A Tactical Planning Model for a Job Shop with Unreliable Work Stations and Capacity Constraints

By

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Abstract

We develop an analytic and a simulation model for a job shop with unreliable work stations and production constraints. In the complex batch manufacturing operation of the factory, smooth production of each work station is required. In the previous work the Tactical Planning Model was proposed for this purpose. In that model, the production of each work station should be proportional to the queueing level of the input work flow. In this paper, the model is extended to the work station with production constraints and with given unreliability of its operation, because in the real world situation, the work stations cannot be operated perfectly without any trouble nor infinite productivity. For the analysis of multiple unreliable work stations, we develop an analytical model and solution. The break down of the work station is modeled as a Bernoulli process. For the analysis with the production constraints with or without the unreliable work station model, we develop a dynamic simulation model. We show some examples of this problem, and show the effect of production constraints and unreliable work stations.

Thesis Supervisor: Dr. Stephen C. Graves
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The contents of the thesis

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My children, Yuichiro and Kentaro, have been a source of my courage and enthusiasm, though sometimes they have been jealous for my job and study.

To my family I dedicated this thesis.

Chapter 1. Introduction

The purpose of this work is to extend an analytical model and to develop a simulation model of a job shop problem. The focus is on the extension of the Tactical Planning model[1] to the unreliable multiple work stations with production constraints.

A job shop is a very flexible production facility composed of several machines or work stations. Unlike the assembly line facility, different jobs with various kinds of work flow and completion times are managed in a job shop[2]. An analytical model of job shop problem was proposed by Stephen C. Graves in his "A Tactical Planning Model for a Job Shop" (1985) [1]. He modeled this job shop problem as a network of queues. The work flow was modeled as the discrete-time, continuous-flow of the tasks. The task units were not expressed in terms of the number of parts to be produced but were expressed as the necessary service time to complete the job. The control law for production of each work station was modeled as proportional to the queueing level of the input work flow and as inversely proportional to the planned lead time. The model provides the steady state distribution of the production levels at each work station, the distribution of queue lengths and the distribution of backlogs. In the Tactical Planning Model, Graves took into account the production capability, the inherent variability and uncertainty of the production requirements.

For the production capability, the production constraints were treated implicitly. When the expected level of the production exceeded the capacity of one facility, he increased the number of the facility to satisfy the requirement. For example if the capacity is 2 units of production by one facility and the requirement of the production is 5, he assumed that there should be three facilities, in parallel, at the work station. In order to make the variation of the production requirements be within the constraints of the production capability, the planned lead time of the heavily-loaded work station was increased, because the increase of the planned lead time will reduce the variation of the production requirements.

In order to analyze more precisely the situation of the job shop, the saturation of the production without sufficient production capacity should be taken into account. One of the purposes of our research is to investigate this point. We allow the model of heavily-loaded work station if the average requirement of the production is within the capability.

The inherent variation of the production was modeled as an additional independent random input to the work station. But the production itself was assumed to operate properly. For the purpose of the extensive analysis of the job shop problem, we have to consider not only the input variation but also the break down of work stations. In the real world situation, the work station cannot be operated perfectly without any trouble nor infinite productivity. Sometime the work station has to stop its operation because of maintenance, even if it has high reliability. When the work station is stopped, the incoming tasks will be delayed and may result in some backlog. For this problem, the case with single input and single output for the single work station was also analyzed by Graves[3]. He showed that the effect of the work station break down causes a longer expected length of queue and a larger variation in the queue length. In this paper, we extend the unreliable job shop problem to the multiple work stations model with production constraints.

In Chapter 2, we review the outline of the Tactical Planning Model proposed by Graves, and show the model with unreliable multiple work stations and capacity constraints.

In Chapter 3, we demonstrate the dynamic simulation model to examine the performance of the system. For the analysis of the complicated net work of queues, a simulation is the only way to solve this problem without any simplification, because of complicated interactions between work stations and nonlinear constraints. In this paper, we use the software named STELLA™+ to model and carry out the simulation. STELLA was designed to make it possible to simulate a System Dynamics model on the personal computer[4]. With the application of the software to this problem, the analysis of the complicated network queue problem is completed without the use of the large computer.

In Chapter 4, we show the result of the simulation and analysis. At first we show the validation of the simulation model with no production constraints and perfectly reliable work stations. We compare the result of the simulation to the result of the analytical work. The same job shop model is used as was used in the Graves' paper. It is the ten work stations job shop model for the production of components for grinding machines. After that we show the example of the simulation of the unreliable work stations with the production constraints.

In Chapter 5, we show the conclusion of this analysis and simulation.

+ STELLA is a trademark licensed to High Performance Systems, Inc.

Chapter 2. Tactical Planning Model and extension

2.1 The Tactical Planning Model

A job shop is a very flexible production facility that consists of a set of versatile machine centers or work stations and is capable of producing a wide variety of jobs. Because of its inherent complexity, it is often difficult to manage the production control in a sophisticated manner. In 1985 Graves[1] proposed a Tactical Planning Model to help address this problem. He developed the problem by making use of the planned lead time for its production.

In this section we would like to describe briefly about the Tactical Planning Model.

At first we would like to note the basic assumption of the model.

(1) The job shop works as discrete time model.

Every transaction is carried out at the specified time, and production is completed within the given period of time.

(2) The work flow is continuous.

At each work station, the arriving jobs to the station, the queue of work at the station, and the production by the station are expressed in terms of the work load on the station. This differs from the usual queueing model in which these variables would be expressed in terms of number of jobs.

(3) The work station does not break down.

100% reliability was assumed.

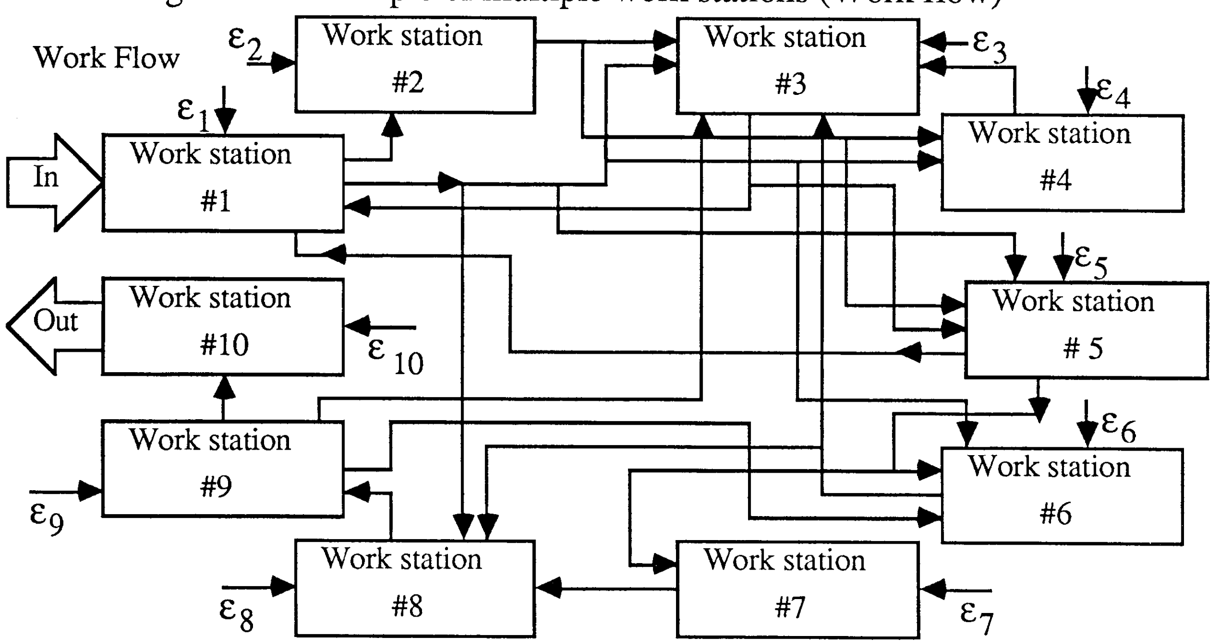
(4) There are no explicit capacity constraints.

Constraints were considered implicitly.

(5) The arrival stream to a work station contains some degree of uncertainty.

The arrival stream to a work station contains two types of input. One consists of the work flow from the other work stations. The other consists of the random noise which simulates the production variation of the system, and includes new work which enters the shop.

Fig 2.1-1 Example of multiple work stations (Work flow)



The above drawing is the example of the multiple work stations model shown in the Graves article. Jobs arrive to work station # 1, and leave from work station # 10. The processing of a job may entail visits to several different work stations.

Second, we would like to describe the relationship and steady state analytical solution of the performance of the model.

In the Tactical Planning model, the production of a work station is set to equal a fraction of the work-in-process (WIP) queue. The control rule is described as follows.

$$P_{i,t} = \alpha_i Q_{i,t}$$

where $P_{i,t}$ is the production level of the work station i, during the time period of t, $Q_{i,t}$ is the WIP queue level, and α_i is the control parameter of the Tactical Planning model. α_i is the inverse of the planned lead time and is restricted to $0<\alpha_i\leq 1$. For example if a planned lead time is 4 periods of time then α_i is 0.25. The balance equation is

$$Q_{i,t}= Q_{i,t-1}- P_{i,t-1}+A_{i,t}$$

where $A_{i,t}$ is the amount of work that arrives at work station i at the start of time period t. These arrivals may come from many other sectors, and the flow from work station j to work station i is modeled by

$$A_{i,j,t}= \phi_{ij}P_{j,t-1} + \varepsilon_{i,j,t}$$

where ϕ_{ij} is the expected number of hours of work generated for work station i by every hour of work completed by work station j, and $\varepsilon_{i,j,t}$ is a random variable with zero mean. The term $\varepsilon_{i,j,t}$ is a noise term which simulates the uncertainty in the arrival stream. It is assumed that the terms of the time series $\{ \varepsilon_{i,j,t} \}$ are i.i.d. The total arrivals to a work station i are the sum over all preceding sectors of equation above.

$$A_{i,t} = \sum_{j=1}^{n} \phi_{i,j} P_{j,t-1} + \varepsilon_{i,t}$$

where

$$\varepsilon_{i,t} = N_{i,t} + \sum_{j=1}^{n} \varepsilon_{i,j,t}$$

$$A_{i,t} = \sum_{j=1}^{n} A_{i,j,t} + N_{i,t}$$

where $N_{i,t}$ is a random variable that represents the work load from new jobs that enter the shop at time t directly from outside. The elements of each time series $\{N_{i,t}\}$ are assumed to be i.i.d. Thus $\varepsilon_{i,t}$ represents those arrivals that are not predictable from the previous history and it consists of random noise and new arrivals. By substituting these equations the following vector-matrix equation can be derived.

$$\mathbf{P}_t = (\mathbf{I} - \mathbf{D} + \mathbf{D}\Phi)\mathbf{P}_{t-1} + \mathbf{D}\,\varepsilon_t$$

where $\mathbf{P}_t$ is the vector of elements $P_{i,t}$, ε_t is the vector of elements $\varepsilon_{i,t}$, $\mathbf{D}$ is the diagonal matrix with the control parameter a_i on the diagonal, and Φ is the matrix whose elements are $\phi_{i,j}$, $\mathbf{I}$ is the identity matrix. By successively substituting the above formula, the equation can be rewritten as the geometric series

$$\mathbf{P}_t = \sum_{s=0}^{\infty} (\mathbf{I} - \mathbf{D} + \mathbf{D}\Phi)^s \mathbf{D}\varepsilon_{t-s}$$

The expectation of production vector, $\rho' = \{\rho_1, \rho_2, \ldots \rho_n\}$ is given by

$$E(\mathbf{P}_t) = \rho = \sum_{s=0}^{\infty} (\mathbf{I} - \mathbf{D} + \mathbf{D}\Phi)^s \mathbf{D}\mu$$

where μ is the expected value of the vector ε_t. It was shown that the geometric series converges, provided that the spectral radius (maximal absolute eigenvalue) of the matrix Φ is less than 1, which is necessary and sufficient for the spectral radius of $(\mathbf{I} - \mathbf{D} + \mathbf{D}\Phi)$ to be less than 1. Then the above equation can be written as the following form.

$$E(\mathbf{P}_t) = \rho = (\mathbf{I} - \Phi)^{-1}\mu$$

The covariance matrix of $\mathbf{P}_t$ is

$$\mathrm{Var}(\mathbf{P}_t) = \sum_{s=0}^{\infty} \mathbf{B}^s \mathbf{D}\,\Sigma\,\mathbf{D}\,\mathbf{B}'^s$$

where $\mathbf{B} = (\mathbf{I} - \mathbf{D} + \mathbf{D}\Phi)$ and Σ is the covariance matrix of the vector ε_t. The expected queue vector and covariance matrix can be written down as follows.

$$\mathbf{Q}_t = \mathbf{D}^{-1}\mathbf{P}_t$$

$$E(\mathbf{Q}_t) = \mathbf{D}^{-1}\rho$$

$$\mathrm{Var}(\mathbf{Q}_t) = \mathbf{D}^{-1}\,[\mathrm{Var}(\mathbf{P}_t)]\,\mathbf{D}^{-1}$$

The queue at each work station is assumed to be served as first in, first out. The oldest input is processed first. The backlog is defined as the portion of the queue that has waited for m periods. Thus, the backlog can be written as the following equation.

$$Q_{it}^{m} = Q_{i,t-1}^{m-1} - P_{i,t-1}$$
$$= Q_{i,t-m} - \sum_{s=1}^{m} P_{i,t-s}$$

where $Q_{it}^{m}<0$ means that paticular backlog is zero, and $Q_{it}^{m}= Q_{it}$ for $m=0$. The first term indicates the queueing level at the period of t-m and the second summation is the total production from the time period of t-m up to the last production. Then in matrix notation , we have

$$\mathbf{Q}_t^{m} = \mathbf{D}^{-1}\mathbf{P}_{t-m} - \sum_{s=1}^{m}\mathbf{P}_{t-s}$$

and expected backlog is as follows.

$$E(\mathbf{Q}_t^{m})=(\mathbf{D}^{-1}-m\mathbf{I})(\mathbf{I}-\boldsymbol{\Phi})^{-1}\mu$$

The covariance of Q_{it}^{m} can also be found from the previous results.

2.2 Unreliable work station

In the previous section, we discussed the model of multiple work stations. There were assumptions that no work station would fail to work, nor would trouble develop. But in the real world situation, the machines in the factory sometimes fail to work, and for some period of time the machines have to be inoperable because of maintenance. The original analysis of the problem was conducted by Graves[3] for a single work station model. In this section, we would like to extend the concept of the unreliable work station to this Tactical Planning Model for multiple work stations.

The definition of work station break down is the condition in which the work station produces nothing for a given period of time. The in-coming material consists of queue in the input side of the work station. The break down of a work station occurs as a Bernoulli process, with the probability p_i. That is, each period work station i fails with probability p_i. When a work station fails, its production for that time period is zero. The state of the work station does not provide any influence on the other work stations, neither at its production level nor at the stage of the production.

Let i be the subscript indicating the work station i. The work station i has the following relations.

$$Q_{i,t} = Q_{i,t-1} - P_{i,t-1} + A_{i,t}$$

$$A_{i,t} = \phi_{i1} P_{1,t-1} + \phi_{i2} P_{2,t-1} + \dots + \phi_{in} P_{n,t-1} + \varepsilon_{i,t}$$

$$P_{i,t} = \alpha_i Q_{i,t} \quad \text{--- with probability} \quad 1-p_i$$

$$= 0.0 \quad \text{--- with probability} \quad p_i$$

$$i = 1, 2, \cdots, n$$

Let $\mathbf{D}_{pt}$ be the diagonal matrix with a_{it}, which is defined as the random variable

$$a_{i,t} = \alpha_i \quad \text{--- With probability } (1-p_i)$$

$$= 0 \quad \text{--- With probability } p_i$$

$$\mathbf{D}_{pt} = \begin{bmatrix} a_{1,t} & 0 & - & 0 \\ 0 & a_{2,t} & - & 0 \\ - & - & - & 0 \\ 0 & 0 & 0 & a_{n,t} \end{bmatrix}$$

p_i denotes the probability of work station break down. This random variable has following characteristics.

$$E(a_{it}) = \alpha_i (1-p_i)$$
$$E(a_{it}^2) = \alpha_i^2 (1-p_i)$$
$$Var(a_{it}) = \alpha_i^2 p_i (1-p_i)$$

$$E(a_{it} a_{jt}) = \alpha_i \alpha_j (1-p_i)(1-p_j) \quad ; i \neq j$$
$$E(a_{it} a_{it-s}) = \alpha_i^2 (1-p_i)^2 \quad ; s \neq 0$$
$$E(a_{it} X_{it}) = \alpha_i (1-p_i) E(X_{it}) \quad ; X_{it} \text{ is independent from } a_{it}$$

$$E(\mathbf{D}_{pt}) = \begin{bmatrix} \alpha_1(1-p_1) & 0 & - & 0 \\ 0 & \alpha_2(1-p_2) & - & 0 \\ - & - & - & - \\ 0 & 0 & - & \alpha_n(1-p_n) \end{bmatrix}$$
$$= \mathbf{D}(\mathbf{I} - \mathbf{P}_r)$$

$\mathbf{P}_r$ denotes a diagonal matrix with elements, $\{p_1 p_2, \ldots p_n\}$, and $\mathbf{D}$ denotes a diagonal matrix with its elements being the of inverse of planned lead time, $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$.

Now the job shop system can be analyzed by the following set of equations:

The control rule is now

$$\mathbf{P}_t = \mathbf{D}_{pt} \mathbf{Q}_t$$

The balance equation is then

$$\mathbf{Q}_t = \mathbf{Q}_{t-1} - \mathbf{P}_{t-1} + \mathbf{A}_t$$
$$= \mathbf{Q}_{t-1} - \mathbf{D}_{pt-1}\mathbf{Q}_{t-1} + \Phi\,\mathbf{D}_{pt-1}\mathbf{Q}_{t-1} + \varepsilon_t$$
$$= \varepsilon_t + \{\mathbf{I} - (\mathbf{I}-\Phi)\,\mathbf{D}_{pt-1}\}\,\mathbf{Q}_{t-1}$$

$$\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} & - & \phi_{1n} \\ \phi_{21} & \phi_{22} & - & \phi_{2n} \\ - & - & - & - \\ \phi_{n1} & \phi_{n2} & - & \phi_{nn} \end{bmatrix}$$

2.2.1 The average and variance of queue

The expectation of queue can be derived by taking expectation of both sides.

$$E(\mathbf{Q}_t) = E(\varepsilon_t) + \{\mathbf{I} - (\mathbf{I}-\Phi)\,E(\mathbf{D}_{pt-1})\}\,E(\mathbf{Q}_{t-1})$$

At the steady state condition, the expectations of $\mathbf{Q}_t$ and $\mathbf{Q}_{t-1}$ are the same. $\{E(\mathbf{Q}_t) = E(\mathbf{Q}_{t-1})\}$ Therefore

$$E(\mathbf{Q}_t) = \{(\mathbf{I}-\Phi)\,E(\mathbf{D}_{pt-1})\}^{-1}\,E(\varepsilon_t)$$
$$= \{(\mathbf{I}-\Phi)\,\mathbf{D}\,(\mathbf{I}-\mathbf{P}_r)\}^{-1}\,\mu$$

The covariance matrix can be obtained by the following manner. At first a quadratic form of $\mathbf{Q}_t$ should be taken. The form consists of NxN matrix.

$$\mathbf{Q}_t\mathbf{Q}_t^T = \begin{bmatrix} Q_{1,t}Q_{1,t} & Q_{1,t}Q_{2,t} & - & Q_{1,t}Q_{n,t} \\ Q_{2,t}Q_{1,t} & Q_{2,t}Q_{2,t} & - & Q_{2,t}Q_{n,t} \\ - & - & - & - \\ Q_{n,t}Q_{1,t} & Q_{n,t}Q_{2,t} & - & Q_{n,t}Q_{n,t} \end{bmatrix}$$

which is expressed as following form.

$$\mathbf{Q}_t\,\mathbf{Q}_t^T = (\varepsilon_t + \{\mathbf{I}-(\mathbf{I}-\Phi)\mathbf{D}_{pt-1}\}\mathbf{Q}_{t-1})\,(\varepsilon_t + \{\mathbf{I}-(\mathbf{I}-\Phi)\mathbf{D}_{pt-1}\}\mathbf{Q}_{t-1})^T$$
$$= \varepsilon_t\,\varepsilon_t^T + \{\mathbf{I}-(\mathbf{I}-\Phi)\,\mathbf{D}_{pt-1}\}\,\mathbf{Q}_{t-1}\,\varepsilon_t^T + \varepsilon_t\,\mathbf{Q}_{t-1}^T\{\mathbf{I}-(\mathbf{I}-\Phi)\mathbf{D}_{pt-1}\}^T$$
$$+ \{\mathbf{I}-(\mathbf{I}-\Phi)\,\mathbf{D}_{pt-1}\}\,\mathbf{Q}_{t-1}\,\mathbf{Q}_{t-1}^T\,\{\mathbf{I}-(\mathbf{I}-\Phi)\,\mathbf{D}_{pt-1}\}^T$$
$$= \mathbf{Q}_{t-1}\,\mathbf{Q}_{t-1}^T - (\mathbf{I}-\Phi)\,\mathbf{D}_{pt-1}\,\mathbf{Q}_{t-1}\mathbf{Q}_{t-1}^T - \mathbf{Q}_{t-1}\,\mathbf{Q}_{t-1}^T\mathbf{D}_{pt-1}{}^T(\mathbf{I}-\Phi)^T$$

$$+(\mathbf{I}-\Phi)\,\mathbf{D}_{pt-1}\,\mathbf{Q}_{t-1}\,\mathbf{Q}_{t-1}^{T}\,\mathbf{D}_{pt-1}^{\;T}\,(\mathbf{I}-\Phi)^{T}$$
$$+\ \varepsilon_t\,\varepsilon_t^{T} + \{\,\mathbf{I}-(\mathbf{I}-\Phi)\,\mathbf{D}_{pt-1}\,\}\,\mathbf{Q}_{t-1}\,\varepsilon_t^{T} + \varepsilon_t\,\mathbf{Q}_{t-1}^{T}\{\,\mathbf{I}-(\mathbf{I}-\Phi)\mathbf{D}_{pt-1}\,\}^{T}$$

The expectation of this quadratic form is following.

$$\begin{aligned}
E(\mathbf{Q}_t\,\mathbf{Q}_t^{T}) = {} & E(\mathbf{Q}_{t-1}\mathbf{Q}_{t-1}^{T}) + E(\varepsilon_t\,\varepsilon_t^{T}) \\
& -(\mathbf{I}-\Phi)\,E(\mathbf{D}_{pt-1}\,\mathbf{Q}_{t-1}\mathbf{Q}_{t-1}^{T}) - E(\mathbf{Q}_{t-1}\mathbf{Q}_{t-1}^{T}\,\mathbf{D}_{pt-1}^{\;T})\,(\mathbf{I}-\Phi)^{T} \\
& +(\mathbf{I}-\Phi)\,E(\mathbf{D}_{pt-1}\,\mathbf{Q}_{t-1}\,\mathbf{Q}_{t-1}^{T}\,\mathbf{D}_{pt-1}^{\;T})\,(\mathbf{I}-\Phi)^{T} + E(\mathbf{Q}_{t-1}\,\varepsilon_t^{T}) \\
& -(\mathbf{I}-\Phi)\,E(\mathbf{D}_{pt-1}\,\mathbf{Q}_{t-1}\varepsilon_t^{T}) + E(\varepsilon_t\mathbf{Q}_{t-1}^{T}) - E(\varepsilon_t\mathbf{Q}_{t-1}^{T}\,\mathbf{D}_{pt-1}^{\;T})(\mathbf{I}-\Phi)^{T}
\end{aligned}$$

As ε_t, $\mathbf{Q}_{t-1}$ and $\mathbf{D}_{pt-1}$ are mutually independent, the expectation of mutual products are the products of mutual expectation.

$$\begin{aligned}
E(\mathbf{Q}_t\,\mathbf{Q}_t^{T}) = {} & E(\mathbf{Q}_{t-1}\mathbf{Q}_{t-1}^{T}) + E(\varepsilon_t\,\varepsilon_t^{T}) \\
& -(\mathbf{I}-\Phi)\,E(\mathbf{D}_{pt-1})\,E(\mathbf{Q}_{t-1}\mathbf{Q}_{t-1}^{T}) - E(\mathbf{Q}_{t-1}\mathbf{Q}_{t-1}^{T})\,E(\mathbf{D}_{pt-1}^{\;T})\,(\mathbf{I}-\Phi)^{T} \\
& +(\mathbf{I}-\Phi)\,E(\mathbf{D}_{pt-1}\,\mathbf{Q}_{t-1}\,\mathbf{Q}_{t-1}^{T}\,\mathbf{D}_{pt-1}^{\;T})\,(\mathbf{I}-\Phi)^{T} + E(\mathbf{Q}_{t-1})E(\varepsilon_t^{T}) + E(\varepsilon_t)\,E(\mathbf{Q}_{t-1}^{T}) \\
& -(\mathbf{I}-\Phi)\,E(\mathbf{D}_{pt-1})\,E(\mathbf{Q}_{t-1})\,E(\varepsilon_t^{T}) - E(\varepsilon_t)\,E(\mathbf{Q}_{t-1}^{T})\,E(\mathbf{D}_{pt-1}^{\;T})(\mathbf{I}-\Phi)^{T}
\end{aligned}$$

On the other hand $E(\mathbf{Q}_t)\,E(\mathbf{Q}_t^{T})$ is as follows.

$$\begin{aligned}
E(\mathbf{Q}_t)\,E(\mathbf{Q}_t^{T}) = {} & E(\mathbf{Q}_{t-1})\,E(\mathbf{Q}_{t-1}^{T}) + E(\varepsilon_t)\,E(\varepsilon_t^{T}) \\
& -(\mathbf{I}-\Phi)\,E(\mathbf{D}_{pt-1}\,\mathbf{Q}_{t-1})\,E(\mathbf{Q}_{t-1}^{T}) - E(\mathbf{Q}_{t-1})\,E(\mathbf{Q}_{t-1}^{T}\,\mathbf{D}_{pt-1}^{\;T})(\mathbf{I}-\Phi)^{T} \\
& +(\mathbf{I}-\Phi)\,E(\mathbf{D}_{pt-1}\,\mathbf{Q}_{t-1})\,E(\mathbf{Q}_{t-1}^{T}\,\mathbf{D}_{pt-1}^{\;T})\,(\mathbf{I}-\Phi)^{T} + E(\mathbf{Q}_{t-1})\,E(\varepsilon_t^{T}) \\
& -(\mathbf{I}-\Phi)\,E(\mathbf{D}_{pt-1}\,\mathbf{Q}_{t-1})\,E(\varepsilon_t^{T}) + E(\varepsilon_t)\,E(\mathbf{Q}_{t-1}^{T}) - E(\varepsilon_t)\,E(\mathbf{Q}_{t-1}^{T}\,\mathbf{D}_{pt-1}^{T})(\mathbf{I}-\Phi)^{T} \\
= {} & E(\mathbf{Q}_{t-1})\,E(\mathbf{Q}_{t-1}^{T}) + E(\varepsilon_t)\,E(\varepsilon_t^{T}) \\
& -(\mathbf{I}-\Phi)\,E(\mathbf{D}_{pt-1})\,E(\mathbf{Q}_{t-1})\,E(\mathbf{Q}_{t-1}^{T}) - E(\mathbf{Q}_{t-1})\,E(\mathbf{Q}_{t-1}^{T})\,E(\mathbf{D}_{pt-1}^{\;T})\,(\mathbf{I}-\Phi)^{T} \\
& +(\mathbf{I}-\Phi)\,E(\mathbf{D}_{pt-1})\,E(\mathbf{Q}_{t-1})\,E(\mathbf{Q}_{t-1}^{T})\,E(\mathbf{D}_{pt-1}^{\;T})\,(\mathbf{I}-\Phi)^{T} \\
& + E(\mathbf{Q}_{t-1})\,E(\varepsilon_t^{T}) - (\mathbf{I}-\Phi)\,E(\mathbf{D}_{pt-1})\,E(\mathbf{Q}_{t-1})\,E(\varepsilon_t^{T}) + E(\varepsilon_t)\,E(\mathbf{Q}_{t-1}^{T}) \\
& - E(\varepsilon_t)\,E(\mathbf{Q}_{t-1}^{T})\,E(\mathbf{D}_{pt-1}^{\;T})(\mathbf{I}-\Phi)^{T}
\end{aligned}$$

The covariance matrix of $\mathbf{Q}_t$ can be calculated as the difference between $E(\mathbf{Q}_t\,\mathbf{Q}_t^T)$ and $E(\mathbf{Q}_t)\,E(\mathbf{Q}_t^T)$. The diagonal elements of the matrix consist of variance of Q_{it} and the other elements consist of covariance of Q_{it} and Q_{jt}.

$$
\begin{aligned}
\mathrm{Var}(\mathbf{Q}_t) &= E(\mathbf{Q}_t\mathbf{Q}_t^T) - E(\mathbf{Q}_t)\,E(\mathbf{Q}_t^T) \\
&= E(\mathbf{Q}_{t-1}\mathbf{Q}_{t-1}^T) + E(\varepsilon_t\,\varepsilon_t^T) \\
&\quad -(\mathbf{I}-\boldsymbol{\Phi})\,E(\mathbf{D}_{pt-1})\,E(\mathbf{Q}_{t-1}\mathbf{Q}_{t-1}^T) - E(\mathbf{Q}_{t-1}\mathbf{Q}_{t-1}^T)\,E(\mathbf{D}_{pt-1}^T)\,(\mathbf{I}-\boldsymbol{\Phi})^T \\
&\quad +(\mathbf{I}-\boldsymbol{\Phi})\,E(\mathbf{D}_{pt-1}\,\mathbf{Q}_{t-1}\,\mathbf{Q}_{t-1}^T\,\mathbf{D}_{pt-1}^T)\,(\mathbf{I}-\boldsymbol{\Phi})^T + E(\mathbf{Q}_{t-1})\,E(\varepsilon_t^T) \\
&\quad -(\mathbf{I}-\boldsymbol{\Phi})\,E(\mathbf{D}_{pt-1})\,E(\mathbf{Q}_{t-1})\,E(\varepsilon_t^T) + E(\varepsilon_t)\,E(\mathbf{Q}_{t-1}^T) \\
&\quad - E(\varepsilon_t)\,E(\mathbf{Q}_{t-1}^T)\,E(\mathbf{D}_{pt-1}^T)(\mathbf{I}-\boldsymbol{\Phi})^T \\
&\quad -\{E(\mathbf{Q}_{t-1})\,E(\mathbf{Q}_{t-1}^T) + E(\varepsilon_t)\,E(\varepsilon_t^T) \\
&\quad -(\mathbf{I}-\boldsymbol{\Phi})\,E(\mathbf{D}_{pt-1})\,E(\mathbf{Q}_{t-1})\,E(\mathbf{Q}_{t-1}^T) - E(\mathbf{Q}_{t-1})\,E(\mathbf{Q}_{t-1}^T)\,E(\mathbf{D}_{pt-1}^T)\,(\mathbf{I}-\boldsymbol{\Phi})^T \\
&\quad +(\mathbf{I}-\boldsymbol{\Phi})\,E(\mathbf{D}_{pt-1})\,E(\mathbf{Q}_{t-1})\,E(\mathbf{Q}_{t-1}^T)\,E(\mathbf{D}_{pt-1}^T)\,(\mathbf{I}-\boldsymbol{\Phi})^T + E(\mathbf{Q}_{t-1})\,E(\varepsilon_t^T) \\
&\quad -(\mathbf{I}-\boldsymbol{\Phi})\,E(\mathbf{D}_{pt-1})\,E(\mathbf{Q}_{t-1})\,E(\varepsilon_t^T) + E(\varepsilon_t)\,E(\mathbf{Q}_{t-1}^T) \\
&\quad - E(\varepsilon_t)\,E(\mathbf{Q}_{t-1}^T)\,E(\mathbf{D}_{pt-1}^T)(\mathbf{I}-\boldsymbol{\Phi})^T\}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathrm{Var}(\mathbf{Q}_{t-1}) + \mathrm{Var}(\varepsilon_t) - (\mathbf{I}-\boldsymbol{\Phi})E(\mathbf{D}_{pt-1})\mathrm{Var}(\mathbf{Q}_{t-1}) - \mathrm{Var}(\mathbf{Q}_{t-1})E(\mathbf{D}_{pt-1}^T)(\mathbf{I}-\boldsymbol{\Phi})^T \\
&\quad +(\mathbf{I}-\boldsymbol{\Phi})\{E(\mathbf{D}_{pt-1}\mathbf{Q}_{t-1}\mathbf{Q}_{t-1}^T\mathbf{D}_{pt-1}^T) - E(\mathbf{D}_{pt-1})E(\mathbf{Q}_{t-1})E(\mathbf{Q}_{t-1}^T)E(\mathbf{D}_{pt-1}^T)\}(\mathbf{I}-\boldsymbol{\Phi})^T
\end{aligned}
$$

The product of $\mathbf{D}_{pt}\mathbf{Q}_t$ and $\mathbf{Q}_t^T\mathbf{D}_{pt}^T$ is expressed by using the following formula.

$$
\mathbf{D}_{pt}\mathbf{Q}_t\mathbf{Q}_t^T\mathbf{D}_{pt}^T =
\begin{bmatrix}
Q_{1,t}^2 a_{1,t}^2 & Q_{1,t}Q_{2,t}a_{1,t}a_{2,t} & - & Q_{1,t}Q_{n,t}a_{1,t}a_{n,t} \\
Q_{2,t}Q_{1,t}a_{2,t}a_{1,t} & Q_{2,t}^2 a_{2,t}^2 & - & Q_{2,t}Q_{n,t}a_{2,t}a_{n,t} \\
- & - & - & - \\
Q_{n,t}Q_{1,t}a_{n,t}a_{1,t} & Q_{n,t}Q_{2,t}a_{n,t}a_{2,t} & - & Q_{n,t}^2 a_{n,t}^2
\end{bmatrix}
$$

Then the expectation of $\mathbf{D}_{pt-1}\mathbf{Q}_{t-1}\mathbf{Q}_{t-1}^T\mathbf{D}_{pt-1}^T$ is

$$E(\mathbf{D}_{pt-1}\mathbf{Q}_{t-1}\mathbf{Q}_{t-1}{}^{T}\mathbf{D}_{pt-1}{}^{T})=$$

$$\begin{bmatrix}
\overline{Q^2_{1,t-1}}\alpha_1^2 p^*_{11} & \overline{Q_{1,t-1}Q_{2,t-1}}\alpha_1\alpha_2 p^*_{12} & - & \overline{Q_{1,t-1}Q_{n,t-1}}\alpha_1\alpha_n p^*_{1n} \\
\overline{Q_{2,t-1}Q_{1,t-1}}\alpha_2\alpha_1 p^*_{21} & \overline{Q^2_{2,t-1}}\alpha_2^2 p^*_{22} & - & \overline{Q_{2,t-1}Q_{n,t-1}}\alpha_2\alpha_n p^*_{2n} \\
- & - & - & - \\
\overline{Q_{n,t-1}Q_{1,t-1}}\alpha_n\alpha_1 p^*_{n1} & \overline{Q_{n,t-1}Q_{2,t-1}}\alpha_n\alpha_2 p^*_{n2} & - & \overline{Q^2_{n,t-1}}\alpha_n^2 p^*_{nn}
\end{bmatrix}$$

where the probability p_{ij}^* denotes the joint probability that the pair of work stations is not broken down.

$$p_{ii}^*=(1-p_i)$$

$$p_{ij}^*=(1-p_i)\,(1-p_j)\ ;\ i\neq j$$

Let $\mathbf{e}_{ij}$ be matrix of only the (i,j) element has value 1,and all other elements are 0.

For example, $\mathbf{e}_{23}$ means

$$\mathbf{e}_{23}=\begin{pmatrix} 0\,0\,0\,0 \\ 0\,0\,1\,0 \\ 0\,0\,0\,0 \\ 0\,0\,0\,0 \end{pmatrix}$$

Then the third term is as follows

$$E(\mathbf{D}_{pt-1}\ \mathbf{Q}_{t-1}\mathbf{Q}_{t-1}{}^{T}\mathbf{D}_{pt-1}{}^{T})-E(\mathbf{D}_{pt-1})E(\mathbf{Q}_{t-1})E(\mathbf{Q}_{t-1}{}^{T})E(\mathbf{D}_{pt-1}{}^{T})$$

$$=\sum_{i=1}^{n}\alpha_i^2\{(1-p_i)\overline{Q^2_{i,t-1}}-(1-p_i)^2\overline{Q}^2_{i,t-1}\}\mathbf{e}_{ii}$$

$$+\sum_{i=1}^{n}\ \sum_{j=1,j\neq i}^{n}\ \alpha_i\alpha_j(1-p_i)(1-p_j)\{\overline{Q_{i,t-1}Q_{j,t-1}}-\overline{Q}_{i,t-1}\overline{Q}_{j,t-1}\}\mathbf{e}_{ij}$$

$$=\sum_{i=1}^{n}\alpha_i^2 p_i(1-p_i)\{Var(Q_{i,t-1})+\overline{Q}^2_{i,t-1}\}\mathbf{e}_{ii}$$

$$+\sum_{i=1}^{n}\{\alpha_i^2(1-p_i)^2 Var(Q_{i,t-1})\mathbf{e}_{ii}+\sum_{j=1,j\neq i}^{n}\alpha_i\alpha_j(1-p_i)(1-p_j)Cov(Q_{i,t-1},Q_{j,t-1})\mathbf{e}_{ij}\}$$

$$=\sum_{i=1}^{n}\alpha_i^2 p_i(1-p_i)\{Var(Q_{i,t-1})+\overline{Q}_{i,t-1}^2\}\mathbf{e}_{ii}+E(\mathbf{D}_{pt})\,Var(\mathbf{Q}_{t-1})E(\mathbf{D}_{pt}^T)$$

$$=\mathbf{D}\,\mathbf{D}\,(\mathbf{I}-\mathbf{P}_r)\mathbf{P}_r\,\mathbf{E}_2(\,\mathbf{Q}_{t-1}\,)+\mathbf{D}(\mathbf{I}-\mathbf{P}_r)\,Var(\mathbf{Q}_{t-1})\,(\mathbf{D}(\mathbf{I}-\mathbf{P}_r)\,)^T$$

$\mathbf{E}_2(\,\mathbf{Q}_{t-1}\,)$ denotes diagonal matrix consists of the expectation of square of $Q_{i,t-1}$.

$$\{\,\mathbf{E}_2(\,\mathbf{Q}_{t-1}\,)\,\}_{ii}=E\,(\,Q_{i,t-1}^2\,)=Var(\,Q_{i,t-1})+\;E\,(\,Q_{i,t-1}\,)^2$$

The covariance matrix of $\mathbf{Q}_t$ is expressed as the following form.

$$\begin{aligned}Var(\mathbf{Q}_t) = \;& Var(\,\varepsilon_t)\\ &+\{\,\mathbf{I}-(\,\mathbf{I}-\Phi)\,\mathbf{D}\,(\mathbf{I}-\mathbf{P}_r)\,\}\,Var(\mathbf{Q}_{t-1})\,\{\,\mathbf{I}-(\,\mathbf{I}-\Phi\,)\,\mathbf{D}\,(\mathbf{I}-\mathbf{P}_r)\,\}^T\\ &+(\,\mathbf{I}-\Phi\,)\,\mathbf{D}\,\mathbf{D}\,(\mathbf{I}-\mathbf{P}_r)\mathbf{P}_r\,\mathbf{E}_2(\,\mathbf{Q}_{t-1}\,)\,(\,\mathbf{I}-\Phi\,)^T\end{aligned}$$

In order to get the steady state solution, let $Var(\mathbf{Q}_t^k)$ be the k-th asymptotical solution. The first approximate solution is

$$Var(\mathbf{Q}_t^1)=\;Var(\varepsilon_t)+(\,\mathbf{I}-\Phi\,)\,\mathbf{D}\,\mathbf{D}\,(\mathbf{I}-\mathbf{P}_r)\mathbf{P}_r\,\mathbf{E}_2(\mathbf{Q}_t^0)\,(\,\mathbf{I}-\Phi\,)^T$$

where $\mathbf{E}_2(\mathbf{Q}_t^0)$ is expressed as

$$\left(\mathbf{E}_2(\mathbf{Q}_{t-1}^0)\right)_{ii}=E(Q_{i,t})^2$$

The k-th asymptotical solution is as follows.

$$\begin{aligned}Var(\mathbf{Q}_t^k) = \;& Var(\,\varepsilon_t)\\ &+\{\,\mathbf{I}-(\,\mathbf{I}-\Phi\,)\,\mathbf{D}\,(\mathbf{I}-\mathbf{P}_r)\,\}\,Var(\mathbf{Q}_t^{k-1})\,\{\,\mathbf{I}-(\,\mathbf{I}-\Phi)\,\mathbf{D}\,(\mathbf{I}-\mathbf{P}_r)\,\}^T\\ &+(\,\mathbf{I}-\Phi\,)\,\mathbf{D}\,\mathbf{D}\,(\mathbf{I}-\mathbf{P}_r)\mathbf{P}_r\,\mathbf{E}_2(\,\mathbf{Q}_t^{k-1}\,)\,(\,\mathbf{I}-\Phi\,)^T\end{aligned}$$

$$\{\,\mathbf{E}_2(\,\mathbf{Q}_t^{k-1}\,)\,\}_{ii}=E\,((\,Q_i^{k-1})^2\,)=Var(\,Q_i^{k-1})+\;E\,(\,Q_{i,t}\,)^2$$

The aymptotical solution will converge if it can be shown that a finite K such that

$$\|\,Var(\mathbf{Q}_t^k)-Var(\mathbf{Q}_t^{k-1})\,\|<\|\,Var(\mathbf{Q}_t^{k-1})-Var(\mathbf{Q}_t^{k-2})\,\|$$

for all k>K.

2.2.2. The average and variance of production

As described above, the relationship between production and queue is defined in the Tactical Planning Model. It is

$$P_{i,t}= \alpha_i Q_{i,t} \quad \text{--- with probability} \quad 1-p_i$$
$$= 0.0 \quad \text{--- with probability} \quad p_i$$

Therefore the average and the square average are calculated without any difficulty.

$$E(P_{i,t})=(1-p_i)\, \alpha_i E(Q_{i,t})$$
$$E(P_{i,t}^2)=(1-p_i)\, (\alpha_i)^2 E((Q_{i,t})^2)$$
$$Var(P_{i,t})= (1-p_i)\, (\alpha_i)^2 [Var (Q_{ii,t}) -p_i E(Q_{i,t})^2]$$

Using the result of the previous section, expectation of production can be written as

$$\begin{aligned} E(\mathbf{P}_t) &= E(\mathbf{D}_{pt}) \{ (\mathbf{I}\text{-}\mathbf{\Phi}) E(\mathbf{D}_{pt}) \}^{-1} E(\varepsilon_t) \\ &= \mathbf{D} (\mathbf{I}\text{-} \mathbf{P}_r) \{ (\mathbf{I}\text{-}\mathbf{\Phi}) \mathbf{D} (\mathbf{I}\text{-} \mathbf{P}_r) \}^{-1} \mu \\ &= (\mathbf{I}\text{-}\mathbf{\Phi})^{-1} \mu \end{aligned}$$

This equation shows that the expectation of the production does not depend on the unreliability of the work station for the long run.

2.3 Capacity Constraints

In the original Tactical Planning Model, the effect of the production constraints were treated as an implicit factor. It can be avoided by setting appropriate planned lead time and resource reallocation. But in the model discussed in the previous section, the explicit involvement of the production control should be taken into account, because after the work station break down, the queue of the station will increase, and it is likely that the production will exceed the production capacity.

Therefore we apply the following production capacity constraint in each work station.

$$P_{it}=min\{\alpha_i Q_{it}, P_{c,it}\}$$

In this case $P_{c,it}$ is the production capability at work center i in time period t. This type of rule was tested by simulation by Cruickshanks, Drescher and Graves (1984)[5] for a simpler case with one production stage. We apply this rule in the multiple work station case and see the effect by the use of dynamic simulation.

Chapter 3. Simulation model.

3.1 Systems dynamics simulation

In this paper, we use a personal computer in order to carry out the analytical calculation and Monte Carlo simulation, because the development of the personal computer has made it possible for the manager to have his/her own tool to evaluate the management problems. As for the analytical approach described in the chapter 2, we use BASIC to model and implement the problem. The detail of the program is described in Appendix B. In this analytical approach, we model the unreliable job shop without the explicit production constraints. Because it is very difficult to apply analytical approach to the model with explicit constraints, we apply the dynamic simulation model to solve this problem.

There are several software packages available to carry out a simulation. For example, Banks and Carson[5] describe GASP, SIMSCRIPT, GPSS, and SLAM as the special purpose simulation language. In this paper, we select the STELLA program to do simulation on the personal computer.

The STELLA, designed by Barry Richmond[3], is a software to simulate the dynamics of social and physical systems. It stands for Structural Thinking, Experiential Learning Laboratory with Animation. It solves systems of differential or difference equations using the following operaters; Stock, Flow & Flow Regulator, Input Link and Converter.

The first operator, Stock, is the variable to be differentiated by time. Let the differential equation be

$$\frac{dx}{dt} = ay - bz$$

where the stock should be x. When we want to represent a difference equation, such as a balance equation

$$X_i = X_{i-1} + A_i - B_i$$

the stock represents X_i.

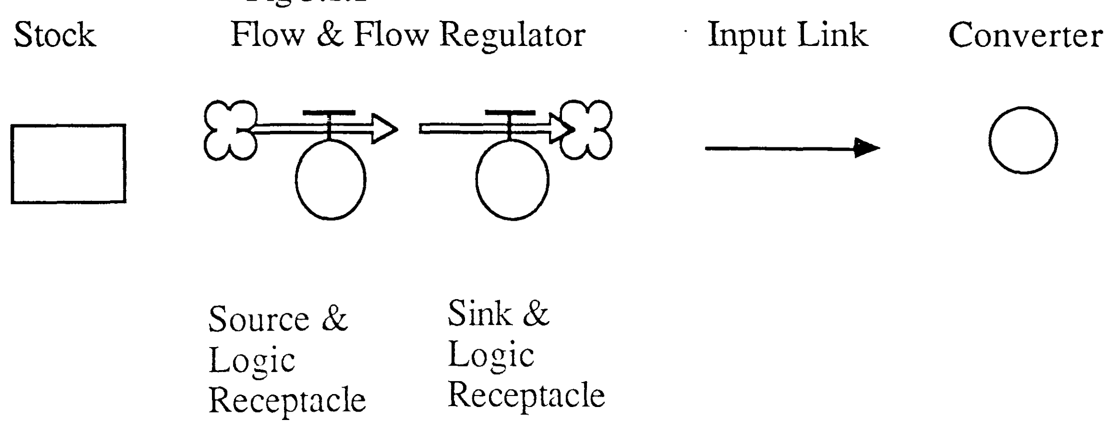
The second operator, Flow and Flow Regulater, has two varieties. One is Source and Logic Receptacle. It represents "ay " of the above differential equation and can be interpretted as the "arrival" rate to the stock. The other is Sink and Logic Receptacle. It represents "bz" and corresponds to the "departure rate" from the stock. The Logic Receptacle works as a control valve of the flow, and we can process the signal to the form of arrival and departure of the differential equations. An infinite amount of flow is assumed in the source. The sink is also assumed to have an infinite capability.

The third operater is Input Link which is the directed arc of the signal flow. In the above differential equation, Input Link brings the output of some other equation "y" and "z" to the Flow and Flow Regulator. In general it indicates signal flow relationship between one operator to another.

The fourth operator is a Converter. It converts several inputs into another form. Mathematical function, some logical operation, random variable generation can be used in this operator.

We can use these operators to make a simulation model of this problem.

Fig 3.1.1

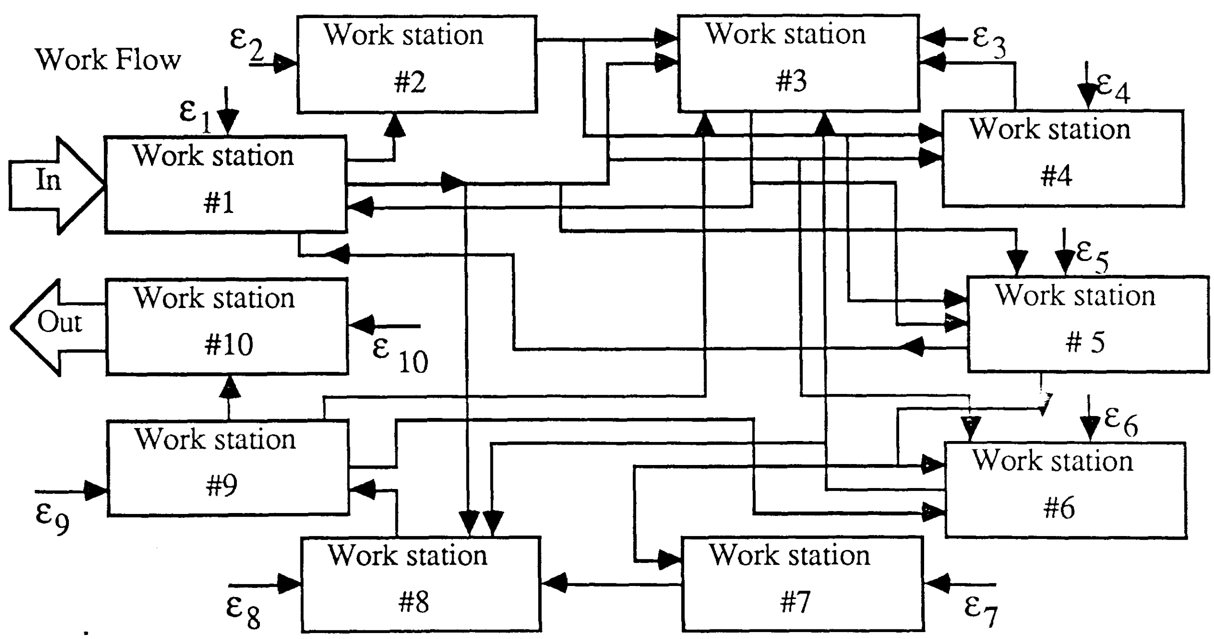


3.2 STELLA model

3.2.1 Multiple work stations model

In this section, we would like to show the multiple work stations simulation model in the Tactical Planning model. The following chart shows the example of work flow between the work stations.

Fig 3.2.1-1 Work Flow example



As described in Chapter 2, the balance equation of the each work station, in this case the # i work station, is as follows

$$Q_{i,t} = Q_{i,t-1} - P_{i,t-1} + A_{i,t}$$

$$A_{i,t} = \phi_{i1} P_{1,t-1} + \phi_{i2} P_{2,t-1} + \dots + \phi_{in} P_{n,t-1} + \varepsilon_{i,t}$$

$$P_{i,t} = \alpha_i Q_{i,t} \quad \text{--- with probability} \quad 1-p_i$$

$$= 0.0 \quad \text{--- with probability} \quad p_i$$

$$i = 1, 2, \cdots, n$$

Each work station has multiple inputs from the other work stations, $A_{i,t}$, and then its output(production: $P_{i,t}$) goes to several work stations.

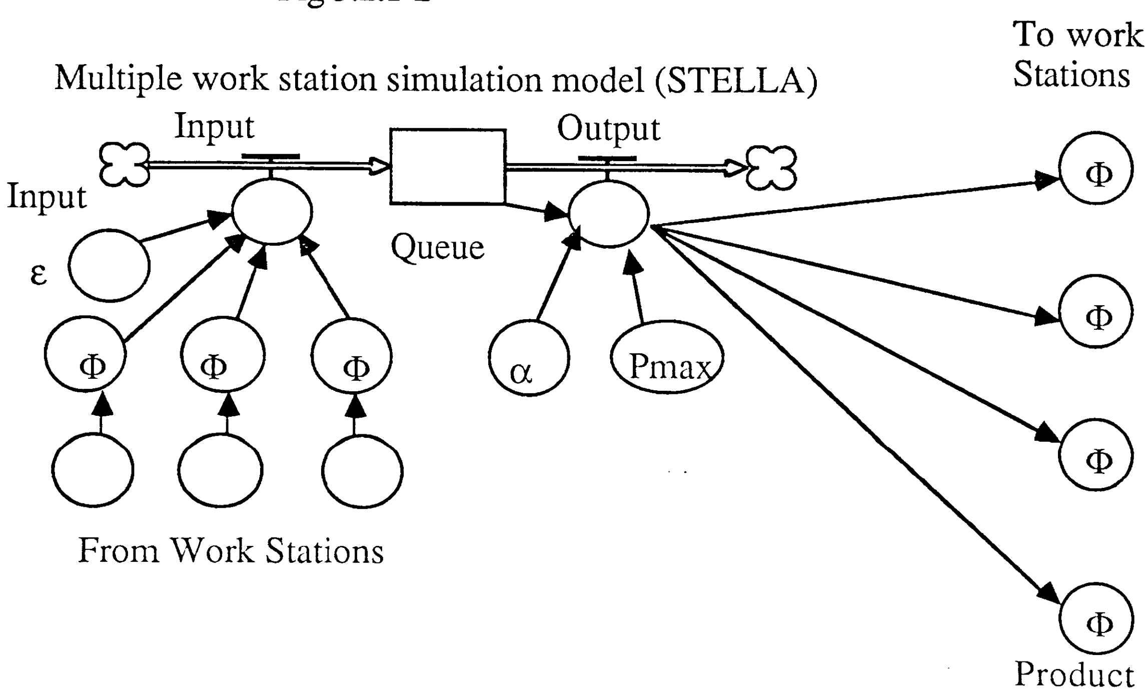
The station receives the input flow with uncertain random inputs.

In this example, work station #1 is the first work station which receives the initial material from the outside of the system. It also receives the work flow from work stations #3 and #5. The work flow goes to the other stations, work stations #2, #3, #4, #5 and #8. After several stages, some of the work load goes back to the work station #1 again.

The following shows the example of the work station model described by the STELLA operators. The noise term of the flow, $\varepsilon_{i,t}$, is modeled as the combination of gaussian normal noise and constant term.

Fig 3.2.1-2

Multiple work station simulation model (STELLA)



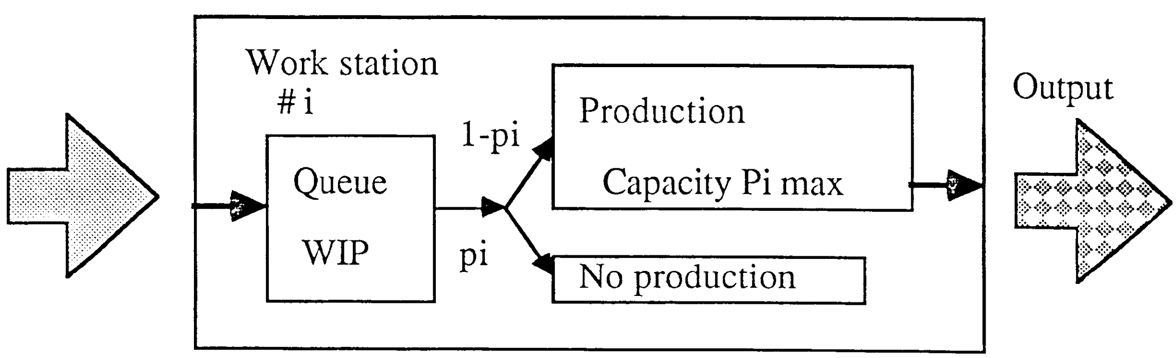
3.2.2 Work station break down and the production constraints model

As discussed in Chapter 2 the unreliability of the work station is the probability of work station break down. When the work station breaks down, the productive activity will stop. The work in process inventory will be increased. The inverse of planned lead time, α_i, is modeled as a Bernoulli random variable $a_{i,t}$ as discussed in 2.2.

$$P_{i,t} = \alpha_i Q_{i,t} \quad \text{--- with probability } 1\text{-}p_i$$
$$= 0.0 \quad \text{--- with probability } p_i$$

$$\leftrightarrow \; P_{i,t} = a_{i,t} Q_{i,t}$$
$$a_{i,t} = \alpha_i \quad \text{--- with probability } 1\text{-}p_i$$
$$= 0.0 \quad \text{--- with probability } p_i$$

Fig 3.2.2.-1 unreliable work station



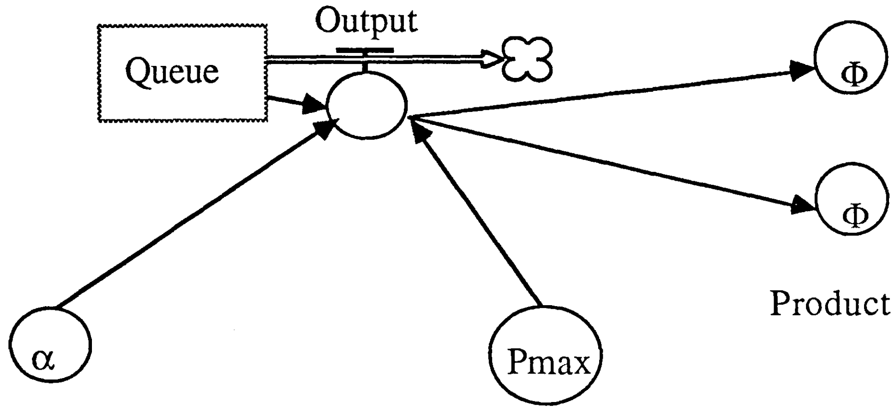
The production constraint is modeled as the simple constraint. If the level of the production given by the control rule exceeds the maximum capacity of the work station, the actual production is set to the maximum capacity.

$$P_{it} = \min\left\{ \alpha_i Q_{it}, P_{c,it} \right\}$$

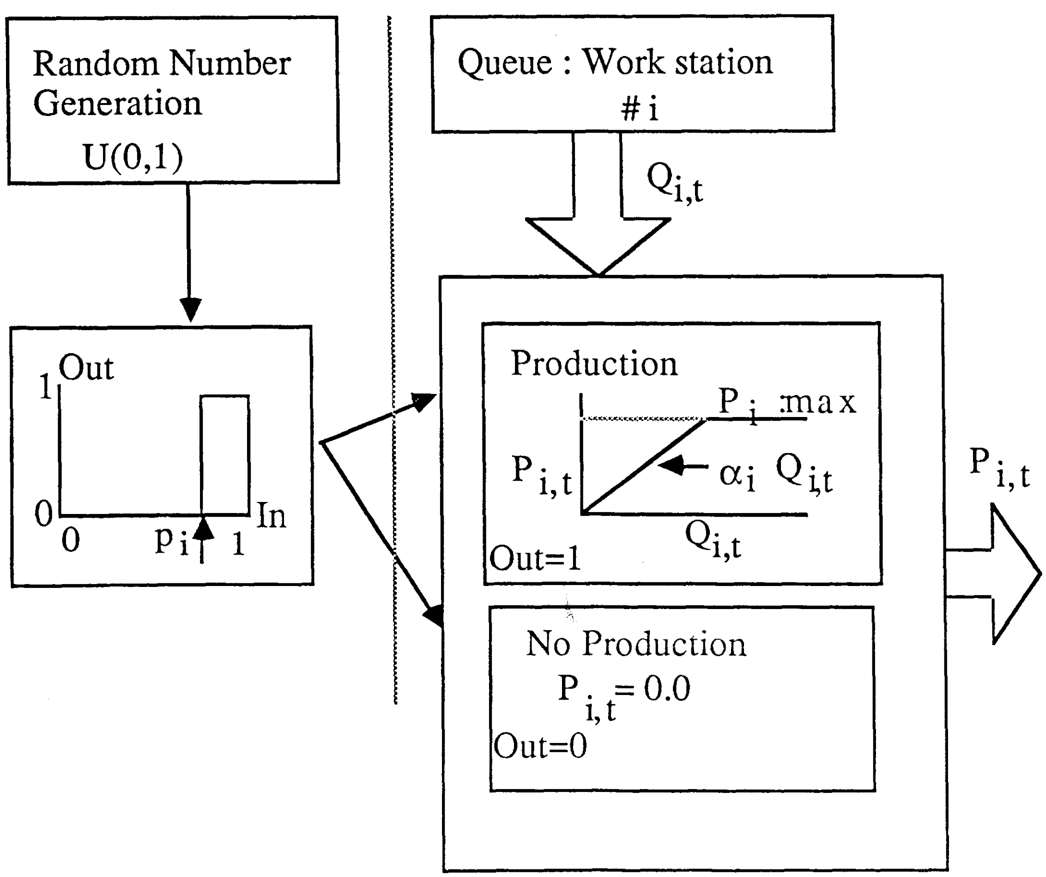
where $P_{c,it}$ is the maximam capacity of production of work station # i.

In the STELLA simulation model, the unreliable work station is modeled by comparing a uniformly distributed random variable to a threshold level to decide working(1) or not working(0). If the random variable is more than p_i, the production is decided to be normal. This process is modeled in the node "Conversion". The production constraint is modeled as an "if" sentence. (IF required Production level is greater than the maximam value, let P be the maximum production.)

Fig3.2.2-2 STELLA model of work station break down and Production constrains



Work station break down Production constraints



3.2.3 Backlog model

The backlog is defined as the amount of the queue that has waited m or more periods, for m=0,1,2,⋯n. For a given m units of time, then the backlog is defined by the following equation.

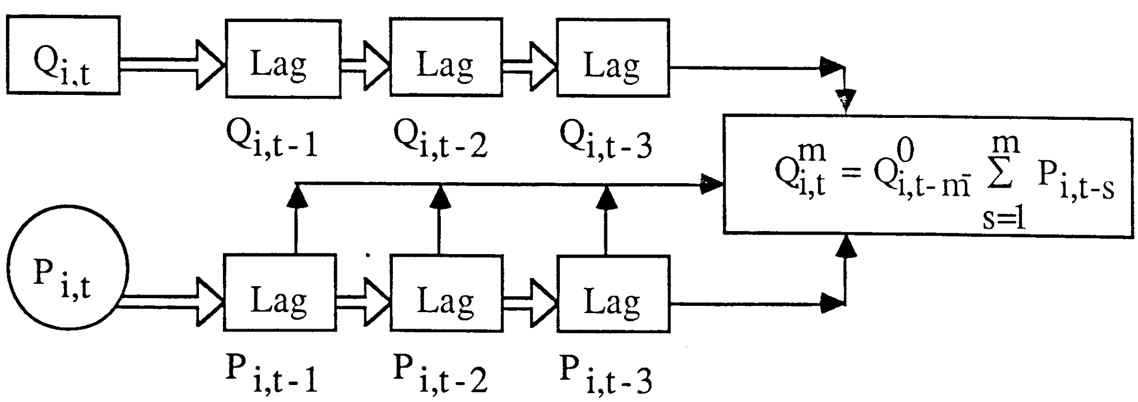
$$Q^{m}_{it} = Q^{m-1}_{i,t-1} - P_{i,t-1}$$
$$= Q_{i,t-m} - \sum_{s=1}^{m} P_{i,t-s}$$

For example, when the planned lead time m is 3, the equation can be written as the following flow chart. It needs $Q_{i,t-3}$, and the past three periods of production level.

Fig. 3.2.3-1

Backlog model

Example m=3



In the STELLA model, the lag element can be modeled by applying a STACK element. As the output of the lag element in time period of t should be the input of the lag element at time *t-1*, the relation can be modeled as follows.

$$Q_{lag(n),i,t} = Q_{lag(n),i,t-1} + (Q_{lag(n-1),i,t-1} - Q_{lag(n),i,t-1})$$

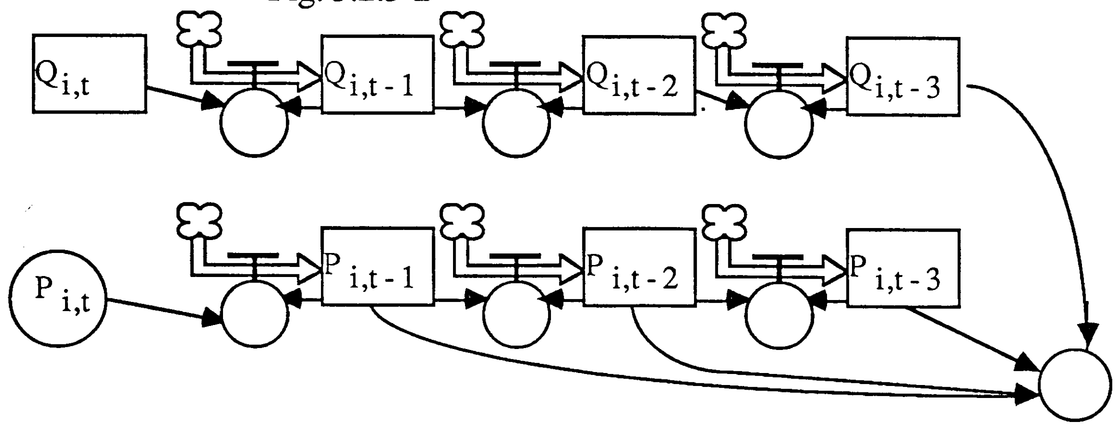
$$Q_{lag(1),i,t} = Q_{lag(1),i,t-1} + (Q_{i,t-1} - Q_{lag(1),i,t-1})$$

$$P_{lag(n),i,t} = P_{lag(n),i,t-1} + (P_{lag(n-1),i,t-1} - P_{lag(n),i,t-1})$$

$$P_{lag(1),i,t} = P_{lag(1),i,t-1} + (P_{i,t-1} - P_{lag(1),i,t-1})$$

Then the flow diagram of the back log is represented as in the next figure.

Fig. 3.2.3-2



3.2.4 Output process

In order to evaluate the performance of the work station, it is necessary to take expectation and variance of the production and queue. Let x be either production or queue, then $E(x)$, the expectation of x , and $Var(x)$, the variance of x , are

$$E(x) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$Var(x) = \frac{1}{n-1}\sum_{i=1}^{n} (x_i - E(x))^2$$

$$= \frac{n}{n-1}(E(x^2) - E(x)^2)$$

where n denotes the number of data. Taking a sample every unit of time, n is equivalent to time of the observation. Let μ_n be the expectation of x up to x_n, $\mu_{2,n}$ be the expectation of x^2 up to x_n^2 and σ_n^2 be the sample variance up to x_n. The above equation can be modeled as the following difference equations.

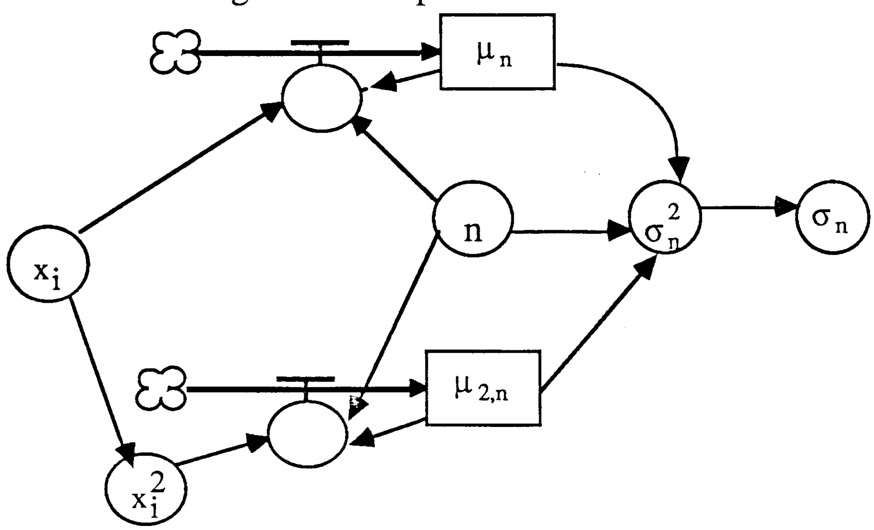
$$\mu_n = \mu_{n-1} + \frac{1}{n}(x_n - \mu_{n-1})$$

$$\mu_{2,n} = \mu_{2,n-1} + \frac{1}{n}(x_n^2 - \mu_{2,n-1})$$

$$\sigma_n^2 = \frac{n}{n-1}(\mu_{2,n-1} - \mu_{n-1})$$

The flow diagram of STELLA is following figure.

Fig 3.2.4- 1 Expectation and variance



The full flow diagrams and equations are shown in Appendix A. And we also show some examples of outputs.

Chapter 4. The result of the analysis and simulation

In this chapter, we show the result of the analysis and simulation. The analysis is based upon the equations derived in Chapter 2. We carry out our calculation using a BASIC program on the personal computer. The detail flow chart and program are described in Appendix A. The simulation is carried out using STELLA as described in Chapter 3. In each simulation, the duration of simulation is 2000 units of time, and the result is obtained from six simulation runs for each case. The full program and operation manual are described in Appendix B.

The applied model in this paper is based upon the examples in Graves's paper. The job shop consists of ten work stations and its work flow is described in Chapter 2 and 3. The flow matrix between each work station, average inputs μ and covariance matrix Σ are in table 4-1.

Table 4-1

From work station (#)	To work station (#) 1	2	3	4	5	6	7	8	9	10
1		0.15	0.04	0.01	0.03	0.24		0.01		
2			0.01	0.04	0.37					
3	0.11				1.36					
4			0.71							
5	0.68					0.15	0.01			
6			0.06					0.22		
7								1.00		
8									3.43	
9			0.07			0.13				1.16
10										
μ	4.0	0	0	0	0	0	0	0	0	0
Σ	4.00	0.01	0.01	0.01	0.04	0.04	0	0.01	0.04	0.04

The work is assumed to flow from one work station to another in two hour periods. The capacities of work stations are 2 units of work, except work station #1 and #10.

As there are there machines are available in #1, the capacity is assumed to be six units of work. Also it is assumed that there are 2.5 units of work capability at work station #10. (One unit of work needs one hour of machine operation.)

4.1 The validation of the simulation model

In this section, we would like to validate the simulation model by comparing its result to the result of Graves's paper. In order to compare the result, we select Case D of his paper. In this case, the planned lead times are given by the following table.

Table 4.1.-1 Planned lead time of case D

W.s.	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10
n(A)	8	1	1	1	2	3	1	2	4	5

At the following tables, "Original model" is the data of Graves's, and "95% High (Low)" is the upper (lower) confidence level of the expected value.

(1) Work station #1 (Planned lead time = 8)

	E(P)	σ_p	E(Q)	$E(Q^m)$
1.Original model	5.01	0.55	40.070	1.050
2.Simualtion (mean)	5.031	0.547	40.312	1.028
95% High	5.074	-	40.702	1.052
95%Low	4.988	-	39.921	1.004
Evaluation	ok	ok	ok	ok

This result shows that the obtained expected production, expected queue and expected backlog are equivalent to the result of the Graves's model with the 95% confidence level[6]. And the standard deviation is effectively equivalent. The evaluation is the same on the following work stations. Therefore we can use the simulation model with confidence[7].

(2) Work station #2 (Planned lead time = 1)

	$E(P)$	σ_p	$E(Q)$	$E(Q^m)$
1.Original model	0.75	0.13	0.75	0.0
2.Simualtion (mean)	0.754	0.128	0.755	0.000
95% High	0.762	-	0.764	0.000
95%Low	0.746	-	0.746	0.000
Evaluation	ok	ok	ok	ok

(3) Work station #3 (Planned lead time = 1)

	$E(P)$	σ_p	$E(Q)$	$E(Q^m)$
1.Original model	0.69	0.14	0.69	0.0
2.Simualtion (mean)	0.697	0.143	0.697	0.000
95% High	0.705	-	0.706	0.000
95%Low	0.688	-	0.688	0.000
Evaluation	ok	ok	ok	ok

(4) Work station #4 (Planned lead time = 1)

	$E(P)$	σ_p	$E(Q)$	$E(Q^m)$
1.Original model	0.36	0.11	0.36	0.0
2.Simualtion (mean)	0.359	0.115	0.359	0.000
95% High	0.364	-	0.363	0.000
95%Low	0.355	-	0.355	0.000
Evaluation	ok	ok	ok	ok

(5) Work station #5 (Planned lead time = 2)

	$E(P)$	σ_p	$E(Q)$	$E(Q^m)$
1.Original model	1.37	0.20	2.74	0.06
2.Simualtion (mean)	1.377	0.200	2.753	0.061
95% High	1.393	-	2.786	0.062
95%Low	1.360	-	2.720	0.061
Evaluation	ok	ok	ok	ok

(6) Work station #6 (Planned lead time = 3)

	$E(P)$	σ_p	$E(Q)$	$E(Q^m)$
1.Original model	1.65	0.18	4.97	0.07
2.Simualtion (mean)	1.659	0.170	4.987	0.066
95% High	1.672	-	5.034	0.067
95%Low	1.646	-	4.940	0.065
Evaluation	ok	ok	ok	ok

(7) Work station #7 (Planned lead time = 1)

	$E(P)$	σ_p	$E(Q)$	$E(Q^m)$
1.Original model	0.14	0.02	0.14	0.0
2.Simualtion (mean)	0.138	0.020	0.138	0.000
95% High	0.140	-	0.140	0.000
95%Low	0.136	-	0.136	0.000
Evaluation	ok	ok	ok	ok

(8) work station #8 (Planned lead time = 2)

	E(P)	σ_p	E(Q)	$E(Q^m)$
1.Original model	0.55	0.08	1.10	0.02
2.Simualtion (mean)	0.554	0.079	1.110	0.024
95% High	0.558	-	1.120	0.024
95%Low	0.550	-	1.099	0.023
Evaluation	ok	ok	ok	ok

(9) Work station #9 (Planned lead time=4)

	E(P)	σ_p	E(Q)	$E(Q^m)$
1.Original model	1.89	0.22	7.56	0.12
2.Simualtion (mean)	1.897	0.214	7.594	0.118
95% High	1.909	-	7.649	0.121
95%Low	1.885	-	7.539	0.116
Evaluation	ok	ok	ok	ok

(10) Work station #10 (Planned lead time = 5)

	E(P)	σ_p	E(Q)	$E(Q^m)$
1.Original model	2.19	0.23	10.96	0.13
2.Simualtion (mean)	2.200	0.226	11.016	0.129
95% High	2.212	-	11.087	0.131
95%Low	2.188	-	10.944	0.127
Evaluation	ok	ok	ok	ok

4.2 Analysis of the unreliable work station model

In 4.1, we validated the simulation model with reliable work stations by comparing the simulation results with the analytic results. In this section, first we validate the simulation of the unreliable work station model with the analytical model discussed in Chapter 2 by comparing both results. Next we show the result of the analysis of unreliable work station without production constraints.

4.2.1 The validation of the analytical model

We choose case D to compare results. The probabilities of break down are set to 0.1 for each work station, except work station #9 whose probability is set to 0.05. In the Table 4.2.1.-1, "work S." denotes work station, "P.L.T." denotes planning lead time, "P(B,d)" denotes probability of break down, "95% H" denotes upper bound of 95% confidence level and "95% L" denotes lower bound of 95% confidence level. Evaluation is based upon whether the result of the analysis is within the 95% confidence level of the result of the simulation. These results show that the analytical solution is within 95% confidence level of the estimated solution derived by the result of the simulation. From the above results, we can conclude that it is appropriate to apply both simulation model and analytical model to evaluate the performance of unreliable work stations.

Table 4.2.1-1

Work S.	1		2		3		4		5	
P.L.T.	8		1		1		1		2	
P(B.d)	0.1		0.1		0.1		0.1		0.1	
Method	Ana.	Sim.	Ana.	Sim.	Ana.	Sim.	Ana.	Sim.	Ana.	Sim.
E(P) m	5.009	4.981	0.751	0.747	0.694	0.69	0.358	0.357	1.372	1.364
95% H	-	5.064	-	0.759	-	0.701	-	0.364	-	1.385
95% L	-	4.897	-	0.734	-	0.679	-	0.349	-	1.343
σ_p	1.805	1.853	0.457	0.462	0.403	0.4000	0.269	0.269	0.617	0.622
E(Q) m.	44.525	44.557	0.835	0.816	0.771	0.766	0.398	0.396	3.048	3.043
95% H.	-	45.715	-	0.865	-	0.777	-	0.406	-	3.097
95% L.	-	43.399	-	0.766	-	0.756	-	0.385	-	2.988
Eval.	ok		ok		ok		ok		ok	

Work S.	6		7		8		9		10	
P.L.T.	3		1		2		4		5	
P(B.d)	0.1		0.1		0.1		0.05		0.1	
Method	Ana.	Sim.	Ana.	Sim.	Ana.	Sim.	Ana.	Sim.	Ana.	Sim.
E(P) m	1.654	1.640	0.137	0.137	0.551	0.546	1.890	1.872	2.193	2.171
95% H	-	1.668	-	0.138	-	0.555	-	1.900	-	2.206
95% L	-	1.611	-	0.135	-	0.538	-	1.844	-	2.137
σ_p	0.650	0.635	0.089	0.089	0.242	0.237	0.556	0.557	0.823	0.799
E(Q) m.	5.512	5.444	0.152	0.151	1.225	1.209	7.958	7.898	12.181	12.002
95% H.	-	5.544	-	0.154	-	1.226	-	8.024	-	12.234
95% L.	-	5.343	-	0.149	-	1.191	-	7.771	-	11.770
Eval.	ok		ok		ok		ok		ok	

4.2.2. The result of analysis

(1) The expectation of production and queue

In Chapter 2, we have shown that the expectation of production and queue can be written as following form .

$$E(\mathbf{P}_t) = (\mathbf{I}-\Phi)^{-1} \mu$$

$$E(Q_{i,t}) = \{(1-p_i)\,\alpha_i\}^{-1} E(P_{i,t})$$

From these two equations, the following facts can be derived.

(a) The unreliability of the work station does not have any influence on the expectation of production.

(b) The expected queueing level of the each work station depends on the planning lead time (α_i^{-1}) and the probability of break down of its own work station. The break down of the other work stations do not affect the expectation of the queue.

These results are based on the fact that because there are infinite capability of the production and queue, the effect of the break down is compensated by large production after recovering from the break down. Thus it is possible to catch up with the production requirement of the system.

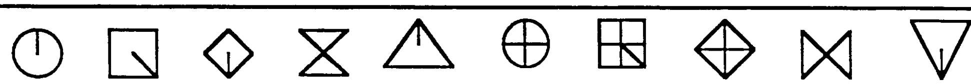
(2) The variance of production and queue

The effect of the break down of the work station increases the variance of the production and queue. As we have shown in Chapter 2, there are mutual and complicated interactions on the variance of production and queue. In order to show the effect of the unreliable work stations, we show the results of the calculation by the analytical model. The model shows the relationship due to increasing the unreliability at work station #1 on the standard deviation of each work station. At first, this is shown as the ratio ($Sr\text{-}1_i$) of the standard deviation and the expected production in Fig. 4.2.2.-1 and Fig. 4.2.2.-2. Second, this is shown as the ratio ($Sr\text{–}2_i$) of standard deviation with unreliable work station and reliable work station in Fig 4.2.2-3 and Fig 4.2.2-4. In order to check the effect of planned lead time, the case A and case D of Graves's example are calculated. The difference between case A and case D is the planned lead time of each work station, as shown in the Table 4.2.2.-2.

Table 4.2.2.-2 Planned lead time

W.s.	# 1	# 2	# 3	# 4	# 5	# 6	# 7	# 8	# 9	# 10
n(A)	1	1	1	1	1	1	1	1	1	1
n(D)	8	1	1	1	2	3	1	2	4	5

In the following figures, the data of the work stations are shown by the following symbols.



Symbols										
# of WS	# 1	# 2	# 3	# 4	# 5	# 6	# 7	# 8	# 9	# 10

The following table is the example of numerical output of the analytical model.

Tabel 4.2.2.-3 Example of numerical output of case D

*** A Tactical Planning Model for a Job Shop *
*** With Unreliable Work Station *

W	N	P -BD	E(Q)	S(Q)	E(P)	S(P)
1	8	0.020	40.890	4.678	5.009	0.920
2	1	0.000	0.751	0.170	0.751	0.170
3	1	0.000	0.694	0.148	0.694	0.148
4	1	0.000	0.358	0.123	0.358	0.123
5	2	0.000	2.743	0.419	1.372	0.209
6	3	0.000	4.961	0.565	1.654	0.188
7	1	0.000	0.137	0.021	0.137	0.021
8	2	0.000	1.102	0.161	0.551	0.080
9	4	0.000	7.561	0.882	1.890	0.221
10	5	0.000	10.963	1.127	2.193	0.225

*** A Tactical Planning Model for a Job Shop *
*** With Unreliable Work Station *

W	N	P -BD	E(Q)	S(Q)	E(P)	S(P)
1	8	0.800	200.362	36.785	5.009	10.227
2	1	0.000	0.751	1.537	0.751	1.537
3	1	0.000	0.694	0.649	0.694	0.649
4	1	0.000	0.358	0.644	0.358	0.644
5	2	0.000	2.743	1.950	1.372	0.975
6	3	0.000	4.961	3.342	1.654	1.114
7	1	0.000	0.137	0.097	0.137	0.097
8	2	0.000	1.102	0.590	0.551	0.295
9	4	0.000	7.561	3.237	1.890	0.809
10	5	0.000	10.963	3.858	2.193	0.772

$$Sr\text{-}1_i = \frac{\text{Standard deviation of production at work station \#i : \#1 unreliable}}{\text{Expected production of work station \#i}}$$

Fig 4.2.2.-1 Sr-1 case A

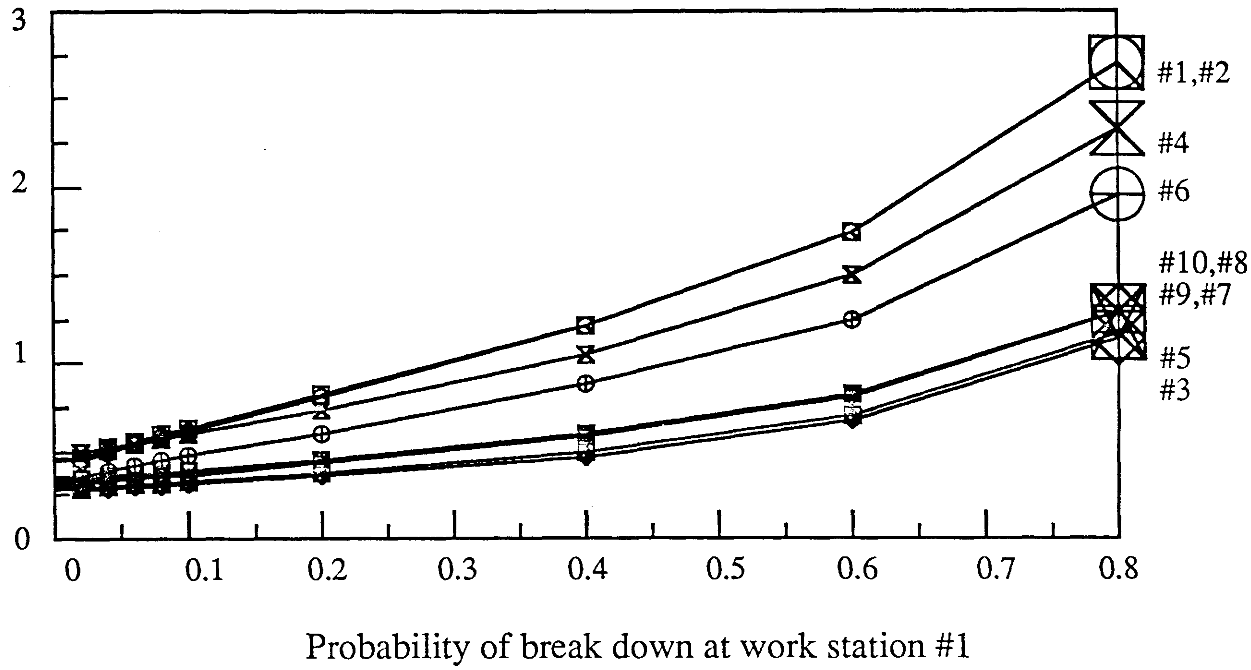
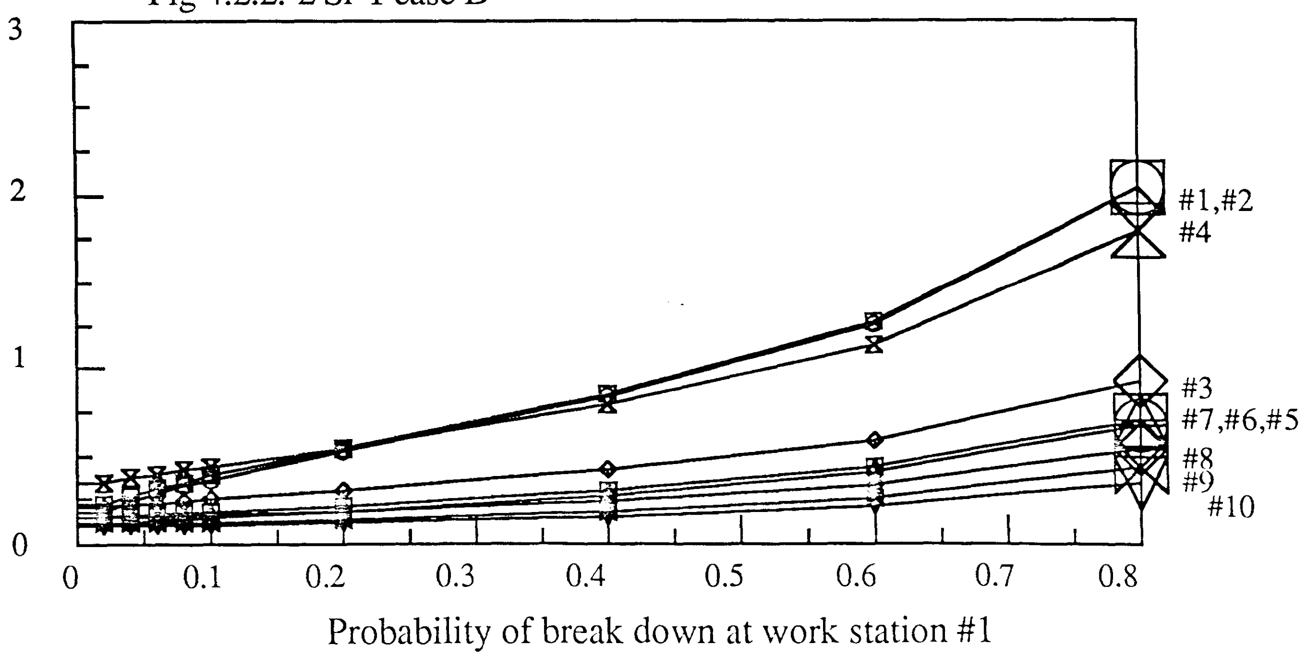


Fig 4.2.2.-2 Sr-1 case D



$$Sr\text{-}2\ _i = \frac{\text{Standard deviation of production at work station \#i : \#1 unreliable}}{\text{Standard deviation of production (case A) at work station \#i : \#1 reliable}}$$

Fig 4.2.2-3 Sr-2 (Case A)

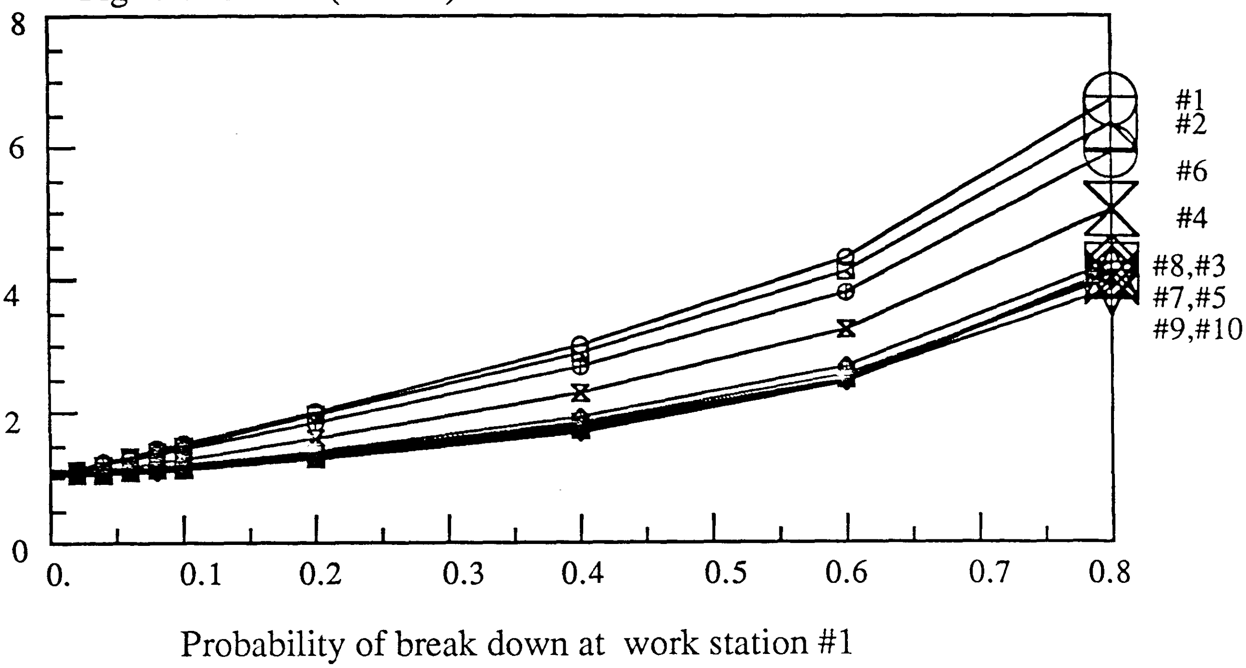
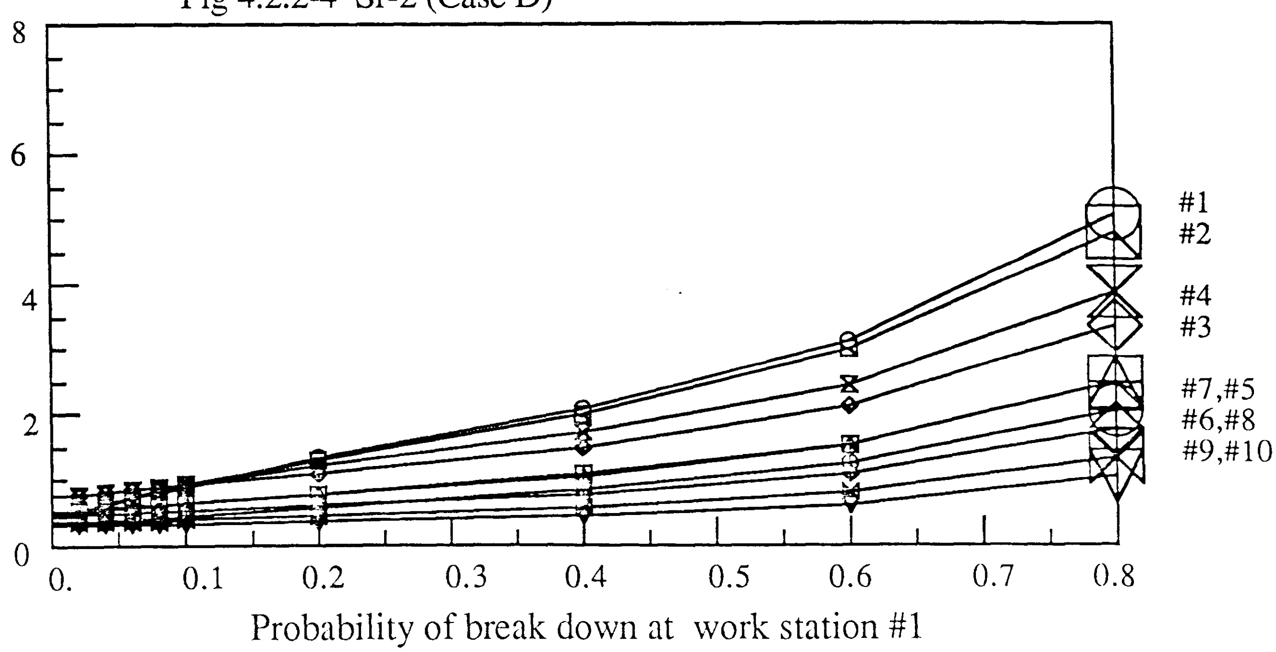


Fig 4.2.2-4 Sr-2 (Case D)



These graphs show the robustness of the case D on the variance of the production. If an appropriate planned lead time is applied to the unreliable job shop, it can make the production smooth. As for the variance of queue, we can derive the relationship from Chapter 2.

$$Var\,(Q_{ii,t})=\{(1-p_i)\,(\alpha_i)^2\}^{-1}[Var(P_{i,t})+p_i\,(1-p_i)^{-1}E(P_{i,t})^2]$$

Thus in the reliable work station, the standard deviation of queue is just proportional to the standard deviation of production and planned lead time.

$$E(Q_{i,t})=\{(1-p_i)\,\alpha_i\}^{-1}\,E(P_{i,t})$$

From the above result, we can conclude following.

(1) If it is required to smooth the level the production, it is necessary to increase the planned lead time for the appropriate work station. This method will work when there are unreliable work stations.

(2) If the work station is unreliable and planned lead time is applied, then the work in process inventory (queue to be produced) will increase and its variation will also increase.

(3) The average production of the work station does not change even if the work station is unreliable.

(4) The average queue of the work station is not affected by the other unreliable work stations. It is affected by the unreliability of the own work station.

We should mention that these conclusions are based upon having no capacity constraints at each work station.

In the following sections, we show the effect of production capacity constraints to this problem.

4.3 Simulation of the capacity constraints

In this section, we would like to show the example of the capacity constraints with reliable work stations. For the infinite capacity case, we applied the analytical approach. But for the finite capacity case, it is necessary to analyze the system with a help of the simulation. In order to show the characteristics of the capacity constraints we apply the model of the Graves where he applied implicit consideration about the constraints. The capacity constraints are shown in the following table.

Table 4.3.-1 Capacity constraints of work stations

W.s #	1	2	3	4	5	6	7	8	9	10
Const	6	2	2	2	2	2	2	2	2	2.5
E(P)	5.01	0.75	0.69	0.36	1.37	1.65	0.14	0.55	1.89	2.19
ρ	0.835	0.375	0.345	0.18	0.685	0.825	0.07	0.275	0.945	0.876
n(A)	1	1	1	1	1	1	1	1	1	1
n(D)	8	1	1	1	2	3	1	2	4	5

where ρ denotes the utilization load factor which is the ratio of expectation of production and capacity constraints. Capacity is defined as the production in the unit time period. At work station #1, it is possible to produce 6 units per time period, and at #10, it is possible to produce 2.5 units per time period. The other work station can produce or process 2 units per time period. E(P) is expected production of work in each work station. If the load factor is greater than one, the work station cannot complete its job within a given period of time. This table shows that work stations #1, #5, #6, #9 and #10 are heavily loaded, especially at work station #9, where the utilization load factor is 0.945. The queueing theory tells us, that there will be large amount of queue at the heavily loaded work station. The planned lead time of the example, case D, was applied mainly for these work stations. In the table above, n(A) and n(D) are planned lead time of simulation case A and case D.

The simulation was carried out and we show the result of the simulation in Fig 4.3.-1 and Table 4.3.-2. Fig 4.3.-1 shows the relationship between load factor and the effect of capacity constraints, and Table 4.3.-2 shows the result of simulation.

Fig 4.3.-1 The effect of the constraints on production variance of reliable work stations

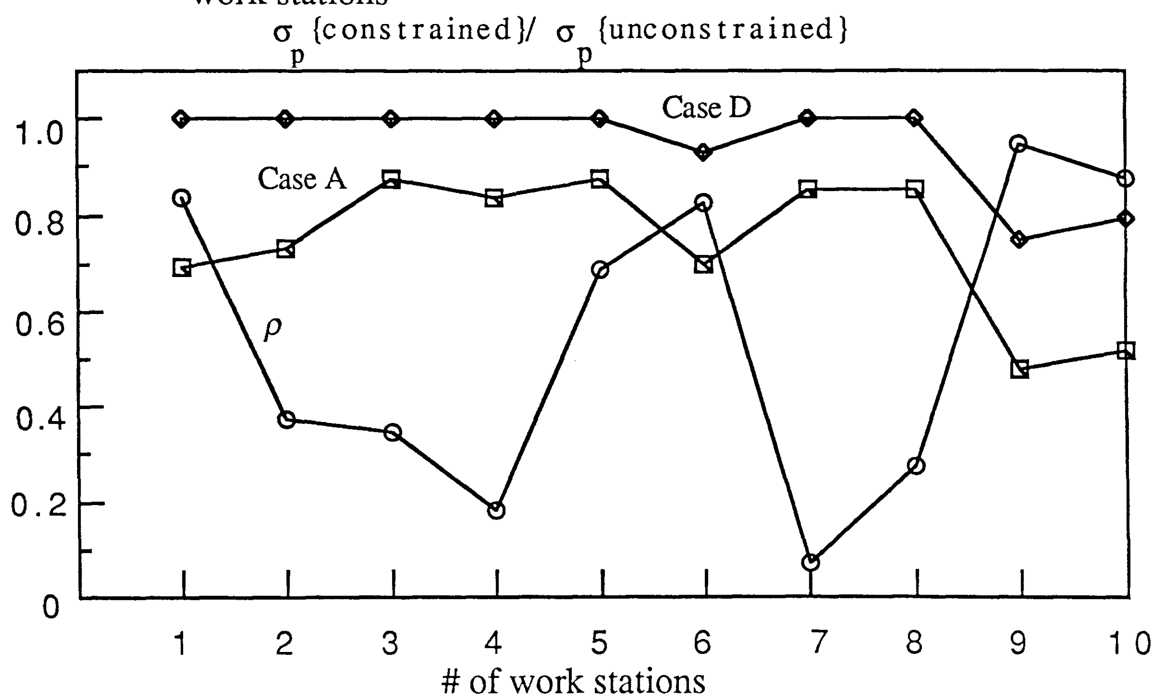


Fig 4.3-1 shows that the capacity constraints give large effect on the case A which uses a planned lead time of one for every work station. The capacity constraints give a smaller effect on the case D. When the appropriate planned lead time is applied, the variance of the production is decreased as shown on the table 4.3.-2. But at the work station #9, the change of the variance of production becomes the most significant, because the load factor at the work station #9 is the heaviest. This work station is the most saturated. On the other hand, there are no significant differences in the other

work stations when the case D is applied. Thus it is not necessary to calculate with constraints in these work stations.

Therefore if proper planned lead time and the capacity are selected, it is possible to avoid the use of a complicated full scale simulation.

Tabel 4.3.-2-A Simulation result of Case A

Work S.	1		2		3		4		5	
P.L.T.	1		1		1		1		1	
P(B.d)	0		0		0		0		0	
Const.	∞	6	∞	2	∞	2	∞	2	∞	6
E(P) m	5.01	4.982	0.75	0.747	0.69	0.69	0.36	0.357	1.37	1.365
95% H	-	5.067	-	0.760	-	0.703	-	0.364	-	1.386
95% L	-	4.896	-	0.735	-	0.678	-	0.349	-	1.343
σ_p	2.02	1.397	0.32	0.234	0.19	0.166	0.17	0.142	0.39	0.340
E(Q) m.	5.01	6.206	0.75	0.747	0.69	0.690	0.36	0.357	1.37	1.368
95% H.	-	6.446	-	0.760	-	0.703	-	0.364	-	1.390
95% L.	-	5.965	-	0.735	-	0.678	-	0.349	-	1.346
$E(Q_n)$m.	0	1.224	0	0	0	0	0	0	0	0.003
95% H.	-	1.418	-	0	-	0	-	0	-	0.004
95% L.	-	1.031	-	0	-	0	-	0	-	0.003

Work S.	6		7		8		9		10	
P.L.T.	1		1		1		1		1	
P(B.d)	0		0		0		0		0	
Const.	∞	2	∞	2	∞	2	∞	2	∞	2.5
E(P) m	1.65	1.641	0.14	0.137	0.55	0.546	1.89	1.876	2.19	2.175
95% H	-	1.670	-	0.139	-	0.555	-	1.905	-	2.212
95% L	-	1.612	-	0.135	-	0.537	-	1.846	-	2.138
σ_p	0.54	0.378	0.04	0.034	0.17	0.144	0.61	0.292	0.74	0.381
E(Q) m.	1.65	1.685	0.14	0.137	0.55	0.546	1.89	4.830	2.19	2.199
95% H.	-	1.718	-	0.139	-	0.555	-	6.172	-	2.239
95% L.	-	1.651	-	0.135	-	0.537	-	3.487	-	2.159
$E(Q_n)$m.	0	0.043	0	0	0	0	0	2.957	0	0.024
95% H.	-	0.051	-	0	-	0	-	4.286	-	0.029
95% L.	-	0.036	-	0	-	0	-	2.159	-	0.020

Tabel 4.3.-2-D Simulation result of Case D

Work S.	1		2		3		4		5	
P.L.T.	8		1		1		1		2	
P(B.d)	0		0		0		0		0	
Const.	∞	6	∞	2	∞	2	∞	2	∞	6
E(P) m	5.01	4.982	0.75	0.747	0.69	0.69	0.36	0.357	1.37	1.365
95% H	-	5.066	-	0.760	-	0.702	-	0.364	-	1.386
95% L	-	4.897	-	0.734	-	0.679	-	0.349	-	1.344
σ_p	0.55	0.554	0.13	0.131	0.14	0.140	0.11	0.116	0.20	0.197
E(Q) m.	40.07	39.977	0.75	0.747	0.69	0.690	0.36	0.357	2.74	2.730
95% H.	-	40.682	-	0.760	-	0.702	-	0.364	-	2.772
95% L.	-	39.271	-	0.734	-	0.679	-	0.349	-	2.687
$E(Q_n)$m.	1.05	1.127	0	0	0	0	0	0	0.06	0.061
95% H.	-	1.213	-	0	-	0	-	0	-	0.063
95% L.	-	1.041	-	0	-	0	-	0	-	0.059

Work S.	6		7		8		9		10	
P.L.T.	3		1		2		4		5	
P(B.d)	0		0		0		0		0	
const.	∞	2	∞	2	∞	2	∞	2	∞	2.5
E(P) m	1.65	1.641	0.14	0.137	0.55	0.546	1.89	1.876	2.19	2.175
95% H	-	1.670	-	0.138	-	0.555	-	1.905	-	2.212
95% L	-	1.613	-	0.135	-	0.537	-	1.846	-	2.139
σ_p	0.18	0.167	0.02	0.020	0.08	0.078	0.22	0.166	0.23	0.181
E(Q) m.	4.97	4.726	0.14	0.137	1.10	1.093	7.56	9.472	10.96	10.878
95% H.	-	5.012	-	0.138	-	1.110	-	10.633	-	11.061
95% L.	-	4.840	-	0.135	-	1.075	-	8.310	-	10.695
$E(Q_n)$m.	0.07	0.067	0	0	0.02	0.023	0.12	2.039	0.13	0.107
95% H.	-	0.068	-	0	-	0.024	-	3.144	-	0.111
95% L.	-	0.066	-	0	-	0.023	-	0.934	-	0.103

4.4 Simulation of unreliable work stations and capacity constraints

In the previous sections, we have shown the individual effect of the unreliable work station and of the capacity constraint. In this section, we show an example of the simulation with both unreliable work stations and capacity constraints. We carry out the simulation for following eight cases which are denoted as A/D(u/c,r/u). A/D indicates case A or case D, u/c indicates whether the capacity is unconstrained or constrained and r/u indicates whether it has reliable work stations or not.

	Unconstrained		Constrained	
	Case A	Case D	Case A	Case D
Reliable	A(u,r)	D(u,r)	A(c,r)	D(c,r)
Unreliable	A(u,u)	D(u,u)	A(c,u)	D(c,u)

The parameters in these simulations are the same as that of previous sections. The unreliable work stations are modeled as having 10% of unreliablity for all but #9 work station. #9 work station is modeled as having 5% of unreliability. The load factor of the unreliable work station is modeled as

$$\rho = \frac{E(P)}{(1-P_b)\,C_p}$$

where P_b denotes the probability of break down, and C_p denotes production capacity. The next table shows the data of unreliable work station model and load factor. We select P_b of #9 work station as 0.05 in order to keep the load factor below 1.0.

Table 4.4-1

W.s #	1	2	3	4	5	6	7	8	9	10
Const	6	2	2	2	2	2	2	2	2	2.5
E(P)	5.01	0.75	0.69	0.36	1.37	1.65	0.14	0.55	1.89	2.19
P_b	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.05	0.1
n(A)	1	1	1	1	1	1	1	1	1	1
n(D)	8	1	1	1	2	3	1	2	4	5
ρ	0.928	0.417	0.383	0.200	0.761	0.917	0.078	0.306	0.995	0.973

Table 4.4-2-A,-D and following figures show the results of these simulation. In order to see the relationship, we choose E(P); expectation of production, σ_p; standard deviation of production, E(Q); expectation of queue, E(Qn); backlog of the production.

Fig 4.4-1-A,D show the standard deviation of production of each work station. The results are normalized by the standard deviation from the unconstrained and reliable cases, that is case A(u,r) or case D(u,r). The figures show that the case with appropriate planned lead time (case D) is insensitive to the capacity constraint. It also provides a smaller standard deviation for both reliable and unreliable work station.

Fig 4.4-2-A,D show the expectation of queue of each work station. The results are normalized by the expectation of queue from the reliable and unconstrained case of case A or case D. Increasing the planned lead time always leads to longer queue. But the longer queue provides a benefit by smoothing the stochastic variation of the input flow. Therefore when we use the planned lead time, we have to trade off the benefit of the smooth production versus the increase in work-in-process inventory.

Fig 4.4-1-A σ_p/σ_p(Case A(u,r)): Standard deviation of the production

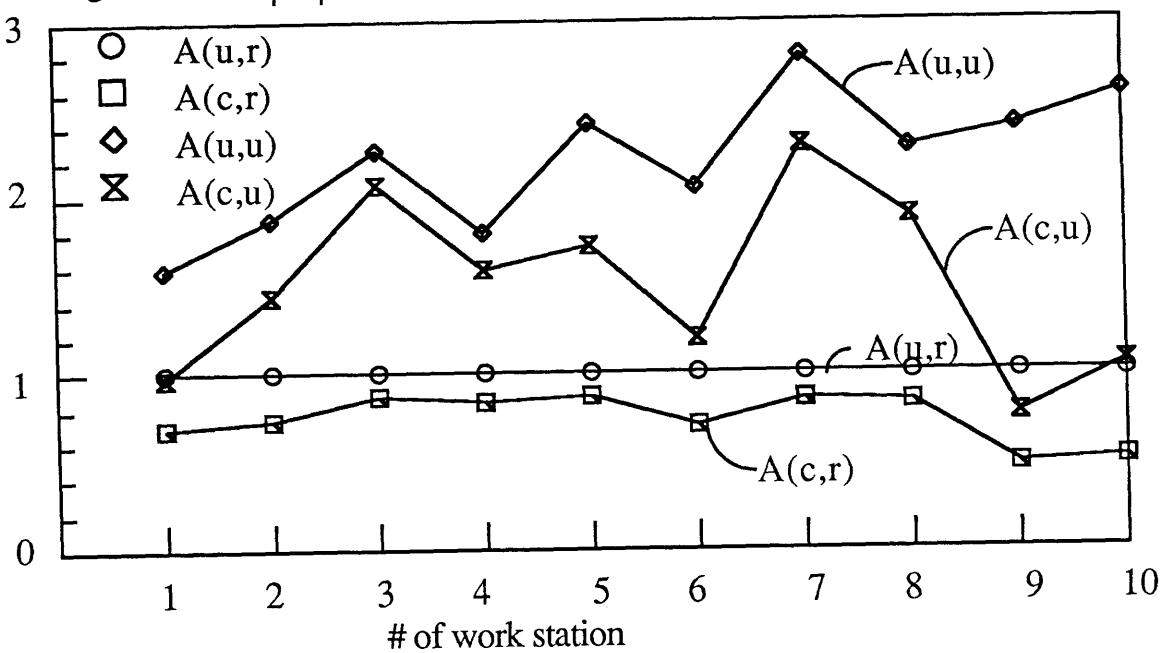


Fig 4.4-1-D σ_p/σ_p(Case D(u,r)): Standard deviation of the production

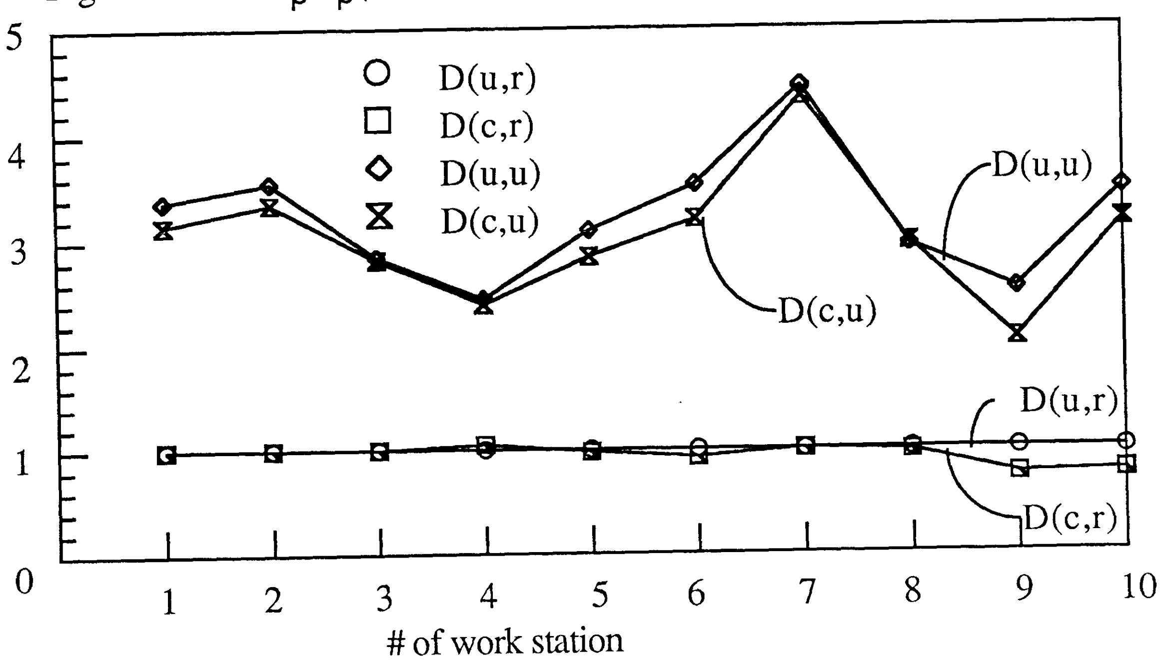


Fig 4.4-2-A E(Q)/E(Q; A(u,r)): Expected queue

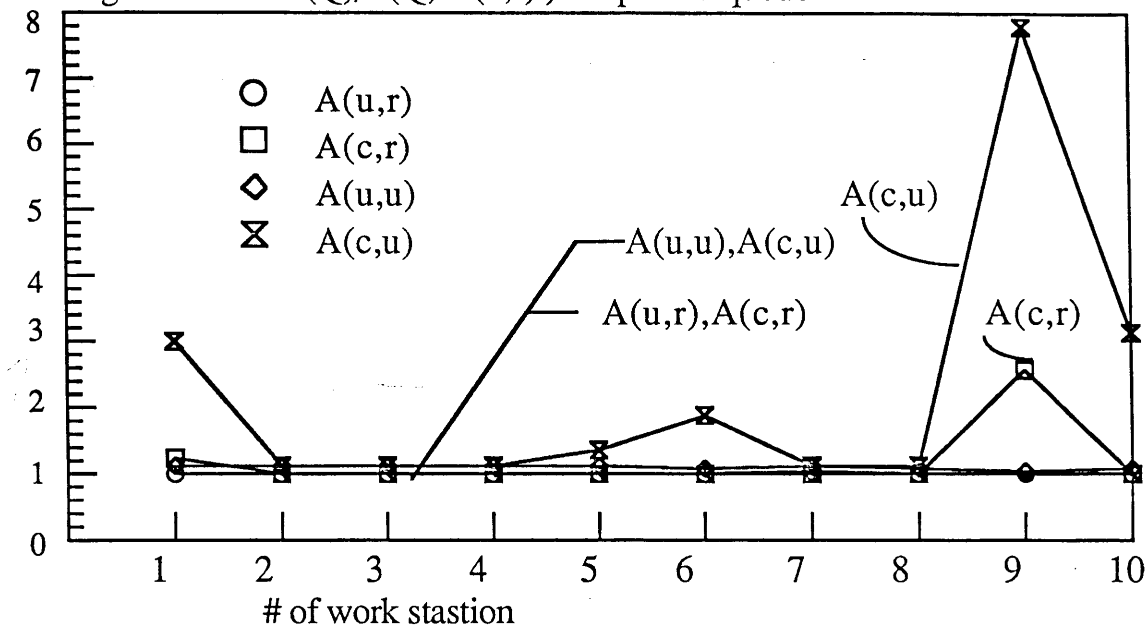


Fig 4.4-2-D E(Q)/E(Q; D(u,r)): Expected queue

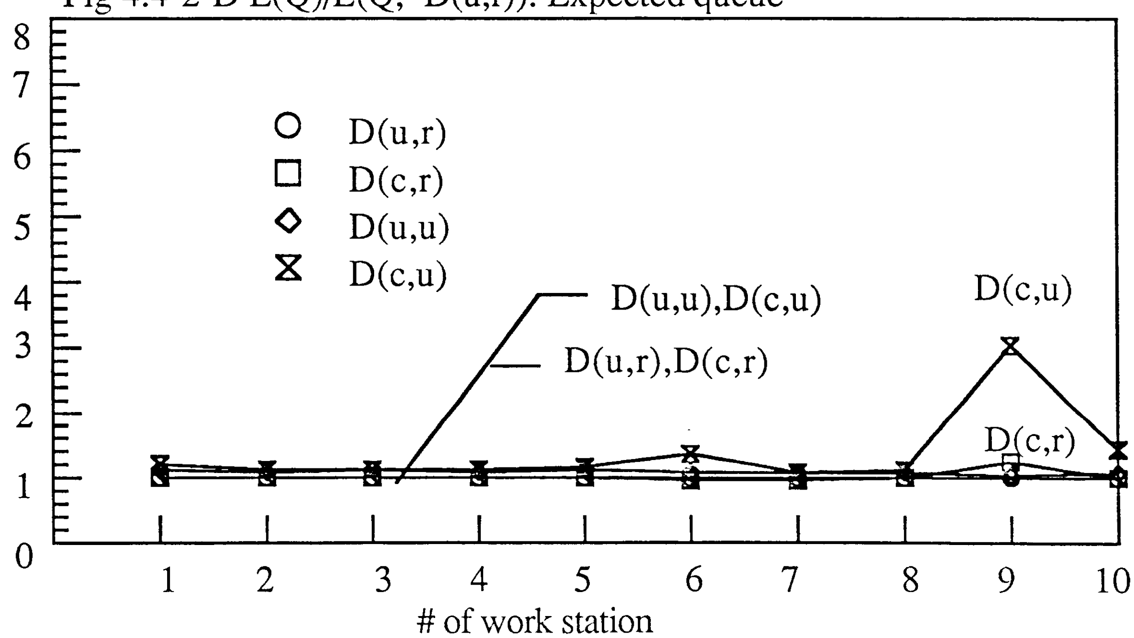


Fig 4.4-3 shows the expectation of the backlog of each work stations. The figure suggests that when the work stations are heavily loaded, the expectation of the back log is large. And when the work stations are unreliable, the back log will be larger.

Fig 4.4-3-A   E(Qn) : Expected backlog

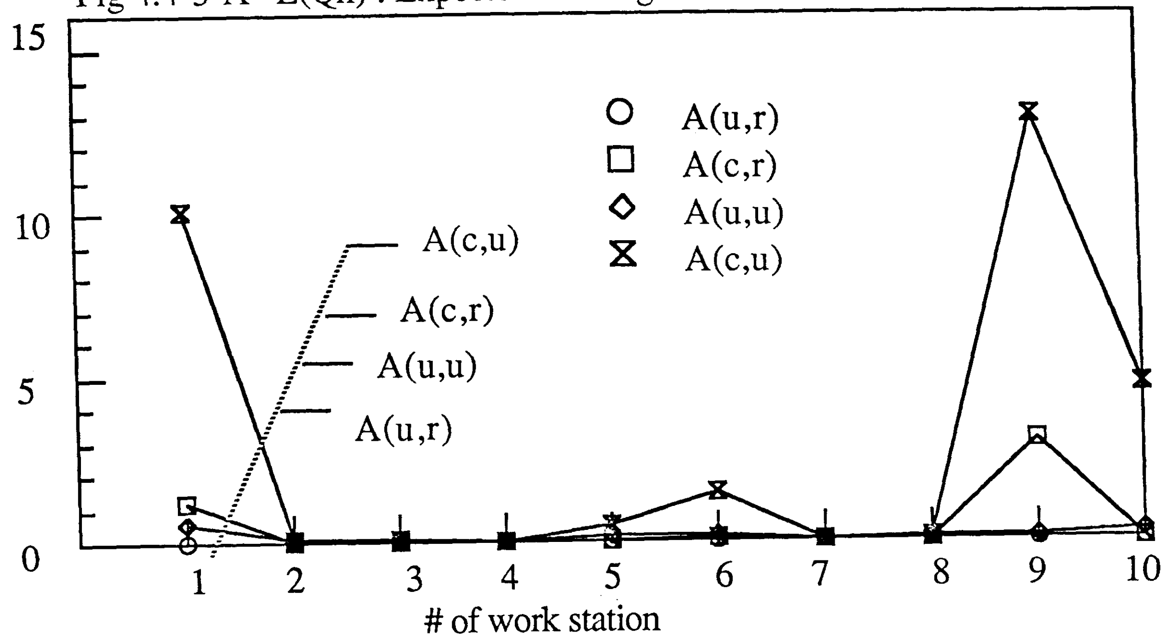


Fig 4.4-3-D   E(Qn) : Expected backlog

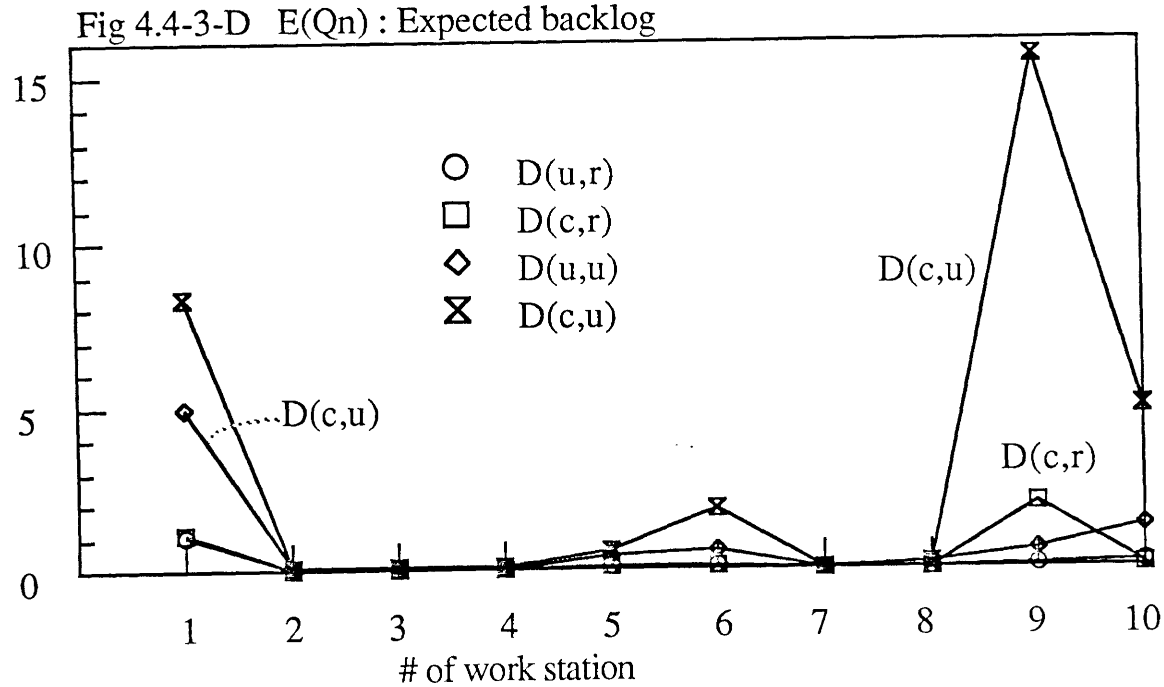


Table 4.4-2-A Simulation result: Unreliable work station Case A

Work S.	1		2		3		4		5	
P.L.T.	1		1		1		1		1	
P(B.d)	0.1		0.1		0.1		0.1		0.1	
Const.	∞	6	∞	2	∞	2	∞	2	∞	6
E(P) m	4.981	4.983	0.747	0.747	0.690	0.690	0.357	0.357	1.364	1.364
95% H	5.066	5.065	0.759	0.759	0.702	0.701	0.364	0.364	1.386	1.384
95% L	4.896	4.901	0.734	0.735	0.678	0.679	0.349	0.350	1.343	1.344
σ_p	3.219	1.968	0.601	0.460	0.430	0.392	0.306	0.270	0.944	0.675
E(Q) m.	5.568	15.038	0.835	0.841	0.766	0.769	0.395	0.395	1.526	1.864
95% H.	5.724	19.221	0.853	0.858	0.777	0.779	0.406	0.406	1.561	1.959
95% L.	5.412	10855	0.816	0.823	0.755	0.759	0.384	0.385	1.490	1.769
E(Q_n)m.	0.587	10.057	0.088	0.093	0.076	0.079	0.039	0.039	0.161	0.499
95% H.	0.687	14.174	0.095	0.100	0.079	0.085	0.044	0.044	0.186	0.586
95% L.	0.487	5.941	0.081	0.086	0.073	0.073	0.034	0.034	0.136	0.412

Work S.	6		7		8		9		10	
P.L.T.	1		1		1		1		1	
P(B.d)	0.1		0.1		0.1		0.05		0.1	
Const.	∞	2	∞	2	∞	2	∞	2	∞	2.5
E(P) m	1.640	1.640	0.137	0.137	0.547	0.546	1.873	1.871	2.172	2.173
95% H	1.669	1.668	0.139	0.138	0.555	0.555	1.903	1.893	2.210	2.200
95% L	1.611	1.612	0.135	0.135	0.538	0.538	1.843	1.850	2.133	2.145
σ_p	1.107	0.650	0.112	0.092	0.387	0.321	1.466	0.471	1.925	0.780
E(Q) m.	1.817	3.097	0.152	0.152	0.605	0.607	1.983	14.775	2.392	6.816
95% H.	1.856	3.535	0.154	0.153	0.616	0.617	2.014	20.085	2.442	8.865
95% L.	1.777	2.659	0.149	0.150	0.595	0.597	1.953	9.466	2.342	4.766
E(Q_n)m.	0.177	1.457	0.015	0.015	0.059	0.058	0.110	12.902	0.222	4.646
95% H.	0.195	1.887	0.016	0.015	0.066	0.063	0.119	18.191	0.247	6.685
95% L.	0.158	1.027	0.014	0.014	0.052	0.053	0.101	7.613	0.197	2.606

Table 4.4-2-D Simulation result: Unreliable work station Case D

Work S.	1		2		3		4		5	
P.L.T.	8		1		1		1		2	
P(B.d)	0.1		0.1		0.1		0.1		0.1	
Const.	∞	6	∞	2	∞	2	∞	2	∞	6
E(P) m	4.981	4.978	0.747	0.746	0.690	0.689	0.357	0.357	1.364	1.364
95% H	5.064	5.023	0.759	0.754	0.701	0.697	0.364	0.360	1.385	1.378
95% L	4.897	4.933	0.734	0.739	0.679	0.681	0.349	0.354	1.343	1.350
σ_p	1.853	1.730	0.462	0.435	0.400	0.394	0.269	0.265	0.622	0.571
E(Q) m.	44.557	47.969	0.816	0.831	0.766	0.773	0.396	0.400	3.043	3.205
95% H.	45.715	49.488	0.865	0.848	0.777	0.783	0.406	0.402	3.097	3.313
95% L.	43.399	46.450	0.766	0.814	0.756	0.764	0.385	0.397	2.988	3.096
$E(Q_n)$m.	4.963	8.323	0.086	0.086	0.076	0.085	0.039	0.043	0.412	0.562
95% H.	5.614	9.585	0.092	0.100	0.081	0.090	0.044	0.048	0.444	0.643
95% L.	4.312	7.062	0.080	0.071	0.072	0.080	0.035	0.038	0.379	0.481

Work S.	6		7		8		9		10	
P.L.T.	3		1		2		4		5	
P(B.d)	0.1		0.1		0.1		0.05		0.1	
const.	∞	2	∞	2	∞	2	∞	2	∞	2.5
E(P) m	1.640	1.642	0.137	0.136	0.546	0.546	1.872	1.866	2.171	2.163
95% H	1.668	1.661	0.138	0.138	0.555	0.552	1.900	1.885	2.206	2.192
95% L	1.611	1.623	0.135	0.135	0.538	0.540	1.844	1.847	2.137	2.133
σ_p	0.635	0.576	0.089	0.087	0.237	0.239	0.557	0.449	0.799	0.731
E(Q) m.	5.444	6.719	0.151	0.152	1.209	1.215	7.898	22.978	12.002	15.711
95% H.	5.544	7.449	0.154	0.154	1.226	1.234	8.024	37.077	12.234	17.980
95% L.	5.343	5.988	0.149	0.150	1.191	1.197	7.771	8.879	11.770	13.442
E(Qn)m.	0.591	1.842	0.015	0.016	0.145	0.150	0.599	15.529	1.251	4.929
95% H.	0.631	2.529	0.016	0.017	0.153	0.159	0.623	29.542	1.355	7.089
95% L.	0.550	1.156	0.014	0.014	0.137	0.140	0.575	1.517	1.147	2.768

Chapter 5. Conclusion

The purpose of this work was to extend an analytical model and to develop a simulation model for a job shop planning problem. The focus was on the extension of the Tactical Planning Model[1] to permit unreliable multiple work stations with production constraints.

The analytical model provides the steady state distribution of the production levels and the work in process inventory at each work station without capacity constraints. The simulation model also provides the steady state distribution of the production levels and the work in process inventory and steady state expectation of backlogs with capacity constraints.

In Chapter 2, we reviewed the Tactical Planning Model proposed by Graves, and extended the analysis to allow unreliable multiple work stations.

In Chapter 3, we showed the dynamic simulation model to examine the performance of the system.

In Chapter 4, we showed the result of the simulation and analysis. We showed the validation of the simulation model and analytical model. After the validation we showed the example of the simulation and analysis of the model with unreliable work stations and with capacity constraints.

We have obtained following results:

(1) If the work station is unreliable, the steady state of the work station will become as follows.

(a) The average production of each work station does not change even if there are unreliable work stations, provided that the load factor of the work station does not exceed one. If the load factor is more than one the system cannot satisfy the requirement.

(b) The variance of production will be larger with the increase of the unreliability.

(c) The variance and expectation of the work in process inventory or queue will increase. The average queue of the work station is not affected by the other unreliable work stations. It is affected by the unreliability of the own work station.

(d) The expectation of the back log will be large. And at the heavily loaded work station, the back log will be larger.

(2) The capacity constraints of the work station makes the variance of the production smaller, especially when the work station is heavily loaded and/or the variance of production is large. Therefore simulation with capacity constraint is essential tool for analysis, when there are some heavily loaded work stations without an appropriate planned lead time and/or high unreliability.

(3) Applying a larger planned lead time makes it possible to analyze the system without the help of complicated and time consuming simulation.

(4) If it is required to make the level the production smooth in case of unreliable work stations, it is necessary to apply appropriate planned lead time especially for the heavily loaded work stations. Larger planned lead time makes the work station insensitive to the variance of input work flow.

(5) If the job shop is highly reliable and the work in process inventory is very costly, tactics like "Just-in-time" system will work because of its low level of queue.

Therefore when we use the planned lead time, we have to trade off the benefit of the smooth production and the cost of the work-in-process inventory.

References

(1) Stephen C. Graves, A Tactical Planning Model for a Job Shop, *Operations Research*, 34, No. 4 July-August 1986 P 522-533

(2) John O. McClain and L. Joseph Thomas, *Operations Management*, Prentice-Hall, Englewood Cliffs, New Jersey,1985

(3) Stephen C. Graves, Further Research in the Organization of Manufacturing Facilities,Research paper in Operations Research Center MIT, 1986

(4) Barry Richmond, *A User's Guide to STELLA*,High-Performance Systems, Inc.,Lyme, New Hampshire, 1985

(5) A. B. Cruickshanks, R. D. Drescher and S. C. Graves, A Study of Production Smoothing in a Job shop Environment , *Management Science*, 30, 368-380, 1984

(6) Richard J. Larsen and Morris L. Marx, *An introduction to Mathematical Statistics and its applications*,Prentice-Hall,Englewood Cliffs, New Jersey,1986

(7) Jerry Banks and John S. Carson,II, *Discrete-Event System Simulation*,, Prentice-Hall, Englewood Cliffs, New Jersey,1984

(8) Mayron R. Rosenthal, *Numerical Methods in Computer Programming,* Richard D.Irwin,Inc., Homewood, Illinois, 1966

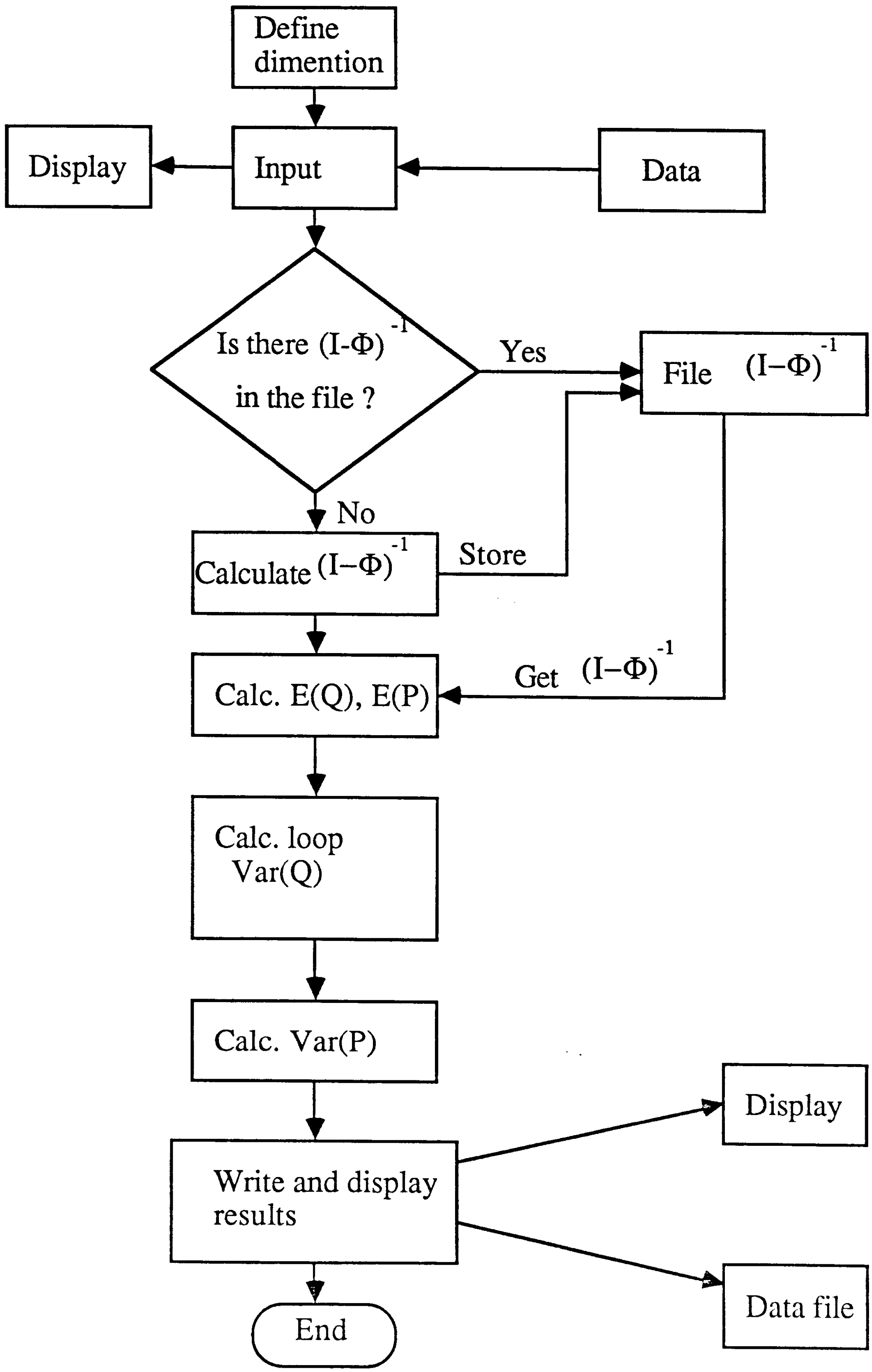
Appendix A. The analytical calculation program

A.1 Flow Chart

This program is desinged to calculate expectation and variance of production and queue of unreliable work station problem without production constraints.

The inputs are stored by the form of DATA sentence of the main program and the file form of outside the program. In order to avoid the time consuming calculation of $(I-\Phi)^{-1}$, it is stored in the outside data file. Therefore this matrix should be calculated at the first time when the new transient matrix Φ comes in. The algorithm of the matrix inversion and determinant are from M.R. Rosenthal[8] (1966).

The outputs are obtained as the display form and data file form. The display form is on the screen of the personal computer, and the data file is on the "CLIP BOARD". The latter form makes it possible to use the output in the word processing software. But these output devices can be changed with ease by changing the assignment sentences.



Define dimention
Display
Input
Data
Is there $(I-\Phi)^{-1}$ in the file ?
Yes
File $(I-\Phi)^{-1}$
No
Calculate $(I-\Phi)^{-1}$
Store
Get $(I-\Phi)^{-1}$
Calc. E(Q), E(P)
Calc. loop Var(Q)
Calc. Var(P)
Display
Write and display results
Data file
End

A.2 Operation manual

(1) Data

The following data sets are in the main program. They should be changed in case of need.

a.Planned lead time

This term is inverse of a_i and the input order is from work station one to ten. For example if it is the case D of Chapter 4, then

DATA 8,1,1,1,2,3,1,2,4,5

b. Probability of the work center break down

This term is the probabilities of break down of each workstation. For example if p_1 is 0.8 and the others are 0 then

DATA 0.80,0,0,0,0,0,0,0,0,0

c.Average input noise

This term is the average input noise. For example

DATA 4,0,0,0,0,0,0,0,0,0

d. Transition matrix

This term is the transition matrix of the work flow. In this paper it was described as Φ (Phai). This matrix is described by ten data sentenses. For example,

DATA 0,0,0.11,0,0.68,0,0,0,0,0

*

*

DATA 0,0,0,0,0,0,0,0,1.16,0

e. Variance of input noise

This term is the diagonal elements of the Covariance matrix of the input noise. For example,

DATA 4.00,0.01,0.01,0.01,0.04,0.04,0,0.01,0.04,0.04

f. Flag for matrix output

This flag is used to command whether matrix elements in the calculation be printed out or not. When it is set as one, the print out of the matrix will come out. For example, when it is unnecessary to print out,

DATA 0

g.Flag for calculate inverse transition matrix.

This flag is used to command whether it is necessary to calculate the inverse of the transition matrix $(I-\Phi)^{-1}$. When the matrix of Φ is changed this matrix has to be recalculated. This flag should be one when calculation is required, else it should be zero. When it is zero stored matrix $(I-\Phi)^{-1}$ can be obtained. For example,

DATA 0

h. Number of iteration of the calculation of Var(Q)

This term command the number of iteration of the calculation block of variance of queue. If it is necessary to calculate 16 times to get the convergence, it should be

DATA 16

(2) Output devices

The outputs of this program is assigned to the screen and data file.

Followings are sentences of the assignment. The first assigns CRT of personal computer, and the second assigns "Clipboard" as a output data file.

```
OPEN "SCRN:" FOR OUTPUT AS #1
OPEN "CLIP:" FOR OUTPUT AS #5
```

A.3 Program list (BASIC)

The applied personal computer is Macintosh ™@

```
'***************************************************************
'******   A Tactical Planning Model for a Job Shop *****************
'******   With Unreliable Work Station              *****************
'***************************************************************
'******     This program was designed to calculate expected value and *
'******    standard deviation of the production and the queue.        *
'******     The work stations were modeled to break down by Bernoulli*
'******    process.                                                ****
'******      This work is a part on the master thesis of the writer. ****
*
'******            November 12, 1987  © Shoichiro Mihara             *
'***************************************************************
' Output port select. Select clipboard as a out put file.
  OPEN "SCRN:" FOR OUTPUT AS #1
   OPEN "CLIP:" FOR OUTPUT AS #5

'Declare of double precision
  DEFDBL A-H,O-Z
'   Definition of dimention of vectors and matrixes
'      Dim  : Input variables
DIM  XNV(10),PV(10),XMUIV(10),PHI(10,10),SIGV(10)

'     Dim   : for convinuence
DIM  EI(10,10),EIMPHI(10,10),REIMPH(10,10),XCHECK(10,10)
DIM  ALPHA(10,10),PHISQ(10,10),EIPPHS(10,10),PMAT(10,10),C(10,10)
'
DIM EIMPMT(10,10),EMPTTD(10,10),REMPTD(10,10),REPDEP(10,10)
DIM QMEAN(10),RALPHA(10,10),PMEAN(10)
'
DIM EIPSPH(10,10),EPSPHD(10,10),SIGMAT(10,10),SQMEAN(10)
DIM EI2(10,10) ,E2MEPD(10,10), R2MEPD(10,10),RPDRPH(10,10)
```

@ Macintosh is a trademark licensed to Apple Computer,Inc.

```
DIM ESPDPD(10,10),ESPPPP(10,10),RPDPPP(10,10),RPPSEQ(10)
DIM R2MVAS(10),VARQUE(10),STAQUE(10),STAPRO(10)
DIM EMPDD(10,10),EMPDDP(10,10),EPDPSQ(10),PDDVAQ(10),VARPRO(10)
DIM EIPHPD(10,10),BBEPDP(10,10),TRBBED(10,10)
DIM  SIGMA(10,10),BN(10,10),TBN(10,10)
DIM  WR1(10,10),WR2(10,10),WR3(10,10)
DIM  SBN(10,10),APEIP(10),ADDSIG(10,10),TREPHI(10,10),ADDVAR(10,10)
DIM ADDWR1(10,10)
'***********************************

'  Input (1) Planned lead time
FOR I=1 TO 10
 READ  XNV(I)
 NEXT I

' Input (2): Probabilities of break down of work centers
  FOR J=1 TO 10
   READ PV(J)
      APEIP(J)=PV(J)*(1-PV(J))/XNV(J)/XNV(J)
  NEXT J
'Input (3): Expected value of inputs into the work centers from outside
'           source. Muiu
  FOR J=1 TO 10
   READ XMUIV(J)
  NEXT J
'Input (4): Work flow  matrix
  FOR K=1 TO 10
   FOR  L=1 TO 10
    READ PHI(K,L)

   NEXT L
  NEXT K

'Input (5): Covariance of inputs into the work centers from outside
'           source. Sigma
  FOR J=1 TO 10
   READ SIGV(J)
  NEXT J

' Debug mode or not/ if Drbug=1 then print many parameters
   READ IDBUG
  '  If Flag is zero output (1-phi)-1 to data file after calculation.
```

```
'               else input data and not calculate.
   READ Flag
   PRINT Flag

  IF Flag=1 THEN    OPEN "InvEmPhi.Data" FOR OUTPUT AS #2
  IF Flag=0 THEN      OPEN "InvEmphi.Data" FOR INPUT AS #2

' Definition of the size of  matrixes
  N=10:N1=1
 ' Input data and title
   GOSUB Titlepage

' Unit matrix
 CALL  UNIT(EI(),N)

 'Equations; (1)  Basic matrix
    ' (1-phi) , (1-phi)-1
       '  If Flag is 1 output (1-phi)-1 to data file after calculation.
  '             0 input data and not calculate.

        CALL MINUS(EI(),PHI(),EIMPHI(),N,N) :'  EIMPHI=1-PHI
        IF Flag = 0 GOTO jumpinv
       CALL INVERS(REIMPH(),EIMPHI(),C(),N)  :'  REIMPH=EIMPHI^-1
       FOR j1=1 TO N
         FOR  i1=1 TO N
            WRITE #2,REIMPH (j1,i1)
         NEXT i1
       NEXT j1
       GOTO outofinv
jumpinv:

       FOR j1=1 TO N
         FOR  i1=1 TO N
            INPUT #2,REIMPH(j1,i1)
         NEXT i1
       NEXT j1
outofinv:
         CLOSE #2
   ' XCHECK=REIMPH*EIMPHI
        'CALL MULTI (REIMPH(),N,N,EIMPHI(),N,N,XCHECK())
       CALL DIAG(XNV(),RALPHA(),N)   :' Trans from vec to diag matrix
```

```
CALL DIAINV(ALPHA(),RALPHA(),N) : 'Alpha = 1/ planned lead time
CALL DIAG(SIGV(),SIGMAT(),N)    :' Trans v to d-mat --- noise var.
CALL DIAG(PV(),PMAT(),N)        :' Trans v to d-mat --- prob. break
  CALL MINUS(EI(),PMAT(),EIMPMT(),N,N)  :' Eipmt=Ei-Pmat
'Empttd=Eimpmt*Alpha
CALL MULTI (EIMPMT(),N,N,ALPHA(),N,N,EMPTTD())
' Eiphpd=(Ei-phi)*D*(1-p)
CALL MULTI(EIMPHI(),N,N,EMPTTD(),N,N,EIPHPD())
' Bbepdp=Ei-Eiphpd
    CALL MINUS(EI(),EIPHPD(),BBEPDP(),N,N)
    CALL TRANS(BBEPDP(),N,N,TRBBED())
    CALL TRANS(EIMPHI(),N,N,TREPHI())

'calculation of expectation( Production and Queue )
   'Remptd=Empttd^-1
    CALL DIAINV(REMPTD(),EMPTTD(),N)
   'Repdep=Remptd*Reimph
    CALL  MULTI(REMPTD(),N,N,REIMPH(),N,N,REPDEP())
   ' E(Q)=Repdep*Xmuiv
    CALL MULTIV(REPDEP(),N,N,XMUIV(),N,QMEAN())
    ' E(P)=Empttd*E(Q)
      CALL MULTIV(EMPTTD(),N,N,QMEAN(),N,PMEAN())

' Output1
    GOSUB Printoutput1

'Print for debug
       IF IDBUG=1 THEN GOSUB Printdebug1

' calculation of variance of queue.
    FOR J=1 TO 10
        SQMEAN(J)=QMEAN(J)*QMEAN(J)      :'  Sqmean(i)= E(Q (i) )^2
    NEXT J

XX=1#
   CALL KMULTMAT(XX,SIGMAT(),SIGMA(),N,N) : ' Sigma=Sigmat
   CALL KMULTMAT(XX,BBEPDP(),BN(),N,N)      :' Bn=Bbepdp
   CALL KMULTMAT(XX,TRBBED(),TBN(),N,N)       :' Tbn= transpose (Bn)

 '  loop of the approximation
  READ Ite
   FOR I=1 TO Ite
```

```
      PRINT USING " I= ##";I
       FOR J=1 TO 10
          ADDSIG(J,J)=APEIP(J)*(SIGMA(J,J)+SQMEAN(J))
       NEXT J

    ' Addvar1=(1-phi)*addsig
         CALL MULTI(EIMPHI(),N,N,ADDSIG(),N,N,ADDWR1())
      'Addvar=Addvar1*(1-phi)T=(1-phi)*addsig*(1-phi)T
        CALL MULTI(ADDWR1(),N,N,TREPHI(),N,N,ADDVAR())
    ' Wr1=Bn*Sigma
        CALL MULTI(BN(),N,N,SIGMA(),N,N,WR1())
     'Wr2=Wr1*Tbn=Bn*Sigma*Tbn
        CALL MULTI(WR1(),N,N,TBN(),N,N,WR2())
     'Wr3= Bn*Sigma*Tbn+Addvar
         CALL PLUS (ADDVAR(),WR2(),WR3(),N,N)
     ' Sigma=Wr3+Sigmat
         CALL PLUS(SIGMAT(),WR3(),SIGMA(),N,N)
    NEXT I

   FOR I=1 TO 10
    VARQUE(I)=SIGMA(I,I)              :' Var(Q)=SIGMA
    STAQUE(I)=SQR(VARQUE(I))          :' Stand. Dev (Q)= Root (Var(Q))
  NEXT I
'End of the calculation 1

' Variance of production and standard deviation
    ' Empdd=Empttd*Alpha
      CALL MULTI (EMPTTD(),N,N,ALPHA(),N,N,EMPDD())
    'Empddp=Empdd*Pmat
      CALL MULTI (EMPDD(),N,N,PMAT(),N,N,EMPDDP())
    'Epdpsq=Empddp*Sqmean
      CALL MULTIV(EMPDDP(),N,N,SQMEAN(),N,EPDPSQ())
    'Pddvaq=Empdd*Varque
        CALL MULTIV(EMPDD(),N,N,VARQUE(),N,PDDVAQ())

  FOR I=1 TO 10
    VARPRO(I)=EPDPSQ(I)+PDDVAQ(I) :'Var(p)=Epdpsq+Pddvaq
     STAPRO(I)=SQR(VARPRO(I)) :' Stand. dev. of p= Root(Var(p))
   NEXT I

'  output
GOSUB Printoutput2
```

```
 GOSUB Printoutput3

 CLOSE #1
 CLOSE # 5

 BEEP:BEEP:BEEP
     END

'   Planned lead time
  DATA  8,1,1,1,2,3,1,2,4,5
   'DATA  1,1,1,1,1,1,1,1,1,1
 '   Probability of the work center break down
    ' DATA 0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.05,0.1
   DATA  0.80,0,0,0,0,0,0,0,0,0
' Average input noise
  DATA  4,0,0,0,0,0,0,0,0,0
   ' Phi  --  Work flow matrix
  DATA  0,0,0.11,0,0.68,0,0,0,0,0
  DATA  0.15,0,0,0,0,0,0,0,0,0
  DATA  0.04,0.01,0,0.71,0,0.06,0,0,0.07,0
  DATA   0.01,0.41,0,0,0,0,0,0,0,0
  DATA  0.03,0.37,1.36,0,0,0,0,0,0,0
  DATA  0.24,0,0,0,0.15,0,0,0,0.13,0
  DATA  0,0,0,0,0.1,0,0,0,0,0
  DATA   0.01,0,0,0,0,0.22,1,0,0,0
  DATA   0,0,0,0,0,0,0,3.43,0,0
  DATA  0,0,0,0,0,0,0,0,1.16,0
'Variance of input noise
  DATA  4.00,0.01,0.01,0.01,0.04,0.04,0,0.01,0.04,0.04
 'Debug or not    (1--- print,   else none)
 DATA 0
 ' calculate inverse of (1-phi)  flag =1  calculate  flag=0 no
 DATA 0
' # of iterations
DATA 32

 SUB DIAINV(A(2),B(2),N) STATIC
   FOR J=1 TO N
      A(J,J)=1/B(J,J)
   NEXT J
   END SUB
```

```
SUB INVERS(A(2),B(2),C(2),N) STATIC
  FOR I=1 TO N
  FOR J=1 TO N
     C(I,J)=B(I,J)
  NEXT J
 NEXT I
    CALL DETM(C(),N,E)
    IF E=0 THEN PRINT #1," NO INVERSE FOR THIS MATRIX" ELSE GOTO 10
        GOTO 20
10      M=N-1
    FOR I=1 TO N
      FOR J=1 TO N
        FOR K=1 TO N
          FOR L=1 TO N
             C(K,L)=B(K,L)
          NEXT L
        NEXT K
         H=1
        FOR L=1 TO N
           C(I,L)=C(N,L)
        NEXT L
        FOR K=1 TO N
           C(K,J)=C(K,N)
        NEXT K
 IF NOT ((N=J AND N=I ) OR (N<>J AND N<>I ) ) THEN H=-1 ELSE H=1
          CALL DETM(C(),M,DET)
             A(J,I)=DET/E*H
     NEXT J
   NEXT I
20 END SUB

SUB DETM(A(2),K,DET) STATIC
  Z=1
 FOR M=2 TO K
  IF A(M-1,M-1) <> 0!  THEN GOTO 50
  FOR I=M TO K
  IF A(M-1,I) <>0 THEN  GOTO 40
    NEXT I

     DET=0!
       GOTO endreturn
40  I3=M-1
     FOR I2=I3 TO K
```

```
      TEMP=A(I2,I3)
          A(I2,I3)=A(I2,I)
        A(I2,I)=TEMP
     NEXT I2
      Z=Z*(-1!)
50 FOR I=M TO K
      R=A(I,M-1)/A(M-1,M-1)
     FOR J=M TO K
          A(I,J)=A(I,J)-A(M-1,J)*R
     NEXT J
     NEXT I
      NEXT M
      DET=1
  FOR I=1 TO K
       DET=DET*A(I,I)
   NEXT I
   DET=DET*Z
   endreturn:
END SUB


SUB  MULTI(A(2),IA,KA,B(2),KB,JB,C(2))  STATIC
IF KA<> KB THEN PRINT #1,"ARGUMENT ERROR" ELSE GOTO LOOPSTART
GOTO LOOPEND

LOOPSTART:  FOR I=1 TO IA
      FOR J=1 TO JB
        S=0
         FOR K=1 TO KA
           S=S+A(I,K)*B(K,J)
         NEXT K
        C(I,J)=S
      NEXT J
     NEXT I
    LOOPEND:
  END SUB
```

```
SUB  MULTIV(A(2),IA,KA,B(1),KB,C(1))  STATIC
  IF KA<>KB THEN PRINT #1, "ARGUMENT ERROR"
 LOOPSTART:
        FOR I=1  TO IA
          S=0
         FOR K=1 TO KA
            S=S+A(I,K)*B(K)
         NEXT K
        C(I)=S
    NEXT I
   LOOPEND:
  END SUB

SUB KMULTMAT(XK,B(2),C(2),N,M) STATIC
  FOR I=1  TO N
   FOR J=1 TO M
       C(I,J)=XK*B(I,J)
   NEXT J
  NEXT I
 END SUB

SUB  PLUS(A(2),B(2),C(2),N,M)  STATIC
  FOR I=1  TO N
   FOR J=1 TO M
       C(I,J)=A(I,J)+B(I,J)
   NEXT J
  NEXT I
 END SUB

SUB  MINUS(A(2),B(2),C(2),N,M)  STATIC
  FOR I=1  TO N
   FOR J=1 TO M
       C(I,J)=A(I,J)-B(I,J)
   NEXT J
  NEXT I
 END SUB
```

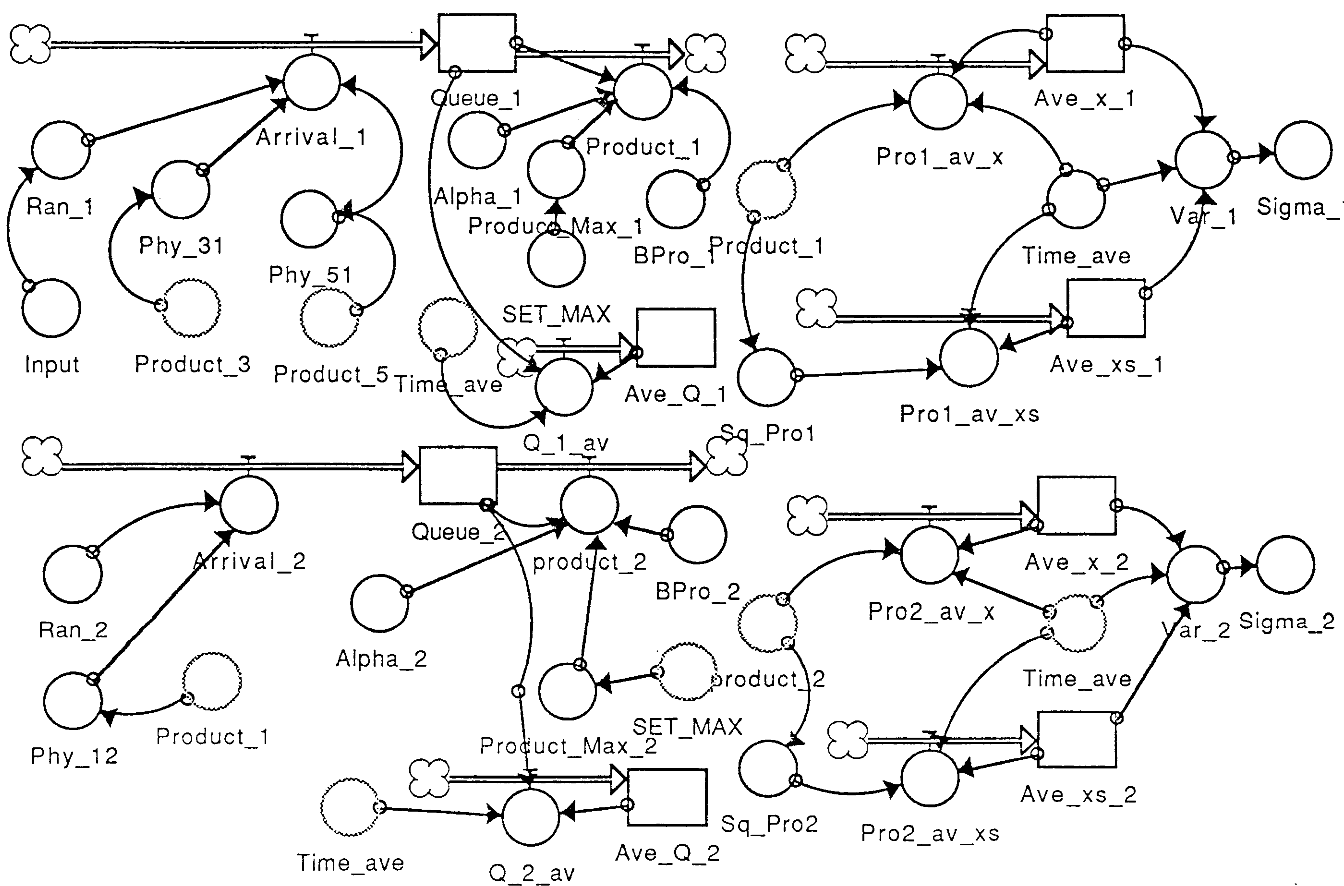
```
SUB CLMAT(A(2),N,M) STATIC
  FOR I=1 TO N
   FOR J=1 TO M
      C(I,J)=0!
   NEXT J
  NEXT I
 END SUB

SUB UNIT(C(2),N) STATIC
  FOR I=1 TO N
       C(I,I)=1#
  NEXT I
END SUB

SUB DIAG(V(1),A(2),N) STATIC
FOR I=1 TO N
    A(I,I)=V(I)
 NEXT I
END SUB

SUB TRANS (A(2),N,M,ATR(2)) STATIC
   FOR I=1 TO N
     FOR J=1 TO M
         ATR(J,I)=A(I,J)
     NEXT J
    NEXT I
 END SUB

 SUB PRINTMAT(A(2),N,M) STATIC
  FOR I=1 TO N
     L=1
      loopprint:
     IF L>10 THEN GOTO ENDLOOP
        PRINT #1, USING  "##.###   " ;A(I,L) ;
      L=L+1
       GOTO loopprint
     endloop:
     PRINT #1
  NEXT I
 PRINT #1, :PRINT #1,
 END SUB
```

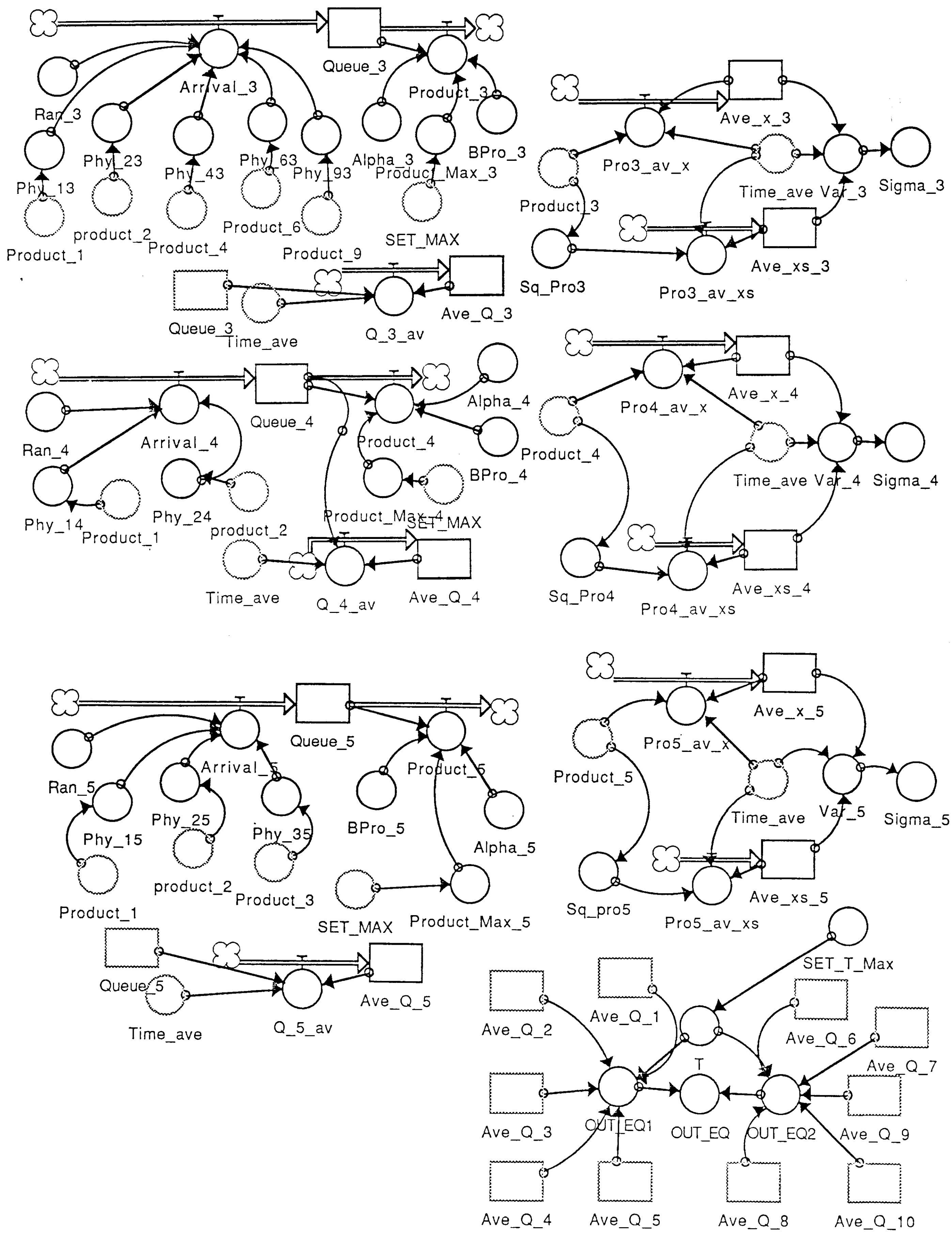
```
SUB PRINTVEC(A(1),N)STATIC
FOR I=1 TO N
     PRINT #1, USING  "##.### " ;A(I);
 NEXT I
 PRINT #1,:PRINT #1,
END SUB

SUB SQELM(A(2),N,M,SA(2)) STATIC
 FOR I=1 TO N
  FOR J=1 TO M
      SA(I,J)=A(I,J)*A(I,J)
   NEXT J
  NEXT I
 END SUB

 Titlepage:
        PRINT   #1,   "*********************************************"
   PRINT #1, "****   MacIntosh version of Tactical Planning Model *"
           PRINT   #1,   "*********************************************"
    PRINT #1, "****   A Tactical Planning Model for a Job Shop  *"
    PRINT #1, "****   With Unreliable Work Station              *"
        PRINT   #1,   "*********************************************"
  PRINT #1, : PRINT #1,
   PRINT #1, "  1.  Input value" :PRINT #1,
   PRINT #1, "  a. Planned lead time"
  CALL  PRINTVEC(XNV(),N)
   PRINT #1, "  b. Probability of break down"
  CALL  PRINTVEC(PV(),N)
   PRINT #1, "  c. Average input noise"
  CALL  PRINTVEC(XMUIV(),N)
   PRINT #1, "  d.Covariance of input noise (diagonal element)"
  CALL  PRINTVEC(SIGV(),N)
RETURN

  Printoutput1:
   PRINT #1, : PRINT #1,
   PRINT #1, "  2.  Output value" :PRINT #1,
PRINT #1, "  a. Average of queue  E(Q)"
    CALL  PRINTVEC(QMEAN(),N)
PRINT #1, "  b. Average of production  E(P)"
    CALL  PRINTVEC(PMEAN(),N)
 RETURN
```

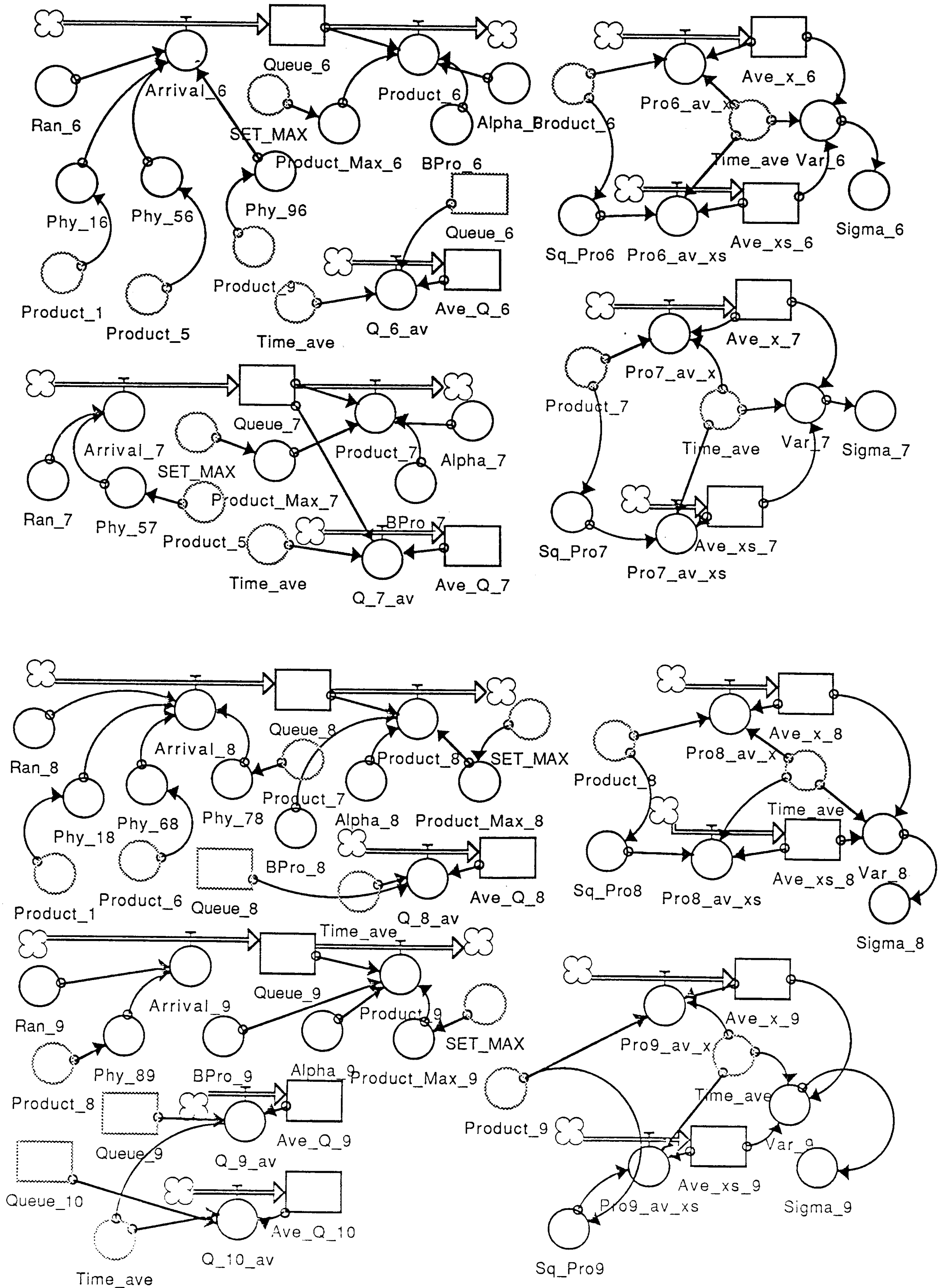
```
Printdebug1:
        PRINT #1, "MATRIX (1-PHI)"
    CALL  PRINTMAT(EIMPHI(),N,N)
       PRINT #1, "MATRIX (1-PHI)^-1*(1-PHI)"
   CALL  PRINTMAT(XCHECK(),N,N)
     PRINT #1, "MATRIX PMAT"
   CALL  PRINTMAT(PMAT(),N,N)
      PRINT #1, "MATRIX  PHI"
   CALL PRINTMAT(PHI(),N,N)
       PRINT #1,
   PRINT #1,
RETURN

 Printoutput2:
  PRINT #1, "  c. Variance of queue   Var(Q)"
   CALL  PRINTVEC(VARQUE(),N)
  PRINT #1, "  d.Standard deviation of queue"
    CALL  PRINTVEC(STAQUE(),N)
  PRINT #1, "  e. Variance of production   Var(P)"
     CALL  PRINTVEC(VARPRO(),N)
    PRINT #1, "  f.Standard deviation of production"
    CALL  PRINTVEC(STAPRO(),N)
    RETURN

 Printoutput3:
       PRINT   #5,   "*******************************************"
    PRINT #5, "***   A Tactical Planning Model for a Job Shop   *"
     PRINT #5, "***   With Unreliable Work Station               *"
          PRINT   #5,   "*******************************************"
 PRINT #5,
   PRINT #5,USING "         Ite=##";Ite
   PRINT #5," W.S. ";"   N  ";" P -BD";"  E(Q)  ";"  S(Q)  ";"  E(P)  ";"  S(P)  "
 FOR I=1 TO 10
      PRINT #5,USING" ##        ##   ##.### ##.### ##.### ##.###
##.###";I;XNV(I);PV(I);QMEAN(I);STAQUE(I);PMEAN(I);STAPRO(I)
   NEXT I
RETURN
```

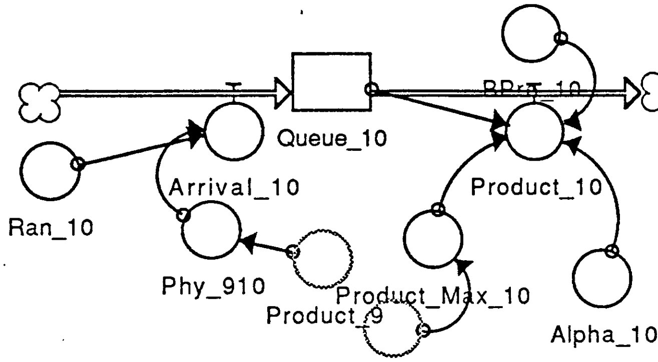
Appendix B. The complete STELLA Model

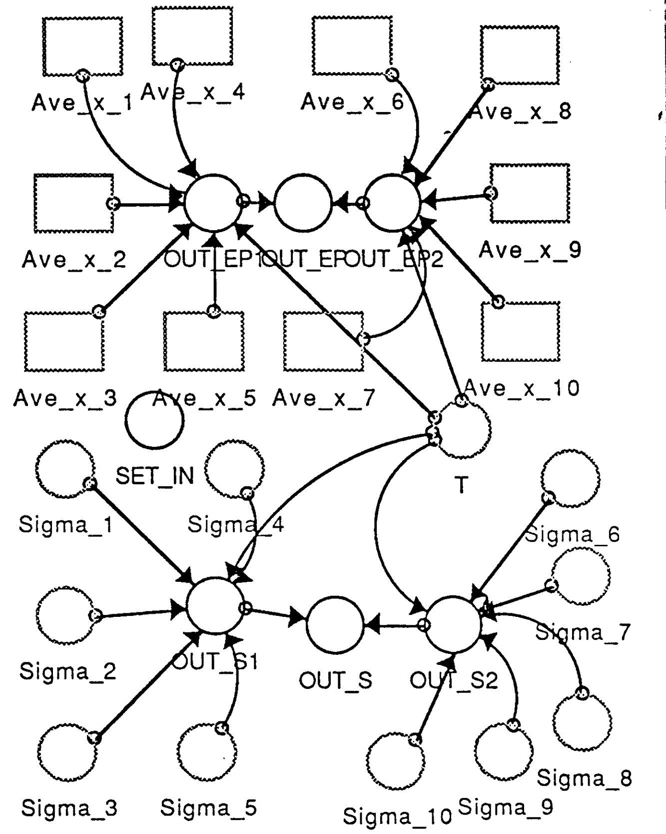
B.1 Flow Diagram

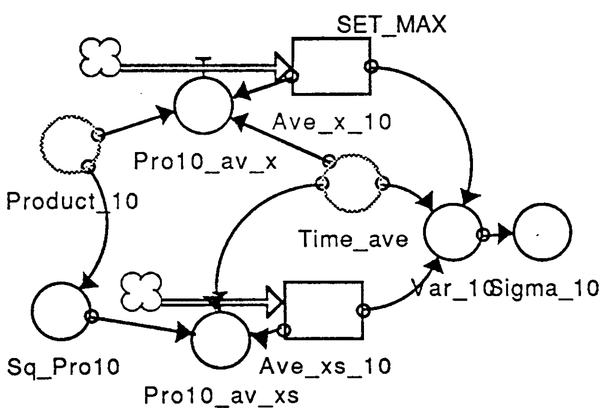


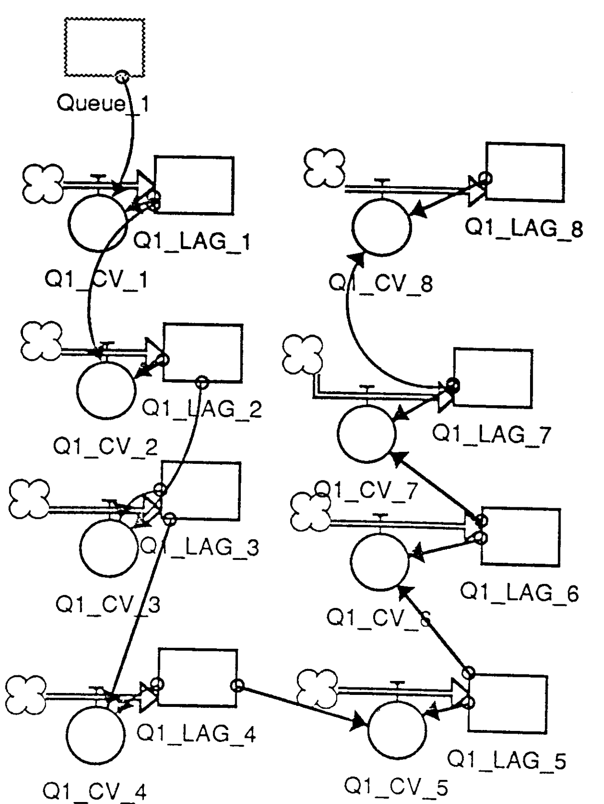

Queue_3
Arrival_3
Product_3
Ran_3
Phy_23
Phy_43
Phy_63
Phy_93
Phy_13
Alpha_3
BPro_3
Product_Max_3
product_2
Product_4
Product_6
Product_1
Product_9
SET_MAX
Queue_3
Time_ave
Q_3_av
Ave_Q_3
Ave_x_3
Pro3_av_x
Time_ave
Var_3
Sigma_3
Product_3
Sq_Pro3
Ave_xs_3
Pro3_av_xs
Queue_4
Alpha_4
Ran_4
Arrival_4
Product_4
BPro_4
Phy_24
Phy_14
Product_1
product_2
Product_Max_4
SET_MAX
Time_ave
Q_4_av
Ave_Q_4
Ave_x_4
Pro4_av_x
Product_4
Time_ave
Var_4
Sigma_4
Sq_Pro4
Ave_xs_4
Pro4_av_xs
Queue_5
Arrival_5
Product_5
Ran_5
Phy_25
Phy_35
Phy_15
BPro_5
Alpha_5
product_2
Product_3
Product_1
SET_MAX
Product_Max_5
Queue_5
Time_ave
Q_5_av
Ave_Q_5
Ave_x_5
Pro5_av_x
Product_5
Time_ave
Var_5
Sigma_5
Sq_pro5
Ave_xs_5
Pro5_av_xs
SET_T_Max
Ave_Q_2
Ave_Q_1
Ave_Q_6
Ave_Q_7
T
Ave_Q_3
OUT_EQ1
OUT_EQ
OUT_EQ2
Ave_Q_9
Ave_Q_4
Ave_Q_5
Ave_Q_8
Ave_Q_10

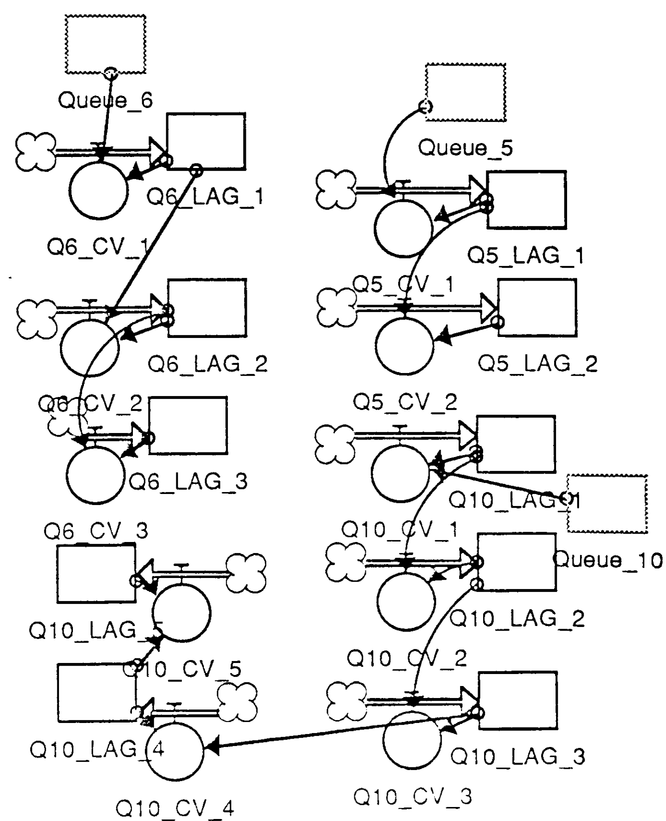

Queue_6
Arrival_6
Ran_6
Product_6
Alpha_Product_6
SET_MAX
Product_Max_6
BPro_6
Phy_16
Phy_56
Phy_96
Product_1
Product_5
Product_9
Queue_6
Ave_Q_6
Q_6_av
Time_ave
Ave_x_6
Pro6_av_x
Time_ave
Var_6
Sigma_6
Sq_Pro6
Pro6_av_xs
Ave_xs_6
Queue_7
Arrival_7
Ran_7
Phy_57
SET_MAX
Product_Max_7
Product_7
Alpha_7
BPro_7
Product_5
Time_ave
Q_7_av
Ave_Q_7
Ave_x_7
Pro7_av_x
Product_7
Time_ave
Var_7
Sigma_7
Sq_Pro7
Pro7_av_xs
Ave_xs_7
Queue_8
Arrival_8
Ran_8
Phy_18
Phy_68
Phy_78
Product_7
Product_8
SET_MAX
Alpha_8
Product_Max_8
Product_1
Product_6
Queue_8
BPro_8
Q_8_av
Ave_Q_8
Time_ave
Ave_x_8
Pro8_av_x
Product_8
Time_ave
Sq_Pro8
Pro8_av_xs
Ave_xs_8
Var_8
Sigma_8
Queue_9
Arrival_9
Ran_9
Phy_89
Product_8
BPro_9
Alpha_9
Product_9
SET_MAX
Product_Max_9
Queue_9
Q_9_av
Ave_Q_9
Queue_10
Q_10_av
Ave_Q_10
Time_ave
Ave_x_9
Pro9_av_x
Product_9
Time_ave
Var_9
Sigma_9
Ave_xs_9
Pro9_av_xs
Sq_Pro9

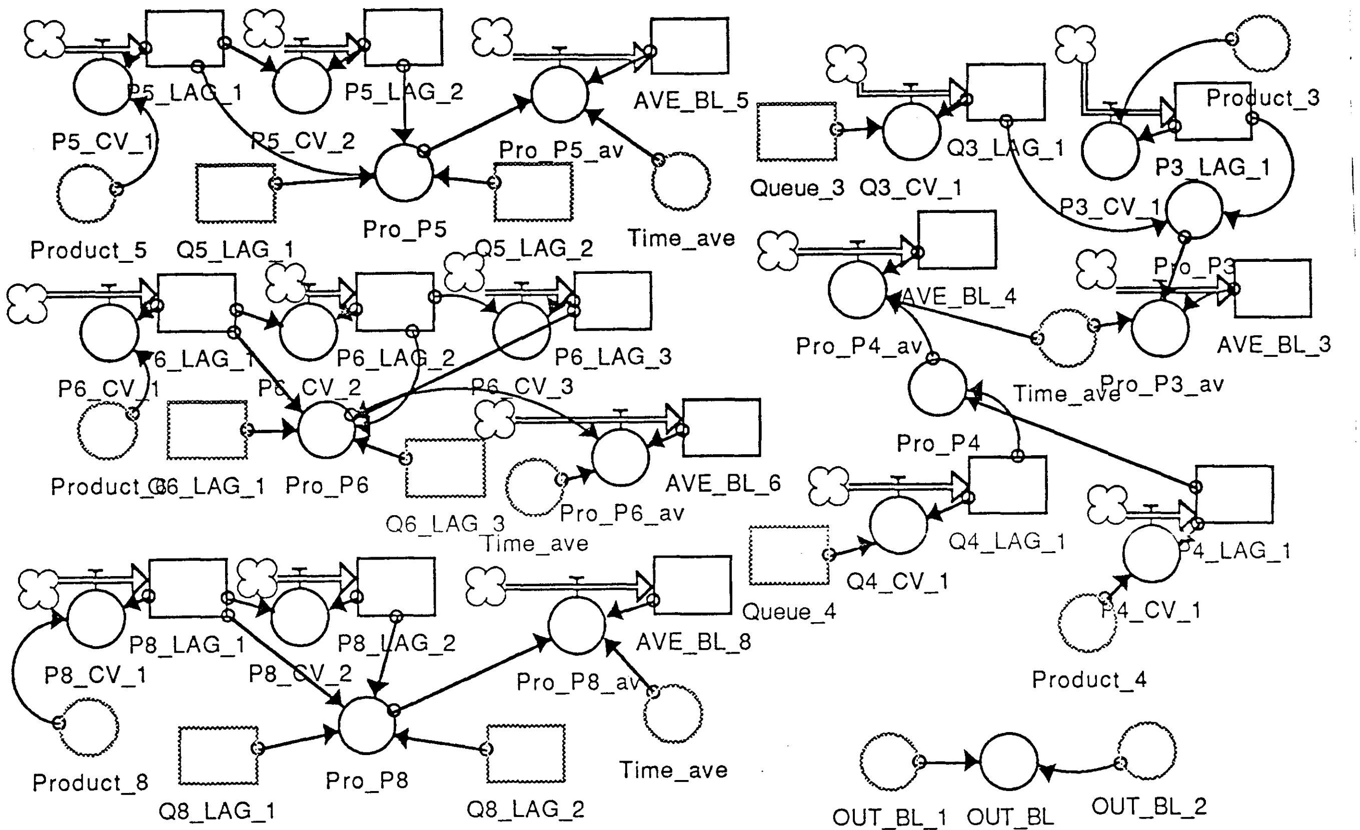

Queue_10
BPro_10
Ran_10
Arrival_10
Product_10
Phy_910
Product_9
Product_Max_10
Alpha_10

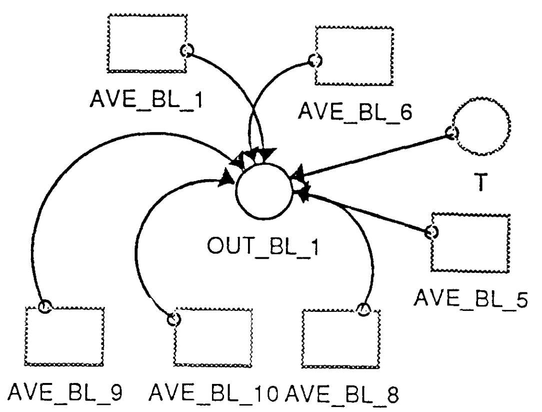

Ave_x_1
Ave_x_4
Ave_x_6
Ave_x_8
Ave_x_2
OUT_EP1
OUT_EP
OUT_EP2
Ave_x_9
Ave_x_3
Ave_x_5
Ave_x_7
Ave_x_10
SET_IN
T
Sigma_1
Sigma_4
Sigma_6
Sigma_7
Sigma_2
OUT_S1
OUT_S
OUT_S2
Sigma_8
Sigma_3
Sigma_5
Sigma_10
Sigma_9

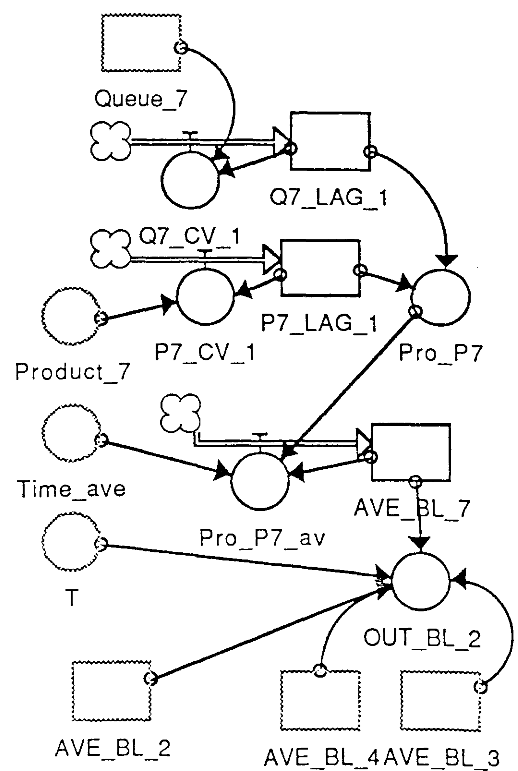

SET_MAX
Ave_x_10
Pro10_av_x
Product_10
Time_ave
Var_10
Sigma_10
Sq_Pro10
Ave_xs_10
Pro10_av_xs

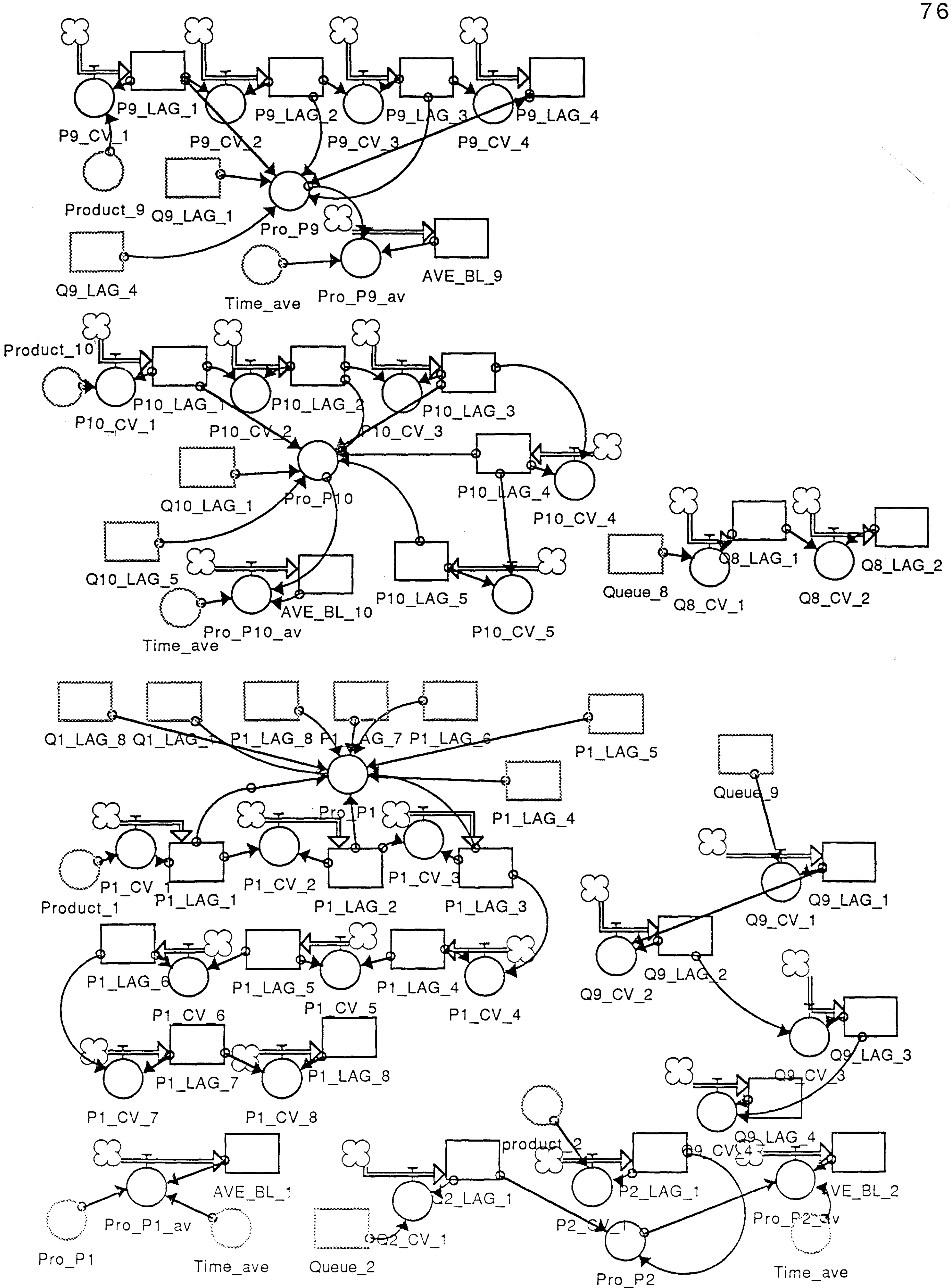

Queue_1
Q1_LAG_1
Q1_CV_1
Q1_LAG_8
Q1_CV_8
Q1_LAG_2
Q1_CV_2
Q1_LAG_7
Q1_CV_7
Q1_LAG_3
Q1_CV_3
Q1_LAG_6
Q1_LAG_4
Q1_CV_4
Q1_LAG_5
Q1_CV_5


Queue_6
Q6_LAG_1
Q6_CV_1
Queue_5
Q5_LAG_1
Q5_CV_1
Q6_LAG_2
Q6_CV_2
Q5_LAG_2
Q5_CV_2
Q6_LAG_3
Q6_CV_3
Q10_LAG_1
Q10_CV_1
Queue_10
Q10_LAG_5
Q10_CV_5
Q10_LAG_2
Q10_CV_2
Q10_LAG_4
Q10_CV_4
Q10_LAG_3
Q10_CV_3


P5_LAG_1
P5_LAG_2
AVE_BL_5
P5_CV_1
P5_CV_2
Pro_P5_av
Product_5
Q5_LAG_1
Pro_P5
Q5_LAG_2
Time_ave
Queue_3
Q3_CV_1
Q3_LAG_1
Product_3
P3_LAG_1
P3_CV_1
Pro_P3
AVE_BL_3
Pro_P3_av
AVE_BL_4
Pro_P4_av
Time_ave
Pro_P4
Q4_LAG_1
Q4_CV_1
Queue_4
P4_LAG_1
P4_CV_1
Product_4
P6_LAG_1
P6_LAG_2
P6_LAG_3
P6_CV_1
P6_CV_2
P6_CV_3
Product_6
Q6_LAG_1
Pro_P6
Q6_LAG_3
AVE_BL_6
Pro_P6_av
Time_ave
P8_LAG_1
P8_LAG_2
AVE_BL_8
P8_CV_1
P8_CV_2
Pro_P8_av
Product_8
Q8_LAG_1
Pro_P8
Q8_LAG_2
Time_ave
OUT_BL_1
OUT_BL
OUT_BL_2


AVE_BL_1
AVE_BL_6
T
OUT_BL_1
AVE_BL_5
AVE_BL_9
AVE_BL_10
AVE_BL_8


Queue_7
Q7_LAG_1
Q7_CV_1
Product_7
P7_CV_1
P7_LAG_1
Pro_P7
Time_ave
AVE_BL_7
Pro_P7_av
T
OUT_BL_2
AVE_BL_2
AVE_BL_4
AVE_BL_3


P9_LAG_1
P9_LAG_2
P9_LAG_4
P9_CV_1
P9_CV_2
P9_CV_3
P9_CV_4
Product_9
Q9_LAG_1
Pro_P9
Q9_LAG_4
AVE_BL_9
Time_ave
Pro_P9_av
Product_10
P10_LAG_2
P10_LAG_3
P10_CV_1
P10_CV_2
Q10_LAG_1
Pro_P10
P10_CV_4
Q10_LAG_5
AVE_BL_10
P10_LAG_5
P10_CV_5
Pro_P10_av
Time_ave
Queue_8
Q8_LAG_1
Q8_LAG_2
Q8_CV_1
Q8_CV_2
Q1_LAG_8
P1_LAG_8
P1_LAG_5
Queue_9
Pro_P1
P1_LAG_4
P1_CV_2
Q9_LAG_1
Product_1
P1_LAG_1
P1_LAG_2
P1_LAG_3
Q9_CV_1
Q9_CV_2
P1_LAG_5
P1_LAG_4
P1_CV_6
P1_CV_5
P1_CV_4
P1_LAG_7
P1_LAG_8
P1_CV_7
P1_CV_8
Q9_LAG_4
product_2
AVE_BL_1
P2_LAG_1
Pro_P1_av
Q2_LAG_1
Pro_P1
Time_ave
Queue_2
Pro_P2
Time_ave

B.2 Equations

- AVE_BL_1=AVE_BL_1+Pro_P1_av
  INIT(AVE_BL_1)=0
- AVE_BL_2=AVE_BL_2+Pro_P2_av
  INIT(AVE_BL_2)=0
- AVE_BL_3=AVE_BL_3+Pro_P3_av
  INIT(AVE_BL_1)=0
- AVE_BL_4=AVE_BL_4+Pro_P4_av
  INIT(AVE_BL_4)=0
- AVE_BL_5=AVE_BL_5+Pro_P5_av
  INIT(AVE_BL_5)=0
- AVE_BL_6=AVE_BL_6+Pro_P6_av
  INIT(AVE_BL_6)=0
- AVE_BL_7=AVE_BL_7+Pro_P7_av
  INIT(AVE_BL_7)=0
- AVE_BL_8=AVE_BL_8+Pro_P8_av
  INIT(AVE_BL_8)=0
- AVE_BL_9=AVE_BL_9+Pro_P9_av
  INIT(AVE_BL_9)=0
- AVE_BL_10=AVE_BL_10+Pro_P10_av
  INIT(AVE_BL_10)=0

- Ave_Q_1=Ave_Q_1+Q_1_av
  INIT(Ave_Q_1)=INIT(Queue_1)
- Ave_Q_2=Ave_Q_2+Q_2_av
  INIT(Ave_Q_2)=INIT(Queue_2)
- Ave_Q_3=Ave_Q_3+Q_3_av
  INIT(Ave_Q_3)=INIT(Queue_3)
- Ave_Q_4=Ave_Q_4+Q_4_av
  INIT(Ave_Q_4)=INIT(Queue_4)
- Ave_Q_5=Ave_Q_5+Q_5_av
  INIT(Ave_Q_5)=INIT(Queue_5)
- Ave_Q_6=Ave_Q_6+Q_6_av
  INIT(Ave_Q_6)=INIT(Queue_6)
- Ave_Q_7=Ave_Q_7+Q_7_av
  INIT(Ave_Q_7)=INIT(Queue_7)
- Ave_Q_8=Ave_Q_8+Q_8_av
  INIT(Ave_Q_8)=INIT(Queue_8)
- Ave_Q_9=Ave_Q_9+Q_9_av

```
  INIT(Ave_Q_9)=INIT(Queue_9)
· Ave_Q_10=Ave_Q_10+Q_10_av
  INIT(Ave_Q_10)=INIT(Queue_10)

· Ave_xs_1=Ave_xs_1+Pro1_av_xs
  INIT(Ave_xs_1)=INIT(Queue_1)*INIT(Queue_1)*Alpha_1*Alpha_1
· Ave_xs_2=Ave_xs_2+Pro2_av_xs
  INIT(Ave_xs_2)=INIT(Queue_2)*INIT(Queue_2)*Alpha_2*Alpha_2
· Ave_xs_3=Ave_xs_3+Pro3_av_xs
  INIT(Ave_xs_3)=INIT(Queue_3)*INIT(Queue_3)*Alpha_3*Alpha_3
· Ave_xs_4=Ave_xs_4+Pro4_av_xs
  INIT(Ave_xs_4)=INIT(Queue_4)*INIT(Queue_4)*Alpha_4*Alpha_4
· Ave_xs_5=Ave_xs_5+Pro5_av_xs
  INIT(Ave_xs_5)=INIT(Queue_5)*INIT(Queue_5)*Alpha_5*Alpha_5
· Ave_xs_6=Ave_xs_6+Pro6_av_xs
  INIT(Ave_xs_6)=INIT(Queue_6)*INIT(Queue_6)*Alpha_6*Alpha_6
· Ave_xs_7=Ave_xs_7+Pro7_av_xs
  INIT(Ave_xs_7)=INIT(Queue_7)*INIT(Queue_7)*Alpha_7*Alpha_7
· Ave_xs_8=Ave_xs_8+Pro8_av_xs
  INIT(Ave_xs_8)=INIT(Queue_8)*INIT(Queue_8)*Alpha_8*Alpha_8
· Ave_xs_9=Ave_xs_9+Pro9_av_xs
  INIT(Ave_xs_9)=INIT(Queue_9)*INIT(Queue_9)*Alpha_9*Alpha_9
· Ave_xs_10=Ave_xs_10+Pro10_av_xs
  INIT(Ave_xs_10)=INIT(Queue_10)*INIT(Queue_10)*Alpha_10*Alpha_10

· Ave_x_1=Ave_x_1+Pro1_av_x
  INIT(Ave_x_1)=INIT(Queue_1)*Alpha_1
· Ave_x_2=Ave_x_2+Pro2_av_x
  INIT(Ave_x_2)=INIT(Queue_2)*Alpha_2
· Ave_x_3=Ave_x_3+Pro3_av_x
  INIT(Ave_x_3)=INIT(Queue_3)*Alpha_3
· Ave_x_4=Ave_x_4+Pro4_av_x
  INIT(Ave_x_4)=INIT(Queue_4)*Alpha_4
· Ave_x_5=Ave_x_5+Pro5_av_x
  INIT(Ave_x_5)=INIT(Queue_5)*Alpha_5
· Ave_x_6=Ave_x_6+Pro6_av_x
  INIT(Ave_x_6)=INIT(Queue_6)*Alpha_6
· Ave_x_7=Ave_x_7+Pro7_av_x
  INIT(Ave_x_7)=INIT(Queue_7)*Alpha_7
· Ave_x_8=Ave_x_8+Pro8_av_x
```

INIT(Ave_x_8)=INIT(Queue_8)*Alpha_8
- Ave_x_9=Ave_x_9+Pro9_av_x
INIT(Ave_x_9)=INIT(Queue_9)*Alpha_9
- Ave_x_10=Ave_x_10+Pro10_av_x
INIT(Ave_x_10)=INIT(Queue_10)*Alpha_10

- P10_LAG_1=P10_LAG_1+P10_CV_1
INIT(P10_LAG_1)=0
- P10_LAG_2=P10_LAG_2+P10_CV_2
INIT(P10_LAG_2)=0
- P10_LAG_3=P10_LAG_3+P10_CV_3
INIT(P10_LAG_3)=0
- P10_LAG_4=P10_LAG_4+P10_CV_4
INIT(P10_LAG_4)=0
- P10_LAG_5=P10_LAG_5+P10_CV_5
INIT(P10_LAG_5)=0
- P1_LAG_1=P1_LAG_1+P1_CV_1
INIT(P1_LAG_1)=0
- P1_LAG_2=P1_LAG_2+P1_CV_2
INIT(P1_LAG_2)=0
- P1_LAG_3=P1_LAG_3+P1_CV_3
INIT(P1_LAG_3)=0
- P1_LAG_4=P1_LAG_4+P1_CV_4
INIT(P1_LAG_4)=0
- P1_LAG_5=P1_LAG_5+P1_CV_5
INIT(P1_LAG_5)=0
- P1_LAG_6=P1_LAG_6+P1_CV_6
INIT(P1_LAG_6)=0
- P1_LAG_7=P1_LAG_7+P1_CV_7
INIT(P1_LAG_7)=0
- P1_LAG_8=P1_LAG_8+P1_CV_8
INIT(P1_LAG_8)=0
- P2_LAG_1=P2_LAG_1+P2_CV_1
INIT(P2_LAG_1)=0
- P3_LAG_1=P3_LAG_1+P3_CV_1
INIT(P3_LAG_1)=0
- P4_LAG_1=P4_LAG_1+P4_CV_1
INIT(P4_LAG_1)=0
- P5_LAG_1=P5_LAG_1+P5_CV_1

```
INIT(P5_LAG_1)=0
˙ P5_LAG_2=P5_LAG_2+P5_CV_2
INIT(P5_LAG_2)=0
˙ P6_LAG_1=P6_LAG_1+P6_CV_1
INIT(P6_LAG_1)=0
˙ P6_LAG_2=P6_LAG_2+P6_CV_2
INIT(P6_LAG_2)=0
˙ P6_LAG_3=P6_LAG_3+P6_CV_3
INIT(P6_LAG_3)=0
˙ P7_LAG_1=P7_LAG_1+P7_CV_1
INIT(P7_LAG_1)=0
˙ P8_LAG_1=P8_LAG_1+P8_CV_1
INIT(P8_LAG_1)=0
˙ P8_LAG_2=P8_LAG_2+P8_CV_2
INIT(P8_LAG_2)=0
˙ P9_LAG_1=P9_LAG_1+P9_CV_1
INIT(P9_LAG_1)=0
˙ P9_LAG_2=P9_LAG_2+P9_CV_2
INIT(P9_LAG_2)=0
˙ P9_LAG_3=P9_LAG_3+P9_CV_3
INIT(P9_LAG_3)=0
˙ P9_LAG_4=P9_LAG_4+P9_CV_4
INIT(P9_LAG_4)=0

˙ Q10_LAG_1=Q10_LAG_1+Q10_CV_1
INIT(Q10_LAG_1)=0
˙ Q10_LAG_2=Q10_LAG_2+Q10_CV_2
INIT(Q10_LAG_2)=0
˙ Q10_LAG_3=Q10_LAG_3+Q10_CV_3
INIT(Q10_LAG_3)=0
˙ Q10_LAG_4=Q10_LAG_4+Q10_CV_4
INIT(Q10_LAG_4)=0
˙ Q10_LAG_5=Q10_LAG_5+Q10_CV_5
INIT(Q10_LAG_5)=0
˙ Q1_LAG_1=Q1_LAG_1+Q1_CV_1
INIT(Q1_LAG_1)=0
˙ Q1_LAG_2=Q1_LAG_2+Q1_CV_2
INIT(Q1_LAG_2)=0
˙ Q1_LAG_3=Q1_LAG_3+Q1_CV_3
INIT(Q1_LAG_3)=0
```

```
• Q1_LAG_4=Q1_LAG_4+Q1_CV_4
INIT(Q1_LAG_4)=0
• Q1_LAG_5=Q1_LAG_5+Q1_CV_5
INIT(Q1_LAG_5)=0
• Q1_LAG_6=Q1_LAG_6+Q1_CV_6
INIT(Q1_LAG_6)=0
• Q1_LAG_7=Q1_LAG_7+Q1_CV_7
INIT(Q1_LAG_7)=0
• Q1_LAG_8=Q1_LAG_8+Q1_CV_8
INIT(Q1_LAG_8)=0
• Q2_LAG_1=Q2_LAG_1+Q2_CV_1
INIT(Q2_LAG_1)=0
• Q3_LAG_1=Q3_LAG_1+Q3_CV_1
INIT(Q3_LAG_1)=0
• Q4_LAG_1=Q4_LAG_1+Q4_CV_1
INIT(Q4_LAG_1)=0
• Q5_LAG_1=Q5_LAG_1+Q5_CV_1
INIT(Q5_LAG_1)=0
• Q5_LAG_2=Q5_LAG_2+Q5_CV_2
INIT(Q5_LAG_2)=0
• Q6_LAG_1=Q6_LAG_1+Q6_CV_1
INIT(Q6_LAG_1)=0
• Q6_LAG_2=Q6_LAG_2+Q6_CV_2
INIT(Q6_LAG_2)=0
• Q6_LAG_3=Q6_LAG_3+Q6_CV_3
INIT(Q6_LAG_3)=0
• Q7_LAG_1=Q7_LAG_1+Q7_CV_1
INIT(Q7_LAG_1)=0
• Q8_LAG_1=Q8_LAG_1+Q8_CV_1
INIT(Q8_LAG_1)=0
• Q8_LAG_2=Q8_LAG_2+Q8_CV_2
INIT(Q8_LAG_2)=0
• Q9_LAG_1=Q9_LAG_1+Q9_CV_1
INIT(Q9_LAG_1)=0
• Q9_LAG_2=Q9_LAG_2+Q9_CV_2
INIT(Q9_LAG_2)=0
• Q9_LAG_3=Q9_LAG_3+Q9_CV_3
INIT(Q9_LAG_3)=0
• Q9_LAG_4=Q9_LAG_4+Q9_CV_4
INIT(Q9_LAG_4)=0
```

```
Q9_LAG_4=Q9_LAG_4+Q9_CV_4
INIT(Q9_LAG_4)=0

Queue_1=Queue_1+Arrival_1-Product_1
  INIT(Queue_1)=5.01/Alpha_1*SET_IN
Queue_2=Queue_2+Arrival_2-Product_2
  INIT(Queue_2)=0.75/Alpha_2*SET_IN
Queue_3=Queue_3+Arrival_3-Product_3
  INIT(Queue_3)=0.69/Alpha_3*SET_IN
Queue_4=Queue_4+Arrival_4-Product_4
  INIT(Queue_4)=0.36/Alpha_4*SET_IN
Queue_5=Queue_5+Arrival_5-Product_5
  INIT(Queue_5)=1.37/Alpha_5*SET_IN
Queue_6=Queue_6+Arrival_6-Product_6
  INIT(Queue_6)=1.65/Alpha_6*SET_IN
Queue_7=Queue_7+Arrival_7-Product_7
  INIT(Queue_7)=0.14/Alpha_7*SET_IN
Queue_8=Queue_8+Arrival_8-Product_8
  INIT(Queue_8)=0.55/Alpha_8*SET_IN
Queue_9=Queue_9+Arrival_9-Product_9
  INIT(Queue_9)=1.89/Alpha_9*SET_IN
Queue_10=Queue_10+Arrival_10-Product_10
  INIT(Queue_10)=2.19/Alpha_10*SET_IN

  Alpha_1=1/8
  Alpha_2=1/1
  Alpha_3=1/1
  Alpha_4=1/1
  Alpha_5=1/2
  Alpha_6=1/3
  Alpha_7=1/1
  Alpha_8=1/2
  Alpha_9=1/4
  Alpha_10=1/5

  Arrival_1=Phy_31+Phy_51+Ran_1
  Arrival_2=Phy_12+Ran_2
  Arrival_3=Phy_13+Phy_23+Phy_43+Phy_63+Phy_93+Ran_3
  Arrival_4=Phy_14+Phy_24+Ran_4
  Arrival_5=Phy_15+Phy_25+Phy_35+Ran_5
```

```
Arrival_6=Phy_16+Phy_56+Phy_96+Ran_6
Arrival_7=Phy_57+Ran_7
Arrival_8=Phy_18+Phy_68+Phy_78+Ran_8
Arrival_9=Phy_89+Ran_9
Arrival_10=Phy_910+Ran_10

BPro_1=0.1
BPro_2=0.1
BPro_3=0.1
BPro_4=0.1
BPro_5=0.1
BPro_6=0.1
BPro_7=0.1
BPro_8=0.1
BPro_9=0.05
BPro_10=0.1

Input=4

OUT_BL=OUT_BL_1+OUT_BL_2
OUT_BL_1= IF T=1 THEN AVE_BL_1 ELSE IF T=5 THEN AVE_BL_5
     ELSE IF T=6 THEN AVE_BL_6 ELSE IF T=8 THEN AVE_BL_8
     ELSE IF T=9 THEN AVE_BL_9 ELSE IF T=10 THEN AVE_BL_10
     ELSE 0
OUT_BL_2= IF T=2 THEN AVE_BL_2 ELSE IF T=3 THEN AVE_BL_3
     ELSE IF T=4THEN AVE_BL_4 ELSE IF T=7 THEN AVE_BL_7
     ELSE 0
OUT_EP=OUT_EP1+OUT_EP2
OUT_EP1= IF T=1 THEN Ave_x_1 ELSE IF T=2 THEN Ave_x_2
     ELSE IF T=3 THEN Ave_x_3 ELSE IF T=4 THEN Ave_x_4
     ELSE IF T=5 THEN Ave_x_5 ELSE 0
OUT_EP2= IF T=6 THEN Ave_x_6 ELSE IF T=7 THEN Ave_x_7
     ELSE IF T=8 THEN Ave_x_8 ELSE IF T=9 THEN Ave_x_9
     ELSE IF T=10 THEN Ave_x_10 ELSE 0
OUT_EQ=OUT_EQ1+OUT_EQ2
OUT_EQ1= IF T=1 THEN Ave_Q_1 ELSE IF T=2 THEN Ave_Q_2
     ELSE IF T=3 THEN Ave_Q_3 ELSE IF T=4 THEN Ave_Q_4
     ELSE IF T=5 THEN Ave_Q_5 ELSE 0
OUT_EQ2= IF T=6 THEN Ave_Q_6 ELSE IF T=7 THEN Ave_Q_7
     ELSE IF T=8 THEN Ave_Q_8 ELSE IF T=9 THEN Ave_Q_9
     ELSE IF T=10 THEN Ave_Q_10 ELSE 0
OUT_S=OUT_S1+OUT_S2
OUT_S1= IF T=1 THEN Sigma_1 ELSE IF T=2 THEN Sigma_2
```

```
     ELSE IF T=3 THEN Sigma_3 ELSE IF T=4 THEN Sigma_4
     ELSE IF T=5 THEN Sigma_5 ELSE 0
OUT_S2= IF T=6 THEN Sigma_6 ELSE IF T=7 THEN Sigma_7
     ELSE IF T=8 THEN Sigma_8 ELSE IF T=9 THEN Sigma_9
     ELSE IF T=10 THEN Sigma_10 ELSE 0


P10_CV_1=Product_10-P10_LAG_1
P10_CV_2=P10_LAG_1-P10_LAG_2
P10_CV_3=P10_LAG_2-P10_LAG_3
P10_CV_4=P10_LAG_3-P10_LAG_4
P10_CV_5=P10_LAG_4-P10_LAG_5
P1_CV_1=Product_1-P1_LAG_1
P1_CV_2=P1_LAG_1-P1_LAG_2
P1_CV_3=P1_LAG_2-P1_LAG_3
P1_CV_4=P1_LAG_3-P1_LAG_4
P1_CV_5=P1_LAG_4-P1_LAG_5
P1_CV_6=P1_LAG_5-P1_LAG_6
P1_CV_7=P1_LAG_6-P1_LAG_7
P1_CV_8=P1_LAG_7-P1_LAG_8
P2_CV_1=Product_2-P2_LAG_1
P3_CV_1=Product_3-P3_LAG_1
P4_CV_1=Product_4-P4_LAG_1
P5_CV_1=Product_5-P5_LAG_1
P5_CV_2=P5_LAG_1-P5_LAG_2
P6_CV_1=Product_6-P6_LAG_1
P6_CV_2=P6_LAG_1-P6_LAG_2
P6_CV_3=P6_LAG_2-P6_LAG_3
P7_CV_1=Product_7-P7_LAG_1
P8_CV_1=Product_8-P8_LAG_1
P8_CV_2=P8_LAG_1-P8_LAG_2
P9_CV_1=Product_9-P9_LAG_1
P9_CV_2=P9_LAG_1-P9_LAG_2
P9_CV_3=P9_LAG_2-P9_LAG_3
P9_CV_4=P9_LAG_3-P9_LAG_4

Phy_12=Product_1*0.15
Phy_13=Product_1*0.04
Phy_14=Product_1*0.01
Phy_15=Product_1*0.03
Phy_16=Product_1*0.24
Phy_18=Product_1*0.01
Phy_23=Product_2*0.01
```

```
Phy_24=Product_2*0.41
Phy_25=Product_2*0.37
Phy_31=Product_3*0.11
Phy_35=Product_3*1.36
Phy_43=Product_4*0.71
Phy_51=Product_5*0.68
Phy_56=Product_5*0.15
Phy_57=Product_5*0.1
Phy_63=Product_6*0.06
Phy_68=Product_6*0.22
Phy_78=Product_7*1.0
Phy_89=Product_8*3.43
Phy_910=Product_9*1.16
Phy_93=Product_9*0.07
Phy_96=Product_9*0.13

Pro10_av_x=(Product_10-Ave_x_10)/Time_ave
Pro10_av_xs=(Sq_Pro_10-Ave_xs_10)/Time_ave
Pro1_av_x=(Product_1-Ave_x_1)/Time_ave
Pro1_av_xs=(Sq_Pro_1-Ave_xs_1)/Time_ave
Pro2_av_x=(Product_2-Ave_x_2)/Time_ave
Pro2_av_xs=(Sq_Pro_2-Ave_xs_2)/Time_ave
Pro3_av_x=(Product_3-Ave_x_3)/Time_ave
Pro3_av_xs=(Sq_Pro_3-Ave_xs_3)/Time_ave
Pro4_av_x=(Product_4-Ave_x_4)/Time_ave
Pro4_av_xs=(Sq_Pro_4-Ave_xs_4)/Time_ave
Pro5_av_x=(Product_5-Ave_x_5)/Time_ave
Pro5_av_xs=(Sq_Pro_5-Ave_xs_5)/Time_ave
Pro6_av_x=(Product_6-Ave_x_6)/Time_ave
Pro6_av_xs=(Sq_Pro_6-Ave_xs_6)/Time_ave
Pro7_av_x=(Product_7-Ave_x_7)/Time_ave
Pro7_av_xs=(Sq_Pro_7-Ave_xs_7)/Time_ave
Pro8_av_x=(Product_8-Ave_x_8)/Time_ave
Pro8_av_xs=(Sq_Pro_8-Ave_xs_8)/Time_ave
Pro9_av_x=(Product_9-Ave_x_9)/Time_ave
Pro9_av_xs=(Sq_Pro_9-Ave_xs_9)/Time_ave

Product_1=IF RANDOM>=BPro_1 THEN MIN(Alpha_1*Queue_1,
        Product_Max_1)   ELSE 0
Product_10=IF RANDOM>=BPro_10 THEN MIN(Alpha_10*Queue_10,
        Product_Max_10)   ELSE 0
Product_2=IF RANDOM>=BPro_2 THEN MIN(Alpha_2*Queue_2,
        Product_Max_2)   ELSE 0
```

```
Product_3=IF RANDOM>=BPro_3 THEN MIN(Alpha_3*Queue_3,
          Product_Max_3)  ELSE 0
Product_4=IF RANDOM>=BPro_4 THEN MIN(Alpha_4*Queue_4,
          Product_Max_4)  ELSE 0
Product_5=IF RANDOM>=BPro_5 THEN MIN(Alpha_5*Queue_5,
          Product_Max_5)  ELSE 0
Product_6=IF RANDOM>=BPro_6 THEN MIN(Alpha_6*Queue_6,
          Product_Max_6)  ELSE 0
Product_7=IF RANDOM>=BPro_7 THEN MIN(Alpha_7*Queue_7,
          Product_Max_7)  ELSE 0
Product_8=IF RANDOM>=BPro_8 THEN MIN(Alpha_8*Queue_8,
          Product_Max_8)  ELSE 0
Product_9=IF RANDOM>=BPro_9 THEN MIN(Alpha_9*Queue_9,
          Product_Max_9)  ELSE 0
Product_Max_1= IF  SET_MAX=1 THEN 6 ELSE 100
Product_Max_10= IF  SET_MAX=1 THEN 2.5 ELSE 100
Product_Max_2= IF  SET_MAX=1 THEN 2 ELSE 100
Product_Max_3= IF  SET_MAX=1 THEN 2 ELSE 100
Product_Max_4= IF  SET_MAX=1 THEN 2 ELSE 100
Product_Max_5= IF  SET_MAX=1 THEN 2 ELSE 100
Product_Max_6= IF  SET_MAX=1 THEN 2 ELSE 100
Product_Max_7= IF  SET_MAX=1 THEN 2 ELSE 100
Product_Max_8= IF  SET_MAX=1 THEN 2 ELSE 100
Product_Max_9= IF  SET_MAX=1 THEN 2 ELSE 100

Pro_P1 =( Q1_LAG_1-P1_LAG_1)*0+(Q1_LAG_8-P1_LAG_1-P1_LAG_2
         -P1_LAG_3-P1_LAG_4-P1_LAG_5-P1_LAG_6-P1_LAG_7
        -P1_LAG_8)*1
     {alpha=1 then 1,0 alpha=1/8 then 0,1}
Pro_P1_av =IF Pro_P1>=0 THEN (  Pro_P1-AVE_BL_4)/Time_ave
             ELSE   (-AVE_BL_1)/Time_ave
Pro_P10 =( Q10_LAG_1-P10_LAG_1)*0+(Q10_LAG_5-P10_LAG_1
         -P10_LAG_2-P10_LAG_3-P10_LAG_4-P10_LAG_5)*1
     {alpha=1 then 1,0 alpha=1/5 then 0,1}
Pro_P10_av =IF Pro_P10>=0 THEN (  Pro_P10-AVE_BL_4)/Time_ave
             ELSE   (-AVE_BL_10)/Time_ave
Pro_P2 =( Q2_LAG_1-P2_LAG_1)
Pro_P2_av =IF Pro_P2>=0 THEN (  Pro_P2-AVE_BL_4)/Time_ave
             ELSE   (-AVE_BL_2)/Time_ave
Pro_P3 =( Q3_LAG_1-P3_LAG_1)
Pro_P3_av =IF Pro_P3>=0 THEN (  Pro_P3-AVE_BL_4)/Time_ave
             ELSE   (-AVE_BL_3)/Time_ave
Pro_P4 =( Q4_LAG_1-P4_LAG_1)
```

```
Pro_P4_av =IF Pro_P4>=0 THEN (  Pro_P4-AVE_BL_4)/Time_ave
                    ELSE   (-AVE_BL_4)/Time_ave
Pro_P5 =( Q5_LAG_1-P5_LAG_1)*0+(Q5_LAG_2-P5_LAG_1
           -P5_LAG_2)*1
         {alpha=1 then 1,0 alpha=1/2 then 0,1}
Pro_P5_av =IF Pro_P5>=0 THEN (  Pro_P5-AVE_BL_4)/Time_ave
                    ELSE   (-AVE_BL_5)/Time_ave
Pro_P6 =( Q6_LAG_1-P6_LAG_1)*0+(Q6_LAG_3-P6_LAG_1-P6_LAG_2
           -P6_LAG_3)*1
         {alpha=1 then 1,0 alpha=1/3 then 0,1}
Pro_P6_av =IF Pro_P6>=0 THEN (  Pro_P6-AVE_BL_4)/Time_ave
                    ELSE   (-AVE_BL_6)/Time_ave
Pro_P7 =( Q7_LAG_1-P7_LAG_1)
Pro_P7_av =IF Pro_P7>=0 THEN (  Pro_P7-AVE_BL_4)/Time_ave
                    ELSE   (-AVE_BL_7)/Time_ave
Pro_P8 =( Q8_LAG_1-P8_LAG_1)*0+(Q8_LAG_2-P8_LAG_1
           -P8_LAG_2)*1
         {alpha=1 then 1,0 alpha=1/2 then 0,1}
Pro_P8_av =IF Pro_P8>=0 THEN (  Pro_P8-AVE_BL_4)/Time_ave
                    ELSE   (-AVE_BL_8)/Time_ave
Pro_P9 =( Q9_LAG_1-P9_LAG_1)*0+(Q9_LAG_4-P9_LAG_1-P9_LAG_2
           -P9_LAG_3-P9_LAG_4)*1
         {alpha=1 then 1,0 alpha=1/4 then 0,1}
Pro_P9_av =IF Pro_P9>=0 THEN (  Pro_P9-AVE_BL_4)/Time_ave
                    ELSE   (-AVE_BL_9)/Time_ave

Q10_CV_1=Queue_10-Q10_LAG_1
Q10_CV_2=Q10_LAG_1-Q10_LAG_2
Q10_CV_3=Q10_LAG_2-Q10_LAG_3
Q10_CV_4=Q10_LAG_3-Q10_LAG_4
Q10_CV_5=Q10_LAG_4-Q10_LAG_5
Q1_CV_1=Queue_1-Q1_LAG_1
Q1_CV_2=Q1_LAG_1-Q1_LAG_2
Q1_CV_3=Q1_LAG_2-Q1_LAG_3
Q1_CV_4=Q1_LAG_3-Q1_LAG_4
Q1_CV_5=Q1_LAG_4-Q1_LAG_5
Q1_CV_6=Q1_LAG_5-Q1_LAG_6
Q1_CV_7=Q1_LAG_6-Q1_LAG_7
Q1_CV_8=Q1_LAG_7-Q1_LAG_8
Q2_CV_1=Queue_2-Q2_LAG_1
Q3_CV_1=Queue_3-Q3_LAG_1
Q4_CV_1=Queue_4-Q4_LAG_1
Q5_CV_1=Queue_5-Q5_LAG_1
```

Q5_CV_2=Q5_LAG_1-Q5_LAG_2
Q6_CV_1=Queue_6-Q6_LAG_1
Q6_CV_2=Q6_LAG_1-Q6_LAG_2
Q6_CV_3=Q6_LAG_2-Q6_LAG_3
Q7_CV_1=Queue_7-Q7_LAG_1
Q8_CV_1=Queue_8-Q8_LAG_1
Q8_CV_2=Q8_LAG_1-Q8_LAG_2
Q9_CV_1=Queue_9-Q9_LAG_1
Q9_CV_2=Q9_LAG_1-Q9_LAG_2
Q9_CV_3=Q9_LAG_2-Q9_LAG_3
Q9_CV_4=Q9_LAG_3-Q9_LAG_4

Q_10_av=(Queue_10-Ave_Q_10)/Time_ave
Q_1_av=(Queue_1-Ave_Q_1)/Time_ave
Q_2_av=(Queue_2-Ave_Q_2)/Time_ave
Q_3_av=(Queue_3-Ave_Q_3)/Time_ave
Q_4_av=(Queue_4-Ave_Q_4)/Time_ave
Q_5_av=(Queue_5-Ave_Q_5)/Time_ave
Q_6_av=(Queue_6-Ave_Q_6)/Time_ave
Q_7_av=(Queue_7-Ave_Q_7)/Time_ave
Q_8_av=(Queue_8-Ave_Q_8)/Time_ave
Q_9_av=(Queue_9-Ave_Q_9)/Time_ave

Ran_1 =NORMAL*2+INPUT
Ran_10=NORMAL*0.2
Ran_2=NORMAL*0.1
Ran_3 =NORMAL*0.1
Ran_4 =NORMAL*0.1
Ran_5=NORMAL*0.2
Ran_6 =NORMAL*0.2
Ran_7 =0
Ran_8 =NORMAL*0.1
Ran_9 =NORMAL*0.2

SET_IN=1 {If it is one the simulation is for steady state.}
SET_MAX=1 {If it is one the limiter will be applied.}
SET_T_MAX=2000 {Maximum simulation time should be set here in order to control out put data.}

Sigma_1= IF Var_1>=0 THEN EXP(0.5*LOGN(Var_1) ELSE 0
Sigma_10= IF Var_10>=0 THEN EXP(0.5*LOGN(Var_10) ELSE 0
Sigma_2= IF Var_2>=0 THEN EXP(0.5*LOGN(Var_2) ELSE 0
Sigma_3= IF Var_3>=0 THEN EXP(0.5*LOGN(Var_3) ELSE 0

Sigma_4= IF Var_4>=0 THEN EXP(0.5*LOGN(Var_4) ELSE 0
Sigma_5= IF Var_5>=0 THEN EXP(0.5*LOGN(Var_5) ELSE 0
Sigma_6= IF Var_6>=0 THEN EXP(0.5*LOGN(Var_6) ELSE 0
Sigma_7= IF Var_7>=0 THEN EXP(0.5*LOGN(Var_7) ELSE 0
Sigma_8= IF Var_8>=0 THEN EXP(0.5*LOGN(Var_8) ELSE 0
Sigma_9= IF Var_9>=0 THEN EXP(0.5*LOGN(Var_9) ELSE 0

Sq_Pro1 =Product_1*Product_1
Sq_Pro10 =Product_10*Product_10
Sq_Pro2 =Product_2*Product_2
Sq_Pro3 =Product_3*Product_3
Sq_Pro4 =Product_4*Product_4
Sq_Pro5 =Product_5*Product_5
Sq_Pro6 =Product_6*Product_6
Sq_Pro7 =Product_7*Product_7
Sq_Pro8 =Product_8*Product_8
Sq_Pro9 =Product_9*Product_9

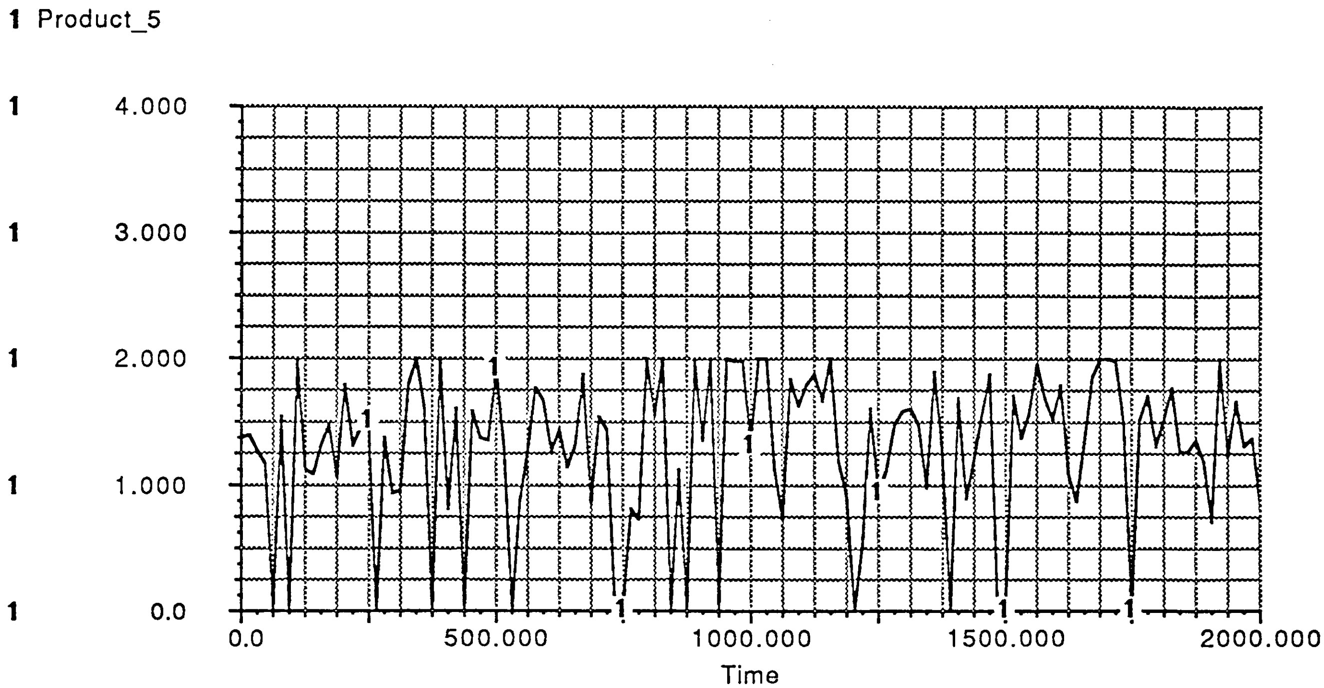
T=IF TIME,(SET_T_MAX-9) THEN 0 ELSE IF TIME>(SET_T_Max) THEN
    0 ELSE TIME-(SET_T_Max-10)
Time_ave= IF TIME<= 500 THEN 1 ELSE TIME-500

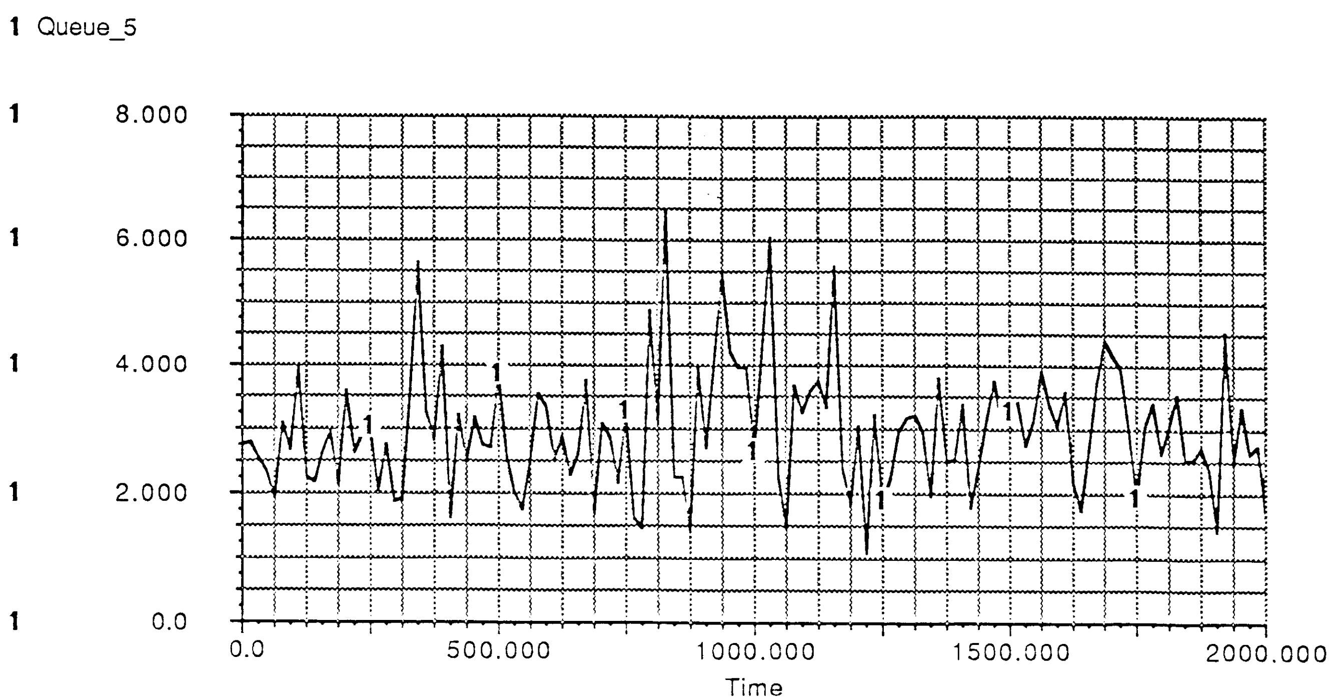
Var_1=IF Time_ave>2 THEN (Ave_xs_1-Ave_x_1*Ave_x_1)*
(Time_ave/(Time_ave-1)) ELSE 0
Var_10=IF Time_ave>2 THEN (Ave_xs_10-Ave_x_10*Ave_x_10)*
(Time_ave/(Time_ave-1)) ELSE 0
Var_2=IF Time_ave>2 THEN (Ave_xs_2-Ave_x_2*Ave_x_2)*
(Time_ave/(Time_ave-1)) ELSE 0
Var_3=IF Time_ave>2 THEN (Ave_xs_3-Ave_x_3*Ave_x_3)*
(Time_ave/(Time_ave-1)) ELSE 0
Var_4=IF Time_ave>2 THEN (Ave_xs_4-Ave_x_4*Ave_x_4)*
(Time_ave/(Time_ave-1)) ELSE 0
Var_5=IF Time_ave>2 THEN (Ave_xs_5-Ave_x_5*Ave_x_5)*
(Time_ave/(Time_ave-1)) ELSE 0
Var_6=IF Time_ave>2 THEN (Ave_xs_6-Ave_x_6*Ave_x_6)*
(Time_ave/(Time_ave-1)) ELSE 0
Var_7=IF Time_ave>2 THEN (Ave_xs_7-Ave_x_7*Ave_x_7)*
(Time_ave/(Time_ave-1)) ELSE 0
Var_8=IF Time_ave>2 THEN (Ave_xs_8-Ave_x_8*Ave_x_8)*
(Time_ave/(Time_ave-1)) ELSE 0
Var_9=IF Time_ave>2 THEN (Ave_xs_9-Ave_x_9*Ave_x_9)*
(Time_ave/(Time_ave-1)) ELSE 0

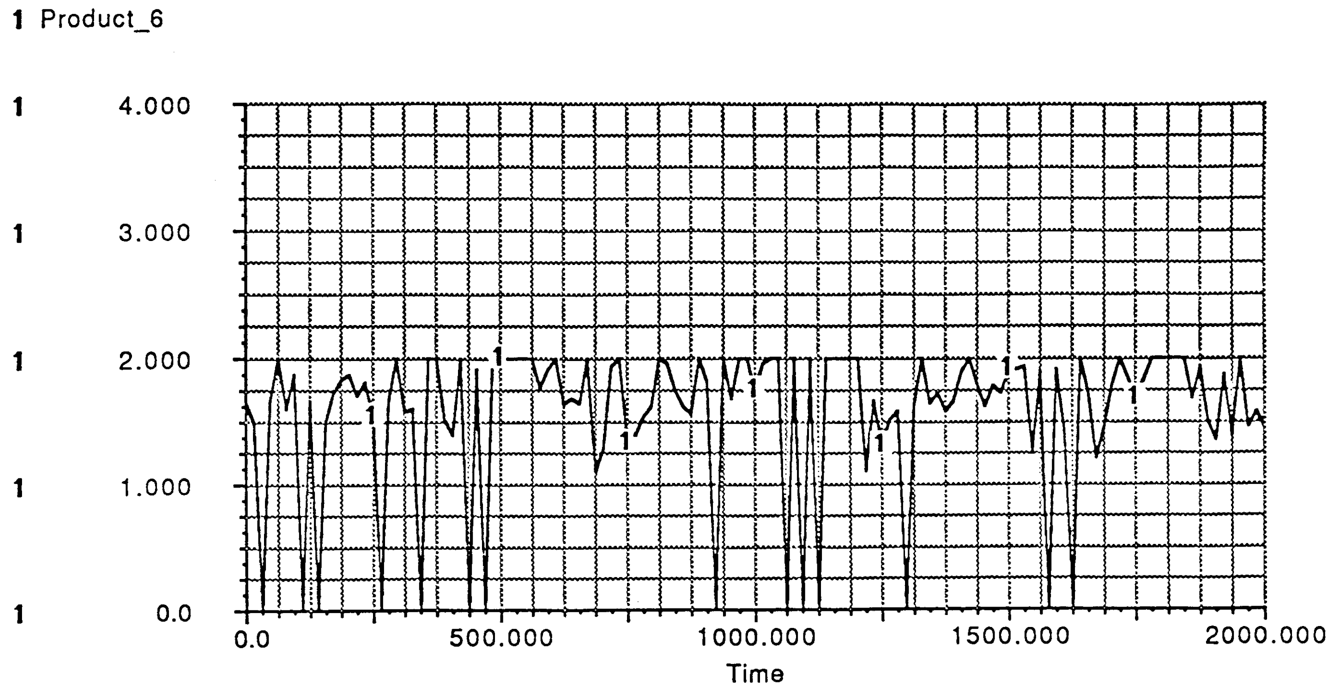
B.3 Example of output
(1) Numerical output

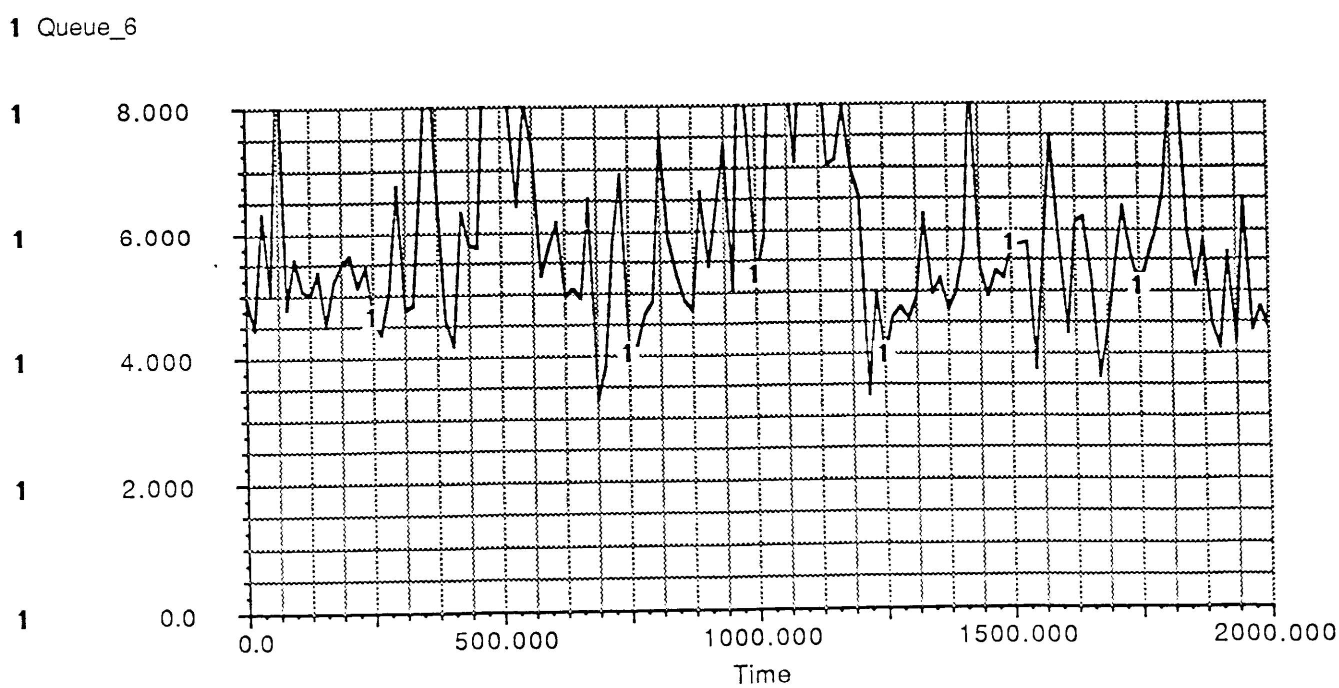
Time	T	OUT_EP	OUT_S	OUT_EQ	OUT_BL
990.000	0.0	0.0	0.0	0.0	0.0
991.000	1.000	4.921	1.653	45.766	6.734
992.000	2.000	0.737	0.390	0.800	0.0637
993.000	3.000	0.683	0.387	0.764	0.0810
994.000	4.000	0.353	0.264	0.401	0.0483
995.000	5.000	1.351	0.577	3.138	0.520
996.000	6.000	1.624	0.545	5.981	1.163
997.000	7.000	0.135	0.0856	0.150	0.0148
998.000	8.000	0.540	0.238	1.201	0.146
999.000	9.000	1.861	0.426	14.194	6.821
2000.000	10.000	2.171	0.689	14.611	3.833

(2) Graphical output






1 Product_6
4.000
3.000
2.000
1.000
0.0
0.0
500.000
1000.000
1500.000
2000.000
Time


1 Queue_6
8.000
6.000
4.000
2.000
0.0
0.0
500.000
1000.000
1500.000
2000.000
Time