes three objectives. Objective function (1) minimizes total costs, including establishing cost of central temporary warehouses, cost of transferring commodities and shortage costs. Objective function (2) minimizes total delivery times. Objective function (3) considers fairness in distribution of commodities to the affected areas by maximizing the minimum of fairness ratio, which is computed through dividing the amount of commodities delivered to each area by its total demand. Objective functions (2) and (3) are incorporated into the model to expedite relief activities and promote equity in the distribution of commodities.

Constraint (4) guarantees that each tour is started from a warehouse and is ended at the same warehouse. Constraint (5) states that when a vehicle enters to an area it must exit there, i.e. discontinuity of tours is not permitted. Constraint (6) states that each area can be visited only by the vehicles belong to the associated warehouse. Constraint (7) implies that transportation of commodities in each period can be only routed via temporary central warehouses that are active in the same period. Constraints (8) and (9) impose limitations on weight and volume capacities of vehicles. Constraints (10) and (11) show the relationship between arrival times to demand points correspond to the same tour and prevent the creation of sub-tours. Constraint (12) restricts the usage of each vehicle in each period and implies that a vehicle must not be used more than its authorized operation time. Constraint (13) indicates the inventory balance of temporary distribution centers for each period.

Constraint (14) determines the amount of unmet demands of each damaged area in each period. Constraint (15) indicates that damaged area j is visited by vehicle m in period t in travel v if vehicle m in travel v enters the area j in period t . Constraint (16) states that the amount of commodities distributed from each supply center in each period is limited to its inventory size. Constraint (17) states that some vehicles can be left unused, i.e. it is not mandatory to use all vehicles. Constraint (18) and (19) represent the continuity of arrival times. Constraint (20) implies that a vehicle starts its journey from its corresponding warehouse. Constraint (21) says, if a vehicle on a travel in a period ends to $n + k + w$ warehouse, it must have begun its tour from warehouse w . Constraint (22) represents the maximum number of temporary central warehouses that can be activate in period t . Constraint (23) ensures that each vehicle carries only commodities that are allowed to transfer. Constraints (24) to (30) define the limits of decision variables.

The first objective function of the model includes triangular fuzzy coefficients. Therefore, it can be converted to the following three objective functions:

$$\min f_{11} = \sum_{t \in T} \sum_{i \in I} \sum_{w \in W} \sum_{c \in C} (c_{i_wct}^u - c_{i_wct}^m) x'_{i_wct} + \sum_{t \in T} \sum_{w \in W} \sum_{m \in M_w} \sum_{j \in I \setminus \{1, 2, \dots, k\}} \sum_{i \in I \setminus \{n+k+1, \dots, n+2k\}} \sum_{v \in TR} (c_{wmijvt}^u - c_{wmijvt}^m) x_{wmijvt} + \quad (31)$$

$$\sum_{w \in W} \sum_{t \in T} \max(0, y_{wt} - \sum_{l=1}^{t-1} y_{wl}) q_w + \sum_{t \in T} \sum_{c \in C} \sum_{i \in I} (de_{ict}) \times p_c$$

$$\min f_{12} = \sum_{t \in T} \sum_{i \in I} \sum_{w \in W} \sum_{c \in C} c_{i_wct}^m x'_{i_wct} + \sum_{t \in T} \sum_{w \in W} \sum_{m \in M_w} \sum_{j \in I \setminus \{1, 2, \dots, k\}} \sum_{i \in I \setminus \{n+k+1, \dots, n+2k\}} \sum_{v \in TR} c_{wmijvt}^m x_{wmijvt} + \quad (32)$$

$$\sum_{w \in W} \sum_{t \in T} \max(0, y_{wt} - \sum_{l=1}^{t-1} y_{wl}) q_w + \sum_{t \in T} \sum_{c \in C} \sum_{i \in I} (de_{ict}) \times p_c$$

$$\max f_{13} = \sum_{t \in T} \sum_{i \in I} \sum_{w \in W} \sum_{c \in C} (c_{i_wct}^m - c_{i_wct}^l) x'_{i_wct} + \sum_{t \in T} \sum_{w \in W} \sum_{m \in M_w} \sum_{j \in I \setminus \{1, 2, \dots, k\}} \sum_{i \in I \setminus \{n+k+1, \dots, n+2k\}} \sum_{v \in TR} (c_{wmijvt}^m - c_{wmijvt}^l) x_{wmijvt} + \quad (33)$$

$$\sum_{w \in W} \sum_{t \in T} \max(0, y_{wt} - \sum_{l=1}^{t-1} y_{wl}) q_w + \sum_{t \in T} \sum_{c \in C} \sum_{i \in I} (de_{ict}) \times p_c$$

Since objective function (3) is nonlinear, it should be linearized before defuzzification. Constraints (10), (11), (14), (15) and (16) are also fuzzy numbers hence requiring a defuzzification procedure. In this paper, the central region method is used as the defuzzification procedure (Jabal-Ameli et al., 2011).

$$\max f_3 = \sum_{t \in T} \sum_{c \in C} y_{tc} \quad (34)$$

$$y_{tc} \leq \frac{\sum_{w \in W} \sum_{m \in M} \sum_{v \in TR} d'_{wicmvt}}{\tilde{d}_{ict}} \quad \forall i \in I, t \in T, c \in C \quad (35)$$

$$y_{tc} \leq \frac{\sum_{w \in W} \sum_{m \in M} \sum_{v \in TR} d'_{wicmvt}}{\frac{(d_{jct}^u - d_{jct}^l) + (d_{jct}^m - d_{jct}^l)}{3} + d_{jct}^l} \quad \forall i \in I, t \in T, c \in C \quad (36)$$

$$time_{wm(n+k+w)(v-1)t} + LT_{wmt} + \frac{(w_{wmwjt}^u - w_{wmwjt}^l) + (w_{wmwjt}^m - w_{wmwjt}^l)}{3} + w_{wmwjt}^l \quad (37)$$

$$+ \sum_{c \in C} \alpha_c d'_{wjcmt} - (1 - x_{wjmvt}) Mbig \leq time_{wmijvt} \quad \forall w \in W, j \in I' \setminus \{1, \dots, k\}, m \in M_w, v \in TR, t \in T \quad (38)$$

$$time_{wmivt} + \frac{(w_{wmijvt}^u - w_{wmijvt}^l) + (w_{wmijvt}^m - w_{wmijvt}^l)}{3} + w_{wmijvt}^l + \sum_{c \in C} \alpha_c d'_{wjcmt} +$$

$$(1 - x_{wjmvt}) Mbig \leq ti_{wmijvt} \quad \forall w \in W, i \in I' \setminus \{n+k+1, \dots, n+2k\}, j \in I' \setminus \{1, \dots, k\}, m \in M_w, v \in TR, t \in T \quad (39)$$

$$de_{ict} = \frac{(d_{jct}^u - d_{jct}^l) + (d_{jct}^m - d_{jct}^l)}{3} + d_{jct}^l + de_{ict-1} - \sum_{w \in W} \sum_{m \in M} \sum_{v \in TR} d'_{wicmvt} \quad \forall i \in I, c \in C, t \in T \quad (39)$$

$$d'_{wjcmt} \leq \left[\frac{(d_{jct}^u - d_{jct}^l) + (d_{jct}^m - d_{jct}^l)}{3} + d_{jct}^l \right] \sum_{i \in I' \setminus \{n+1\}} x_{ijmvt} \quad \forall w \in W, j \in I, c \in C, m \in M_w, v \in TR, t \in T \quad (40)$$

$$\sum_{w \in W} x'_{i_wct} \leq \frac{(inv_{i_wct}^u - inv_{i_wct}^l) + (inv_{i_wct}^m - inv_{i_wct}^l)}{3} + inv_{i_wct}^l \quad \forall t \in T, c \in C, i_1 \in I_1 \quad (41)$$

In constraints (8) and (9) the term $x_{wijmvt}d'_{wjcmt}$ is nonlinear. To make it linear, it is replaced by an alternative variable dy_{wijcmt} while the following three constraints are added.

$$dy_{wijcmt} \leq d'_{wjcmt} \tag{42}$$

$$\forall t \in T, w \in W, m \in M_w, i \in I \setminus \{k+n+1, \dots, k+n+k\}, j \in I \setminus \{1, \dots, k\}, c \in C$$

$$dy_{wijcmt} \leq Mbig \cdot x_{wijmvt} \tag{43}$$

$$\forall t \in T, w \in W, m \in M_w, i \in I \setminus \{k+n+1, \dots, k+n+k\}, j \in I \setminus \{1, \dots, k\}, c \in C$$

$$dy_{wijcmt} \geq d'_{wjcmt} - Mbig(1 - x_{wijmvt}) \tag{44}$$

$$\forall t \in T, w \in W, m \in M_w, i \in I \setminus \{k+n+1, \dots, k+n+k\}, j \in I \setminus \{1, \dots, k\}, c \in C$$

Constraint (13) is also linearized as mentioned above. Thus, a multi-objective linear programming model is obtained as below.

MOLP

$$\min Z = (f_{11}, f_{12}, -f_{13}, f_2, -f_3) \tag{45}$$

$$\text{s.t } (4)-(9), (12), (13), (17)-(30), (36)-(44)$$

3.1 Global Criterion method for solving multi-objective problems

Consider the following multi-objective model. Each objective function is considered independently and its optimal solution subjected to the given constraints are obtained. Suppose that $f_i^*(x)$ is the optimal value of i^{th} independent objective function. According to Global Criterion method, the following combined objective function is developed:

$$\min F(x) = \left[\sum_{i=1}^n w_i \left| \frac{f_i^*(x) - f_i(x)}{f_i^*(x)} \right|^r \right]^{\frac{1}{r}} \tag{46}$$

$$g_i(x) \geq 0 \tag{47}$$

In the above model, w_i shows the importance of i^{th} objective function which is determined by the decision maker. Instead of solving the original multi-objective model, the above single-objective formulation can be used (Rao, 1996).

Comprehensive measure method which is a suitable approach for solving multi-objective models is used to solve the designed model with three objectives of different importance. Since the proposed model developed in this section is NP-hard in nature, two metaheuristic algorithms are proposed.

4. The proposed genetic algorithm

The proposed genetic algorithm (GA) is elaborated in this section. The performance of genetic algorithm depends on its solution representation, crossover and mutation operators, sampling mechanism, elitism strategy and termination criterion. These salient components are delineated in detail in the following subsections.

4.1 Definition of chromosomes

Five types of matrices with integer values have been used to display chromosomes. The first matrix is Location matrix which is defined as follows:

$$Location = \begin{bmatrix} L_{11} & L_{12} & \dots & L_{1W} \\ L_{21} & L_{22} & \dots & L_{2W} \\ \vdots & \vdots & \ddots & \vdots \\ L_{T1} & L_{T2} & \dots & L_{TW} \end{bmatrix}_{|T| \times |W|}$$

Rows and columns correspond to periods and distribution centers, respectively. When an element is one, it means that the distribution center is active during that period while zero indicates that the related distribution center is inactive.

The second matrix is Deli_depot matrix that corresponds to each commodity. A composite matrix consists of |T| sub-matrices with dimensions of $|I_1| \times |W|$. Each element of these sub-matrices shows the amount of commodity that is sent from the corresponding temporary warehouse. Start with allocating commodities from supply centers to active temporary warehouses in order to initialize this matrix.

$$Deli_depot_c = \left[\begin{bmatrix} i_{11} & i_{12} & \dots & i_{1|W|} \\ i_{21} & i_{22} & \dots & i_{2|W|} \\ \dots & \dots & \dots & \dots \\ i_{|I_1|1} & i_{|I_1|2} & \dots & i_{|I_1||W|} \end{bmatrix}^1 \dots \begin{bmatrix} i_{11} & i_{12} & \dots & i_{1|W|} \\ i_{21} & i_{22} & \dots & i_{2|W|} \\ \dots & \dots & \dots & \dots \\ i_{|I_1|1} & i_{|I_1|2} & \dots & i_{|I_1||W|} \end{bmatrix}^{|T|} \right]$$

The third matrix is a compound matrix for each distribution center. For example, we consider a Tour_1 compound matrix for distribution center where for each period M_1 we have a sub-matrix that each of them is associated with a vehicle and each row is related to a travel and elements of row determine the path of vehicle in travel.

$$M_1 = \max_{t \in T} \{ |M_{1t}| \}$$

$$Tour_W = \left[\begin{bmatrix} i_{11} & i_{12} & \dots & i_{1|I|} \\ i_{21} & i_{22} & \dots & i_{2|I|} \\ \vdots & \vdots & \ddots & \vdots \\ i_{v1} & i_{v2} & \dots & i_{v|I|} \end{bmatrix}^1, \dots, \begin{bmatrix} i_{11} & i_{12} & \dots & i_{1|I|} \\ i_{21} & i_{22} & \dots & i_{2|I|} \\ \vdots & \vdots & \ddots & \vdots \\ i_{v1} & i_{v2} & \dots & i_{v|I|} \end{bmatrix}^{M_w} \dots \begin{bmatrix} i_{11} & i_{12} & \dots & i_{1|I|} \\ i_{21} & i_{22} & \dots & i_{2|I|} \\ \vdots & \vdots & \ddots & \vdots \\ i_{v1} & i_{v2} & \dots & i_{v|I|} \end{bmatrix}^1, \dots, \begin{bmatrix} i_{11} & i_{12} & \dots & i_{1|I|} \\ i_{21} & i_{22} & \dots & i_{2|I|} \\ \vdots & \vdots & \ddots & \vdots \\ i_{v1} & i_{v2} & \dots & i_{v|I|} \end{bmatrix}^{M_w} \right]^{|T|}$$

We consider C compound matrices for each of the above matrices; for example, we consider the following matrices corresponding to matrix Tour_1 where each one correspond to a particular commodity.

Deli_demand_1_C

$$= \left[\begin{bmatrix} d_{11} & d_{12} & \dots & d_{1|I|} \\ d_{21} & d_{22} & \dots & d_{2|I|} \\ \vdots & \vdots & \ddots & \vdots \\ d_{v1} & d_{v2} & \dots & d_{v|I|} \end{bmatrix}^1, \dots, \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1|I|} \\ d_{21} & d_{22} & \dots & d_{2|I|} \\ \vdots & \vdots & \ddots & \vdots \\ d_{v1} & d_{v2} & \dots & d_{v|I|} \end{bmatrix}^{M_1} \dots \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1|I|} \\ d_{21} & d_{22} & \dots & d_{2|I|} \\ \vdots & \vdots & \ddots & \vdots \\ d_{v1} & d_{v2} & \dots & d_{v|I|} \end{bmatrix}^1, \dots, \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1|I|} \\ d_{21} & d_{22} & \dots & d_{2|I|} \\ \vdots & \vdots & \ddots & \vdots \\ d_{v1} & d_{v2} & \dots & d_{v|I|} \end{bmatrix}^{M_1} \right]^{|T|}$$

Each matrix shows total volume of the corresponding commodities transported from the related distribution center in the related period. For instance, deli_demand_1_1 matrix shows commodities type 1 that are carried from the first distribution center by different vehicles in different trips and in the related period.

4.2 Initial population

According to the maximum number of active distribution centers, a number of distribution centers are randomly activated and Location matrix is formed. Subsequently, the Roulette Wheel logic will be used to generate Deli_depot matrices; this means that in each period, active distribution centers are identified based on Location matrix and a vector with dimensions of supply centers for each active distribution center w and each commodity c in period t.

$$\left[\frac{\sum_{i \in I_1} \tilde{c}'_{1ciwt}}{\tilde{c}_{1c1wt}}, \frac{\sum_{i \in I_1} \tilde{c}'_{1ciwt}}{\tilde{c}_{1c2wt}}, \dots, \frac{\sum_{i \in I_1} \tilde{c}'_{1ciwt}}{\tilde{c}_{1d_{|I|}wt}} \right]$$

Naturally, supply centers with lower allocation costs are more likely to be selected. Then, a sequence with a length equal to supply centers in that period is produced by roulette wheel which is formed based on the above vector to activate distribution center w in period t that are chosen randomly and consider it $List_{depot} = (d_1, d_2, \dots, d_{|I|})$. Now, for generating wth column under the tth matrix of Deli_depot_c matrix, the received commodity C of distribution center w from supply center d₁ is obtained as a random value between $D_{d_1wt} = [0, \tilde{inv}_{d_1ct}]$ and then inventory of supply center

is updated. Afterwards, the received commodity C of distribution center w from supply center d_2 is obtained as a random value between $D_{d_2wct} = \left[0, \min(i\tilde{v}_{d_2ct}, D_{d_2wct}) \right]$ and then inventory of supply center d_2 is updated. The above procedure will be continued to determine all amounts of w^{th} column under the t^{th} matrix of Deli_depot_c . This process is repeated until all Deli_depot matrices are initialized. This method enables the distribution centers to provide more inventories from supply centers.

In order to initialize tour_w and Deli_demand_w_c matrices the following instruction is followed. Firstly, given that commodity distribution must be fair, the commodity coverage for different areas are determined. To do so, the amount of available commodities and total demands in each period must be determined. As a result, the level of coverage is calculated as follows.

$$\frac{\sum_{i \in I} d_{i11}}{\sum_{i \in I_1} \sum_{w \in W} x'_{iw11}}$$

For each distribution center, a list of the damaged areas based on their distances from the distribution center is formed such that the nearest one is located on top of the list and the others are sorted accordingly.

Sub-matrices related to tour_w and Deli_demand_w_c matrices are generated as follows. Given a commodity and an arbitrarily chosen active distribution center, a vehicle from distribution center is randomly selected and a roulette wheel based on the list of damaged areas is created. Assume that there are four relevant areas and the obtained list is $[2, 4, 3, 1]$. The intended roulette wheel is divided into 10 equal parts from which four parts belong to 2, three parts belong to 4, two parts are assigned to 1 and the remaining part belongs to 3.

Afterwards, a region is randomly selected using the designed roulette wheel, a vehicle goes there and delivers the maximum demand coverage. Subsequently, the remaining demand is updated in order to determine the extent to which the commodities are to be delivered to that area. This update takes place according to the demand of that area, fairness requirements in the distribution of commodities and inventory capacity of the vehicle. If the vehicle still carries a load of commodities, a new list of the remaining areas is created, the roulette wheel is accordingly updated and the next area is randomly selected. Again, according to the regional demand and inventory capacity of the vehicle the amount of delivery rates is determined. This process is repeated until all areas are visited, the vehicle is fully unloaded or the available time is over. Under any of these conditions the vehicle returns to the distribution center. When the vehicle is returned, the remaining demand of the damaged areas is computed again. In case of existing unmet demands, a vehicle is again selected randomly and all the above-mentioned steps are repeated until either the distribution center runs out of inventory or there are no unmet demands. When the demands of all areas are met, all elements of two tour_w and Deli_demand_w_c matrices are identified.

4.3 Genetic Operators

The crossover and mutation operators of the proposed genetic algorithm are explained in the following.

4.3.1 Crossover operator

In the crossover operator, a temporary warehouse is randomly replaced with another temporary warehouse in the same period. In other words, a random number is considered as elective period from interval $[1 T]$ and is replaced with the row corresponding to that period in Location matrix of two responses. Also, the values corresponding selected period are replaced in Deli_demand_w_c , Deli_depot and Tour_w matrixes and then updated.

4.3.2 Mutation operators

Four different modes are considered for the mutation operator of the genetic algorithm. In the first mode, mutation operator is implemented on the central core of Location matrix. It considers two temporary warehouses in each period and changes their corresponding elements. As a result, Tour_w , Deli_depot_c and Deli_demand_w_c matrices are re-initialized considering supply constraints in the supply centers and demand rate of the damaged areas. In the second mutation operator, a temporary warehouse is considered in each period and its corresponding value becomes one if it is zero and vice versa. Since the establishment conditions of distribution centers are changed, Tour_w , Deli_depot_c and Deli_demand_w_c matrices become updated according to mentioned method. In the third mutation operator, without changing location matrix, a vehicle is considered in Tour_w matrix, the two different routes related to the vehicle are changed and the corresponding data of Deli_depot_c and Deli_demand_w_c matrices are swapped simultaneously. In the fourth mutation operator, keeping the location matrix unchanged, a vehicle is considered in Tour_w matrix, two areas in the vehicle path are interchanged and the corresponding data of Deli_depot_c and Deli_demand_w_c matrices are transferred at the same time.

5. The proposed simulated annealing

Simulated annealing performance is affected by its principal factors including initial temperature, final temperature, the number of iterations at each temperature and cooling rate. The algorithm mechanism is elaborated in the following.

- Solution representation: solution representation for the proposed SA is the same as that of GA.
- Neighborhood search mechanism: To produce neighborhoods, four methods based on the mutation operators introduced for the proposed GA are employed.

Table 1. Values of the genetic algorithm factors

Problem size	Levels	Population size (P_s)	Number of generation (N_g)	Crossover rate (C_r)	Mutation rate (M_r)
Small	1	50	1000	0.7	0.1
	2	75	1250	0.8	0.05
	3	100	1500	0.9	0.01
large	1	150	2000	0.7	0.1
	2	175	2500	0.8	0.05
	3	200	3000	0.9	0.1

Table 2. Values of the simulated annealing factors

Problem size	Levels	A	B	C	D
		Cooling rate (α)	Number of repetition in each temperature (N)	Final temperature (T_f)	Initial temperature (T_0)
Small	1	0.98	80	0.000001	25000
	2	0.99	100	0.00001	40000
	3	0.995	120	0.0005	50000
Large	1	0.99	120	0.000001	35000
	2	0.995	150	0.00001	60000
	3	0.999	200	0.0005	85000

Different values and levels of parameters related to the developed genetic algorithm and simulation annealing are presented in Tables 1 and 2. It is worth noting that Taguchi method is employed in order to determine efficient parameter values for the two algorithms. To do so, a table of orthogonal arrays, L9 mode in Table 3 is selected. Two issues with small and large sizes are considered and genetic algorithm and simulated annealing was separately executed 10 times on each issue.

Table 3. L9 orthogonal array

Experiments	A	B	C	D	Experiments	A	B	C	D
1	A (1)	B (1)	C (1)	D (1)	6	A (2)	B (3)	C (1)	D (2)
2	A (1)	B (2)	C (2)	D (2)	7	A (3)	B (1)	C (3)	D (2)
3	A (1)	B (3)	C (3)	D (3)	8	A (3)	B (2)	C (1)	D (3)
4	A (2)	B (1)	C (2)	D (3)	9	A (3)	B (3)	C (2)	D (1)
5	A (2)	B (2)	C (3)	D (1)					

Table 4. Characteristics of small sample problems

Instance Name	Number of periods	Number of Supply center	Number of damaged area	Maximum number of distribution centers in periods	Candidate area for establish distribution center	Number of vehicle in distribution centers
I1	3	(1,1,1)	(7,7,7)	(1,1,1)	(3,4,4)	(1,1,1) (1,1,1) (1,1,1)
I2	3	(1,1,1)	(9,9,9)	(1,2,2)	(3,4,4)	(1,1,1) (1,1,1) (1,1,1)
I3	3	(3,3,3)	(9,9,9)	(3,4,4)	(5,5,5)	(1,1,2,2,2) (1,2,2,2,2) (1,2,2,2,2)

Table 4. Continued

Instance Name	Number of periods	Number of Supply center	Number of damaged area	Maximum number of distribution centers in periods	Candidate area for establish distribution center	Number of vehicle in distribution centers
I4	3	(3,3,3)	(9,9,10)	(3,4,4)	(6,6,8)	(1,1,1,2,2,2) (1,1,2,2,2,2) (1,1,2,2,2,2,2)
I5	3	(3,3,3)	(10,10,12)	(4,4,4)	(8,8,8)	(1,1,2,2,2,2,2,2) (1,2,2,2,2,2,2,2) (2,2,2,2,2,2,2,2)
I6	3	(3,3,3)	(12,12,12)	(4,4,5)	(8,10,10)	(2,2,2,2,2,2,2,2) (1,1,2,2,2,2,2,2,2,2) (1,2,2,2,2,2,2,2,2,2)
I7	5	(1,1,2,2,2)	(5,5,7,7,7)	(1,1,2,2,2)	(4,4,5,5,5)	(1,1,1,1,1) (1,1,1,1,1) (1,1,1,1,1,1) (1,1,1,1,1,2) (1,1,1,1,1,2)
I8	5	(1,1,2,2,2)	(7,7,9,9,9)	(2,2,3,3,3)	(5,5,5,6,6)	(1,1,1,1,1,2) (1,1,1,1,1,2) (1,1,1,1,1,2) (2,1,1,1,1,2) (2,1,1,1,1,2)

Table 5. Characteristics of large sample problems

Instance Name	Number of periods	Number of Supply center	Number of damaged area	Maximum number of distribution centers in periods	Candidate area for establish distribution center
I9	5	(5,5,5,6,6)	(30,30,30,35,35)	(4,4,4,5,5)	(12,12,12,14,14)
I10	5	(5,5,5,6,6)	(30,30,35,35,35)	(4,4,4,5,5)	(12,12,12,14,14)
I11	5	(5,5,6,6,6)	(35,35,35,40,40)	(5,5,6,6,6)	(12,12,14,14,14)
I12	5	(6,6,8,8,8)	(40,40,40,35,35)	(6,6,6,6,6)	(12,12,14,14,14)
I13	5	(6,6,8,8,8)	(45,45,45,40,40)	(6,6,8,8,8)	(14,14,14,16,16)
I14	5	(8,8,8,8,8)	(40,40,40,45,45)	(6,6,8,8,8)	(16,16,16,18,18)
I15	5	(8,8,10,10,10)	(50,50,50,45,45)	(8,8,10,10,10)	(20,20,20,25,25)
I16	3	(8,10,10)	(50,50,50)	(10,10,10)	(25,25,30)
I17	3	(8,10,10)	(50,60,60)	(10,10,12)	(25,25,30)
I18	3	(10,10,10)	(60,70,70)	(10,12,12)	(25,30,30)
I19	3	(10,10,10)	(70,70,80)	(12,12,12)	(25,30,30)
I20	3	(10,10,10)	(80,80,80)	(12,12,12)	(30,35,35)
I21	3	(10,12,12)	(80,90,90)	(12,12,12)	(30,35,35)
I22	3	(10,12,12)	(90,90,90)	(12,14,14)	(35,35,40)
I23	3	(10,12,12)	(90,100,100)	(12,14,14)	(35,35,40)
I24	3	(12,12,12)	(100,100,100)	(12,14,14)	(40,40,40)
I25	3	(12,12,12)	(100,120,120)	(12,14,14)	(40,40,40)

After determining the appropriate parameters for the proposed GA and SA, different small size instances are generated according to Table 4. Subsequently, these instances were solved by Lingo 11 software and the proposed GA and SA and the obtained comparative results are provided in Table (7). Investigating the obtained results, it is seen that both algorithms produce satisfactory outcomes in reasonable processing times.

As stated before, Taguchi method was used to determine the optimal levels for different factors of both GA and SA. To do so, S/N values for different levels of each factor are computed. The graphs of mean rate S/N and mean objective function of the proposed GA for the small size of problem are shown in figures (1) and (2), respectively. The obtained optimal values of GA and SA parameters, extracted from Taguchi method, are shown in Table 6. Results indicate that in seven cases, GA outperformed SA in terms of average solutions while from the best solution view point, both algorithms were able to reach the optimal solution each in one case. The mean standard deviation for the genetic algorithm is less than that of the SA which indicates that the genetic algorithm solutions are less dispersed compared to those of the simulated annealing algorithm. Also, Genetic algorithm shows higher performance in terms of processing times.

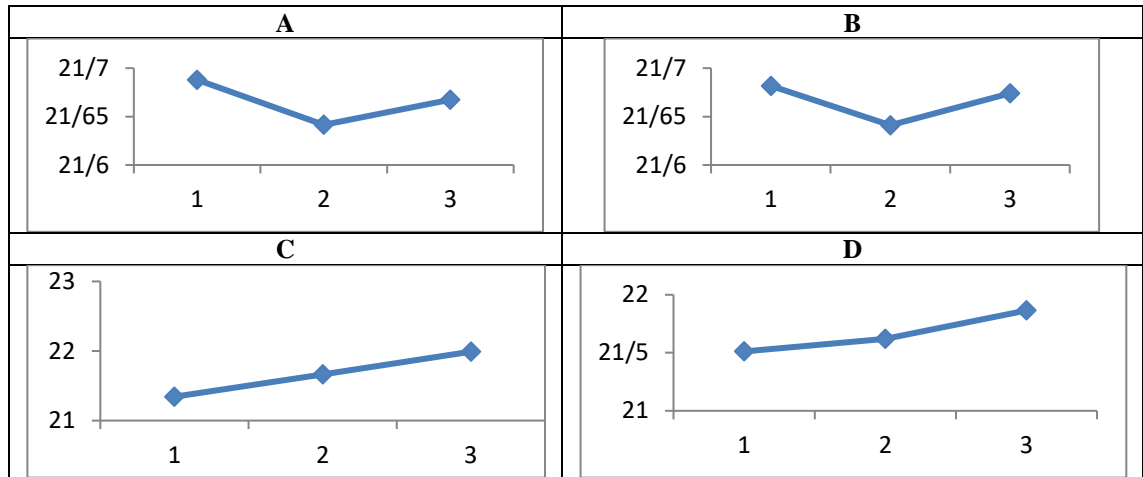


Figure 1. The mean rate of S/ N of the GA for small size problem

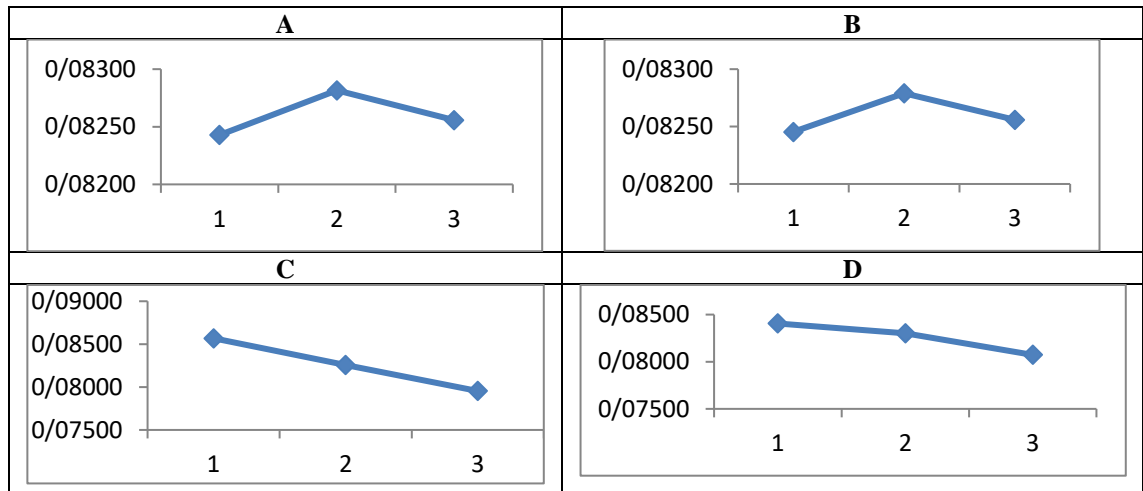


Figure 2. The mean objective function of the GA for small size problem

Table 6. Optimal values of factors for the designed algorithms

Problem size	A	B	C	D
Small	0.8	0.02	120	8000
Large	0.9	0.02	200	12000
Small	0.97	80	0.00001	5500
Large	0.99	120	0.0001	8500

Characteristics of large size problems are provided in Table (5). Since it takes a long time to solve such kinds of problems by Lingo 11 software, they were only solved by the proposed GA and SA and the corresponding results are given in Table (8). Considering the data presented in Table (8), it is observed that in all cases GA is better than SA in terms of average solutions while simulated annealing provided better solutions in 3 of 17 cases.

PRBS% and PRAS% indicators are used to compare the two algorithms which are obtained using the following equations.

$$PRBS\% = \frac{SA_{Best} - GA_{Best}}{SA_{Best}} \times 100\%$$

$$PRAS\% = \frac{SA_{Average} - GA_{Average}}{SA_{Average}} \times 100\%$$

With respect to PRBS% and PRAS% indicators, genetic algorithm shows higher performance compared to its rival.

Table 7. Comparison of genetic algorithm, simulated annealing and Lingo outcomes for problems of small size

Problem NO.	LINGO		GA in 10 iteration					
	Exact solution	Time	Average solution	Average time	Standard deviation	Best solution	The best solution error	Error percent average
1	0.06791	67327	0.068061	645	0.000106	0.06791	0	0.22
2	0.07708	117639	0.077845	697	0.000933	0.07708	0	0.99
3	0.02802	138794	0.029012	727	0.000659	0.02821	0.678	3.54
4	0.04192	164368	0.043332	791	0.001243	0.04219	0.64	3.37
5	0.10127	254737	0.102971	835	0.002201	0.10127	0	1.68
6	0.06181	342768	0.064156	897	0.002671	0.06189	0.13	3.78
7	0.10778	489457	0.112585	974	0.004215	0.10778	0	4.4
8	0.082	653784	0.08625	1073	0.003269	0.0825	0.61	5.51
		278609		830	0.001912		0.26	2.9

Problem NO.	SA in 10 iteration						PRAS%	PRBS%
	Average solution	Average time	Standard deviation	Best solution	The best solution error	Error percent average		
1	0.068111	579	0.000159	0.06791	0	0.296	0.073	0
2	0.078353	667	0.001609	0.07716	0.104	1.652	0.65	0.103
3	0.02942	708	0.000928	0.02821	0.678	4.99	1.41	0
4	0.04363	779	0.001406	0.04217	0.596	4.08	0.687	-0.05
5	0.103071	859	0.002472	0.10127	0	1.78	0.097	0
6	0.065466	931	0.002678	0.06199	0.291	5.91	2.04	0.161
7	0.111235	1004	0.002475	0.10785	0.065	3.2	-1.15	0.065
8	0.8742	1102	0.003564	0.0835	1.829	6.61	1.04	0.198
			0.001911		0.4455	3.57	0.61	0.185

Table 8. Comparison of GA and SA for large instances

Problem NO.	SA in 10 iteration				GA in 10 iteration				PRA S%	PRB S%
	Average solution	Average time	Standard deviation	Best solution	Average solution	Average time	Standard deviation	Best solution		
9	0.0115	3698	0.000861	0.01091	0.01099	3583	0.000644	0.01024	4.74	6.14
10	0.0262	3785	0.001535	0.02431	0.02641	3671	0.001096	0.02349	3.05	3.37
11	0.0994	3879	0.002968	0.09569	0.0976	3705	0.002399	0.09503	1.8	0.69
12	0.0838	3985	0.003187	0.07891	0.0821	3796	0.002964	0.07701	2.02	2.41
13	0.0182	4098	0.001716	0.01628	0.0177	3874	0.001133	0.01631	2.96	-0.18
14	0.1213	4176	0.003537	0.1167	0.1194	3974	0.003301	0.1167	1.57	0
15	0.0672	4285	0.0053	0.06114	0.0656	4098	0.003828	0.06122	2.31	-0.13
16	0.0724	3986	0.004152	0.06791	0.0672	3869	0.003453	0.06099	7.2	10.19
17	0.0965	4048	0.005247	0.09071	0.0923	3958	0.004263	0.0812	4.36	10.48
18	0.0953	4178	0.007976	0.0849	0.0892	4079	0.005365	0.08001	6.48	5.76
19	0.1046	4308	0.004854	0.09572	0.099	4158	0.007836	0.0891	5.83	6.92
20	0.0489	4427	0.003917	0.04262	0.0456	4217	0.002524	0.04268	6.93	-0.14
21	0.0585	4584	0.002832	0.05448	0.0507	4298	0.004955	0.04327	13.3	20.57
22	0.1457	4718	0.014042	0.13139	0.1356	4374	0.008786	0.12035	6.93	8.4
23	0.0873	4896	0.010885	0.0732	0.0793	4439	0.008251	0.0701	9.17	4.23
24	0.1016	4987	0.006592	0.09008	0.0926	4637	0.00713	0.07998	8.95	11.21
25	0.0831	5083	0.0154	0.06778	0.0723	4739	0.008151	0.06014	13.01	11.27
Average		4301	0.006285			4086	0.004475		5.92	5.95

6. Experimental results

6.1 The designed procedure

The following steps were done in order to solve the designed model.

In the first step, the multi-objective programming problem considering triangular possibility distributions for uncertain data was formulated. Then, uncertain constraints were defuzzified and formed their crisp counterparts. In the next step, the developed multi-objective model was converted to a single-objective model using a comprehensive measure and finally, the single-objective model was solved using Lingo 11 software.

It should be pointed out that the model was solved using Lingo 11 software on a personal computer with intel (r) core (tm) i3 cpu M330@2.13Ghz and 4G RAM. Each objective function was solved independently and its optimal value was determined in order to use the comprehensive criteria and then the model was converted to a single-objective one considering coefficient value of 0.1 for first objective function, 0.45 for the second objective function and 0.45 for the third one; values that were assigned according to the experts' judgments. Parameter $r=1$ is also considered. The values of coefficients imply that the time taken to reach the damaged areas and fairness in the distribution are more important than total costs.

The model was validated through solving a small scale instance of the problem. In order to evaluate the applicability of the model, the model was used based on the data driven from Southern Khorasan province which is one of the most disaster-prone provinces of the country.

6.2 Case study and numerical results

In this case, the data driven from Southern Khorasan is considered. According to geographical locations and population dispersions across the province, three cities of the province (Birjand, Qaen and Ferdows) are considered as candidate areas for establishing temporary distribution centers. Noteworthy, temporary distribution centers receive commodities from supply centers and deliver them to the damaged areas.

There are a number of vehicles with different characteristics in each city and the maximum number of central warehouses that can be active at any time is considered 2. Nine damaged areas are assumed as the demand points. The information includes the transfer time between different areas based on the real data and demand rate of damaged areas and inventories of central warehouses are estimated in different periods. It must be mentioned that they are considered as triangular fuzzy numbers due to the uncertain nature of the data. Table (9) presents the number of vehicles available in central warehouse with their volumes and weight capacities. Table (10) presents transfer times between different regions in the form of triangular fuzzy numbers and transfer costs are computed proportional to transfer times. Table (11) shows unit costs of allocating commodities of supply centers to temporary warehouses. In this study, two temporary warehouses are to be established in order to transfer commodities from supply centers to the damaged areas. Also, planning horizon includes three 72-hour periods. A maximum of 3960 minutes is considered as the permitted time of using a vehicle during each period. The demand of commodities in the damaged areas are shown in Table (12). Table (13) shows the volume, weight and size of relief commodities and Table (14) shows inventories of each commodity in supply centers for different periods which are shown in the model of triangular fuzzy numbers.

Numerical results are presented through Tables (15) to (17). Table (15) shows the optimal allocated commodities to temporary distribution centers or temporary warehouse centers in different periods.

As can be seen, for example, 14200 units of commodity 1 and 9400 units of commodity 2 are allocated from supply center Kerman to temporary distribution center Birjand in Period 1. Likewise, 19800 units of commodity 1 and 12000 units of commodity 2 are transferred from supply center Zahedan to temporary distribution center Birjand in the same period. On the other hand, temporary distribution center Qaen is only assigned to supply center Mashhad which receives 21000 units of commodity 1 and 17000 units of commodity 2 in period 1. The obtained results show that temporary distribution centers try to acquire their needed items from their nearest supply center hence reducing total system costs and expediting the procurement process.

The detailed results of commodities carriage are presented in Table (16) where for each vehicle, its detailed traveling path containing the corresponding central depot and the visited areas are illustrated. For example, vehicle 2 departs from its central depot (Birjand temporary distribution center), goes to Asadiyeh and returns back to the origin during its first journey. The second journey of vehicle 2 starts from the same depot visiting Asadiyeh and Zirkoh and finishing again at the original depot.

Table (17) shows the demand coverage percentage of regional depots per period. These data can be used as a guide in order to determine the amount of commodities that must be delivered to each region in each period. For example, the amount of commodities type 1 that must be delivered to Sarbishe in the first period is 57.9% of the region's demand which is approximately equal to 1042 units. Analogously, the amount of commodities of type 2 that must be delivered to Sarbishe in the same period is 55.6% of the region's demand that is equal to 1112 units. The other values provided in Table (17) can be also interpreted similarly. Given that inventories at supply centers are less than demand of the damaged areas, the model has balanced the delivery quantities such that the maximum possible fairness in distribution could be achieved.

The obtained solutions indicate that fairness in distribution, speed in delivery along with cost minimization are addressed in a satisfactory way; the three objectives that formed the basis of the conducted study.

7. Conclusions

Logistic activities in relief chain management are one of the crucial steps in relief fuzzy response chain management and careful planning in this area can increase the efficiency of attempts confronting with disasters. In this study, a multi-period multi-objective formulation was developed to optimally locate temporary central depots and determine vehicle routes in disaster relief logistics considering several central depots and multiple commodities. Three basic objectives of the model included minimizing total costs, minimizing total time to reach the damaged areas and maximizing the minimum ratio of met demands as a fairness indicator. Moreover, in order to cope with the uncertainties, triangular fuzzy numbers were considered. Afterwards, the model was defuzzified and converted to a single-model formulation. Since the formulation was NP-hard, two well-known GA and SA metaheuristics were developed to enable the model to solve large-scale problems. The model was then validated using Lingo 11 software and applied to a real case study. The case study was carried out in Southern Khorasan, one the most disaster-prone provinces of Iran.

Examining the results of the designed model, one can see that establishment of temporary distribution centers and allocation of commodities and vehicle routes was done such that the best compromised solutions regarding the considered objectives be attained. Also, it was found that, in terms of solution times and solution qualities, the genetic algorithm method represented higher performance compared to the simulated annealing approach. Given the multi-mode of vehicle and adding a delay penalty and speed reward in return and flexibility of destination warehouses, can be considered as a generalization of the findings of the current model and is recommended for future researches.

Table 9. The number and capacity of vehicles at central temporary depots in different periods

Period	Central depots	Vehicles	Vehicle weight capacity (Kg)	Vehicle volume capacity (cubic meters)
1	1	1	24000	80
		2	15000	60
	2	1	24000	80
		2	15000	60
	3	1	15000	60
		2	15000	60
2	1	1	24000	80
		2	15000	60
	2	1	24000	80
		2	15000	60
	3	1	15000	60
		2	15000	60
3	1	1	24000	80
		2	15000	60
		3	24000	80
	2	1	24000	80
		2	15000	60
		3	24000	80
	3	1	15000	60
		2	15000	60
		3	15000	60

Table 10. Travel time between central depot and damaged areas (min)

	1-Birjand	2-Qaen	3-Zirkoh	4-Nehbandan	5-Sarbisheh
Central depot 1	(10,12,15)	(96,105,115)	(125,143,150)	(173,190,220)	(57,70,83)
Central depot 2	(76,85,99)	(10,15,20)	(46,53,61)	(184,203,218)	(96,105,120)
Central depot 3	(120,129,148)	(98,114,125)	(170,181,194)	(260,279,290)	(170,181,195)
Damaged area 1	0	(91,100,114)	(120,138,145)	(183,200,230)	(67,80,93)
Damaged area 2	(91,100,114)	0	(61,68,76)	(199,218,233)	(111,120,135)
Damaged area 3	(120,138,145)	(61,68,76)	0	(260,285,298)	(171,188,200)
Damaged area 4	(183,200,230)	(199,218,233)	(260,285,298)	0	(89,98,109)
Damaged area 5	(67,80,93)	(111,120,135)	(171,188,200)	(89,98,109)	0
Damaged area 6	(97,116,127)	(90,98,107)	(170,189,190)	(250,263,277)	(156,165,176)
Damaged area 7	(80,90,108)	(112,130,144)	(62,68,76)	(142,160,178)	(47,50,61)
Damaged area 8	(211,225,240)	(190,206,225)	(259,274,288)	(352,371,388)	(261,274,293)
Damaged area 9	(130,139,158)	(108,124,135)	(180,191,204)	(270,289,300)	(180,191,205)

	6-Sarayan	7-Asadiyeh	8-Boshroyeh	9-Ferdows
Central depot 1	(102,121,132)	(75,85,103)	(216,230,245)	(135,144,163)
Central depot 2	(75,83,92)	(97,115,129)	(175,191,210)	(93,109,120)
Central depot 3	(12,16,20)	(156,170,187)	(88,93,98)	(8,10,12)
Damaged area 1	(97,116,127)	(80,90,108)	(211,225,240)	(130,139,158)
Damaged area 2	(90,98,107)	(112,130,144)	(190,206,225)	(108,124,135)
Damaged area 3	(170,189,190)	(62,68,76)	(259,274,288)	(180,191,204)
Damaged area 4	(250,263,277)	(142,160,178)	(352,371,388)	(270,289,300)
Damaged area 5	(156,165,176)	(47,50,61)	(261,274,293)	(180,191,205)
Damaged area 6	0	(164,175,186)	(100,109,121)	(22,26,30)
Damaged area 7	(164,175,186)	0	(250,263,277)	(166,180,197)
Damaged area 8	(100,109,121)	(250,263,277)	0	(78,83,88)
Damaged area 9	(22,26,30)	(166,180,197)	(78,83,88)	0

Table 11. Allocation cost of each unit of commodities from supplier centers to temporary central depots

		Central depot 1 (Birjand)	Central depot 2 (Qaen)	Central depot 3 (Ferdows)
Supply central 1	Commodity 1	(330,345,357)	(280,294,302)	(285,295,300)
	Commodity 2	(680,704,728)	(520,528,539)	(535,546,559)
Supply central 2	Commodity 1	(300,320,335)	(367,376,390)	(385,398,409)
	Commodity 2	(620,650,670)	(720,724,731)	(750,761,796)
Supply central 3	Commodity 1	(365,378,389)	(425,435,458)	(430,442,449)
	Commodity 2	(720,732,748)	(810,817,829)	(825,834,846)

Table 12. The demand of commodities in damaged areas (100 *)

period	Commodity	1-Birjand	2-Qaen	3-Zirkoh	4-Nehbandan
1	1	(160,165,170)	(170,180,198)	(220,230,245)	(25,27,30)
	2	(110,115,125)	(140,145,155)	(150,160,175)	(28,30,33)
2	1	(150,153,178)	(150,160,165)	(220,240,250)	(28,30,31)
	2	(100,105,112)	(120,127,138)	(160,165,175)	(32,35,38)
3	1	(180,187,189)	(190,195,205)	(215,225,235)	(23,25,26)
	2	(115,120,124)	(140,142,146)	(145,153,160)	(25,27,28)

Table 12. Continued

period	Commodity	5-Sarbisheh	6-Sarayan	7-Asadiyeh	8-Boshroyeh	9-Ferdows
1	1	(18,18,20)	(28,30,33)	(216,225,235)	(23,25,30)	(37,40,43)
	2	(19,20,24)	(26,30,34)	(145,149,158)	(25,27,30)	(40,40,45)
2	1	(19,20,20)	(30,30,33)	(185,195,200)	(28,30,32)	(40,40,42)
	2	(20,25,27)	(30,32,36)	(140,144,149)	(31,33,36)	(40,45,47)
3	1	(17,18,18)	(25,27,30)	(220,224,232)	(20,20,24)	(32,35,38)
	2	(19,20,22)	(25,28,30)	(152,155,162)	(22,25,30)	(34,38,40)

Table 13. Weight, volume and time of unloading

commodity	Weight(kg)	Volume(M ³)	Download time (min)
Commodity 1	0.5	0.0015	0.02
Commodity 2	1.5	0.004	0.02

Table 14. The stock of supplier centers in various periods (100 *)

		Supply center 1 Mashhad	Supply center 2 Zahedan	Supply center 3 Kerman
Period 1	Commodity 1	(204,211,215)	(189,195,209)	(137,142,148)
	Commodity 2	(165,170,175)	(115,119,126)	(89,94,98)
Period 2	Commodity 1	(260,265,270)	(242,251,259)	(112,124,137)
	Commodity 2	(225,237,247)	(131,137,146)	(92,98,102)
Period 3	Commodity 1	(376,393,398)	(237,245,254)	(141,149,160)
	Commodity 2	(278,285,290)	(153,161,168)	(101,108,118)

Table 15. The amount of transported commodities from supplier centers to temporary distribution centers (100 *)

			Birjand	Qaen	Ferdows
Period 1	Commodity 1	Kerman	142		
		Zahedan	198		
		Mashhad		210	
	Commodity 2	Kerman	94		
		Zahedan	120		
		Mashhad		170	
Period 2	Commodity 1	Kerman	124		
		Zahedan	251		
		Mashhad		265	
	Commodity 2	Kerman	97		
		Zahedan	138		
		Mashhad		236	
Period 3	Commodity 1	Kerman	150		
		Zahedan	245		
		Mashhad		389	
	Commodity 2	Kerman	109		
		Zahedan	161		
		Mashhad		284	

Table 16. The detailed results of commodities carriage

Period	Central depots	vehicles	travel	Path	The amount of commodities 1 in vehicle (100 *)	The amount of commodities 2 in vehicle (100 *)
1	1	1	1	1-4-5-1-1	123	88
		2	1	1-7-1	131	56
		2	2	1-3-7-1	86	70
	2	1	1	2-8-9-6-2-2	57	130
		2	1	2-3-2-2	153	40
2	1	1	1	1-4-5-7-1	151	109
		2	1	1-1-1	113	62
		2	2	1-3-7-1-1	111	64
	2	1	1	2-3-2-2	80	101
		2	1	2-2-2	112	62
		2	2	2-8-9-6-2	73	73
3	1	1	1	1-1-1	151	93
		2	1	1-4-5-7-1	186	38
		3	1	1-3-7-1	58	139
	2	1	1	2-2-2	161	106
		2	1	2-8-9-6-2-2	69	77
		3	1	2-3-2	159	101

Table 17. Demand coverage percentage of regional depots in different periods

Period	Commodity	Area 1 Birjand	Area 2 Qaen	Area 3 Zirkoh	Area 4 Nehbandan	Area 5 Sarbisheh	Area 6 Sarayan	Area 7 Asadiyeh	Area 8 Boshroyeh	Area 9 Ferdows
1	1	58.2%	57.9%	57.8%	59.3%	57.9%	60%	57.8%	57.7%	60%
	2	53%	52.4%	53.1%	53.3%	55.6%	53.3%	53%	55.6%	52.4%

Table 17. Continued

Period	Commodity	Area 1 Birjand	Area 2 Qaen	Area 3 Zirkoh	Area 4 Nehbandan	Area 5 Sarbisheh	Area 6 Sarayan	Area 7 Asadiyeh	Area 8 Boshroyeh	Area 9 Ferdows
2	1	70.6%	79.9%	70.5%	73.3%	75%	71%	70.5%	73.3%	70.7%
	2	66%	66.4%	65.3%	65.7%	68.8%	66.7%	66%	66.7%	65.9%
3	1	81.6%	81.7%	81.8%	84%	83.3%	81.5%	81.8%	85.7%	82.9%
	2	77.5%	77.6%	77.8%	77.8%	80%	78.6%	77.6%	80.8%	78.4%

References

- Ahmadi, M., Seifi, A., Tootoni, B., (2015). A humanitarian logistics model for disaster relief operation considering network failure and standard relief time: A case study on San Francisco district. *Transportation Research Part E: Logistics and Transportation Review*, Vol. 45, pp. 145-163.
- Barbarosoglu, G., Arda, Y. (2004). A Two-Step Stochastic Programming Framework for Transportation Planning in Disaster Response. *Journal of Operational Research Society*, Vol. 55, pp. 43–53.
- Barbarosoglu, G., Ozdamar, L., Cevik, A. (2000). An Interactive Approach for Hierarchical Analysis of Helicopter Logistics in Disaster Relief Operations. *European Journal of Operation Research*, Vol. 140, pp. 118–133.
- Barzinpour, F., Saffarian, M., Makoui, A., Teimoury, E., (2014). Metaheuristic Algorithm for Solving Biobjective Possibility Planning Model of Location-Allocation in Disaster Relief Logistics. *Journal of Applied Mathematics*, ID: 239868.
- Beltrami, E.J., Bodin, L.D. (1974). Networks and Vehicle Routing for Municipal Waste collection. *Networks*, Vol. 4(1), pp. 65-94.
- Berkoune, D., Renaud, J., Rekik, M, Ruiz, A. (2012). Transportation in Disaster Response Operations. *Socio-Economic Planning Sciences*, Vol. 46, pp. 23-32.
- Chiappetta Jabbour, C. J., Sobreiro, V. A., Lopes de Sousa Jabbour, A. B., de Souza Campos, L. M., Mariano, E. B., & Renwick, D. W. S. (2017). An analysis of the literature on humanitarian logistics and supply chain management: paving the way for future studies. *Annals of Operations Research*. doi:10.1007/s10479-017-2536-x
- Christofides, N., Beasley, J.E. (1984). The Period Routing Problem. *Networks*, Vol. 14(2), pp. 237 – 256.
- Eshghi, K., Najafi, M. (2013). A Logistics Planning Model to Improve the Response Phase of Earthquake, *International Journal of Industrial Engineering & Production Management*, Vol. 23, pp. 401-416.
- Golabi, M., Shavarani, S. M., and Izbirak, G. (2017). An edge-based stochastic facility location problem in UAV-supported humanitarian relief logistics: a case study of Tehran earthquake. *Natural Hazards*, Vol. 87(3), pp. 1545-1565.
- Jabal-Ameli, M.S., Bozorgi-Amiri, A., Heydari, M., (2011). A Multi-Objective Possibilistic Programming Model for Relief Logistics Problem. *International Journal of Industrial Engineering & Production Management*, Vol. 22, pp. 65-75.
- Jha, A., Acharya, D., and Tiwari, M. K. (2017). Humanitarian relief supply chain: a multi-objective model and solution. *Sādhanā*, Vol. 42(7), pp. 1167-1174.
- John, L. (2018). *Review of Empirical Studies in Humanitarian Supply Chain Management: Methodological Considerations, Recent Trends and Future Directions*. In G.
- Kovács, K. Spens, and M. Moshtari (Eds.), *The Palgrave Handbook of Humanitarian Logistics and Supply Chain Management* (pp. 637-673). London: Palgrave Macmillan UK.

- Ke, L., Feng, Z. (2013). A two-phase metaheuristic for the cumulative capacitated vehicle routing problem. *Computers & Operations Research*, Vol. 40, pp. 633–638.
- Knott, R., (1988). Vehicle Routing for Emergency Relief Management: A Knowledge - Based Approach. *Disaster*, Vol. 12, pp. 285–293.
- Lin, Y.H, Batta, R., Rogerson, A.P. Blatt, A., Flanigan, M. (2011). A logistics model for emergency supply of critical items in the aftermath of a disaster, *Socio-Economic Planning Sciences*, Vol. 45, pp. 132-145
- Ngueveu, S.U. , Prins, C., Calvo, R.W. (2010). An effective memetic algorithm for the cumulative capacitated vehicle routing problem. *Computers & Operations Research*, Vol. 37, pp. 1877-1885.
- Nolz, P.C., Semet, F., Doerner, K.F. (2011). Risk approaches for delivering disaster relief supplies, *OR Spectrum*, Vol. 33, pp. 543–569.
- Oh, S., Haghani, A. (1996). Formulation and Solution of a Multi-Commodity, Multi-Modal Network Flow Model for Disaster Relief Operations. *Transport. Res.*, Vol. 30, pp. 231–250.
- Ozdamar, L., Ekinci, E., Kucukyazici, B. (2004). Emergency Logistics Planning in Natural Disasters. *Annals of Operations Research*, Vol. 129, pp. 217–245.
- Pishvae, M.S., Torabi, S.A. (2010). A Possibilistic Programming Approach for Closed-Loop Supply Chain Network Design under Uncertainty. *Fuzzy Sets and Systems*, Vol. 161(20), pp. 2668-2683.
- Rao, S.S. (1996). *Engineering optimization: theory and practice*, 3rd ed. John Wiley & Sons, New Jers.
- Rath, S., Gutjahr, W.J. (2014). A math-heuristic for the warehouse location–routing problem in disaster relief. *Computers & Operations Research*, Vol. 42, pp. 25-39.
- Saffarian, M., Barzinpour, F., Eghbali, M.A., (2015). A robust programming approach to bi-objective optimization model in the disaster relief logistics response phase, *International Journal of Supply and Operations Management*, Vol. 2(1), pp. 595-616.
- Tofghi, S., Torabi, S.A., ansouri, S.A., (2016). Humanitarian logistics network design under mixed uncertainty. *European Journal of Operational Research*, Vol. 250, pp. 239-250.
- Thomas, A.S., Kopczak, L.R. (2005). From logistics to supply chain management: the path forward in the humanitarian sector. <http://www.fritzinstitute.org/PDFs/WhitePaper/FromLogisticsto.pdf>.
- Uslu, A., Cetinkaya, C., & İŞLEYEN, S. K. (2017). Vehicle routing problem in post-disaster humanitarian relief logistics: a case study in Ankara. *Sigma Journal of Engineering and Natural Sciences-SIGMA MUHENDISLIK VE FEN BILIMLERI DERGISI*, Vol. 35(3), pp. 481-499.
- Van Wassenhove, L.N. (2006). Humanitarian aid logistics: supply chain management in high gear. *Journal of the Operational Research Society*, Vol. 57, pp. 475–489.
- Van Wassenhove, L.N., Pedraza Martinez A.J. (2010). Using OR to adapt supply chain management best practices to humanitarian logistics. *International Transactions in operational Research*, Vol. 19, pp. 307-322.
- Wang, H., Du, L., Ma, S., (2014). Multi-objective open location-routing model with split delivery for optimized relief distribution in post-earthquake. *Transportation Research Part E: Logistics and Transportation Review*, Vol. 69, pp. 160-179.