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ABSTRACT

A frequency domain (harmonic) finite element model is developed for the numerical prediction of depth average circulation within small embayments. Such embayments are often characterized by irregular boundaries and bottom topography and large gradients in velocity. Previously developed finite element based time domain models require high eddy viscosity coefficients and small time steps to insure numerical stability, making application to small bays infeasible. Application of the harmonic method in conjunction with finite elements overcomes these problems. The model TEA, for Tidal Embayment Analysis, solves the linearized problem and is the core of a fully nonlinear code presently under development.

This report discusses in detail both the theory behind TEA and program usage. Furthermore the versatility of TEA is demonstrated in several prototype examples.

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1. INTRODUCTION

In the past few decades there has been considerable advancement in the predictive capabilities for tidally induced circulation in waterbodies. Many of the more recent numerical models have applied the finite element method [11,12]. The principal advantage of finite element methods over the more traditional finite difference methods is the greater versatility allowed in grid discretization, both in terms of fitting the often highly irregular coastal geometry and in terms of refining the grid in critical areas. Most of these models have been time based, the time dependence being resolved either by using some type of numerical integration scheme or by applying a combined space-time finite element scheme. However, these time domain models have been plagued with requirements for small time steps necessary to insure numerical stability. The maximum allowable time step decreases along with decreasing element size, making application to small bays infeasible because of the extreme costs involved. Furthermore, these models have been criticized for their dependence on high eddy viscosity coefficients, required for numerical damping of small wave length noise, to achieve reasonable and smooth solutions [3]. Finally, these time domain codes have the additional shortcoming of requiring start up periods in order to allow initial transients to be damped out.

To overcome these inherent difficulties associated with time domain discretizations, investigators have lately applied the harmonic method in conjunction with finite elements [7,8,10]. Because of the periodic nature of the tidal phenomenon, the harmonic method is an intrinsically more natural solution procedure and was one of the traditional methods for analysis before the advent of finite difference and finite element methods [2]. There are no time stepping limitations as with this procedure a set of quasi-steady (or time independent) equations are generated. Furthermore, there is no more need for artificially high eddy viscosity coefficients. One difficulty which arises, however, when implementing this frequency domain method, is that non linear terms can not be included as a part of the solution but must be generated by iterative superpositioning of several frequencies.

Present research is aimed at advancing the capacities of this harmonic method by developing a code specifically geared towards small scale geometries, where the most formidable problems occur with the time domain codes now available.

This report discusses a highly efficient linear code which shall be used as the core of a full non linear iterative program presently under development. This linear code has been specifically tailored to fulfill the characteristics which are desirable when applying this frequency domain method to small bays where the non linearities are often quite substantial. However, for many applications this linearized finite element frequency domain model may be quite sufficient and very useful. In the following two chapters we shall briefly discuss the theory and numerical techniques involved in the development of this linear model, such that the user will have a clearer and more comprehensive understanding of the potentials, limitations and requirements of the computer code TEA (Tidal Embayment Analysis) developed.

2. GOVERNING EQUATIONS

The equations to be solved are the depth averaged forms of the continuity and Navier-Stokes equations. With the assumptions of constant density fluid, hydrostatic pressure distribution, negligible horizontal momentum dispersion (or eddy viscosity) and constant pressure at the air-water interface, these equations are:

$$\eta_{,t} + [u(h + \eta)]_{,x} + [v(h + \eta)]_{,y} = 0 \tag{2.1}$$

$$u_{,t} + uu_{,x} + vu_{,y} = -g\, \eta_{,x} + fv + \frac{1}{\rho h} \left(\frac{h}{H}\right) (\tau_x^s - \tau_x^b) \tag{2.2a}$$

$$v_{,t} + uv_{,x} + vv_{,y} = -g\, \eta_{,y} - fu + \frac{1}{\rho h} \left(\frac{h}{H}\right) (\tau_y^s - \tau_y^b) \tag{2.2b}$$

where u,v are the depth averaged velocities in the x and y coordinate directions, respectively, η is surface elevation above mean water level, h is mean water depth, t is time, $H = h + \eta$ is total depth, g is acceleration due to gravity, ρ is the density of water, f is the Coriolis parameter, τ_x^s and τ_y^s are surface stresses in the x and y directions, and finally τ_x^b and τ_y^b are the bottom stresses in the x and y directions. These equations are readily linearized by neglecting the finite amplitude components of flux in the continuity equation (i.e., the $u\eta$ and $v\eta$ terms), by neglecting the convective acceleration terms in the momentum equations, by approximating $\frac{h}{H} = \frac{h}{h+\eta} \approx 1$, and by linearizing bottom friction to yield the following set of equations:

$$\eta_{,t} + (uh)_{,x} + (vh)_{,y} = 0 \tag{2.3}$$

$$u_{,t} + g\eta_{,x} - fv - \frac{1}{\rho h}(\tau_x^s - \tau_x^{b,lin}) = 0 \qquad (2.4a)$$

$$v_{,t} + g\eta_{,y} + fu - \frac{1}{\rho h}(\tau_y^s - \tau_y^{b,lin}) = 0 \qquad (2.4b)$$

Bottom friction has been linearized as:

$$\frac{\tau_x^{b,lin}}{\rho} = \lambda u \quad \text{and} \quad \frac{\tau_y^{b,lin}}{\rho} = \lambda v \qquad (2.5)$$

where

$\lambda = c_f U$ = linearized friction coefficient

$$c_f = \text{friction factor} = \begin{cases} \frac{1}{8} f_{DW} & \text{Darcy-Weisbach} \\ \frac{g}{c^2} & \text{Chezy} \\ \frac{n^2 g}{h^{1/3}} & \text{Manning} \end{cases}$$

U = representative velocity of flow

For the tidal case, when the linearization is done on an equivalent work over a tidal cycle basis, λ has the following form

$$\lambda = U_{max} \cdot \frac{8}{3\pi} \cdot c_f \qquad (2.6)$$

where

U_{max} = representative maximum velocity during a tidal cycle

Hence, this fully linear set of equations describe tidal wave propagation with coriolis, surface wind stress and bottom friction effects. Their validity are subject to the relative magnitude of the non-linearities being small, which implies that both the long wave and small amplitude assumptions are satisfied, hence;

$$\frac{a}{h} \ll 1 \qquad \text{and} \qquad \frac{h}{L} \ll 1$$

where

a = wave amplitude

$L = \sqrt{gh}\, T$ = wave length for shallow water waves

$T = \frac{2\pi}{\omega}$ = tidal wave period

The linearity of this set of partial differential equations permits superpositioning of solutions, hence allowing different frequency tides (eg., M_2,...) in addition to any steady state components to be solved for seperately and then to have the results superimposed.

The boundary conditions associated with our linearized governing equations are the elevation prescribed and normal flux prescribed conditions which are respectively expressed as,

$$\eta(x,y,t) = \eta^{*}(x,y,t) \text{ on } \Gamma_{\eta} \tag{2.7a}$$

and

$$Q_{\eta}(x,y,t) = Q_{\eta}^{*}(x,y,t) \text{ on } \Gamma_{Q} \tag{2.7b}$$

where

Γ_{η} = elevation prescribed boundary

Γ_{Q} = flux prescribed boundary

Normal flux is expressed as:

$$Q_n = \alpha_{nx} Q_x + \alpha_{ny} Q_y \tag{2.8}$$

where

Q_x, Q_y are flux components in the x and y direction

α_{nx}, α_{ny} are the direction cosines on the boundary

In order to be consistent with our previous linearization we must define the flux components as,

$$Q_x = uh \tag{2.9a}$$

$$Q_y = vh \tag{2.9b}$$

A further modification of our equations which is applied in order to increase the accuracy of computation, specifically in order to reduce round off error, is the splitting of elevation into a base component η_o, which is representative of tidal elevation amplitude of the entire bay, and a local value η_1, which describes the amplitude deviation from the base value. Hence the actual elevation, η, is now expressed as:

$$\eta(x,y,t) = \eta_o(t) + \eta_1(x,y,t) \tag{2.10}$$

Substituting into our linearized governing equations we have,

$$\eta_{o,t} + \eta_{1,t} + (uh)_{,x} + (vh)_{,y} = 0 \tag{2.11}$$

$$u_{,t} + g\eta_{1,x} - fv - \frac{1}{\rho h}(\tau_x^s - \tau_x^{b,lin}) = 0 \qquad (2.12a)$$

$$v_{,t} + g\eta_{1,y} + fu - \frac{1}{\rho h}(\tau_y^s - \tau_y^{b,lin}) = 0 \qquad (2.12b)$$

The boundary conditions become:

$$\eta_1(x,y,t) = \eta_1^*(x,y,t) = \eta(x,y,t) - \eta_o(t) \quad \text{on } \Gamma_\eta \qquad (2.13a)$$

and

$$Q_\eta(x,y,t) = Q_\eta^*(x,y,t) \quad \text{on } \Gamma_Q \qquad (2.13b)$$

Our equations now have the primary variables η_1, u and v, and an additional loading term η_o.

The known loading may be thought of as a pumping mode which forces the entire basin surface up and down with the base elevation η_o. From this point on we shall refer to η_o as the base or pumping mode and the additional local response, η_1, as elevation. Hence as shown in Eq. 2.10 the actual total response equals the base loading, η_o, plus the computed elevation, η_1. The program determines the value of the base loading by averaging the prescribed total elevation boundary loadings (Eq. 2.7a). We note that in order to be effective in improving computational accuracy, the difference between η_o and η_1 must be at least several orders of magnitude. This is indeed the case for small scale embayments where the difference is typically quite large (e.g., 2 or 3 orders of magnitude).

We shall now eliminate the time dependence from our equations by reducing them to the frequency domain. The basic assumption in this reduction is that both forcings on the system and responses of the system in elevation and velocity are of the same periodicity, ω. The assumed periodic response in velocity and total elevation are expressed as:

$$u(x,y,t) = \hat{u}(x,y)\, e^{i\omega t} \tag{2.14a}$$

$$v(x,y,t) = \hat{v}(x,y)\, e^{i\omega t} \tag{2.14b}$$

$$\eta(x,y,t) = \hat{\eta}(x,y)\, e^{i\omega t} \tag{2.14c}$$

It follows that our base loading and elevation are:

$$\eta_o(t) = \hat{\eta}_o\, e^{i\omega t} \tag{2.15a}$$

$$\eta_1(x,y,t) = \hat{\eta}_1(x,y)\, e^{i\omega t} \tag{2.15b}$$

It is noted that all quantities with ^ superscripts are complex spatially varying amplitudes which include both a magnitude and a phase shift. In order for these assumed responses to be valid both boundary and surface forcings must be periodic with frequency, ω. For the flux and elevation boundary forcings this is expressed as:

$$Q^*_\eta\, (x,y,t)\, \Big|_{\Gamma_Q} = \hat{Q}^*_\eta\, (x,y)\, \Big|_{\Gamma_Q} e^{i\omega t} \tag{2.16a}$$

$$\eta^*_1\, (x,y,t)\, \Big|_{\Gamma_\eta} = \hat{\eta}^*_1\, (x,y)\, \Big|_{\Gamma_\eta} e^{i\omega t} \tag{2.16b}$$

Similarly wind forcings are assumed periodic such that:

$$\tau^s_x = \hat{\tau}^s_x\, e^{i\omega t} \tag{2.17a}$$

$$\tau^s_y = \hat{\tau}^s_y\, e^{i\omega t} \tag{2.17b}$$

We note that it is the wind forcing and not the wind velocity that is assumed periodic. The associated wind stress amplitude factors are then expressed as:

$$\hat{\tau}_x^s = \rho_{air}\, C_D\, |\hat{U}_{10}|\, \hat{U}_{10}\, \cos\theta_w \tag{2.18a}$$

$$\hat{\tau}_y^s = \rho_{air}\, C_D\, |\hat{U}_{10}|\, \hat{U}_{10}\, \sin\theta_w \tag{2.18b}$$

where

$\hat{U}_{10}$ is maximum wind speed amplitude

ρ_{air} is density of air

C_D is a wind drag coefficient

θ_w is the wind approach angle

Finally the periodicity of the linearized bottom friction concurs with the form of the velocity response and hence is expressed as:

$$\frac{\tau_x^{b,lin}}{\rho} = \lambda \hat{u}\, e^{i\omega t} \tag{2.19a}$$

$$\frac{\tau_y^{b,lin}}{\rho} = \lambda\, \hat{v}\, e^{i\omega t} \tag{2.19b}$$

As previously mentioned these assumed response amplitudes include both a magnitude and a phase shift which allows reference times to be handled. This may be somewhat more easily seen by expanding out these periodic responses/functions as follows:

$$A(x,y,t) = \text{Re}\,[\hat{A}(x,y)\,e^{i\omega t}]$$

$$= \text{Re}\,\left|\hat{A}(x,y)\right| e^{i(\omega t+\phi)}$$

$$= \left|\hat{A}(x,y)\right| \cos(\omega t + \phi) \tag{2.20}$$

We note that the real part of our complex responses is always implied. Furthermore, an amplitude;

$$\hat{A}(x,y) = \left|\hat{A}(x,y)\right| e^{i\phi} \tag{2.21}$$

contains both information about the magnitude $\left|\hat{A}(x,y)\right|$ and about the time phase shift, ϕ.

It is readily seen that the steady state case is represented by specifying the frequency, ω, as zero. Furthermore, we note that for this case phase shifts are not applicable and must always be specified as $\phi = 0$. Finally, we note that for $\omega = 0$, our wind forcing equations (Eq. 2.18) reduce to their customary form.

Substituting in the assumed forcings and responses into our governing partial differential equations and boundary conditions, taking appropriate time derivatives and noting that $e^{i\omega t}$ terms drop from all equations, the following set of quasi-steady equations is generated:

$$i\omega\hat{\eta}_1 + i\omega\hat{\eta}_o + (\hat{u}h)_{,x} + (\hat{v}h)_{,y} = 0 \tag{2.22}$$

$$i\omega\hat{u} + g\hat{\eta}_{1,x} - f\hat{v} - \frac{1}{h}\left(\frac{\hat{\tau}^s_x}{\rho} - \lambda\hat{u}\right) = 0 \tag{2.23a}$$

$$i\omega\hat{v} + g\hat{\eta}_{1,y} + f\hat{u} - \frac{1}{h}\left(\frac{\hat{\tau}^s_y}{\rho} - \lambda\hat{v}\right) = 0 \qquad (2.23b)$$

with boundary conditions:

$$\hat{Q}_\eta(x,y)\Big|_{\Gamma_Q} = \hat{Q}^*_o(x,y) \qquad (2.24a)$$

$$\hat{\eta}_1(x,y)\Big|_{\Gamma_\eta} = \hat{\eta}^*_1(x,y) \qquad (2.24b)$$

We note that these equations are time independent and hence we have eliminated any time stepping restrictions in solving the equations.

We now wish to develop some numerical technique to solve these partial differential equations for an arbitrary geometry. We shall use the finite element method to do this and shall solve for the spatial variation of the amplitudes $\hat{\eta}_1(x,y)$, $\hat{u}(x,y)$ and $\hat{v}(x,y)$. The magnitudes and phase shifts having been solved for using the finite element method program developed, we can readily generate the entire time history using Eqs. 2.14. Moreover, if we choose to linearly superimpose solutions we superimpose the time histories generated to resolve the total time history.

3. WEIGHTED RESIDUAL AND FINITE ELEMENT METHOD FORMULATIONS

In order to apply the finite element method we must first develop a weighted residual formulation. The basic idea behind the weighted residual method is to establish a set of equations which minimize the weighted error which is incurred between the exact solution and our approximation to the solution. Before proceeding with the weighted residual method we multiply through the momentum equations by h in order to induce a certain symmetry between the continuity and momentum equations and hence allow for similar matrices (for the derivative terms of both equations) to be generated, thereby reducing storage requirements. Hence we have:

$$i\omega\hat{\eta}_o + i\omega\hat{\eta}_1 + (\hat{u}h)_{,x} + (\hat{v}h)_{,y} = 0 \tag{3.1}$$

$$i\omega h\hat{u} + gh\hat{\eta}_{1,x} - fh\hat{v} - \left(\frac{\hat{\tau}^s_x}{\rho} - \lambda\hat{u}\right) = 0 \tag{3.2a}$$

$$i\omega h\hat{v} + gh\hat{\eta}_{1,y} + fh\hat{u} - \left(\frac{\hat{\tau}^s_y}{\rho} - \lambda\hat{v}\right) = 0 \tag{3.2b}$$

In order to avoid a singular formulation, we shall in later manipulations substitute the momentum equations into the continuity equation. This dictates that the continuity equation be used to establish the symmetrical weak weighted residual form. Formulating the fundamental weak form in this fashion leads to elevation amplitude prescribed boundary conditions being essential and the normal flux amplitude prescribed boundary conditions being natural. We note that it is therefore mandatory to prescribe elevation on at least one point of the boundary. This is required in order to ensure uniqueness of the solution, that is a reference depth must be specified.

Specifically applying Galerkin's method to establish the fundamental weak form the error in the continuity equation is weighted by the variation in elevation, $\delta\hat{\eta}_1$, and is integrated over the interior domain. In addition, the natural boundary error must be accounted for by weighting it with $\delta\hat{\eta}_1$, and integrating over the natural boundary. It is required that the combined integrated and properly weighted interior and natural boundary errors vanish and the following expression results:

$$\iint_{\Omega}\{i\omega\hat{\eta}_1 + i\omega\hat{\eta}_o + (\hat{u}h)_{,x} + (\hat{v}h)_{,y}\}\delta\hat{\eta}_1 d\Omega + \int_{\Gamma_Q}\{-\hat{Q}_{\eta} + \hat{Q}^{*}_{\eta}\}\delta\hat{\eta}_1 d\Gamma = 0 \qquad (3.3)$$

Applying Gauss' theorem in order to eliminate derivatives on the flux terms and combining with boundary relationships leads to the desired symmetrical weighted residual weak form:

$$\iint_{\Omega}\{i\omega\hat{\eta}_1\delta\hat{\eta}_1 + i\omega\hat{\eta}_o\delta\hat{\eta}_1 - \hat{u}h(\delta\hat{\eta}_1)_{,x} - \hat{v}h(\delta\hat{\eta}_1)_{,y}\}d\Omega + \int_{\Gamma_Q}\hat{Q}^{*}_{\eta}\delta\hat{\eta}_1 d\Gamma = 0 \qquad (3.4)$$

The weighted residual forms for the momentum equations are obtained by weighting the associated errors with residual velocities and integrating over the interior domain, resulting in:

$$\iint_{\Omega}\{i\omega h\hat{u} + gh\hat{\eta}_{1,x} - fh\hat{v} - (\frac{\hat{\tau}^{s}_{x}}{\rho} - \lambda\hat{u})\}\delta\hat{u}d\Omega = 0 \qquad (3.5a)$$

$$\iint_{\Omega}\{i\omega h\hat{v} + gh\hat{\eta}_{1,y} + fh\hat{u} - (\frac{\hat{\tau}^{s}_{y}}{\rho} - \lambda\hat{v})\}\delta\hat{v}d\Omega = 0 \qquad (3.5b)$$

The functional continuity requirements imposed upon variables are that $\hat{\eta}_1$ and $\delta\hat{\eta}_1$ be continuous over the domain, requiring at least a linear finite element expansion, and that $\hat{u}$, $\hat{v}$, $\delta\hat{u}$ and $\delta\hat{v}$ be finite over the domain, requiring only constant expansions. For the full non linear form of the equations, however, $\hat{u}$, $\hat{v}$, $\delta\hat{u}$ and $\delta\hat{v}$ also require linear expansions.

Before proceeding with our numerical scheme we rearrange the weighted residual equations somewhat such that all non variable load terms such as the pumping mode, boundary flux and wind stress loadings appear on the right hand side. With these modifications our equations now appear as:

$$\iint_{\Omega}\{i\omega\hat{\eta}_1\delta\hat{\eta}_1 - \hat{u}h(\delta\hat{\eta}_1)_{,x} - \hat{v}h(\delta\hat{\eta}_1)_{,y}\}d\Omega = \iint_{\Omega} i\omega\hat{\eta}_o\delta\hat{\eta}_1 d\Omega - \int_{\Gamma_Q} Q_{\eta}^{*}\delta\hat{\eta}_1 d\Gamma \quad (3.6)$$

$$\iint_{\Omega}\{i\omega h\hat{u} + \lambda\hat{u} - fh\hat{v} + gh\hat{\eta}_{1,x}\}\delta\hat{u}d\Omega = \iint_{\Omega}\frac{\tau_x^s}{\rho}\,\delta\hat{u}d\Omega \quad (3.7a)$$

$$\iint_{\Omega}\{i\omega h\hat{v} + \lambda\hat{v} + fh\hat{u} + gh\hat{\eta}_{1,y}\}\delta\hat{v}d\Omega = \iint_{\Omega}\frac{\tau_y^s}{\rho}\,\delta\hat{v}d\Omega \quad (3.7b)$$

To generate a system of algebraic equations from the previous integral equations the finite element method is applied. This involves dividing the global domain, Ω, into element sub-domains, Ω_e, and representing the variables within each element by a polynominal expansion. Contributions from all elements are summed and inter-element functional continuity requirements are taken into account in order to generate a global system of equations. In our particular case we select triangular elements and the lowest possible order polynomial mandated for the full non linear form of the equations. All variables, $\hat{\eta}_1$, $\hat{u}$, $\hat{v}$, $\delta\hat{\eta}_1$, $\delta\hat{u}$, and $\delta\hat{v}$, are therefore represented by linear expansion over each element, yielding variables which are defined at the nodes. Furthermore, linear element expansions are used for mean water level depths h. In this manner elevations, velocities and depths are defined at nodes such that inter-element fluxes are both continuous and clearly defined. It may be easily shown that the application of the finite element method to the final weighted residual equations yields the following set of global algebraic matrix equations:

$$i\omega \underline{\underline{M}}_{\eta} \hat{\underline{\eta}}_1 - \underline{\underline{D}} \hat{\underline{U}} = - \hat{\underline{P}}_P - \hat{\underline{P}}_Q^{lin} \tag{3.8}$$

$$(i\omega \underline{\underline{M}}_U + \underline{\underline{M}}_F + \underline{\underline{M}}_C)\hat{\underline{U}} + g \underline{\underline{D}}^T \hat{\underline{\eta}}_1 = \hat{\underline{P}}_W \tag{3.9}$$

where

$\hat{\underline{\eta}}$ is the global elevation vector (1 elevation per node)

$\hat{\underline{U}}$ is the global velocity vector (2 velocities per node)

$\underline{\underline{M}}_{\eta}$, $\underline{\underline{M}}_U$ are global mass matrices

$\underline{\underline{M}}_F$ is the global linearized bottom friction matrix

$\underline{\underline{M}}_C$ is the coriolis matrix

$\underline{\underline{D}}$ is the derivative matrix

$\hat{\underline{P}}_P = i\omega M_{\eta} \hat{\eta}_o$ is the base elevation vector

$\hat{\underline{P}}_Q$ is the prescribed boundary flux loading vector

$\hat{\underline{P}}_W$ is the wind loading vector

We note that all matrices are efficiently banded, the bandwidth depending on the nodal numbering scheme used. In order to reduce computational effort and storage requirements we use lumped mass, bottom friction and Coriolis matrices. The lumping for the symmetric mass and friction matrices involves combining all terms on a row onto the diagonal and for the skew symmetric Coriolis matrix combining rows into off-diagonal terms. The lumping procedure is in effect a slight redistribution between the nodes of mass and certain forcings. We have found that results are quite insensitive to these lumpings.

Combining the matrices commonly multiplying the velocity vector in the momentum equation as:

$$\hat{\underline{\underline{M}}}_T = i\omega \underline{\underline{M}}_U + \underline{\underline{M}}_F + \underline{\underline{M}}_C \qquad (3.10)$$

yields a tri-diagonal matrix which can be very economically inverted when solving for $\hat{\underline{U}}$. Hence solving for $\hat{\underline{U}}$ using the momentum equation we have

$$\hat{\underline{U}} = \hat{\underline{\underline{M}}}_T^{-1}(\hat{\underline{P}}_W - g\underline{\underline{D}}^T\hat{\eta}_1) \qquad (3.11)$$

Substituting for $\hat{\underline{U}}$ into the continuity equation and rearranging somewhat yields:

$$(i\omega \underline{\underline{M}}_\eta + g\underline{\underline{D}}\,\hat{\underline{\underline{M}}}_T^{-1}\,\underline{\underline{D}}^T)\hat{\underline{\eta}}_1 = \hat{\underline{P}}_P - \hat{\underline{P}}_Q^{lin} + \underline{\underline{D}}\,\hat{\underline{\underline{M}}}_T^{-1}\,\hat{\underline{P}}_W \qquad (3.12)$$

The left hand side matrix, which is non-symmetrically banded and complex is then decomposed using an LU decomposition. The right hand side is combined into a single loading vector and we solve for $\hat{\underline{\eta}}_1$. Then using Eq. 3.11 we directly solve for $\hat{\underline{U}}$. Finally in order to obtain the total elevation $\hat{\underline{\eta}}$ we simply add back in our base component, $\hat{\eta}_o$.

4. PROGRAM USAGE

In this chapter we shall briefly comment on usage of program TEA. The exact required input information is shown in Appendix 1.2.

4.1 Grid Layout

The layout and degree of refinement used for the grid depends on the amount of detail desired and also on the expected flow pattern. Generally a finer grid is used in areas of rapidly varying flow, near sharp boundaries and near regions where the boundary type changes (i.e., from elevation prescribed to normal flux prescribed). For tidal forcing cases grid size should be small enough such that;

$$\frac{\Delta \ell}{L} << \frac{1}{4}$$

where $\Delta \ell$ = representative element length scale

L = tidal wavelength = $\sqrt{gh}\,T$

This requirement stems from the nature of our linear element expansion and the form of a wave. Regarding grid shapes an attempt should be made to keep the interior angles of triangular grid cells larger than 30°.

Once the grid layout has been completed, nodes should be numbered sequentially. Node numbering should be done such that the arithmetic difference between node numbers of connecting nodes is minimized. This should be done since the matrix bandwidth and hence the efficiency of the code is determined by the maximum nodal point difference. Program RENUMB, a bandwidth optimization code (Appendix 2), is available to renumber the nodes in input files of program TEA. Node connectivity for each element must be entered counterclockwise. We note that the sequential element numbering doesn't effect program efficiency.

Friction factors estimated according to Eq. 2.6 are entered as constant for each element. For zero frequency cases, it is necessary that either friction or Coriolis be specified as non-zero in order to avoid a singular formulation.

4.2 Units

The gravitational constant should be consistent with other units used. Hence if lengths are entered in meters, frequency in sec^{-1}, then g must be entered as 9.81 m/sec^2 and output will be meters for elevation and m/sec for velocity.

The forcing frequency is $\omega = 2\pi/T$ where T is the tidal period. For steady cases $T = \infty$ and hence $\omega = 0$.

The Coriolis parameter is entered in the same time units as frequency and is calculated as:

$$f = 2\omega_e \sin\phi_\ell$$

where ω_e = phase velocity of Earth's rotation (in rad/time)

ϕ_ℓ = degrees latitude.

Wind velocity should be entered in the same units as water velocity is being calculated. The wind friction factor, C_D, may be calculated according to standard formulae (e.g. see Ref. 14) and typically has a value around 0.001.

4.3 Specified Phase Shifts

Phase shifts must be applied in order to specify time lags in boundary conditions and other forcings. Phase shifts should be carefully and consistently specified in order to avoid a meaningless solution. They are specified in rad/sec and are calculated as:

$$\phi = \omega \, t_p$$

where t_p is the phase shift time (in seconds if ω in sec^{-1}).

4.4 Boundary Conditions

The boundary is segmented into two boundary types, the elevation prescribed boundary, Γ_η, and the normal flux prescribed boundary, Γ_Q. The elevation prescribed boundary is the essential boundary condition and must be specified at at least one point on the boundary in order to establish a reference elevation. The entire boundary may be specified as being elevation prescribed. At each node on Γ_η we must specify an elevation amplitude and the associated phase shift. Typically elevation prescribed boundaries are ocean boundaries where tidal forcing exists. Also elevation prescribed boundaries are used for a constant surface lake boundary which is connected to a river or estuary where we would specify zero elevation. Essential boundaries are exactly satisfied and hence elevation at these nodes will coincide exactly with our input boundary conditions.

The normal flux prescribed boundary is the natural boundary condition. Any part of the boundary not specified as essential is automatically taken to be natural. Unless otherwise specified, program TEA will assume that the normal flux boundary has zero normal flux across it, that is it assumes a land boundary. When a non-zero normal flux exists across a natural boundary segment, for either the case of a known tidal flux or a river discharge, both the segments across which the flow occurs and the value of the flow at the nodes involved must be specified. The segment end node numbers must be specified in clockwise order for shoreline boundaries and in counterclockwise fashion for islands. The x and y components of flux per unit length (i.e., velocity × depth) is then specified at each of the nodes on the non-zero flux boundaries. The user can either specify the x and y components of the normal flux vector or the x and y components of the actual flux vector and TEA will find the required normal components. Because specified flux is a natural boundary condition the computed flux on Γ_Q will be satisfied exactly only in the limit (as grid refinement is increased).

4.5 Output

Program TEA outputs information onto both tape 6 and tape 8. Tape 6 output is a formated descriptive listing of both inputs and results. Tape 8 outputs plotting (grids and results) and/or time history generating information. For details on outputs see Appendix 1.4.

When dealing with a set of tidal and steady state components, program TEA must be run separately for each different forcing frequency. The results for each frequency are then superimposed for each node to generate time history information as follows:

$$A_{node}(t) = \left|A_{node,\omega_1}\right| \cos(\omega_1 t + \phi_{node,\omega_1}) + \left|A_{node,\omega_2}\right| \cos(\omega_2 t + \phi_{node,\omega_2}) + \cdot$$

where

t is time at which result is desired

A_{node,ω_1} is amplitude (for elevation or velocity) at node at frequency ω_1

ϕ_{node,ω_1} is corresponding phase shift

5. APPLICATION

Program TEA has been verified by comparing computed results with exact analytical solutions for both the case of a straight channel with linear friction [5] and the case of a rectangular basin with a slot [1]. In this chapter, however, we shall focus on the application of program TEA to case studies of several bays and coastal inlets. Comparisons to the full non linear time domain finite element code CAFE [12] shall be shown and some of the differences shall be discussed.

5.1 Massachusetts Bay

The first example is the calculation of the circulation pattern of Massachusetts Bay. The grid used (Fig. 5.1) is identical to one applied with CAFE [12]. The circulation is driven both by a tidal fluctuation and a steady coastal current. The tidal forcing is simply specified by giving tidal elevation values at ocean nodes and driving the system at a frequency corresponding to a period of T = 12 hours. Tidal elevations vary linearly between Cape Ann and Cape Cod and no phase shifts are applied. The steady coastal current is simulated by imposing a linear elevation gradient along the ocean boundary and driving the system at zero frequency. Since there are no fluxes from any land boundary segment, no natural boundary condition need be specified in either case. The particular boundary values used were selected to be identical to the boundary forcing values used for CAFE. Program TEA input for both the tidal forcing component and the steady current component are shown in Appendices 3.1 and 3.2 and program TEA outputs are shown in Appendices 3.3 and 3.4.

In order to find the resulting currents, velocity results for both the tidal and the steady state components are superimposed at any desired

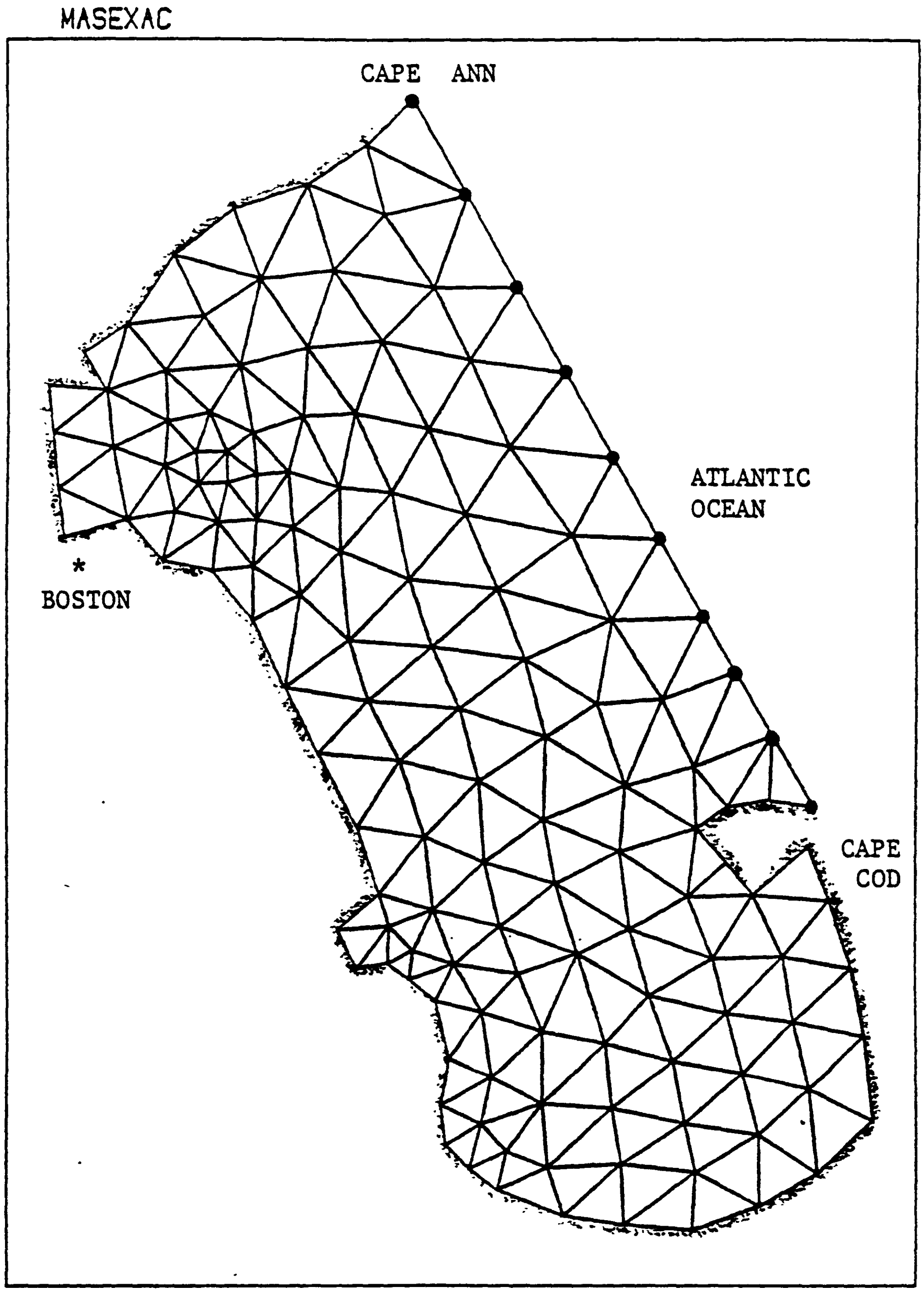

Fig. 5.1 Finite Element Grid of Massachusetts Bay

time. The resulting circulation patterns at several stages of the tide are shown in Figs. 5.2, 5.3 and 5.4. Predictions from the non-linear CAFE at T/6 after low tide [12] are shown in Figs 5.5 and can be compared with corresponding output from TEA (Fig. 5.4). Note that large values of eddy viscosity were needed in CAFE for numerical damping and are most likely unrealistically high. Non-linearities are probably not very important due to the large depth of Massachusetts Bay in most areas. We note that at some points along the shoreline TEA allows a certain amount of leakage onto land. This is due to the fact that TEA treats flux boundary conditions as natural and hence allows some error. Due to the solution methodology of CAFE, flux boundary conditions are essential and hence the no flux boundary conditions are strictly enforced. However, if it is important in a particular application to conserve total mass, this boundary leakage problem of TEA may be readily resolved by doing some grid refinement in these boundary areas, especially near sharp corners and high gradient regions.

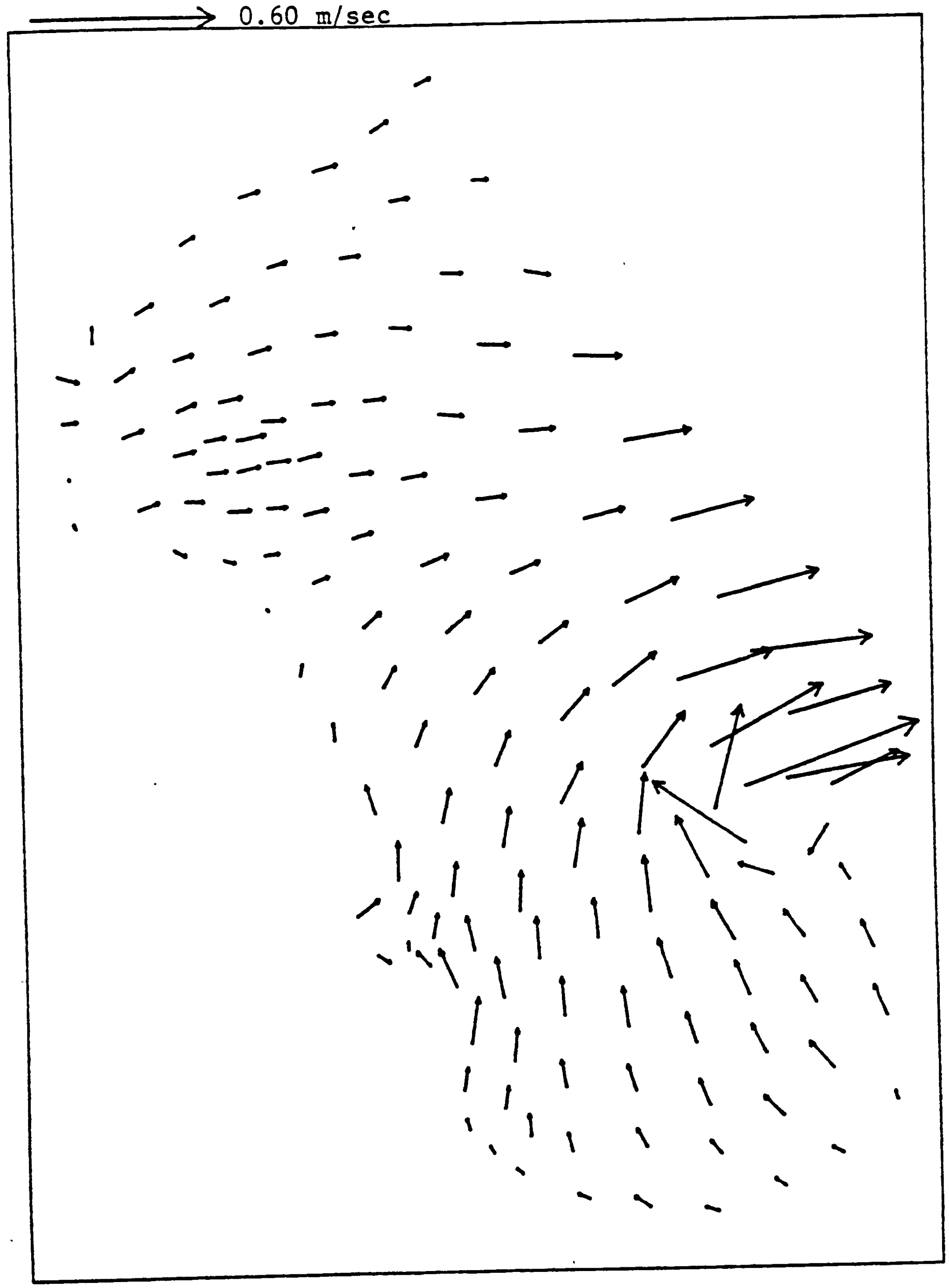

Fig. 5.2 Tidal Circulation Computed by TEA at T/6 after High Tide

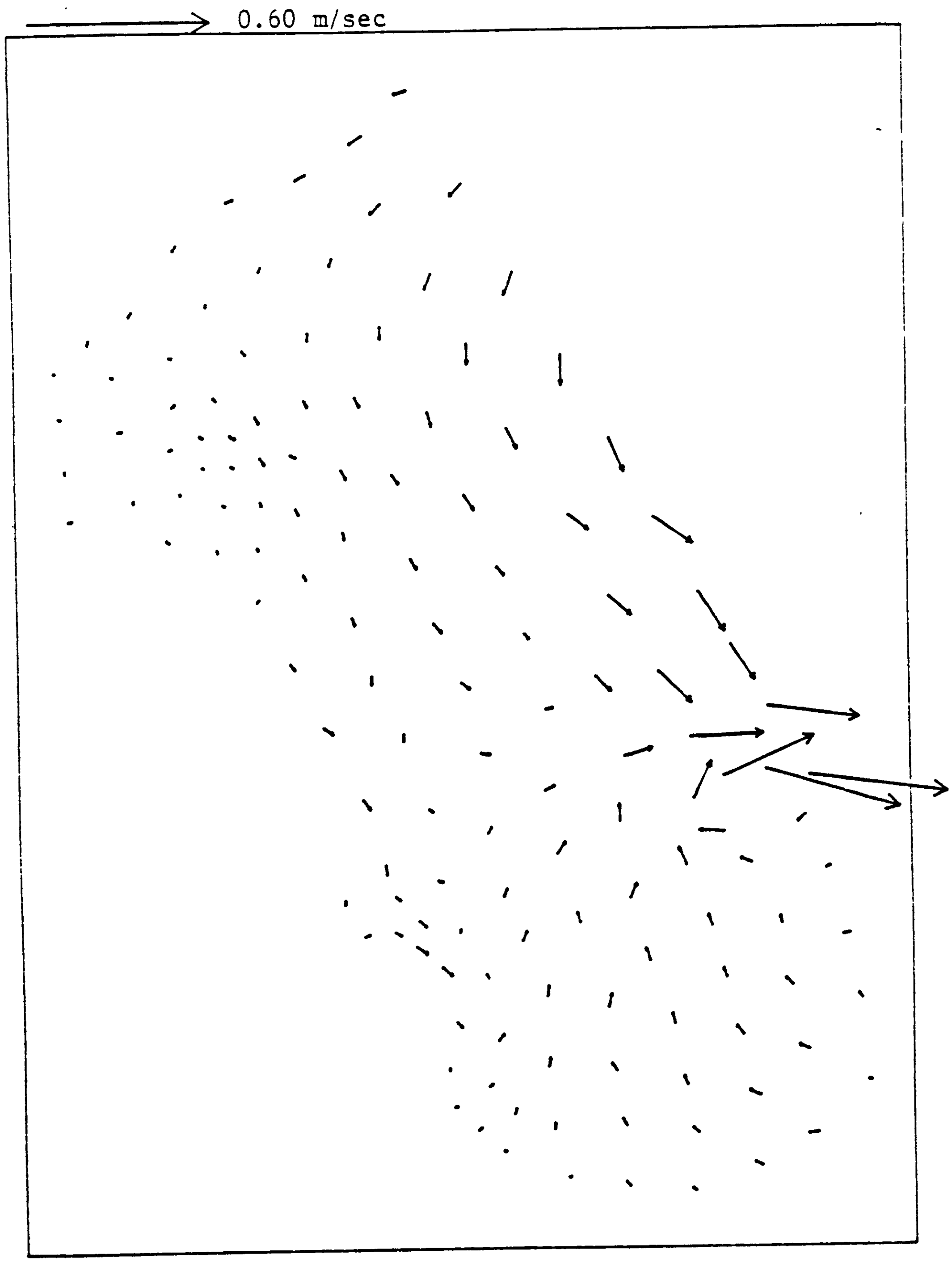

Fig. 5.3 Tidal Circulation Computed by TEA at Low Tide

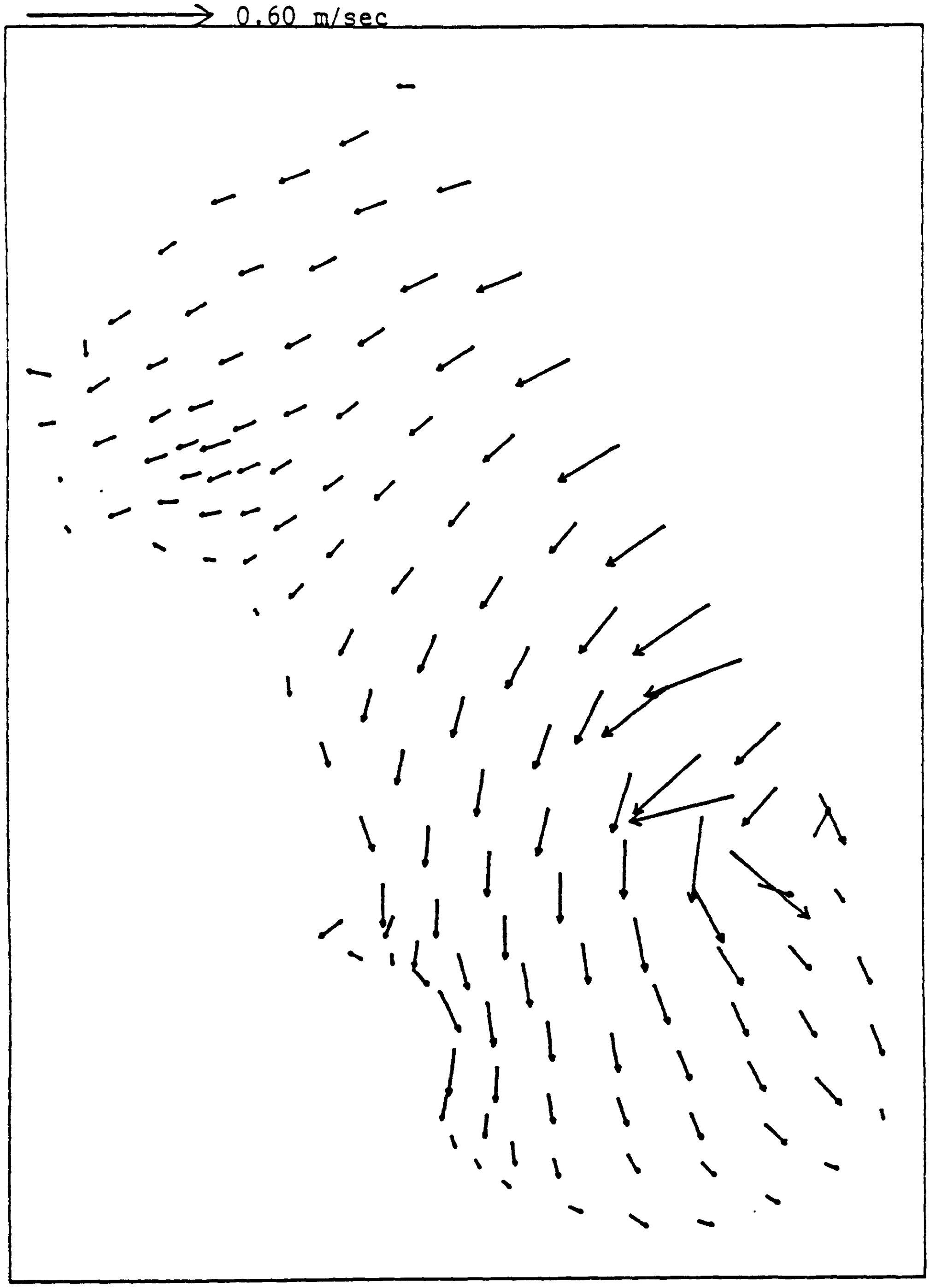

Fig. 5.4 Tidal Circulation Computed by TEA at T/6 after Low Tide

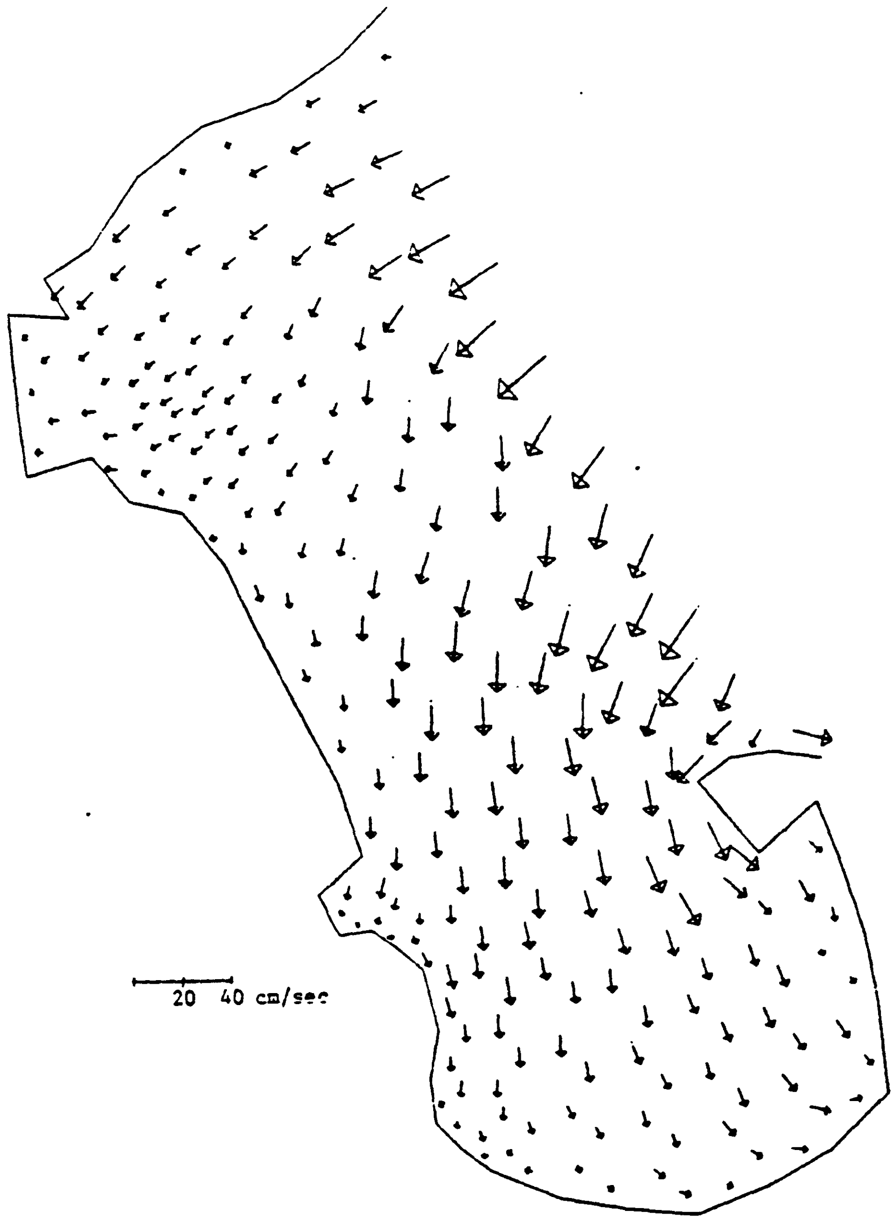

Fig. 5.5 Tidal Circulation Computed by CAFE at T/6 after Low Tide (from Ref. 12)

5.2 Niantic Bay

The second case study involves computing the tidally induced circulation in Niantic Bay, in the eastern part of Long Island Sound (Fig. 5.6). Specifically we desire to resolve the circulation around the Millstone point, the site of the Millstone Nuclear Power Station, and therefore the grid used (Fig. 5.7) shows a great deal of resolution in that vicinity. Tidal forcing boundary conditions were simulated by specifying a constant tidal elevation at ocean boundary nodes with a phase lag which increases along the boundary as shown in Fig. 5.7. It is stressed that the user must be very cautious in applying correct boundary condition values and phase lags. TEA is quite sensitive to incorrect boundary conditions and unrealistic solutions and/or boundary problems may develop due to incorrect boundary specifications. These types of problems will appear much more dramatically for TEA than for CAFE since TEA has no eddy viscosity to dampen unrealistic velocity gradients and furthermore does not allow boundary fluxes to be forced to zero in the manner that CAFE does.

The resulting circulation computed by TEA at two tidal stages are shown in Figs. 5.8 and 5.9. Results from CAFE [9] corresponding to Fig. 5.9 are shown in Fig. 5.10. Details for the flow around Millstone Point at maximum flood are shown in Figs. 5.11 and 5.12. Comparisons of velocity amplitudes and directions to both CAFE and measurements made at seven sites shown in Fig. 5.13 are presented in Fig. 5.14.

We note that, in general, velocity amplitude results from TEA compare better to measured results than those of CAFE, which substantially underpredicts flows at all points. We note especially the dramatic

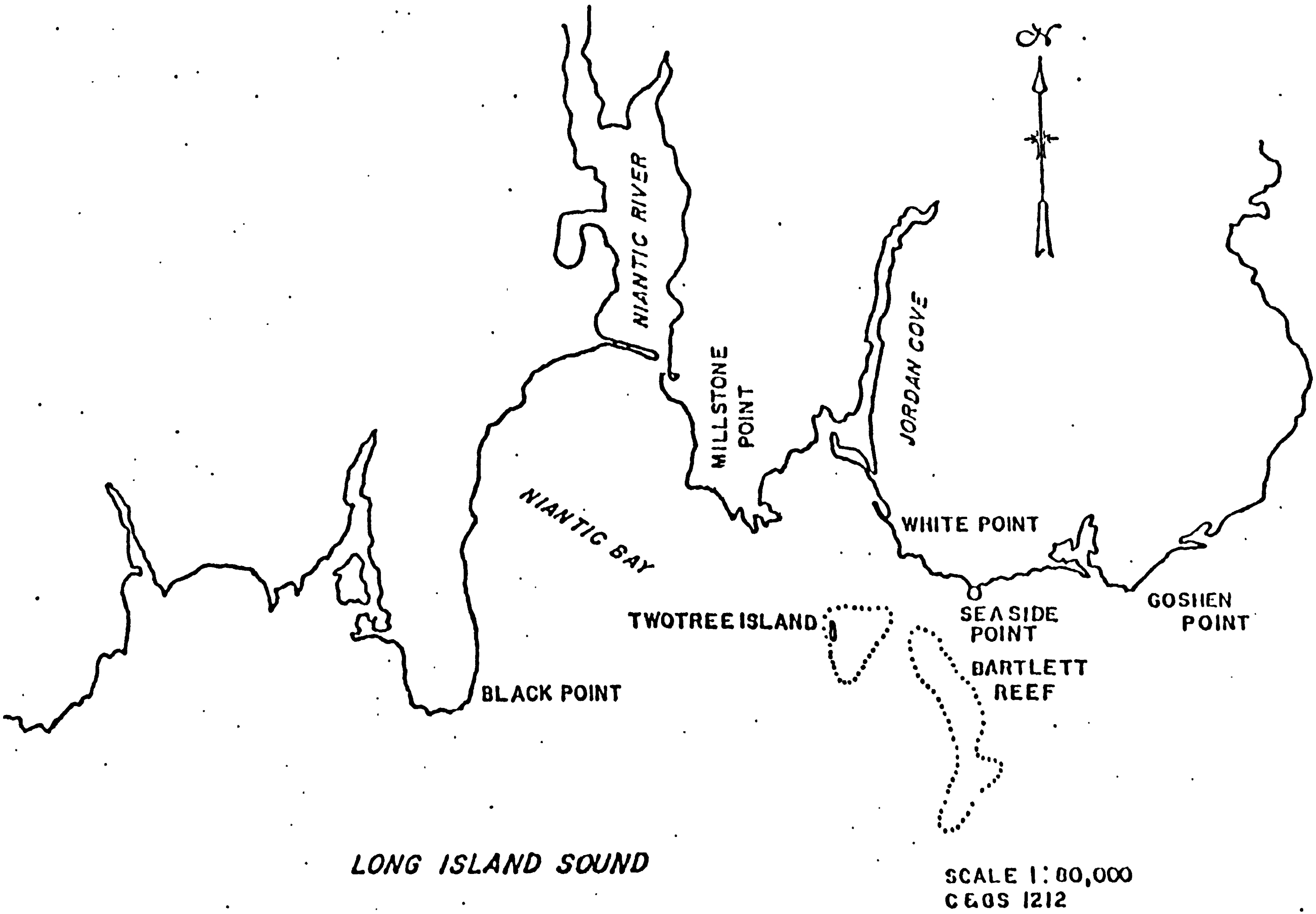

Fig. 5.6 Niantic Bay and Location of Millstone Nuclear Power Station (from Ref. 9)

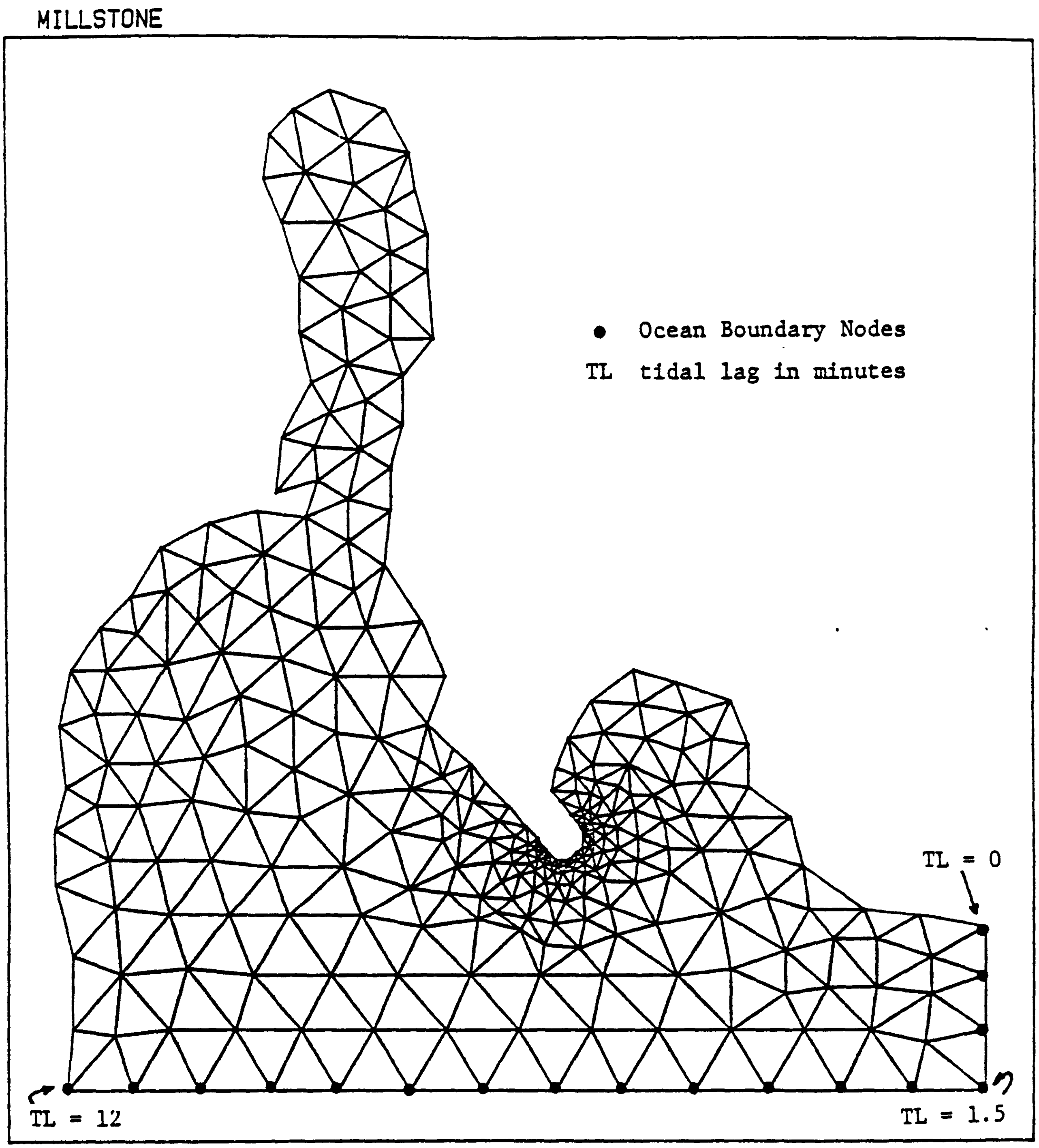

Fig. 5.7 Finite Element Grid Discretization of Niantic Bay and Millsone Point

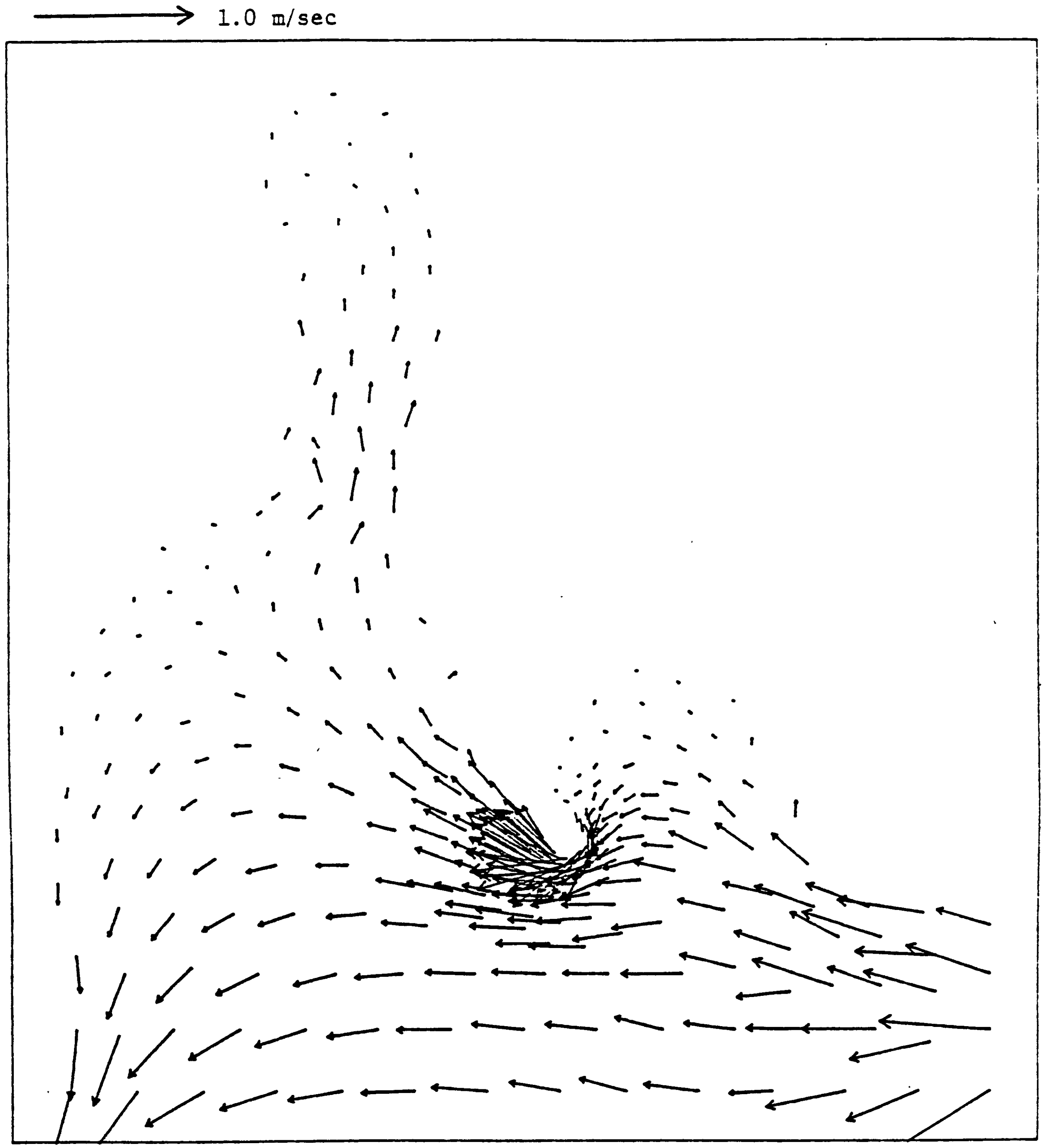

Fig. 5.8 Computed Circulation by TEA at Maximum Flood

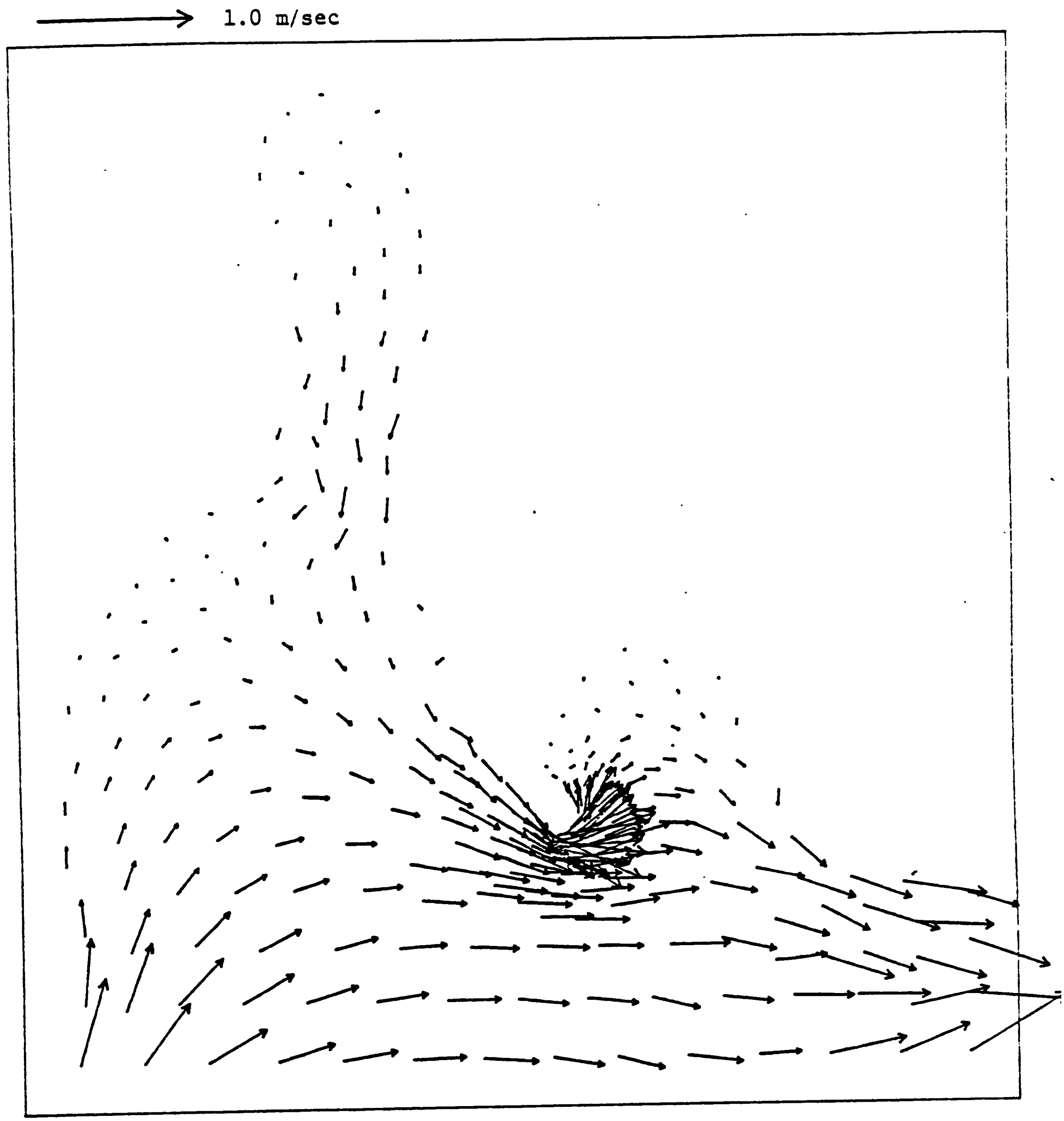

Fig. 5.9 Computed Circulation by TEA at Maximum Ebb

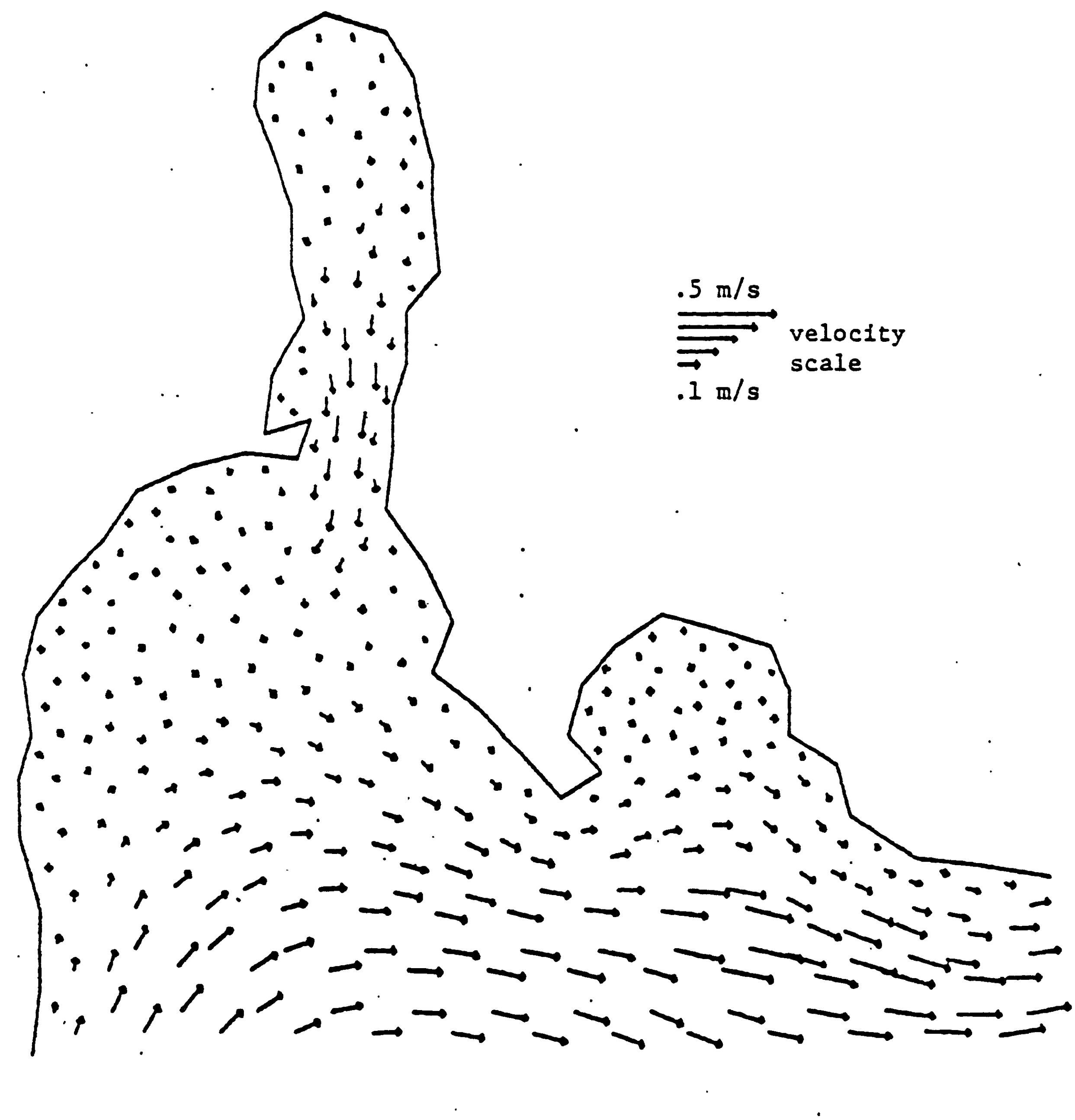

Fig. 5.10 Computed Circulation by CAFE at Maximum Ebb (from Ref. 9)

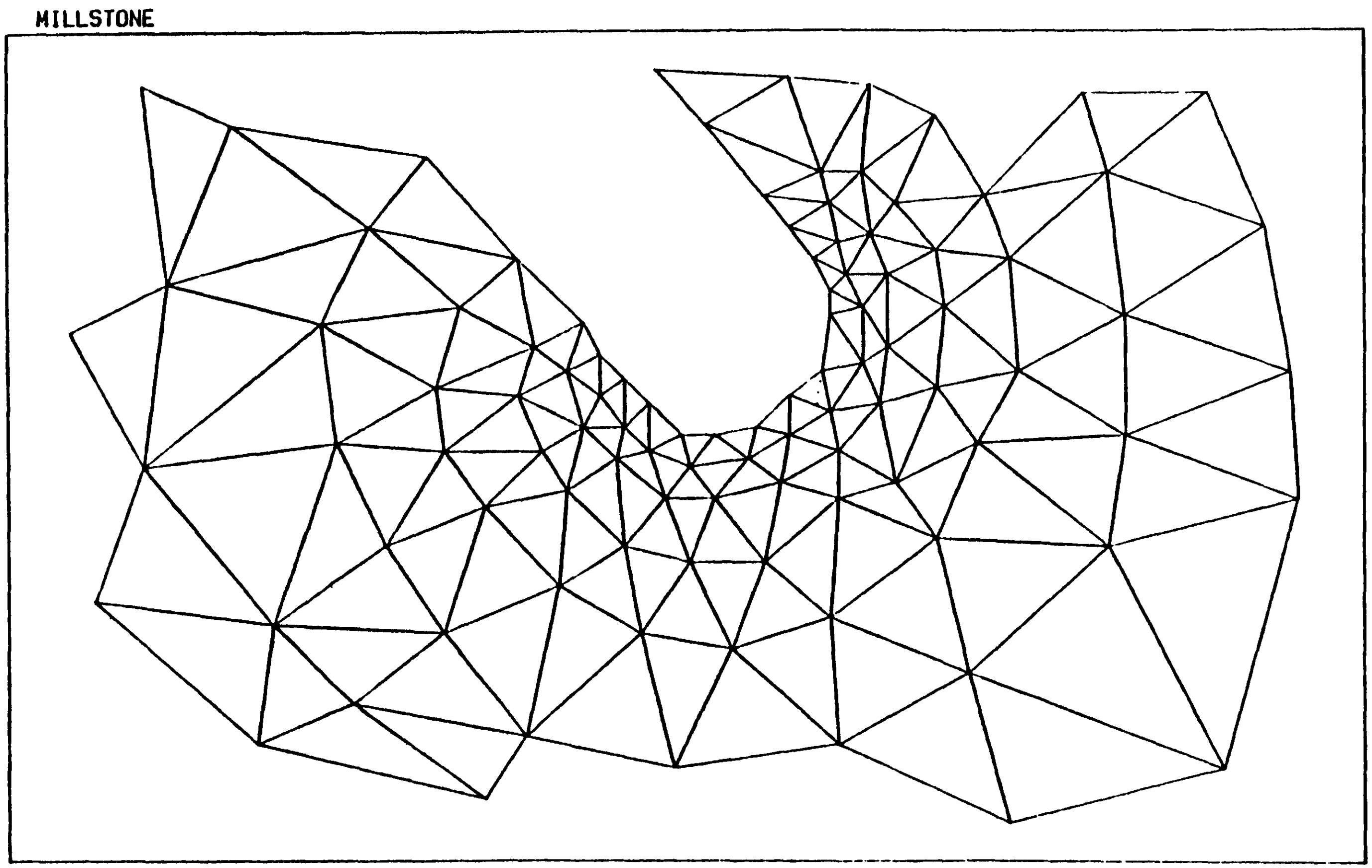

Fig. 5.11 Detail of Grid around Millstone Point

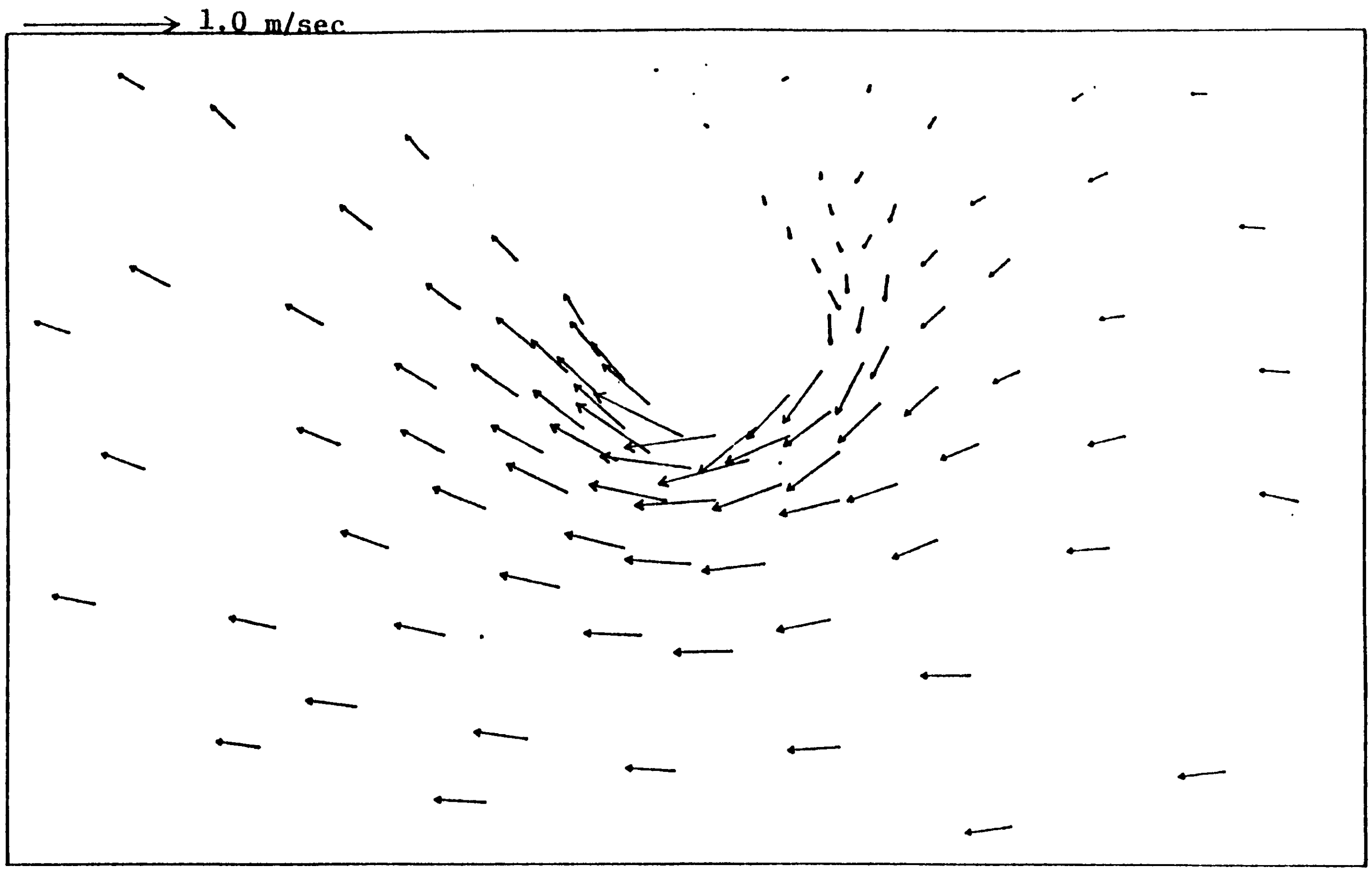

Fig. 5.12 Detail of Flow around Millstone Point Computed by TEA at Maximum Flood

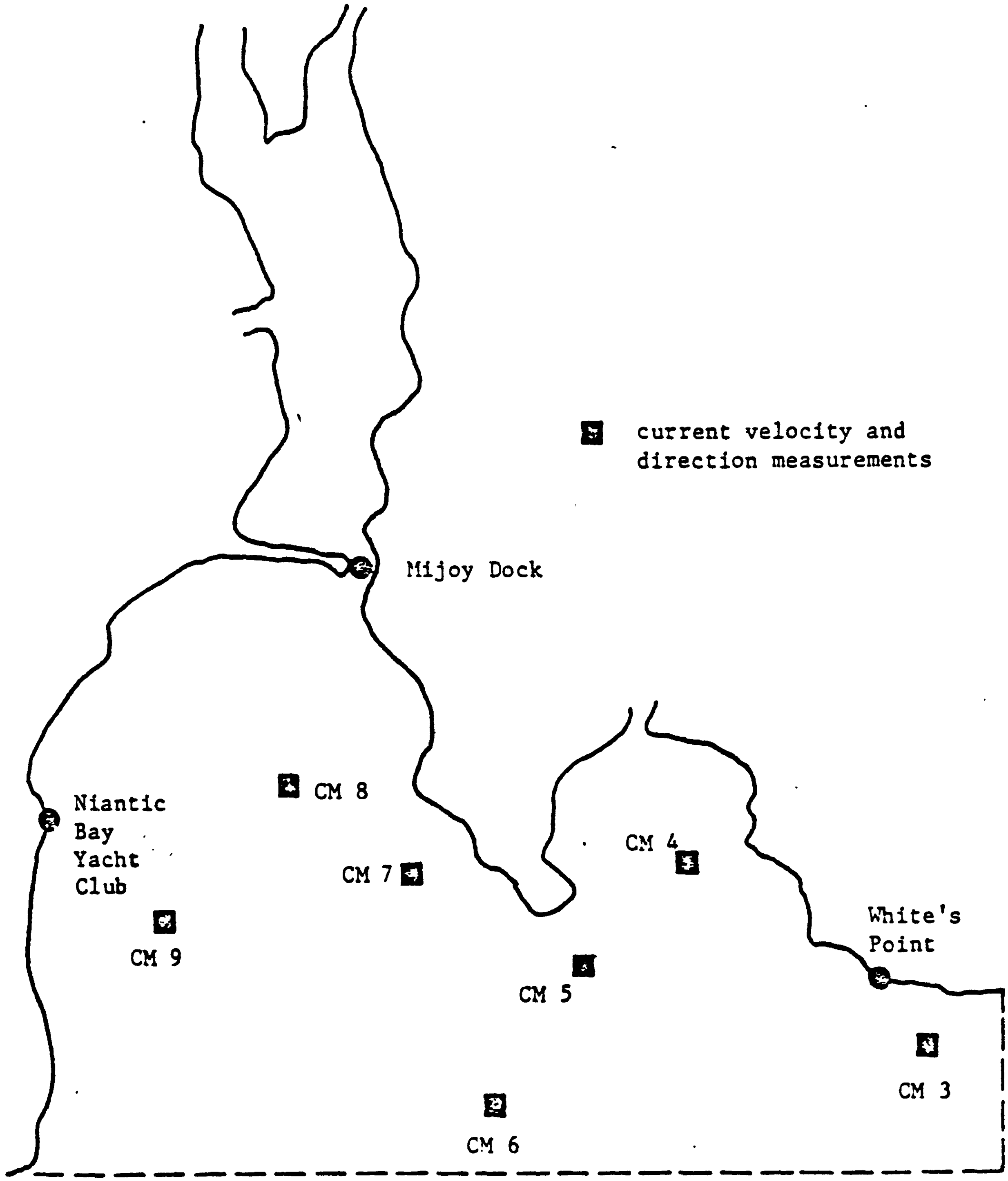

Fig. 5.13 Location of Current Measurement Sites in Niantic Bay (from Ref. 9)

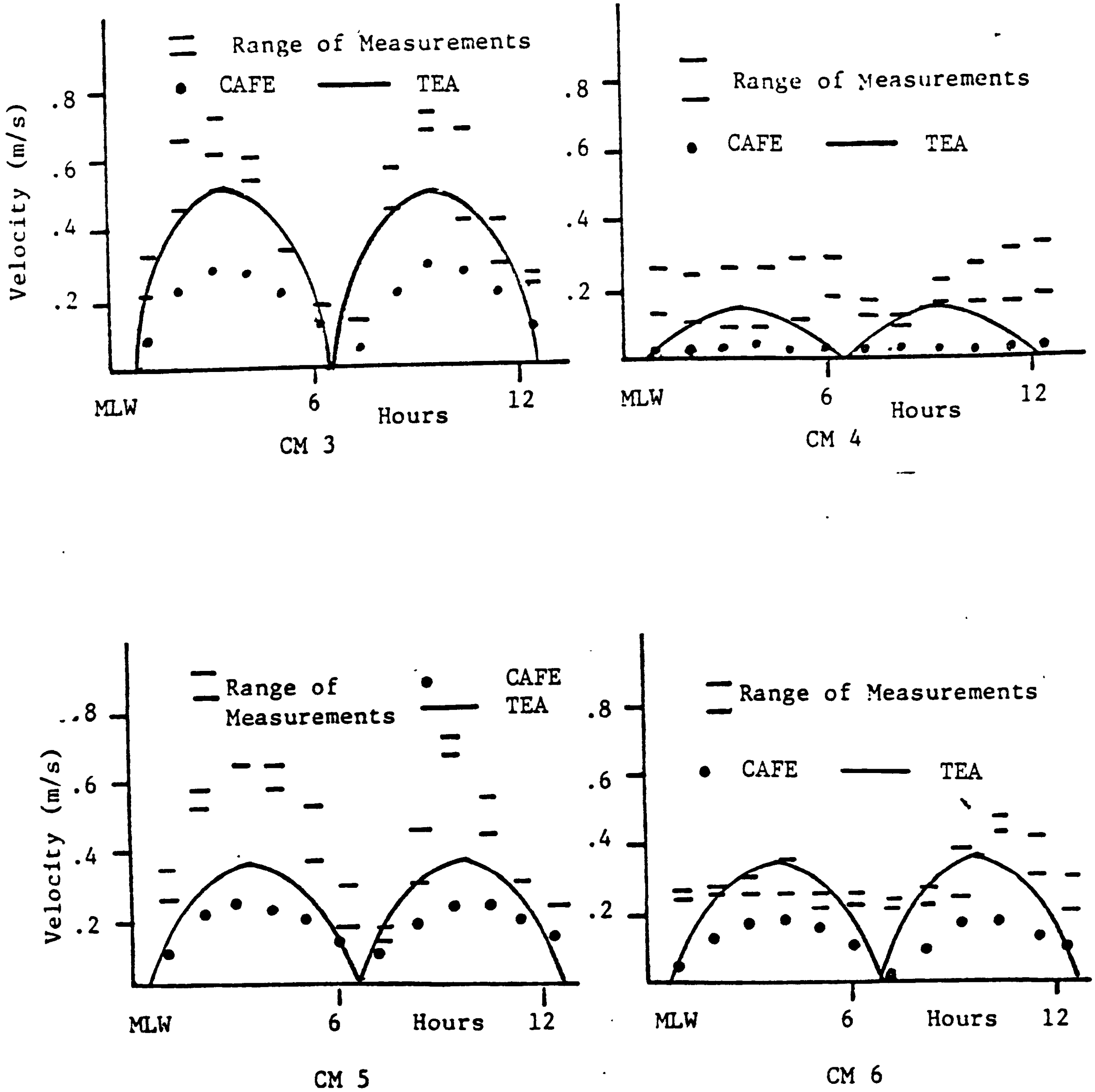

Fig. 5.14 a-d Comparison of Current Measurements and Predictions by CAFE and TEA at various locations

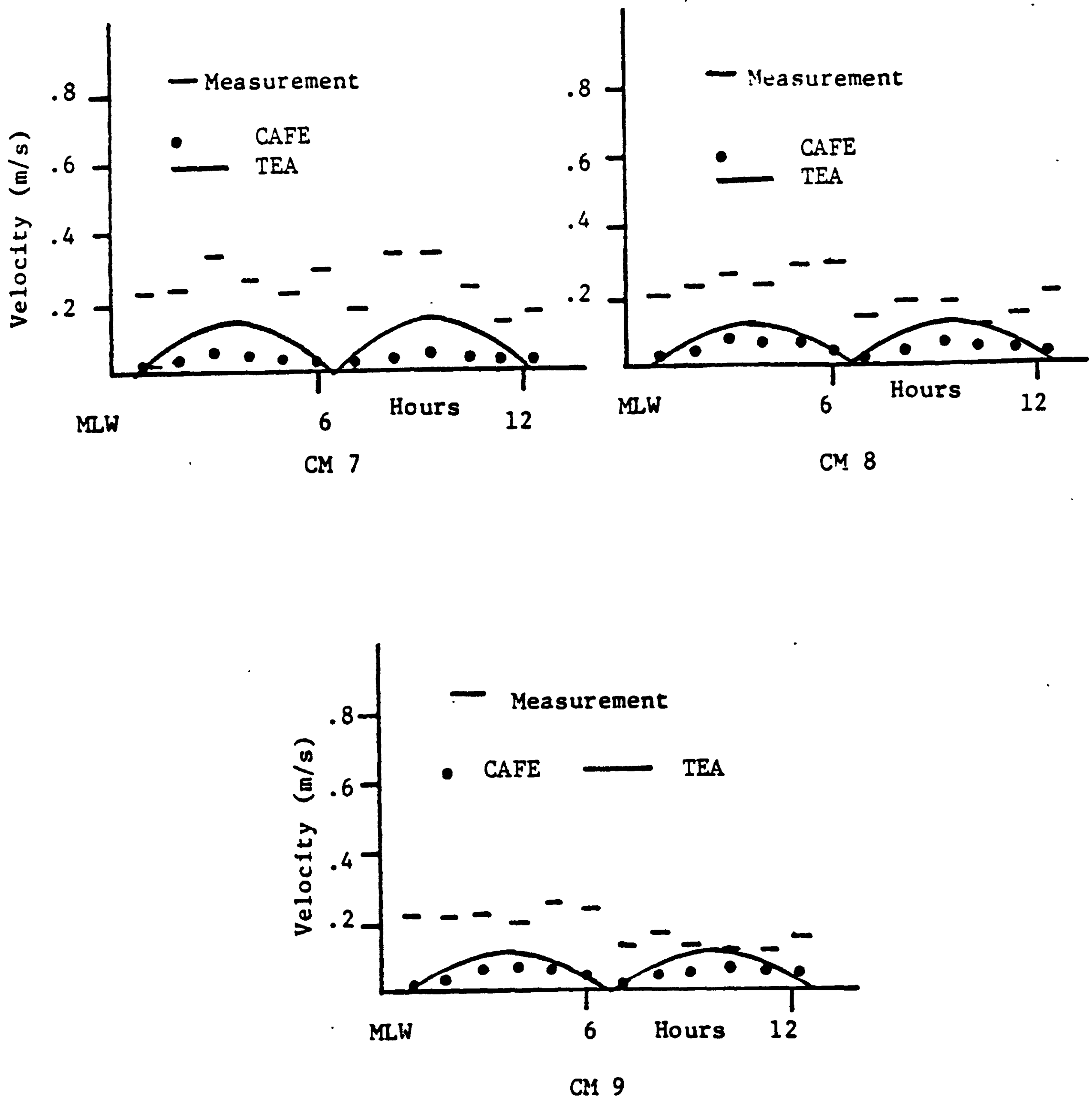

Fig. 5.14 e-g Comparison of Current Measurements and Predictions by CAFE and TEA at various locations

improvement of currents around Millstone Point which is shown in the comparison at point 5. Even though velocities predicted by TEA at the measurement point itself are not quite high enough, velocities do tend to increase close to Millstone Point, a behavior not exhibited by CAFE. Again results for Niantic Bay are justified because, for the most part the bay is quite deep (8 - 15 meters) and hence the linear solution is applicable. Furthermore velocity amplitudes tend to be higher than those calculated by CAFE since there is no need for artificially high eddy viscosity values which tend to dampen and smooth out velocities.

Finally there is a dramatic difference in run times between CAFE and TEA. For this grid, a run with CAFE required approximately 20 hours of CPU times on a Honeywell level 68/DPS computer. This excessive amount of CPU time resulted from the maximum time step for numerical stability of 2 seconds required due to the small grid sizes. For the same grid, program TEA required only 2.5 minutes of CPU time running on a VAX 11/780, which is of comparable speed as the Honeywell.

5.3 Brayton Point

Our final example deals with predicting the combined circulation due to both tides and a power plant discharge at Brayton Point in Mount Hope Bay, Somerset, Massachusetts (Fig. 5.15). Again the main region of interest is around the power plant and hence the finite element grid discretization reflects this with a large amount of refinement in this region as shown in Figs. 5.16 and 5.17. The particular case run with program TEA examines the upper layer circulation pattern produced by tides and a three unit power plant operation and is analogous to a case study previously done with CAFE [6].

To simulate upper layer circulation, the depths used were the estimated upper layer depths which ranged between 1.5 and 3.5 meters. Again the tide and the steady state discharge must be handled by two separate runs of TEA. In the first run, the tidal component of circulation is calculated by prescribing tidal amplitude at the ocean boundary, as shown in Fig. 5.16, and forcing the system at a frequency corresponding to T = 12.4 hours. The second run simulates steady state power operation and natural river inflow. We attempt to simulate a jet by prescribing flow conditions at the near field - far field jet interface. Hence we prescribed large normal fluxes representing the outgoing jet and the in-going re-entrainment fluxes surrounding the jet as shown in Fig. 5.17. We note the very refined region necessary near the jet due to the large gradient and rapid turning expected in this region. Power plant intake flow is simulated by specifying a normal flux along the east side of the discharge peninsula. Additional withdrawals representing downwelling to the lower layer are simulated by specifying normal fluxes through elements along the southeastern edge of the domain. Taunton River

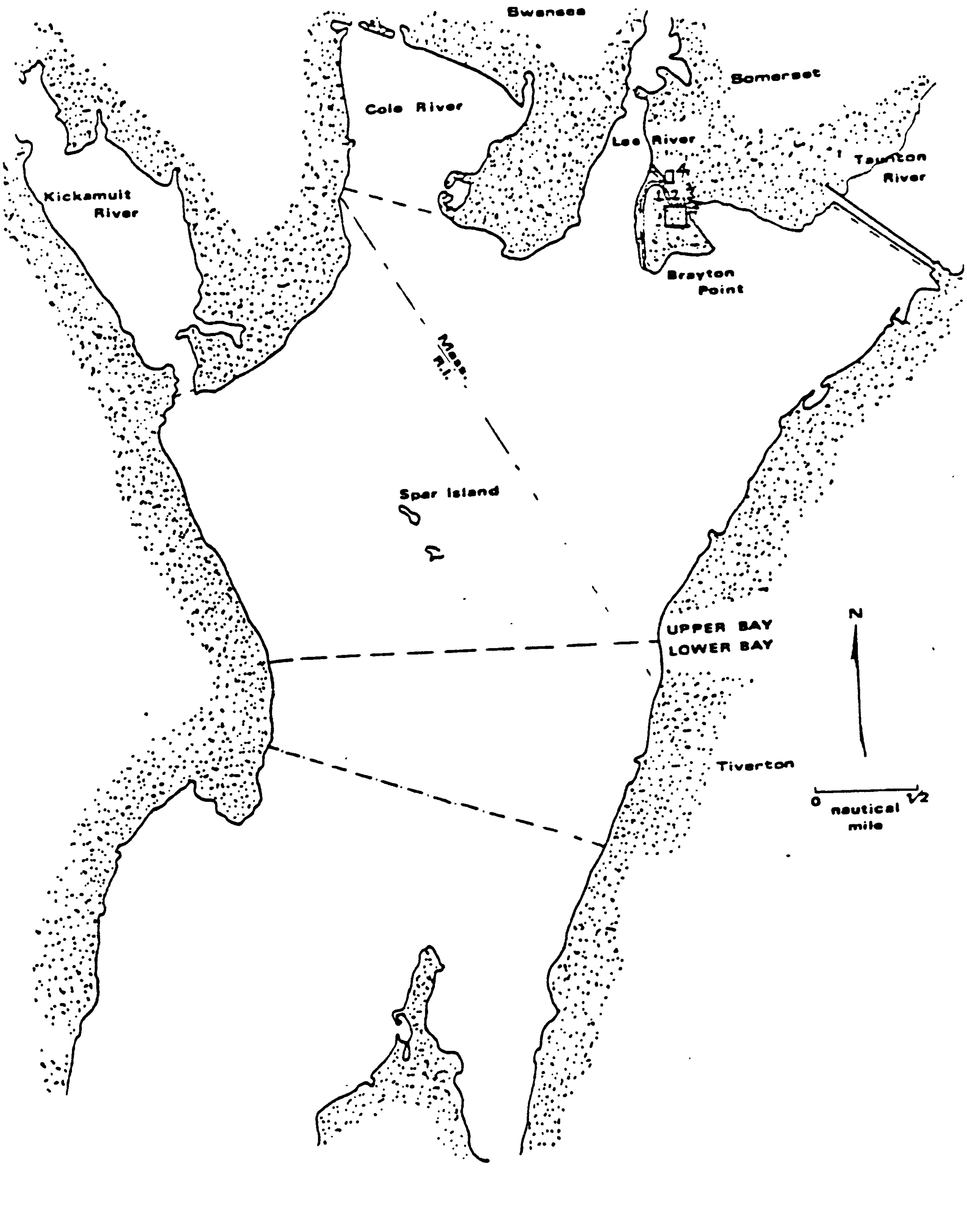

Fig. 5.15 Site of Brayton Point Generating Station located in Mount Hope Bay, Somerset, Massachusetts (from Ref. 6)

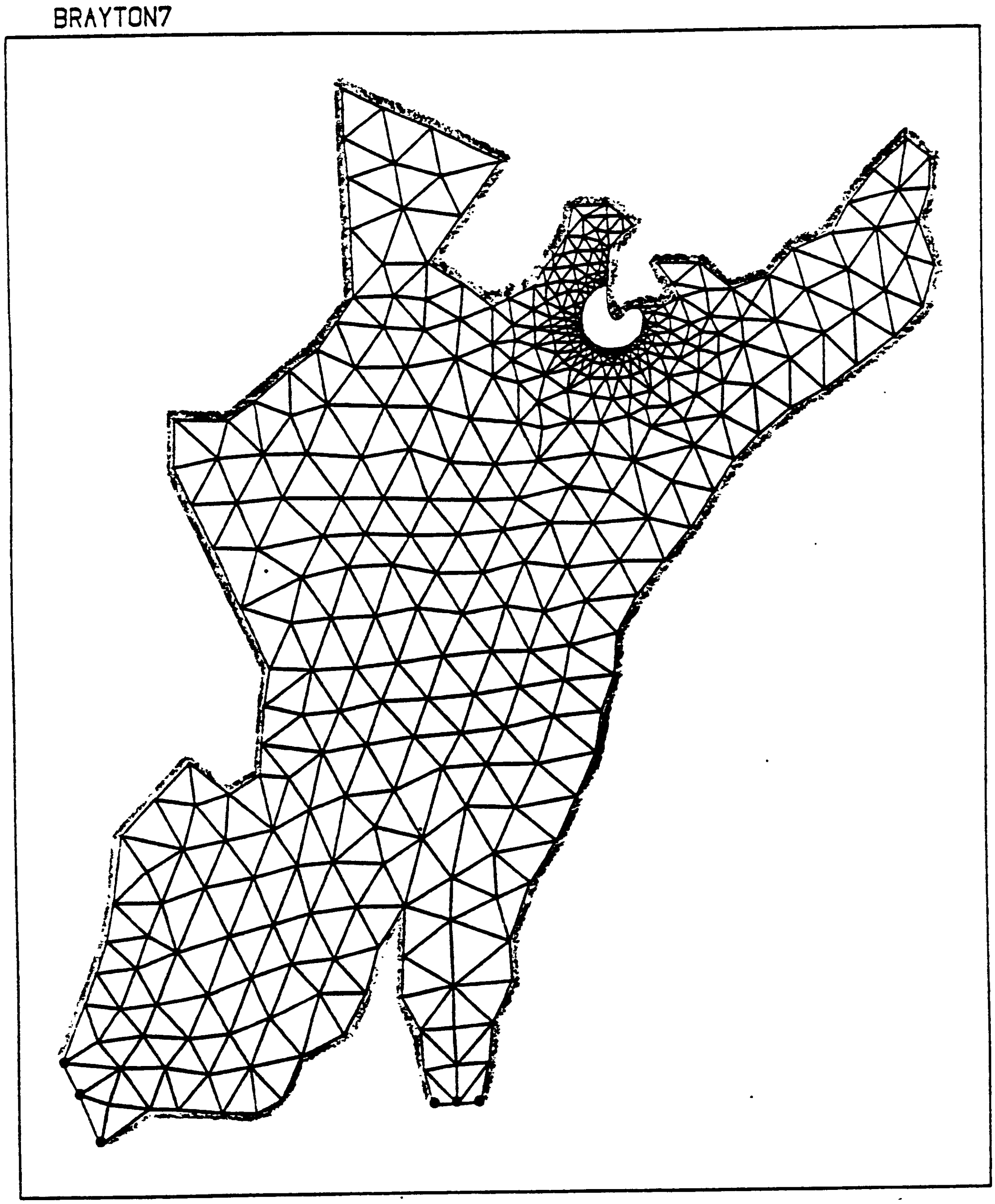

Fig. 5.16 Finite Element Grid Discretization of Mount Hope Bay

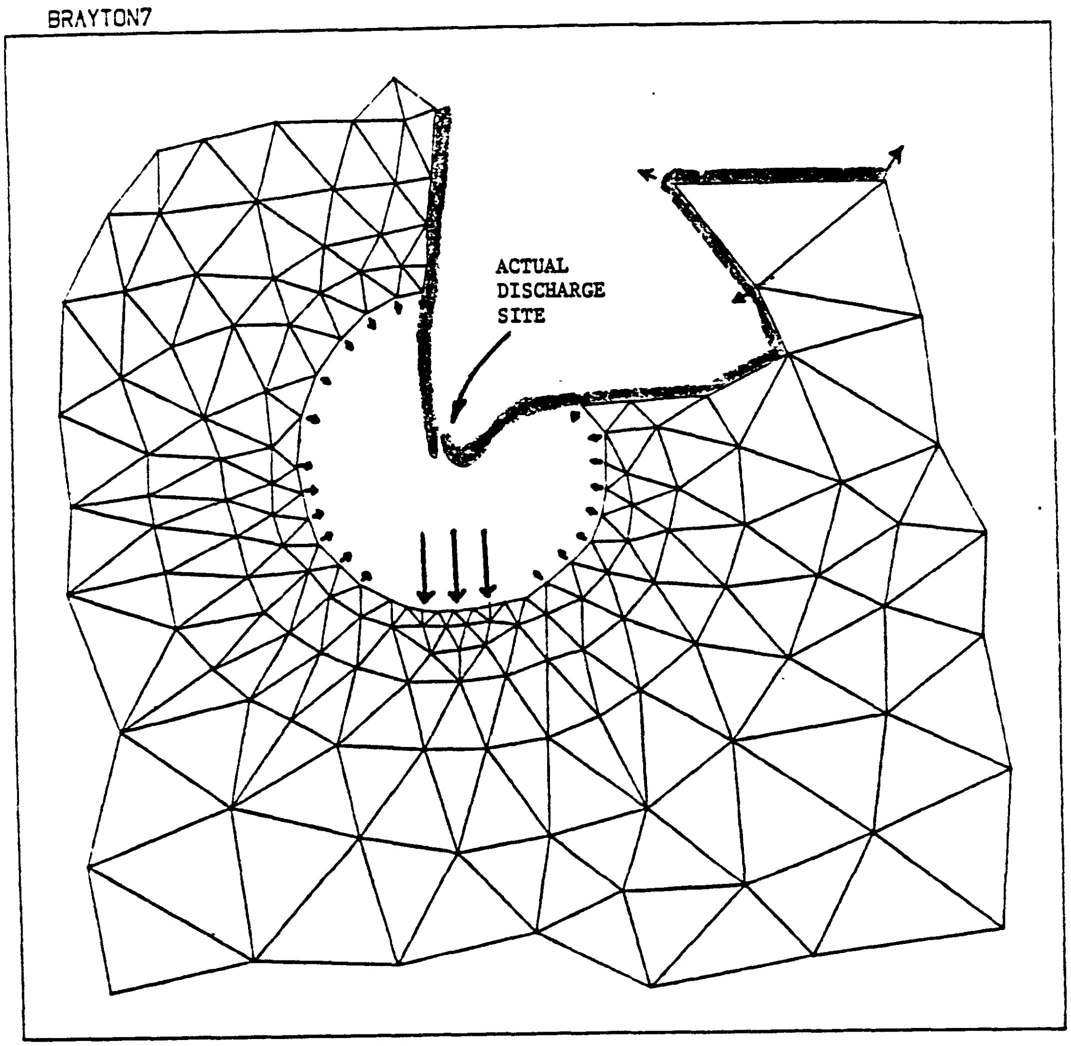

Fig. 5.17 Detail of Grid at Discharge Region at Brayton Point

flow is specified as a normal flux into the domain through elements in the northeast corner. All normal flux information is provided to TEA by specifying the end nodes, with a correct orientation (see Chapter 4.4), of each segment through which normal flow occurs and by specifying the x and y components of the normal flux at the nodes included in these flux segments. Finally, zero elevation amplitude is prescribed at the ocean boundary nodes and the system is driven at zero frequency.

Superimposed flow circulation patterns at two stages of the tide are shown in Figs. 5.18 and 5.19. Details of the flow around Brayton Point, including the discharge and intake locations, are shown in Figs. 5.20, 5.21 and 5.22. A result from CAFE corresponding to Fig 5.22 is shown in Fig. 5.23. When comparing results from TEA and CAFE we note that the jet is not as well simulated by TEA. This is due mainly to the fact that TEA does not include the non-linear momentum terms needed to simulate jet physics. TEA drives the discharge only by elevation gradients which accounts for the rapid spreading of the jet, best exhibited in Fig. 5.22. Hence we may conclude that a code which includes non-linearities is superior for simulating the circulation close to jet discharges. However program TEA will simulate discharges quite adequately if one is not interested in the region near the discharge where the momentum effects are important. Furthermore, a full non-linear version of TEA is presently under development.

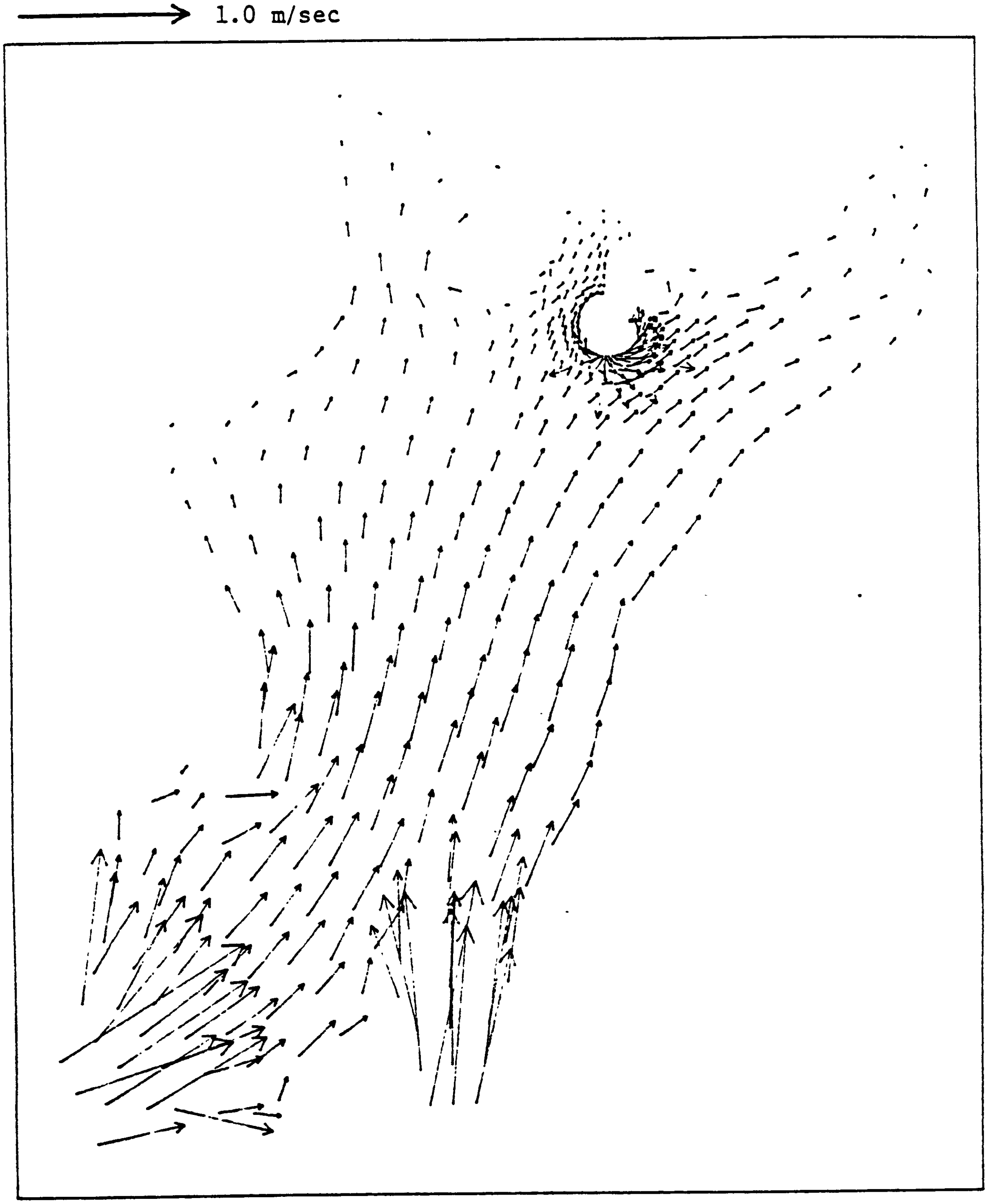

Fig. 5.18 Circulation Computed by TEA at Maximum Flood

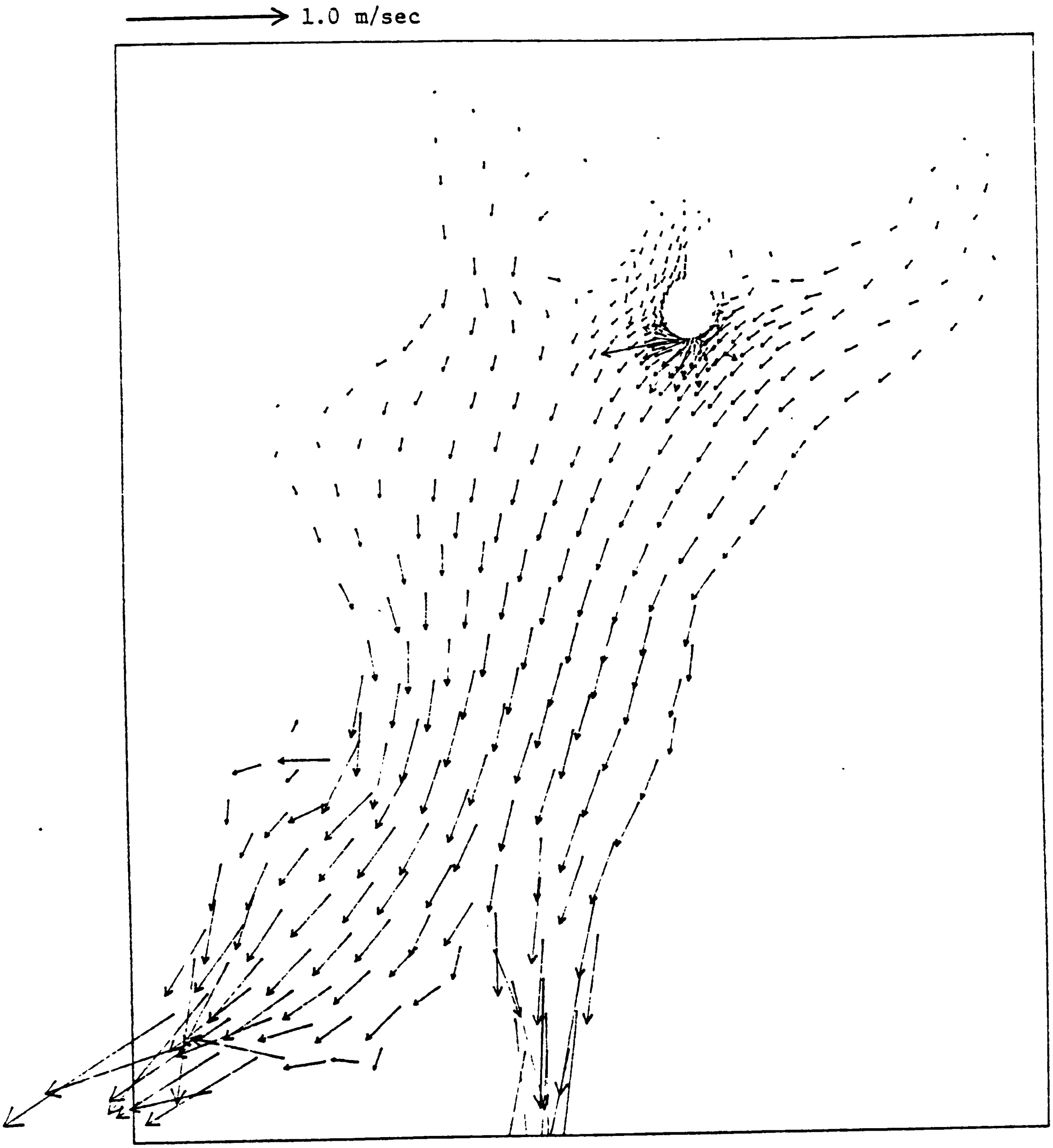

Fig. 5.19 Circulation Computed by TEA at Maximum Ebb

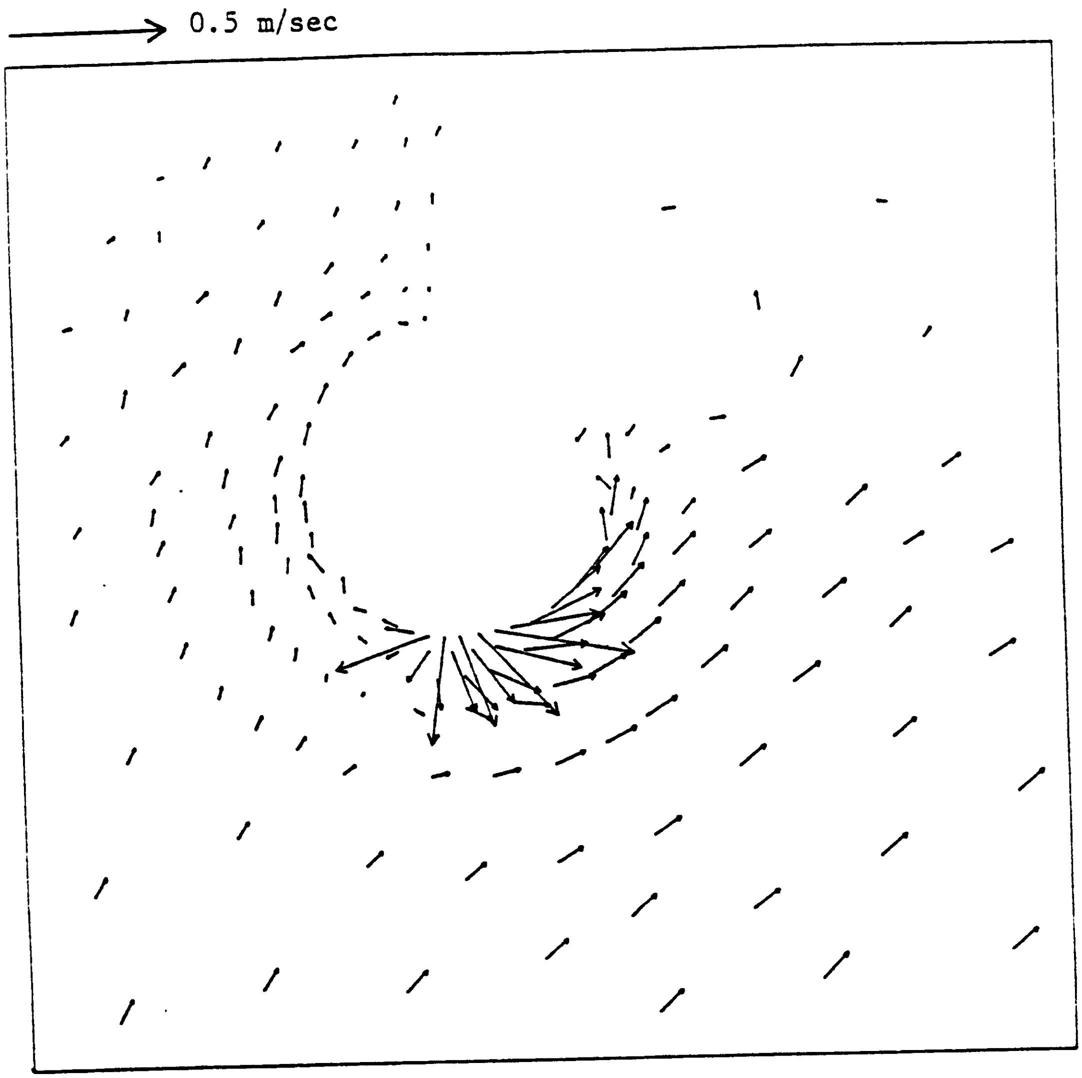

Fig. 5.20 Detail of Circulation at Brayton Point Computed by TEA at Maximum Flood

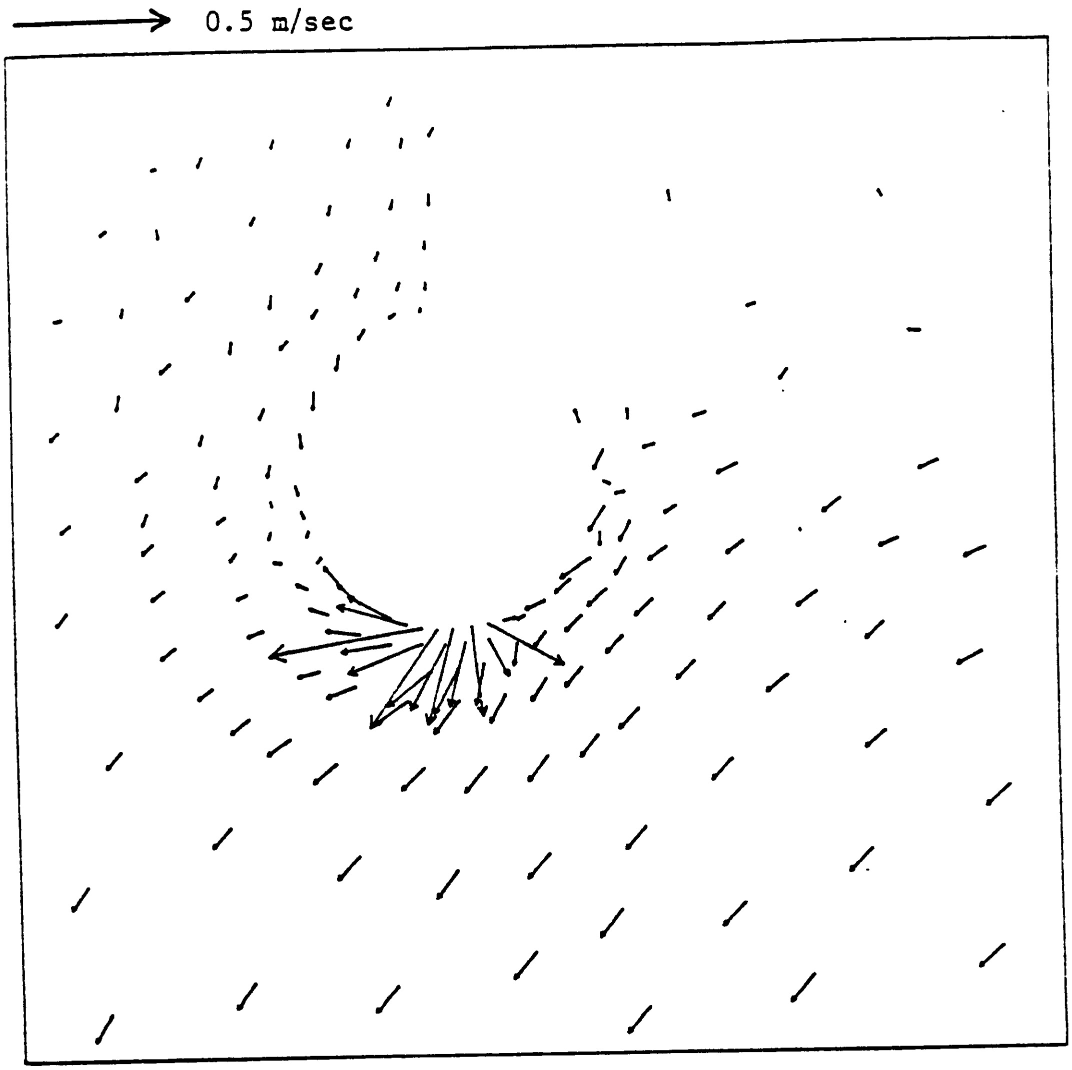

Fig. 5.21 Detail of Circulation at Brayton Point Computed by TEA at Maximum Ebb

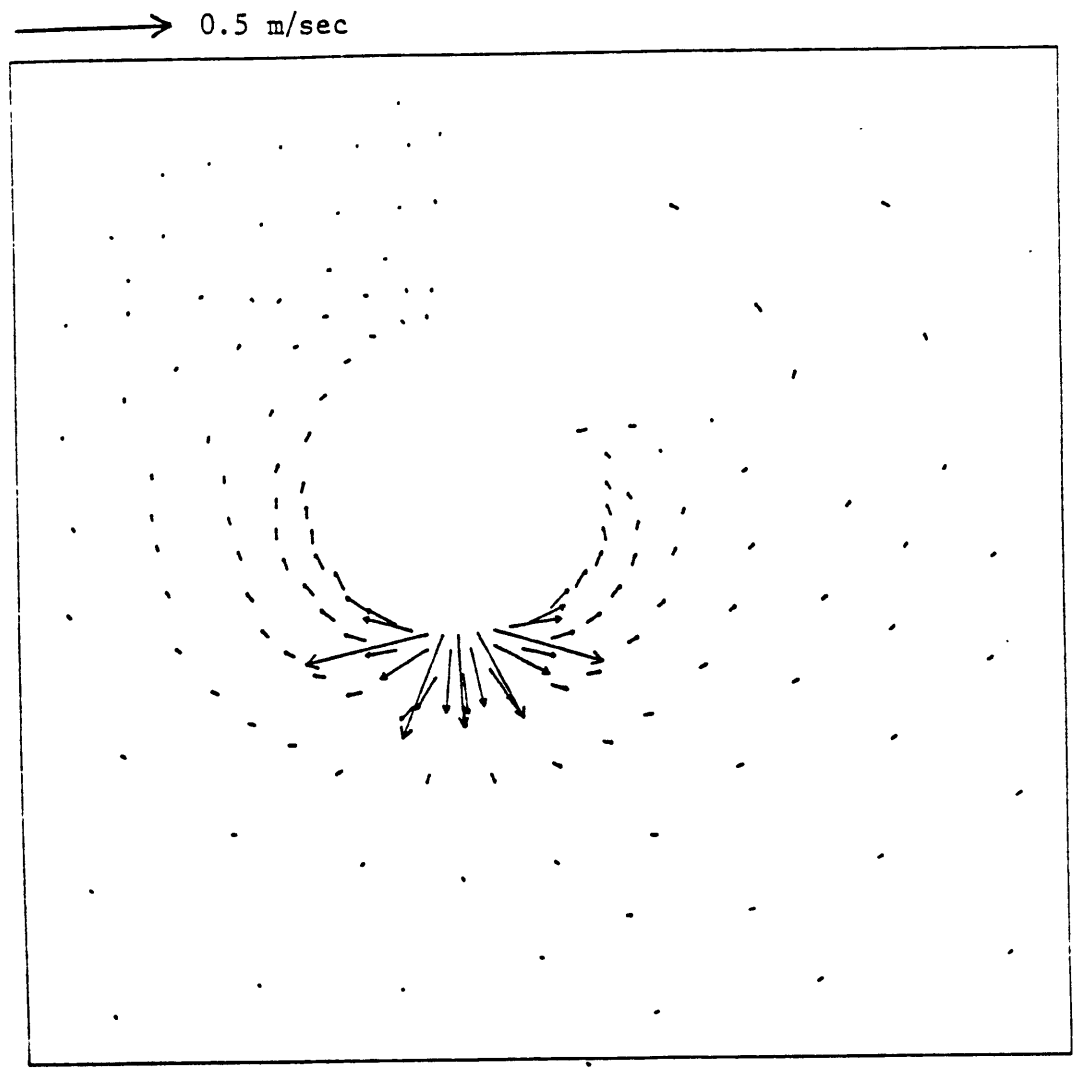

Fig. 5.22 Detail of Circulation at Brayton Point Computed by TEA at High Slack

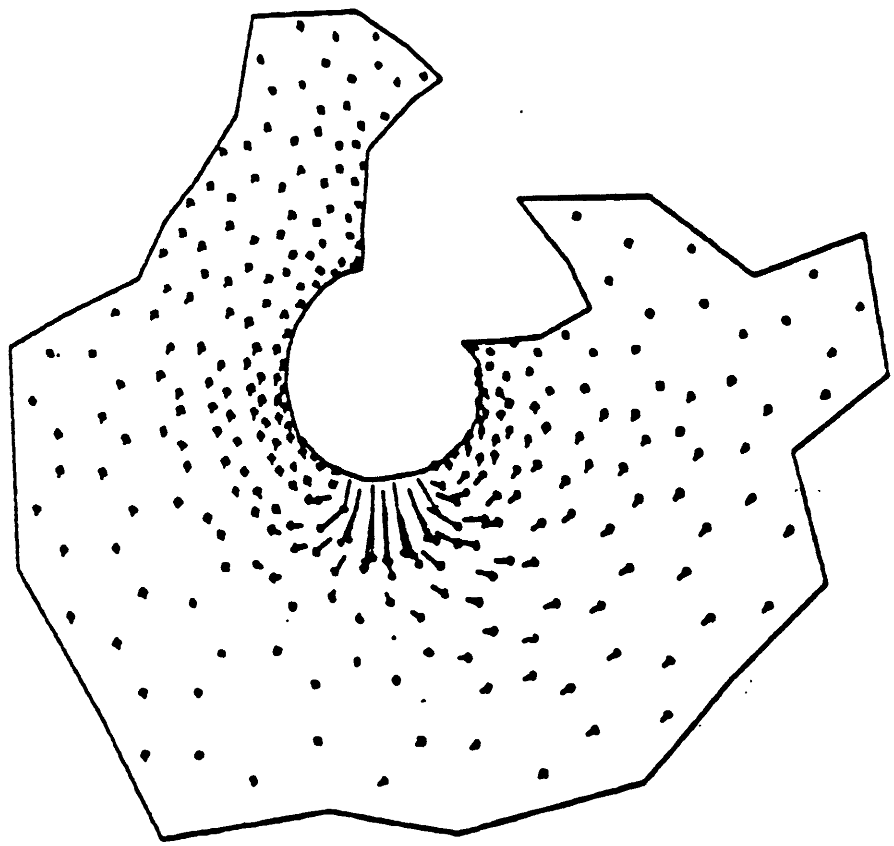

Fig. 5.23 Detail of Circulation at Brayton Point Computed by CAFE at High Slack (from Ref. 6)

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Appendix 1: Program TEA

1.1 input format

1.2 output format

1.3 example problem

1.4 listing

Appendix 1.1

Input for TEA is free formated and information should be read as follows:

card 1: 10 character alphanumeric geometry identification

card 2: 10 character alphanumeric run number identification

card 3: number of elements; number of nodes

card group 4: node number; x-coord; y-coord; and nodal depth

card group 5: element number; element node connectivity given by 3 element node numbers (counterclockwise); element linear friction factor

card 6: gravitational constant in consistent units

card 7: forcing frequency

card 8: Coriolis factor

card 9: amplitude of wind speed; wind phase shift (in radians); wind direction (in degrees); wind drag coefficient

Prescribed normal flux section:

card 10: number of boundary segments which have non-zero normal flux prescribed (if this is zero skip inputs 10a, 10b, 10c to card 11 input)

card 10a: end nodes of boundary segments with non-zero normal flux prescribed with correct orientation (clockwise for land boundary and counterclockwise for islands)

card group 10b: total number of different nodes inluded in non-zero normal flux load segments

card group 10c: node number; modulus of x-direction flux at node; phase shift for x-direction (in radians); modulus of y-direction flux; phase shift for y-direction (in radians)

Prescribed elevation section:

card 11: number of nodes where elevation is prescribed

card group 11a: node number; modulus of prescribed elevation at node; phase shift at node (in radians)

Appendix 1.2

As discussed in Section 4.5 program TEA outputs information onto TAPE 6 and TAPE 8. TAPE 6 information is a formated descriptive output and consists of three main sections:

(1) Print statements which ask for required input in the order needed by TEA. This allows the program to be run in an interactive fashion. Furthermore these prints may also aid the user in checking to make sure that the input is in proper order. Finally error messages concerning dimensioning of matrices may appear here.

(2) A labeled echo print of all input information.

(3) Results of calculations consisting of elevation amplitude and phase shifts at nodes and velocities in the x and y direction with corresponding phase shifts at nodes.

Example outputs are shown in Appendices 3.3 and 3.4.

TAPE 8 output consists of information required for plotting of the grid and results and/or generation of time histories. TAPE 8 output is free-formated and consists of the following information groups:

- 10 character alphanumeric geometry identificaton
- 10 character boundary condition identification
- number of elements, number of nodes
- node number, x-coordinate, y-coordinate
- element number, three corresponding node numbers

- frequency
- node number, elevation amplitude, elevation phase shift
- node number, x-velocity, x-phase shift, y-velocity, y-phase shift

Appendix 1.3.1

Here we discuss the boundary condition input requirements for program TEA with a case which shows a variety of features. The grid used is very coarse and is used only for illustrative purposes.

We consider a tidal inlet with an island and several steady state discharges. The geometry and the coarse grid used are shown in Fig. A1. The grid is shown again in Fig. A2 with element and node numbers included. The bay is subject to a 12.4 hour tidal forcing of unit amplitude and zero phase shift along the ocean boundary. Furthermore we note the steady state discharge on the land boundary and the steady intake on the island.

To simulate this case with TEA we perform two runs with two sets of boundary conditions. The first run calculates the tidal circulation component. The required boundary condition information consists only of the specification of ocean boundary node numbers and their corresponding tidal elevation amplitude and phase shift. TEA input for this run is shown in Appendix 1.3.2.

The second run calculates the steady state circulation component (hence zero frequency) and we must input the end nodes of the boundary segments through which flow occurs (in clockwise order for the land segments and in counterclockwise order for island segments) and the x and y components of the normal flow at the nodes involved. We note that phase shifts do not apply to the zero frequency case and must be specified as zero. Finally we provide information about the open ocean boundary by specifying zero amplitude at these nodes, allowing the total flow to balance. In this case since there is a net inflow into the bay, there will be an outflow into the ocean. TEA input for this run is shown in

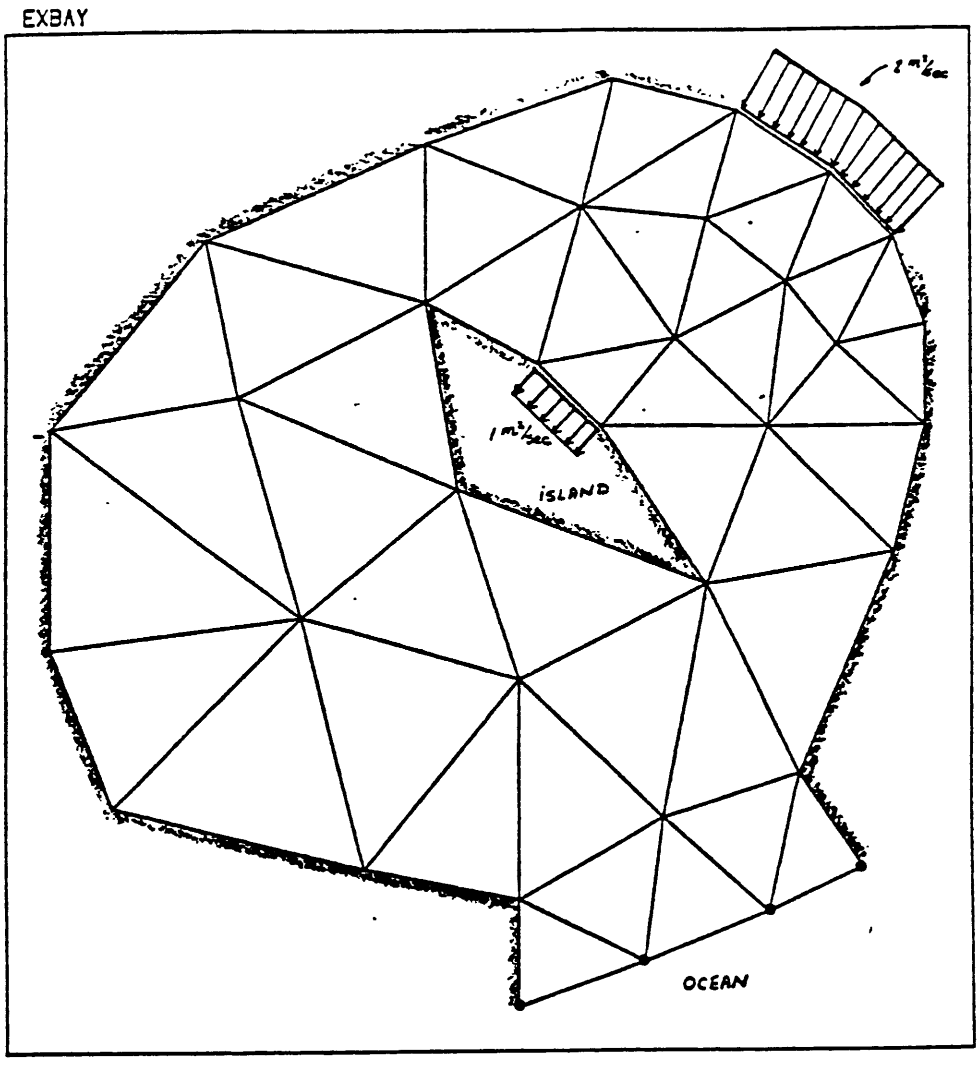

Fig. A1 Example Finite Element Grid

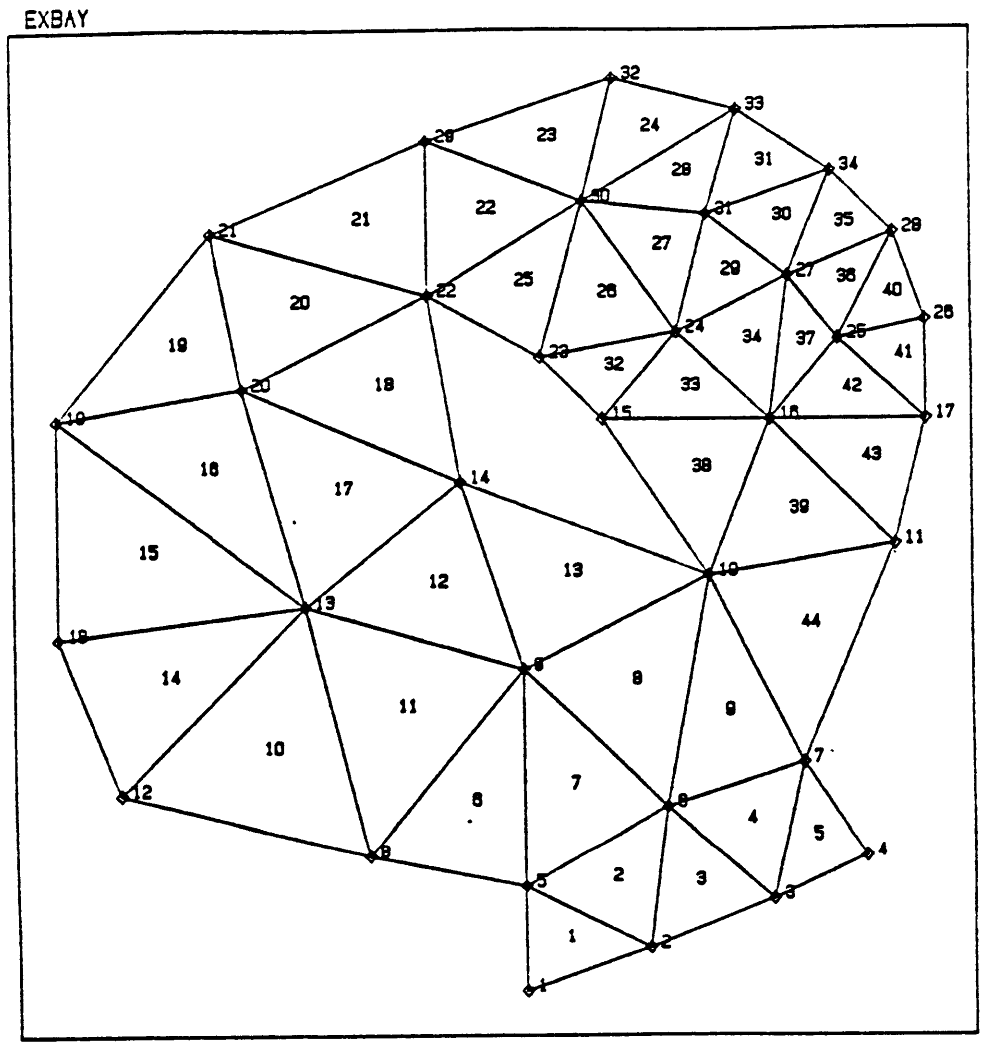

Fig. A2 Example Finite Element Grid with Node and Element Numbers Included

Appendix 1.3.3. We note that for a totally enclosed waterbody we would only specify one elevation at one boundary node. In that case then there would only be flows through specified flux (natural) boundary segments. Furthermore we would have to specify fluxes such that the total flow in and out of the boundary balanced in order to satisfy continuity.

Outputs for the two sample calculations are shown in Appendices 1.3.4 and 1.3.5. Since we have solved two linear problems we may now superimpose the results as discussed in Section 4.5.

Appendix 1.3.2

```
EXBAY1
BC-01
44, 34
      1    7500.      800.        25.
      2    9500.     1500.        25.
      3   11500.     2300.        25.
      4   13000.     3000.        25.
      5    7500.     2500.        25.
      6    9800.     3800.        25.
      7   12000.     4500.        25.
      8    5000.     3000.        25.
      9    7500.     6000.        25.
     10   10500.     7500.        25.
     11   13500.     8000.        25.
     12    1000.     4000.        25.
     13    4000.     7000.        25.
     14    6500.     9000.        25.
     15    8800.    10000.        25.
     16   11500.    10000.        25.
     17   14000.    10000.        25.
     18       0.     6500.        25.
     19       0.    10000.        25.
     20    3000.    10500.        25.
     21    2500.    13000.        25.
     22    6000.    12000.        25.
     23    7800.    11000.        25.
     24   10000.    11400.        25.
     25   12600.    11300.        25.
     26   14000.    11600.        25.
     27   11800.    12300.        25.
     28   13500.    13000.        25.
     29    6000.    14500.        25.
     30    8500.    13500.        25.
     31   10500.    13300.        25.
     32    9000.    15500.        25.
     33   11000.    15000.        25.
     34   12500.    14000.        25.
       1       1       2       5   0.00100
       2       2       6       5   0.00100
       3       2       3       6   0.00100
       4       3       7       6   0.00100
       5       3       4       7   0.00100
       6       8       5       9   0.00100
       7       5       6       9   0.00100
       8       6      10       9   0.00100
       9       6       7      10   0.00100
      10       8      13      12   0.00100
      11       8       9      13   0.00100
      12      13       9      14   0.00100
      13       9      10      14   0.00100
      14      12      13      18   0.00100
      15      18      13      19   0.00100
      16      19      13      20   0.00100
      17      13      14      20   0.00100
      18      20      14      22   0.00100
      19      19      20      21   0.00100
      20      20      22      21   0.00100
      21      21      22      29   0.00100
      22      29      22      30   0.00100
      23      29      30      32   0.00100
      24      30      33      32   0.00100
      25      22      23      30   0.00100
      26      23      24      30   0.00100
```

27	30	24	31	0.00100
28	30	31	33	0.00100
29	24	27	31	0.00100
30	31	27	34	0.00100
31	31	34	33	0.00100
32	23	15	24	0.00100
33	15	16	24	0.00100
34	24	16	27	0.00100
35	27	28	34	0.00100
36	27	25	28	0.00100
37	16	25	27	0.00100
38	15	10	16	0.00100
39	10	11	16	0.00100
40	25	26	28	0.00100
41	25	17	26	0.00100
42	16	17	25	0.00100
43	16	11	17	0.00100
44	10	7	11	0.00100

9.81
0.00014075
0.00010000
0.00, 0.00, 0.00, 0.00
0
4
1, 1.00, 0.00
2, 1,00, 0.00
3, 1.00, 0.00
4, 1.00, 0.00

Appendix 1.3.3

```
EXBAY2
BC-01
44, 34
     1     7500.       800.        25.
     2     9500.      1500.        25.
     3    11500.      2300.        25.
     4    13000.      3000.        25.
     5     7500.      2500.        25.
     6     9800.      3800.        25.
     7    12000.      4500.        25.
     8     5000.      3000.        25.
     9     7500.      6000.        25.
    10    10500.      7500.        25.
    11    13500.      8000.        25.
    12     1000.      4000.        25.
    13     4000.      7000.        25.
    14     6500.      9000.        25.
    15     8800.     10000.        25.
    16    11500.     10000.        25.
    17    14000.     10000.        25.
    18        0.      6500.        25.
    19        0.     10000.        25.
    20     3000.     10500.        25.
    21     2500.     13000.        25.
    22     6000.     12000.        25.
    23     7800.     11000.        25.
    24    10000.     11400.        25.
    25    12600.     11300.        25.
    26    14000.     11600.        25.
    27    11800.     12300.        25.
    28    13500.     13000.        25.
    29     6000.     14500.        25.
    30     8500.     13500.        25.
    31    10500.     13300.        25.
    32     9000.     15500.        25.
    33    11000.     15000.        25.
    34    12500.     14000.        25.
       1         1         2         5   0.00100
       2         2         6         5   0.00100
       3         2         3         6   0.00100
       4         3         7         6   0.00100
       5         3         4         7   0.00100
       6         8         5         9   0.00100
       7         5         6         9   0.00100
       8         6        10         9   0.00100
       9         6         7        10   0.00100
      10         8        13        12   0.00100
      11         8         9        13   0.00100
      12        13         9        14   0.00100
      13         9        10        14   0.00100
      14        12        13        18   0.00100
      15        18        13        19   0.00100
      16        19        13        20   0.00100
      17        13        14        20   0.00100
      18        20        14        22   0.00100
      19        19        20        21   0.00100
      20        20        22        21   0.00100
      21        21        22        29   0.00100
      22        29        22        30   0.00100
      23        29        30        32   0.00100
      24        30        33        32   0.00100
      25        22        23        30   0.00100
      26        23        24        30   0.00100
```

```
  27        30        24        31    0.00100
  28        30        31        33    0.00100
  29        24        27        31    0.00100
  30        31        27        34    0.00100
  31        31        34        33    0.00100
  32        23        15        24    0.00100
  33        15        16        24    0.00100
  34        24        16        27    0.00100
  35        27        28        34    0.00100
  36        27        25        28    0.00100
  37        16        25        27    0.00100
  38        15        10        16    0.00100
  39        10        11        16    0.00100
  40        25        26        28    0.00100
  41        25        17        26    0.00100
  42        16        17        25    0.00100
  43        16        11        17    0.00100
  44        10         7        11    0.00100
9.81
0.00000000
0.00010000
0.00, 0.00, 0.00, 0.00
3
33, 34
34, 28
15, 23
5
33,   -0.93,   0.00,   -1.80,   0.00
34,   -1.21,   0.00,   -1.60,   0.00
28,   -1.53,   0.00,   -1.27,   0.00
23,   -0.69,   0.00,   -0.74,   0.00
15,   -0.69,   0.00,   -0.74,   0.00
4
1,  0.00, 0.00
2,  0.00, 0.00
3,  0.00, 0.00
4,  0.00, 0.00
```

Appendix 1.3.4

```
$

PROGRAM TEA
DEPT OF CIVIL ENGINEERING, M. I. T.
2-D LINEAR FINITE ELEMENT FREQUENCY DOMAIN ANALYSIS OF TIDAL WAVES FOR  SMALL SCALE GEOMETRY.

ENTER 10 DIGIT F.E. GRID GEOMETRY IDENTIFICATION

ENTER 10 DIGIT I.D. CODE FOR B.C. VERSION

INPUT NUMBER OF ELEMENTS AND NUMBER OF NODE POINTS

INPUT THE NODE NO.,X-COORD.,Y-COORD., AND NODAL M.S.L. DEPTH, ONE NODE/LINE

INPUT ELEMENT NUMBER, THREE NODE NUMBERS FOR THE ELEMENTAND THE ELEMENT LINEAR FRICTION FACTOR

MAX. NODAL POINT DIFFERENCE =    11


INPUT FREQUENCY IN RADIANS SEC.

INPUT CORIOLIS PARAMETER

INPUT AMPLITUDE OF WIND SPEED, WIND PHASE SHIFT, ANGLE OF WIND DIRECTION  (IN DEGREES)  AND WIND DRAG COEFFICIENT

INPUT HOW MANY BOUNDARY SEGMENTS HAVE NON-ZERO NORMAL FLUX PRESCRIBED


INPUT NO. OF NODES WHERE ELEVATION IS PRESCRIBED
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE

-----------------------------------------------------------------------------------------------------------------


PROGRAM TEA
DEPT OF CIVIL ENGINEERING,  M. I. T.
2-D LINEAR FINITE ELEMENT FREQUENCY DOMAIN ANALYSIS OF TIDAL WAVES FOR  SMALL SCALE GEOMETRY.


F.E. GRID GEOMETRY IDENTIFICATION : EXBAY1
BOUNDARY CONDITION IDENTIFICATION  : BC-01


FREQUENCY = 0.00014075
CORIOLIS PARAMETER = 0.00010000


WIND SPEED AT 10 METERS ABOVE SURFACE =      0.0000
WIND PHASE SHIFT =    0.00000 RADIANS
WIND DIRECTION =    0.00000 DEGREES
WIND DRAG COEFFICIENT =   0.00000000
```

NO. OF NODE POINTS = 34
NO. OF ELEMENTS = 44

MAX. NODAL POINT DIFFERENCE = 11

************************************ NODAL COORDINATES AND DEPTHS ************************************

NODE	X-COORD.	Y-COORD.	DEPTH
1	7500.00	800.00	25.00
2	9500.00	1500.00	25.00
3	11500.00	2300.00	25.00
4	13000.00	3000.00	25.00
5	7500.00	2500.00	25.00
6	9800.00	3800.00	25.00
7	12000.00	4500.00	25.00
8	5000.00	3000.00	25.00
9	7500.00	6000.00	25.00
10	10500.00	7500.00	25.00
11	13500.00	8000.00	25.00
12	1000.00	4000.00	25.00
13	4000.00	7000.00	25.00
14	6500.00	9000.00	25.00
15	8800.00	10000.00	25.00
16	11500.00	10000.00	25.00
17	14000.00	10000.00	25.00
18	0.00	6500.00	25.00
19	0.00	10000.00	25.00
20	3000.00	10500.00	25.00
21	2500.00	13000.00	25.00
22	6000.00	12000.00	25.00
23	7800.00	11000.00	25.00
24	10000.00	11400.00	25.00
25	12600.00	11300.00	25.00
26	14000.00	11600.00	25.00
27	11800.00	12300.00	25.00
28	13500.00	13000.00	25.00
29	6000.00	14500.00	25.00
30	8500.00	13500.00	25.00
31	10500.00	13300.00	25.00
32	9000.00	15500.00	25.00
33	11000.00	15000.00	25.00
34	12500.00	14000.00	25.00

************************************ ELEMENT ARRAY ************************************

ELEMENT	I	J	K	ELEMENT FRICTION FACTOR
1	1	2	5	0.0010000
2	2	6	5	0.0010000
3	2	3	6	0.0010000
4	3	7	6	0.0010000
5	3	4	7	0.0010000
6	8	5	9	0.0010000
7	5	6	9	0.0010000
8	6	10	9	0.0010000
9	6	7	10	0.0010000

10	8	13	12	0.0010000
11	8	9	13	0.0010000
12	13	9	14	0.0010000
13	9	10	14	0.0010000
14	12	13	18	0.0010000
15	18	13	19	0.0010000
16	19	13	20	0.0010000
17	13	14	20	0.0010000
18	20	14	22	0.0010000
19	19	20	21	0.0010000
20	20	22	21	0.0010000
21	21	22	29	0.0010000
22	29	22	30	0.0010000
23	29	30	32	0.0010000
24	30	33	32	0.0010000
25	22	23	30	0.0010000
26	23	24	30	0.0010000
27	30	24	31	0.0010000
28	30	31	33	0.0010000
29	24	27	31	0.0010000
30	31	27	34	0.0010000
31	31	34	33	0.0010000
32	23	15	24	0.0010000
33	15	16	24	0.0010000
34	24	16	27	0.0010000
35	27	28	34	0.0010000
36	27	25	28	0.0010000
37	16	25	27	0.0010000
38	15	10	16	0.0010000
39	10	11	16	0.0010000
40	25	26	28	0.0010000
41	25	17	26	0.0010000
42	16	17	25	0.0010000
43	16	11	17	0.0010000
44	10	7	11	0.0010000

****************************** BOUNDARY CONDITIONS ******************************

NO. OF BOUNDARY SEGMENTS WHICH HAVE NON-ZERO NORMAL FLUX PRESCRIBED = 0

NO. OF NODES WHERE ELEVATION IS PRESCRIBED = 4

NODE	PRESCRIBED ELEVATION	PHASE
1	1.0000	0.00000
2	1.0000	0.00000
3	1.0000	0.00000
4	1.0000	0.00000

****************************** RESULTS OF COMPUTATIONS ******************************

NODAL ELEVATIONS

NODE	MODULUS	PHASE
1	1.00000000	0.00000
2	1.00000000	0.00000
3	1.00000000	0.00000
4	1.00000000	0.00000
5	1.00100239	-0.00380
6	1.00395713	-0.00162
7	1.00278606	0.00096
8	1.00874233	-0.00752
9	1.00641567	-0.00327
10	1.00663572	-0.00264
11	1.01004628	-0.00109
12	1.00906940	-0.00451
13	1.00869093	-0.00488
14	1.00748832	-0.00262
15	1.01123755	-0.00493
16	1.00958349	-0.00294
17	1.00940425	-0.00247
18	1.00925525	-0.00581
19	1.01037390	-0.00474
20	1.01013314	-0.00480
21	1.01006339	-0.00446
22	1.01083832	-0.00400
23	1.01085458	-0.00333
24	1.01030048	-0.00331
25	1.01040624	-0.00249
26	1.01058496	-0.00212
27	1.01058154	-0.00314
28	1.01057978	-0.00297
29	1.01116096	-0.00415
30	1.01110617	-0.00372
31	1.01107183	-0.00351
32	1.01132440	-0.00343
33	1.01111392	-0.00330
34	1.01081104	-0.00271

NODAL VELOCITIES

NODE	X-DIRECTION MODULUS	X-DIRECTION PHASE	Y-DIRECTION MODULUS	Y-DIRECTION PHASE
1	0.16149733	-0.89457	0.25987739	0.70683
2	0.07789793	-0.51438	0.17803156	1.20267
3	0.08442079	-0.11300	0.16461041	1.77377
4	0.11026195	0.59346	0.17433353	2.29168
5	0.08233856	-1.14076	0.14247863	1.28004
6	0.03004363	-0.84828	0.10528989	1.47924
7	0.02408071	-0.29799	0.11304919	1.55365
8	0.06198340	-1.37087	0.02835620	1.66106
9	0.05060869	-1.61163	0.04258825	1.45694
10	0.01192018	0.28291	0.07564032	1.54913
11	0.04900782	1.75537	0.05858621	1.63318
12	0.02279296	0.52544	0.02081520	2.76747
13	0.02453620	-1.91804	0.00478852	2.96110
14	0.03264711	-1.58872	0.03211532	1.40126
15	0.03512620	-1.54573	0.04109161	1.60012
16	0.00825685	-2.06721	0.04024705	1.32265
17	0.01892373	-0.91485	0.03553314	0.98635
18	0.01753154	-1.62605	0.01699655	1.05706
19	0.00709762	0.87872	0.02494912	[illegible]

20	0.01466212	-1.28589	0.02755656	1.55791
21	0.02232717	2.59118	0.01686409	-2.19821
22	0.00326947	2.16991	0.02221997	1.57475
23	0.01037866	-1.99476	0.00320050	-1.69148
24	0.01250223	-1.81539	0.00870381	2.04171
25	0.00517681	2.60651	0.02616408	1.64628
26	0.00386064	2.40496	0.02558402	1.59382
27	0.00470081	-1.59448	0.02237984	1.46158
28	0.01544576	-1.57270	0.02540191	0.57953
29	0.01557799	2.69347	0.00362244	-2.46895
30	0.00543193	-1.66514	0.01173477	1.21762
31	0.00795269	-1.88447	0.01525586	1.40359
32	0.01306114	-3.06925	0.00655764	-2.67862
33	0.01552919	-2.40697	0.00316643	-0.65023
34	0.02074723	-2.24350	0.00385084	0.59141

FORTRAN STOP

Appendix 1.3.5

```
PROGRAM TEA
DEPT OF CIVIL ENGINEERING, M. I. T.
2-D LINEAR FINITE ELEMENT FREQUENCY DOMAIN ANALYSIS OF TIDAL WAVES FOR  SMALL SCALE GEOMETRY.

ENTER 10 DIGIT F.E. GRID GEOMETRY IDENTIFICATION

ENTER 10 DIGIT I.D. CODE FOR B.C. VERSION

INPUT NUMBER OF ELEMENTS AND NUMBER OF NODE POINTS

INPUT THE NODE NO.,X-COORD.,Y-COORD., AND NODAL M.S.L. DEPTH, ONE NODE/LINE

INPUT ELEMENT NUMBER, THREE NODE NUMBERS FOR THE ELEMENTAND THE ELEMENT LINEAR FRICTION FACTOR

MAX. NODAL POINT DIFFERENCE =    11


INPUT FREQUENCY IN RADIANS/SEC.

INPUT CORIOLIS PARAMETER

INPUT AMPLITUDE OF WIND SPEED, WIND PHASE SHIFT, ANGLE OF WIND DIRECTION  (IN DEGREES)  AND WIND DRAG COEFFICIENT

INPUT HOW MANY BOUNDARY SEGMENTS HAVE NON-ZERO NORMAL FLUX PRESCRIBED
INPUT END NODES OF BOUNDARY SEGMENT    1 WITH CORRECT ORIENTATION
INPUT END NODES OF BOUNDARY SEGMENT    2 WITH CORRECT ORIENTATION
INPUT END NODES OF BOUNDARY SEGMENT    3 WITH CORRECT ORIENTATION
INPUT TOTAL NO. OF DIFFERENT NODES INCLUDED IN NON-ZERO NORMAL FLUX SEGMENTS
INPUT NODE NO.,MODULUS AND PHASE SHIFT (IN RAD) FOR X AND Y DIRECTIONS
INPUT NODE NO.,MODULUS AND PHASE SHIFT (IN RAD) FOR X AND Y DIRECTIONS
INPUT NODE NO.,MODULUS AND PHASE SHIFT (IN RAD) FOR X AND Y DIRECTIONS
INPUT NODE NO.,MODULUS AND PHASE SHIFT (IN RAD) FOR X AND Y DIRECTIONS
INPUT NODE NO.,MODULUS AND PHASE SHIFT (IN RAD) FOR X AND Y DIRECTIONS


INPUT NO. OF NODES WHERE ELEVATION IS PRESCRIBED
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE

----------------------------------------------------------------------------------------------------------------


PROGRAM TEA
DEPT OF CIVIL ENGINEERING, M. I. T.
2-D LINEAR FINITE ELEMENT FREQUENCY DOMAIN ANALYSIS OF TIDAL WAVES FOR  SMALL SCALE GEOMETRY.


F.E. GRID GEOMETRY IDENTIFICATION : EXBAY2
BOUNDARY CONDITION IDENTIFICATION  : BC-01


FREQUENCY = 0.00000000
```

CORIOLIS PARAMETER = 0.00010000

WIND SPEED AT 10 METERS ABOVE SURFACE = 0.0000
WIND PHASE SHIFT = 0.00000 RADIANS
WIND DIRECTION = 0.00000 DEGREES
WIND DRAG COEFFICIENT = 0.00000000

NO. OF NODE POINTS = 34
NO. OF ELEMENTS = 44

MAX. NODAL POINT DIFFERENCE = 11

################################### NODAL COORDINATES AND DEPTHS ###################################

NODE	X-COORD.	Y-COORD.	DEPTH
1	7500.00	800.00	25.00
2	9500.00	1500.00	25.00
3	11500.00	2300.00	25.00
4	13000.00	3000.00	25.00
5	7500.00	2500.00	25.00
6	9800.00	3800.00	25.00
7	12000.00	4500.00	25.00
8	5000.00	3000.00	25.00
9	7500.00	6000.00	25.00
10	10500.00	7500.00	25.00
11	13500.00	8000.00	25.00
12	1000.00	4000.00	25.00
13	4000.00	7000.00	25.00
14	6500.00	9000.00	25.00
15	8800.00	10000.00	25.00
16	11500.00	10000.00	25.00
17	14000.00	10000.00	25.00
18	0.00	6500.00	25.00
19	0.00	10000.00	25.00
20	3000.00	10500.00	25.00
21	2500.00	13000.00	25.00
22	6000.00	12000.00	25.00
23	7800.00	11000.00	25.00
24	10000.00	11400.00	25.00
25	12600.00	11300.00	25.00
26	14000.00	11600.00	25.00
27	11800.00	12300.00	25.00
28	13500.00	13000.00	25.00
29	6000.00	14500.00	25.00
30	8500.00	13500.00	25.00
31	10500.00	13300.00	25.00
32	9000.00	15500.00	25.00
33	11000.00	15000.00	25.00
34	12500.00	14000.00	25.00

################################### ELEMENT ARRAY ###################################

ELEMENT	I	J	K	ELEMENT FRICTION FACTOR

1	1	2	5	0.0010000
2	2	6	5	0.0010000
3	2	3	6	0.0010000
4	3	7	6	0.0010000
5	3	4	7	0.0010000
6	8	5	9	0.0010000
7	5	6	9	0.0010000
8	6	10	9	0.0010000
9	6	7	10	0.0010000
10	8	13	12	0.0010000
11	8	9	13	0.0010000
12	13	9	14	0.0010000
13	9	10	14	0.0010000
14	12	13	18	0.0010000
15	18	13	19	0.0010000
16	19	13	20	0.0010000
17	13	14	20	0.0010000
18	20	14	22	0.0010000
19	19	20	21	0.0010000
20	20	22	21	0.0010000
21	21	22	29	0.0010000
22	29	22	30	0.0010000
23	29	30	32	0.0010000
24	30	33	32	0.0010000
25	22	23	30	0.0010000
26	23	24	30	0.0010000
27	30	24	31	0.0010000
28	30	31	33	0.0010000
29	24	27	31	0.0010000
30	31	27	34	0.0010000
31	31	34	33	0.0010000
32	23	15	24	0.0010000
33	15	16	24	0.0010000
34	24	16	27	0.0010000
35	27	28	34	0.0010000
36	27	25	28	0.0010000
37	16	25	27	0.0010000
38	15	10	16	0.0010000
39	10	11	16	0.0010000
40	25	26	28	0.0010000
41	25	17	26	0.0010000
42	16	17	25	0.0010000
43	16	11	17	0.0010000
44	10	7	11	0.0010000

****************************** BOUNDARY CONDITIONS ******************************

NO. OF BOUNDARY SEGMENTS WHICH HAVE NON-ZERO NORMAL FLUX PRESCRIBED = 3

SEGMENTS WITH NON-ZERO NORMAL FLUX PRESCRIBED ARE:

SEGMENT NO.	BEGINING NODE NO.	END NODE NO.
1	33	34
2	34	28
3	15	23

PRESCRIBED FLOW VALUES FOR NODES ON SEGMENTS ARE:

NODE NO.	X-DIRECTION		Y-DIRECTION		DEPTH
	FLUX	PHASE	FLUX	PHASE	
33	-0.9300	0.00000	-1.8000	0.00000	
34	-1.2100	0.00000	-1.6000	0.00000	
28	-1.5300	0.00000	-1.2700	0.00000	
23	-0.6900	0.00000	-0.7400	0.00000	
15	-0.6900	0.00000	-0.7400	0.00000	

NO. OF NODES WHERE ELEVATION IS PRESCRIBED = 4

NODE	PRESCRIBED ELEVATION	PHASE
1	1.0000	0.00000
2	1.0000	0.00000
3	1.0000	0.00000
4	1.0000	0.00000

****************************** RESULTS OF COMPUTATIONS ******************************

------------------------------ NODAL ELEVATIONS ------------------------------

NODE	MODULUS	PHASE
1	1.00000000	0.00000
2	1.00000000	0.00000
3	1.00000000	0.00000
4	1.00000000	0.00000
5	1.00277521	0.00000
6	1.00020943	0.00000
7	0.99953257	0.00000
8	1.00179901	0.00000
9	1.00135058	0.00000
10	1.00212289	0.00000
11	1.00114398	0.00000
12	1.00225619	0.00000
13	1.00220921	0.00000
14	0.99977815	0.00000
15	1.00182866	0.00000
16	1.00128145	0.00000
17	1.00082184	0.00000
18	1.00197919	0.00000
19	1.00254823	0.00000
20	1.00191389	0.00000
21	1.00239231	0.00000
22	1.00246658	0.00000
23	1.00199375	0.00000
24	1.00250935	0.00000
25	1.00193067	0.00000
26	1.00123397	0.00000
27	1.00164488	0.00000
28	1.00106324	0.00000
29	1.00332668	0.00000
30	1.00267349	0.00000
31	1.00267472	0.00000
32	1.00348645	0.00000

33	1.00421317	0.00000
34	1.00317041	0.00000

NODAL VELOCITIES

	X-DIRECTION		Y-DIRECTION	
NODE	MODULUS	PHASE	MODULUS	PHASE
1	0.11872876	3.14159	0.10354253	3.14159
2	0.02610023	3.14159	0.07462922	3.14159
3	0.00984074	0.00000	0.00439589	3.14159
4	0.01634801	0.00000	0.01742001	0.00000
5	0.01702290	0.00000	0.04220178	3.14159
6	0.00641079	3.14159	0.04607911	3.14159
7	0.02053215	3.14159	0.04340251	3.14159
8	0.00574442	0.00000	0.00234294	3.14159
9	0.01998268	0.00000	0.00337163	3.14159
10	0.01021991	3.14159	0.01700479	3.14159
11	0.00509576	3.14159	0.03441213	3.14159
12	0.00007838	3.14159	0.00274338	3.14159
13	0.01497699	0.00000	0.01432924	3.14159
14	0.01887863	0.00000	0.01610249	3.14159
15	0.00627862	3.14159	0.01509755	3.14159
16	0.00237505	0.00000	0.02056481	3.14159
17	0.00828776	3.14159	0.02908004	3.14159
18	0.00653017	3.14159	0.00232953	0.00000
19	0.00135094	3.14159	0.01065585	3.14159
20	0.00190716	3.14159	0.02869783	3.14159
21	0.01979741	3.14159	0.00837544	3.14159
22	0.03540456	3.14159	0.01914364	3.14159
23	0.02567951	3.14159	0.00300588	3.14159
24	0.01943137	3.14159	0.02257412	3.14159
25	0.00369105	3.14159	0.01881449	3.14159
26	0.01701885	0.00000	0.04196155	3.14159
27	0.02198353	3.14159	0.04287280	3.14159
28	0.00146240	3.14159	0.04930760	3.14159
29	0.03065666	3.14159	0.01331787	3.14159
30	0.03112854	3.14159	0.00823502	3.14159
31	0.04612814	3.14159	0.02989864	3.14159
32	0.03615336	3.14159	0.00071217	3.14159
33	0.06219012	3.14159	0.00729552	3.14159
34	0.07614501	3.14159	0.05998277	3.14159

FORTRAN STOP

Appendix 1.4

```
C***************************************************************************
C                                                                          *
C       PROGRAM TEA                                                        *
C       LINEAR FREQUENCY DOMAIN FINITE ELEMENT MODEL FOR TIDAL CIRCULATION *
C                                                                          *
C       COPYRIGHT :                                                        *
C       DEPARTMENT OF CIVIL ENGINEERING                                    *
C       MASSACHUSETTS INSTITUTE OF TECHNOLOGY                              *
C                                                                          *
C       FORMULATION : ELEVATION/VELOCITY                                   *
C       EFFECTS : TIDAL MOTION, WIND STRESS, LINEARIZED BOTTOM FRICTION,   *
C                 AND CORIOLIS                                             *
C       SOLUTION : COMPACT DIRECT                                          *
C       SOURCE CODE : BAYOUF3V9                                            *
C                                                                          *
C***************************************************************************

        COMMON /SOL1/ SYSM2C,P,ELEV
        DIMENSION ICON(600,3),M(600),NN(450),IFLSEG(600,2),ISVHT(450)
        REAL*8  XORD(450),YORD(450),H(450),ELFRIC(600),SVHT(450,2),
     &  SYSNULC(450),SYSDC(450,150),A(3),B(3),HS(3),SVFLP(450,4),
     &  ISVFLP(450)
        COMPLEX*16 PW(900),ELEVPR(450),FLOW(2),PN(450),SYSM1C(450,150),
     &  SYSM2C(450,150),PT(900),SYSKLC(900,3),P(450),ELEV(450),U(900)
        COMPLEX*16 QX,QY,TEMWINXEL,TEMWINYEL,T3,T8,ELEV0,
     &  TEMWIN,TEMWINX,TEMWINY,TEMP1C,TEMP2C,ZERCOM,UNICOM
        REAL*8 AREA,DEPTH,DX1,DX2,DX3,
     &  DY1,DY2,DY3,ELIMAG,ELMOD,ELREAL,G,OMEGA,PHASE,PI,
     &  TEMP,TXORD,TYORD,X1,X2,X3,XIMAG,XMOD,XREAL,Y1,Y2,
     &  Y3,HTPR,LEN,XDIF,YDIF,NX,NY,QXR,QXI,QYR,QYI,ZERO,UNITY,
     &  DEP,WIND10,WINPHAS,WINDIR,WINDRAG,T1,T2,T4,T5,T6,T7,CORFAC
        CHARACTER*10 ALPHID,BCV
        MXANPD=25
        MXEL=600
        MXNP=450
        WRITE(6,6010)
 6010   FORMAT(/,' PROGRAM TEA ',/,
     &  ' DEPT OF CIVIL ENGINEERING,   M. I. T.',/,
     &  ' 2-D LINEAR FINITE ELEMENT FREQUENCY DOMAIN ANALYSIS ',
     &  'OF TIDAL WAVES FOR  SMALL SCALE GEOMETRY.',/)
        PI=3.14159265
        ZERO=0.0
        UNITY=1.0
        ZERCOM=DCMPLX(ZERO,ZERO)
        UNICOM=DCMPLX(ZERO,UNITY)
C
C...... READ THE INPUT VARIABLES
C
        WRITE(6,6020)
 6020   FORMAT(' ENTER 10 DIGIT F.E. GRID GEOMETRY IDENTIFICATION',/)
        READ(5,57) ALPHID
   57   FORMAT(A10)
        WRITE(6,6030)
 6030   FORMAT(' ENTER 10 DIGIT I.D. CODE FOR B.C. VERSION',/)
        READ(5,57) BCV
        WRITE(8,7010) ALPHID,BCV
 7010   FORMAT(1X,A10)
        WRITE(6,6040)
 6040   FORMAT(' INPUT NUMBER OF ELEMENTS AND NUMBER OF NODE POINTS',/)
        READ(5,*) NMEL,NMNP
        WRITE(8,7701) NMEL,NMNP
 7701   FORMAT(1X,I3,1H,,I3)
        IF(NMEL.GT.MXEL) GOTO 115
        IF(NMNP.GT.MXNP) GOTO 117
        GOTO 119
  115   WRITE(6,6050) MXEL
```

```
6050   FORMAT(/,' NO. OF ELEMENTS SPECIFIED EXCEEDS ARRAY DIMENSIONS ',
     & 'WHICH ALLOWS FOR ONLY ',I5,' ELEMENTS',/,' PROGRAM EXECUTION '
     & 'IS STOPPED; MODIFY PROGRAM DIMENSIONING')
       GOTO 1295
 117   WRITE(6,6055) MXNP
6055   FORMAT(/,' NO. OF NODES SPECIFIED EXCEEDS ARRAY DIMENSIONING',
     &  ' WHICH ALLOWS FOR ONLY ',I5,' NODES',/,' PROGRAM EXECUTION '
     &  'IS STOPPED; MODIFY PROGRAM DIMENSIONING')
       GOTO 1295
 119   CONTINUE
       WRITE(6,6060)
6060   FORMAT(' INPUT THE NODE NO.,X-COORD.,Y-COORD., AND NODAL',
     &  ' M.S.L. DEPTH, ONE NODE/LINE',/)
       DO 130 I=1,NMNP
          READ(5,*) NN(I),TXORD,TYORD,DEP
          L1=NN(I)
          XORD(L1)=TXORD
          YORD(L1)=TYORD
          H(L1)=DEP
          WRITE(8,7702) L1,TXORD,TYORD
7702      FORMAT(1X,I4,1H,,F10.2,1H,,F10.2)
 130   CONTINUE
       WRITE(6,6070)
6070   FORMAT(' INPUT ELEMENT NUMBER, THREE NODE NUMBERS FOR THE ELEMENT'
     &  ,'AND THE ELEMENT LINEAR FRICTION FACTOR',/)
       DO 145 I=1,NMEL
       READ(5,*) N,(ICON(N,J),J=1,3),ELFRIC(N)
       WRITE(8,7703) N,(ICON(N,J),J=1,3)
7703   FORMAT(1X,3(I4,1H,),I4)
       M(I)=N
 145   CONTINUE
C
C...... CHECK FOR EXCESSIVE BANDWIDTH
C
       IFLAG=0
       MXNPD=0
       DO 160 N=1,NMEL
       DO 160 I=1,2
       I2=I+1
       DO 155 J=I2,3
       MM=IABS(ICON(N,J)-ICON(N,I))
       IF(MM.GT.MXNPD) MXNPD=MM
       NEXCEED=MXNPD-MXANPD
       IF(MM.GT.MXANPD) WRITE(6,6080) MXANPD,NEXCEED,N
6080   FORMAT(' MAX. ALLOWABLE NODAL POINT DIFFERENCE OF ',I3,' IS ',
     &  'EXCEEDED BY',I4,' AT ELEMENT ',I4,/,' ARRAY DIMENSIONING',
     &  'MUST BE MODIFIED : PROGRAM EXECUTION IS TERMINATED')
       IF(MM.GT.MXANPD) IFLAG=1
 155   CONTINUE
 160   CONTINUE
       WRITE(6,6090) MXNPD
6090   FORMAT(' MAX. NODAL POINT DIFFERENCE =',I6,//)
       IF(IFLAG.EQ.1) GO TO 1295
C
C.......IBW=BANDWIDTH OF FULL MATRIX WITH SINGLE DEGREE OF FREEDOM PER NODE
C
       IBW=2*MXNPD+1
       IBW2=2*IBW
 180   FORMAT(' INPUT GRAVITATIONAL CONSTANT IN  CONSISTENT UNITS',/)
       READ(5,*) G
       WRITE(6,6100)
6100   FORMAT(' INPUT FREQUENCY IN RADIANS/SEC.',/)
       READ(5,*) OMEGA
       WRITE(8,7705) OMEGA
7705   FORMAT(1X,F20.10)
       WRITE(6,6110)
```

```
 6110   FORMAT(' INPUT CORIOLIS PARAMETER',/)
        READ(5,*) CORFAC
        WRITE(6,6120)
 6120   FORMAT(' INPUT AMPLITUDE OF WIND SPEED, WIND PHASE SHIFT, ',
     &   'ANGLE OF WIND DIRECTION  (IN DEGREES) ',
     &   ' AND WIND DRAG COEFFICIENT',/)
        READ(5,*) WIND10,WINPHAS,WINDIR,WINDRAG
        WINDIR=(PI/180.0)*WINDIR
        TEMWIN=0.0012*WINDRAG*WIND10**2*(COS(WINPHAS)+UNICOM*
     &   SIN(WINPHAS))
        TEMWINY=TEMWIN*SIN(WINDIR)
        TEMWINX=TEMWIN*COS(WINDIR)
        WRITE(6,6130)
 6130   FORMAT(' INPUT HOW MANY BOUNDARY SEGMENTS HAVE NON-ZERO NORMAL',
     &   ' FLUX PRESCRIBED')
        READ(5,*) NBSF
C
C...... INITIALIZE THE LOAD VECTOR FOR THE CONTINUITY EQUATION
C
        DO 192 I=1,NMNP
  192   PN(I)=ZERCOM
C
C...... LOAD VECTOR LOADING SECTION FOR NON-ZERO PRESCRIBED NORMAL FLUXES
C
        IF(NBSF.EQ.0) GOTO 226
C
C...... READ IN REQUIRED INFO FOR SEGMENTS WITH NON-ZERO NORMAL FLUX PRESCRIBED
C
        DO 196 I=1,NBSF
           WRITE(6,6140) I
 6140      FORMAT(' INPUT END NODES OF BOUNDARY SEGMENT',I5,' WITH ',
     &        'CORRECT ORIENTATION')
           READ(5,*) IFLSEG(I,1),IFLSEG(I,2)
  196   CONTINUE
        WRITE(6,6145)
 6145   FORMAT(' INPUT TOTAL NO. OF DIFFERENT NODES INCLUDED IN NON-',
     &   'ZERO NORMAL FLUX SEGMENTS')
        READ(5,*) NBNF
        DO 200 I=1,NBNF
           WRITE(6,6148)
 6148      FORMAT(' INPUT NODE NO.,MODULUS AND PHASE SHIFT (IN RAD) ',
     &        'FOR X AND Y DIRECTIONS ')
           READ(5,*) NODE,(SVFLP(NODE,J),J=1,4)
           ISVFLP(I)=NODE
  200   CONTINUE
        DO 225 I=1,NBSF
           M1=IFLSEG(I,1)
           M2=IFLSEG(I,2)
           XDIF=XORD(M2)-XORD(M1)
           YDIF=YORD(M2)-YORD(M1)
           LEN=(XDIF**2+YDIF**2)**0.5
           NX=-YDIF/LEN
           NY= XDIF/LEN
           DO 210 K=1,2
              NODE=IFLSEG(I,K)
              QXR=SVFLP(NODE,1)*COS(SVFLP(NODE,2))
              QXI=SVFLP(NODE,1)*SIN(SVFLP(NODE,2))
              QYR=SVFLP(NODE,3)*COS(SVFLP(NODE,4))
              QYI=SVFLP(NODE,3)*SIN(SVFLP(NODE,4))
              QX=DCMPLX(QXR,QXI)
              QY=DCMPLX(QYR,QYI)
              FLOW(K)=QX*NX+QY*NY
  210      CONTINUE
C
C.........LOAD THE NON-ZERO FLUX SEGMENT CONTRIBUTIONS INTO GLOBAL LOAD VECTOR
C
```

```
         TEMP=LEN/6.0
         PN(M1)=PN(M1)+TEMP*(2*FLOW(1)+FLOW(2))
         PN(M2)=PN(M2)+TEMP*(FLOW(1)+2*FLOW(2))
  225   CONTINUE
  226   CONTINUE
C
C.......READ IN ELEVATION LOADING INFORMATION
C
        ELEVO=ZERCOM
        WRITE(6,6200)
 6200   FORMAT(//,' INPUT NO. OF NODES WHERE ELEVATION IS PRESCRIBED')
        READ(5,*) NBL
        IF(NBL.EQ.0) GOTO 240
        DO 230 I=1,NBL
        WRITE(6,6210)
 6210   FORMAT(' INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND',
     &  ' PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE')
        READ(5,*) NODE,HTPR,PHASE
        ISVHT(I)=NODE
        SVHT(I,1)=HTPR
        SVHT(I,2)=PHASE
        XREAL=HTPR*COS(PHASE)
        XIMAG=HTPR*SIN(PHASE)
        ELEVPR(I)=DCMPLX(XREAL,XIMAG)
        ELEVO=ELEVO+ELEVPR(I)
  230   CONTINUE
        ELEVO=ELEVO/NBL
  240   CONTINUE
        DO 250 I=1,NBL
  250   ELEVPR(I)=ELEVPR(I)-ELEVO
C
C.....  PRINT INPUT VALUES
C
        WRITE(6,6300)
 6300   FORMAT(//,130('-'),/)
        WRITE(6,6010)
        WRITE(6,6310) ALPHID,BCV
 6310   FORMAT(//,1X,'F.E. GRID GEOMETRY IDENTIFICATION : ',A10,
     &  /,1X,'BOUNDARY CONDITION IDENTIFICATION  : ',A10,//)
        WRITE(6,6320) OMEGA
 6320   FORMAT(1X,'FREQUENCY = ',F10.8)
        WRITE(6,6322) CORFAC
 6322   FORMAT(1X,'CORIOLIS PARAMETER = ',F10.8,//)
        WRITE(6,6330) WIND10,WINPHAS,WINDIR,WINDRAG
 6330   FORMAT(//,' WIND SPEED AT 10 METERS ABOVE SURFACE = ',F10.4,
     &  /,' WIND PHASE SHIFT = ',F10.5,' RADIANS',
     &  /,' WIND DIRECTION = ',F10.5,' DEGREES',
     &  /,' WIND DRAG COEFFICIENT = ',F12.8,//)
        WRITE(6,6340) NMNP,NMEL
 6340   FORMAT(' NO. OF NODE POINTS =',I3,/,' NO. OF ELEMENTS =',I3,/)
        WRITE(6,6090) MXNPD
        WRITE(6,6350)
 6350   FORMAT(/,1X,37('*'),10X,'NODAL COORDINATES AND DEPTHS',10X,
     &  35('*'))
        WRITE(6,6352)
 6352   FORMAT(/,38X,'NODE',9X,'X-COORD.',8X,'Y-COORD.',9X,'DEPTH',/)
        DO 270 I=1,NMNP
        L1=NN(I)
        WRITE(6,6355) L1,XORD(L1),YORD(L1),H(L1)
 6355   FORMAT(36X,I5,6X,F10.2,6X,F10.2,6X,F10.2)
  270   CONTINUE
        WRITE(6,6360)
 6360   FORMAT(6(/),1X,37('*'),10X,'ELEMENT ARRAY',10X,40('*'),
     &  //,35X,'ELEMENT',8X,'I',5X,'J',5X,'K',12X,'ELEMENT',
     &  ' FRICTION FACTOR',/)
        DO 287 N=1,NMEL
```

```
      I=M(N)
      WRITE(6,6365) I,(ICON(I,J),J=1,3),ELFRIC(I)
 6365 FORMAT(37X,I3,8X,I3,3X,I3,3X,I3,13X,F13.7)
  287 CONTINUE
      WRITE(6,6400)
 6400 FORMAT(6(/),1X,30('*'),10X,'BOUNDARY CONDITIONS',10X,30('*'))
      WRITE(6,6410) NBSF
 6410 FORMAT(///,1X,'NO. OF BOUNDARY SEGMENTS WHICH HAVE NON-ZERO ',
     & 'NORMAL FLUX PRESCRIBED =',I4)
      IF(NBSF.EQ.0) GOTO 321
      WRITE(6,6415)
 6415 FORMAT(//,1X,'SEGMENTS WITH NON-ZERO NORMAL FLUX PRESCRIBED ',
     & 'ARE:',/,4X,'SEGMENT NO.',5X,'BEGINING NODE NO.',5X,
     & 'END NODE NO.',/)
      DO 305 I=1,NBSF
  305 WRITE(6,6420) I,(IFLSEG(I,J),J=1,2)
 6420 FORMAT(7X,I3,2(16X,I4))
      WRITE(6,6430)
 6430 FORMAT(//,1X,'PRESCRIBED FLOW VALUES FOR NODES ON',
     & ' SEGMENTS ARE:')
      WRITE(6,6435)
 6435 FORMAT(//,6X,'NODE NO.',9X,'X-DIRECTION',19X,'Y-DIRECTION',15X,
     & /,20X,2(' FLUX',6X,'PHASE',14X),'DEPTH',/)
      DO 316 I=1,NBNF
         NODE=ISVFLP(I)
         WRITE(6,6436) NODE,(SVFLP(NODE,J),J=1,4)
 6436    FORMAT(6X,I4,5X,2(F11.4,2X,F9.5,8X))
  316 CONTINUE
  321 CONTINUE
      WRITE(6,6440) NBL
 6440 FORMAT(5(/),' NO. OF NODES WHERE ELEVATION IS PRESCRIBED =',I3)
      IF(NBL.EQ.0) GOTO 2741
      WRITE(6,6442)
 6442 FORMAT('0',25X,'NODE',10X,'PRESCRIBED ELEVATION',10X,
     & 'PHASE',/)
      DO 2740 I=1,NBL
      WRITE(6,6445) ISVHT(I),SVHT(I,1),SVHT(I,2)
 6445 FORMAT(26X,I3,8X,F15.4,11X,F13.5)
 2740 CONTINUE
 2741 CONTINUE
C
C.....FORM THE ELEMENT MATRICES IN GLOBAL COORDINATE SYSTEM
C
      NT2=NMNP*2
      ILCB=(IBW-1)/2.0
      ICB=ILCB+1
      DO 325 I=1,NT2
      DO 325 J=1,3
  325 SYSKLC(I,J)=ZERCOM
      DO 326 I=1,NMNP
  326 SYSNULC(I)=0.0
      DO 327 I=1,NMNP
      DO 327 J=1,IBW2
  327 SYSDC(I,J)=0.0
      DO 328 I=1,NT2
  328 PW(I)=ZERCOM
      DO 500 N=1,NMEL
         NR1=ICON(N,1)
         NR2=ICON(N,2)
         NR3=ICON(N,3)
         X1=XORD(NR1)
         X2=XORD(NR2)
         X3=XORD(NR3)
         Y1=YORD(NR1)
         Y2=YORD(NR2)
         Y3=YORD(NR3)
```

```
          A(1)=X3-X2
          A(2)=X1-X3
          A(3)=X2-X1
          B(1)=Y2-Y3
          B(2)=Y3-Y1
          B(3)=Y1-Y2
          AREA=0.5*(B(1)*A(2)-B(2)*A(1))
          IF(AREA.LE.0.0) WRITE(6,6500) N
 6500     FORMAT(' AREA OF ELEMENT',I2,' IS LESS THAN ZERO,',
     &     ' PROGRAM EXECUTION IS TERMINATED')
          IF(AREA.LE.0.0) STOP
          HS(1)=2*H(NR1)+H(NR2)+H(NR3)
          HS(2)=H(NR1)+2*H(NR2)+H(NR3)
          HS(3)=H(NR1)+H(NR2)+2*H(NR3)
C
C...... LOAD ELEMENT CONTRIBUTION INTO GLOBAL MATRIX SYSNULC (IN LUMPED-
C....... COMPACT STORAGE MODE)
C
          T1=AREA/3.0
          SYSNULC(NR1)=SYSNULC(NR1)+T1
          SYSNULC(NR2)=SYSNULC(NR2)+T1
          SYSNULC(NR3)=SYSNULC(NR3)+T1
C
C...... GENERATE GLOBAL LUMPED-COMPACT STIFFNESS MATRIX FOR MOMENTUM EQUATION,
C....... INCLUDES COMBINED MASS,FRICTION AND CORIOLIS MATRICES
C
          T2=AREA/12.0
          DO 460 I=1,3
             IR=ICON(N,I)
             IR2=2*IR
             IR1=IR2-1
             T3=T2*DCMPLX(4*ELFRIC(N),OMEGA*HS(I))
             T4=T2*CORFAC*HS(I)
             SYSKLC(IR1,2)=SYSKLC(IR1,2)+T3
             SYSKLC(IR1,3)=SYSKLC(IR1,3)-T4
             SYSKLC(IR2,1)=SYSKLC(IR2,1)+T4
             SYSKLC(IR2,2)=SYSKLC(IR2,2)+T3
  460     CONTINUE
C
C...... LOAD ELEMENT CONTRIBUTIONS INTO GLOBAL MATRIX SYSD (IN COMPACT
C....... NON-SYMMETRIC STORAGE MODE)
C
          T6=1.0/24.0
          DO 450 I=1,3
             IR=ICON(N,I)
             DO 440 J=1,3
                JNCOMP=ICON(N,J)
                JCOMP=JNCOMP+ICB-IR
                T7=T6*HS(I)
                SYSDC(IR,2*JCOMP-1)=SYSDC(IR,2*JCOMP-1)+B(I)*T7
                SYSDC(IR,2*JCOMP)=SYSDC(IR,2*JCOMP)+A(I)*T7
  440        CONTINUE
  450     CONTINUE
C
C...... ADD ELEVO COMPONENT TO PN LOADING VECTOR
C
          T8=T1*UNICOM*OMEGA*ELEVO
          DO 465 I=1,3
          IR=ICON(N,I)
  465     PN(IR)=PN(IR)+T8
C
C...... LOAD PW TO CONTAIN WIND STRESS LOADINGS
C
          TEMWINXEL=TEMWINX*T1
          TEMWINYEL=TEMWINY*T1
          DO 470 I=1,3
```

```
            IR=ICON(N,I)
            IR2=2*IR
            IR1=IR2-1
            PW(IR1)=PW(IR1)+TEMWINXEL
            PW(IR2)=PW(IR2)+TEMWINYEL
  470     CONTINUE
  500   CONTINUE
C
C...... INVERT THE MATRIX SYSKLC
C
        DO 910 IR=1,NMNP
          IR2=2*IR
          IR1=IR2-1
          TEMP1C=SYSKLC(IR1,2)*SYSKLC(IR2,2)-SYSKLC(IR2,1)
     &     *SYSKLC(IR1,3)
          TEMP2C=SYSKLC(IR1,2)
          SYSKLC(IR1,2)=SYSKLC(IR2,2)/TEMP1C
          SYSKLC(IR2,2)=TEMP2C/TEMP1C
          SYSKLC(IR2,1)=-SYSKLC(IR2,1)/TEMP1C
          SYSKLC(IR1,3)=-SYSKLC(IR1,3)/TEMP1C
  910   CONTINUE
C
C...... FORM THE PRODUCT PT=SYSKLC*PW
C
        DO 915 IR=1,NT2
          TEMP1C=ZERCOM
          DO 912 K=1,3
            KSUM=IR-2+K
            IF((KSUM.LE.0).OR.(KSUM.GT.NT2)) GOTO 912
            TEMP1C=TEMP1C+SYSKLC(IR,K)*PW(KSUM)
  912     CONTINUE
          IF(IR.GT.1) PT(IR-1)=TEMP2C
          TEMP2C=TEMP1C
  915   CONTINUE
        PT(NT2)=TEMP2C
C
C...... FORM THE PRODUCT P=SYSDC*PT-PN
C
        DO 920 IR=1,NMNP
          JSHIFT=2*(IR-ICB)
          TEMP1C=ZERCOM
          DO 918 JCOCL=1,IBW2
            JNCOCL=JCOCL+JSHIFT
            IF((JNCOCL.LE.0).OR.(JNCOCL.GT.NT2)) GOTO 918
            TEMP1C=TEMP1C+SYSDC(IR,JCOCL)*PT(JNCOCL)
  918     CONTINUE
          P(IR)=TEMP1C-PN(IR)
  920   CONTINUE
C
C...... FORM PRODUCT SYSM1C=SYSKLC*SYSDC(TRANSPOSED):SYSM1C IS STORED SIDEWAYS
C
        DO 930 IR=1,NMNP
          JSHIFT=2*(IR-ICB)
          DO 925 JCOCL=1,IBW2
            JNCOCL=JCOCL+JSHIFT
            SYSM1C(IR,JCOCL)=ZERCOM
            IF((JNCOCL.LE.0).OR.(JNCOCL.GT.NT2)) GOTO 925
            KSUM=JCOCL-2
            DO 924 KADD=1,3
              KSUM=KSUM+1
              IF((KSUM.LE.0).OR.(KSUM.GT.IBW2)) GOTO 924
              SYSM1C(IR,JCOCL)=SYSM1C(IR,JCOCL)+SYSKLC(JNCOCL,KADD)*
     &         SYSDC(IR,KSUM)
  924       CONTINUE
  925     CONTINUE
  930   CONTINUE
```

```
C
C...... FORM THE PRODUCT SYSM2C=1*OMEGA*SYSNULC+G*SYSDC*SYSM1C
C
        IBW3=IBW2-1
        ILCB3=IBW-1
        ICB3=ILCB3+1
        DO 935 IR=1,IBW
        DO 935 JCOM=1,IBW-IR
  935   SYSM2C(IR,JCOM)=ZERCOM
        DO 937 IR=NMNP+2-IBW,NMNP
        DO 937 JCOM=NMNP+2+IBW,IBW3
  937   SYSM2C(IR,JCOM)=ZERCOM
        DO 950 IR1=1,NMNP
           IRBEG=IR1-IBW+1
           IF(IRBEG.LE.0) IRBEG=1
           IREND=IR1+IBW-1
           IF(IREND.GT.NMNP) IREND=NMNP
           DO 945 IR2=IRBEG,IREND
              KSHIFT=2*(IR1-IR2)
              KBEG=1
              IF(KSHIFT.LT.0) KBEG=1-KSHIFT
              KEND=2*IBW-KSHIFT
              IF(KSHIFT.LT.0) KEND=IBW2
              TEMP1C=ZERCOM
              DO 943 K1=KBEG,KEND
                 K2=K1+KSHIFT
                 TEMP1C=TEMP1C+SYSDC(IR1,K1)*SYSM1C(IR2,K2)
  943         CONTINUE
              JCOMP=IR2-IR1+IBW
              SYSM2C(IR1,JCOMP)=G*TEMP1C
              IF(JCOMP.NE.ICB3) GOTO 945
              SYSM2C(IR1,JCOMP)=SYSM2C(IR1,JCOMP)+UNICOM*OMEGA
     &         *SYSNULC(IR1)
  945      CONTINUE
  950   CONTINUE
C
C...... INCLUDE PRESCRIBED ELEVATIONS IN FINAL SYSTEM OF EQUATIONS FOR
C....... CALCULATING ELEVATIONS,  SYSM2C*ELEV=P
C
        IF(NBL.EQ.0) GOTO 990
        DO 980 I=1,NBL
        NODE=ISVHT(I)
C
C...... ZERO OUT THE ROW FOR THE PRESCRIBED NODE
C
        DO 960 JCOM=1,IBW3
  960   SYSM2C(NODE,JCOM)=ZERCOM
C
C...... SET DIAGONAL EQUAL TO 1
C
        SYSM2C(NODE,ICB3)=DCMPLX(UNITY,ZERO)
C
C...... SUBSTITUTE PRESCRIBED ELEVATION INTO P
C
        P(NODE)=ELEVPR(I)
  980   CONTINUE
  990   CONTINUE
C
C...... SOLVE THE SYSTEM OF EQUATIONS SYSM2C*ELEV=P
C
        CALL EFICSOL(NMNP,IBW3)
C
C...... CALCULATE NODAL VELOCITIES VECTOR U FROM ELEV
C
        DO 1070 I=1,NT2
 1070   U(I)=ZERCOM
```

```
      DO 1100 IRCOM=1,NMNP
        KSHIFT=2*(IRCOM-1)-2*ILCB
        DO 1080 JCOM=1,IBW2
          IRNCOM=JCOM+KSHIFT
          IF((IRNCOM.LE.0).OR.(IRNCOM.GT.NT2)) GOTO 1080
          U(IRNCOM)=U(IRNCOM)+G*SYSM1C(IRCOM,JCOM)*ELEV(IRCOM)
1080    CONTINUE
1100  CONTINUE
      DO 1102 IR=1,NT2
1102  U(IR)=PT(IR)-U(IR)
      WRITE(6,6600)
6600  FORMAT(///,1X,30('*'),10X,'RESULTS OF COMPUTATIONS',10X,30('*'))
C
C...... CALCULATE AND PRINT MODULUS AND PHASE FOR ELEVATION
C
      WRITE(6,6610)
6610  FORMAT(////,1X,31('-'),8X,'NODAL ELEVATIONS',7X,31('-'),
     &  //,28X,'   NODE   ',14X,'MODULUS  ',18X,'PHASE  ',/)
C
C...... ADD BACK IN ELEVO TO TOTAL ELEVATION
C
      DO 1246 I=1,NMNP
1246  ELEV(I)=ELEV(I)+ELEVO
      DO 1250 I=1,NMNP
        ELREAL=DREAL(ELEV(I))
        ELIMAG=DIMAG(ELEV(I))
        ELMOD=CDABS(ELEV(I))
        IF((ELREAL.EQ.0.0).AND.(ELIMAG.EQ.0.0)) GOTO 1235
        PHASE=ATAN2(ELIMAG,ELREAL)
        GOTO 1240
1235    PHASE=0.0
1240    CONTINUE
        WRITE(6,6620) I,ELMOD,PHASE
6620    FORMAT(30X,I3,14X,F12.8,14X,F9.5)
        WRITE(8,7706) I,ELMOD,PHASE
7706    FORMAT(1X,I4,1H,,F20.10,1H,,F15.10)
1250  CONTINUE
C
C...... CALCULATE AND PRINT THE MODULUS AND PHASE FOR VELOCITY
C
      WRITE(6,6630)
6630  FORMAT(////,1X,30('-'),10X,'NODAL VELOCITIES',10X,30('-'),
     &  //,31X,'X-DIRECTION',28X,'Y-DIRECTION',/,
     &  8X,'NODE',12X,'MODULUS',11X,'PHASE',16X,
     &  'MODULUS',12X,'PHASE',/)
      DO 1290 I=1,NMNP
      L1=NN(I)*2-1
      DX1=CDABS(U(L1))
      DX2=DIMAG(U(L1))
      DX3=DREAL(U(L1))
      IF(DX2.EQ.0.0.AND.DX3.EQ.0.0) GO TO 1276
      DX2=ATAN2(DX2,DX3)
1280  L1=L1+1
      DY1=CDABS(U(L1))
      DY2=DIMAG(U(L1))
      DY3=DREAL(U(L1))
      IF(DY2.EQ.0.0.AND.DY3.EQ.0.0) GO TO 1278
      DY2=ATAN2(DY2,DY3)
      GO TO 1282
1276  DX2=0.0
      GO TO 1280
1278  DY2=0.0
1282  WRITE(6,6650) NN(I),DX1,DX2,DY1,DY2
6650  FORMAT(8X,I3,10X,F12.8,6X,F9.5,12X,F12.8,7X,F9.5)
      WRITE(8,7709) NN(I),DX1,DX2,DY1,DY2
7709  FORMAT(1X,I4,1H,,F20.10,1H,,F15.10,1H,,F20.10,1H,,F15.10)
```

```
 1290   CONTINUE
 1295   STOP
        END


        SUBROUTINE EFICSOL(NDM,IBW)
        COMMON /SOL1/ B,C,X
        COMPLEX*16 B(450,150),C(450),X(450)
        COMPLEX*16 TEMSOL1,TEMSOL2
        REAL*8 ZERO,UNITY
        ZERO=0.0
        UNITY=1.0
        ILCB=(IBW-1)/2
        ICB=ILCB+1
        IRCB=ILCB+2
C
C...... PERFORM GAUSS ELIMINATION ON COMPACT MATRIX AND LOAD VECTOR
C
        DO 100 IRW=1,NDM
           TEMSOL1=B(IRW,ICB)
           C(IRW)=C(IRW)/TEMSOL1
           B(IRW,ICB)=DCMPLX(UNITY,ZERO)
           DO 10 JNMR=IRCB,IBW
              B(IRW,JNMR)=B(IRW,JNMR)/TEMSOL1
   10      CONTINUE
           IRWB=IRW+1
           IRWE=IRW+ILCB
           DO 90 IRWZ=IRWB,IRWE
              IRMVE=ICB-(IRWZ-IRW)
              TEMSOL2=B(IRWZ,IRMVE)
              B(IRWZ,IRMVE)=DCMPLX(ZERO,ZERO)
              IROCH=IRMVE+1
              DO 40 JCOZ=IROCH,IBW
                 B(IRWZ,JCOZ)=B(IRWZ,JCOZ)-TEMSOL2*(
     &             B(IRW,JCOZ+(IRWZ-IRW)))
   40         CONTINUE
              C(IRWZ)=C(IRWZ)-TEMSOL2*C(IRW)
   90      CONTINUE
  100   CONTINUE
C
C...... SOLVE FOR UNKNOWNS USING BACK-SUBSTITUTION
C
        ISV=NDM
  200   CONTINUE
        X(ISV)=C(ISV)
        DO 300 JSV=1,ILCB
           KSV=ISV+JSV
           IF(KSV.GT.NDM) GOTO 301
           X(ISV)=X(ISV)-B(ISV,ICB+JSV)*X(KSV)
  300   CONTINUE
  301   CONTINUE
        ISV=ISV-1
        IF(ISV.GE.1) GOTO 200
        RETURN
        END
```

Appendix 2: Program RENUMB

2.1. inputs/outputs

2.2 listing

Appendix 2.1

Program RENUMB renumbers grid node numbers in an efficient manner in order to optimize the maximum nodal point difference between connecting nodes, allowing program TEA to be run more efficiently. RENUMB should be run after a grid has been set up or after some grid modification has been done.

Input requirements for RENUMB are exactly the same as for program TEA. Hence a complete input file set up for TEA can be run with RENUMB. The un-renumbered grid input file is read in on TAPE 5. The renumbered output file will be output onto TAPE 6. We note that only node numbers change when running RENUMB and that element numbers remain the same. Finally RENUMB outputs renumbering information on TAPE 8. The example shown in Appendix 1.3.3 has been run with RENUMB and is listed here.

```
EXBAY2
BC-01
44, 34
  1,  11000.00,  15000.00,    25.0000
  2,   8500.00,  13500.00,    25.0000
  3,   9000.00,  15500.00,    25.0000
  4,  10500.00,  13300.00,    25.0000
  5,  12500.00,  14000.00,    25.0000
  6,   6000.00,  14500.00,    25.0000
  7,   6000.00,  12000.00,    25.0000
  8,   7800.00,  11000.00,    25.0000
  9,  10000.00,  11400.00,    25.0000
 10,  11800.00,  12300.00,    25.0000
 11,  13500.00,  13000.00,    25.0000
 12,   2500.00,  13000.00,    25.0000
 13,   3000.00,  10500.00,    25.0000
 14,   6500.00,   9000.00,    25.0000
 15,   8800.00,  10000.00,    25.0000
 16,  11500.00,  10000.00,    25.0000
 17,  12600.00,  11300.00,    25.0000
 18,  14000.00,  11600.00,    25.0000
 19,      0.00,  10000.00,    25.0000
 20,   4000.00,   7000.00,    25.0000
 21,   7500.00,   6000.00,    25.0000
 22,  10500.00,   7500.00,    25.0000
 23,  13500.00,   8000.00,    25.0000
 24,  14000.00,  10000.00,    25.0000
 25,      0.00,   6500.00,    25.0000
 26,   5000.00,   3000.00,    25.0000
 27,   1000.00,   4000.00,    25.0000
 28,   7500.00,   2500.00,    25.0000
 29,   9800.00,   3800.00,    25.0000
 30,  12000.00,   4500.00,    25.0000
 31,   7500.00,    800.00,    25.0000
 32,   9500.00,   1500.00,    25.0000
 33,  11500.00,   2300.00,    25.0000
 34,  13000.00,   3000.00,    25.0000
  1,  31,   32,   28,  0.00100000
  2,  32,   29,   28,  0.00100000
  3,  32,   33,   29,  0.00100000
  4,  33,   30,   29,  0.00100000
  5,  33,   34,   30,  0.00100000
  6,  26,   28,   21,  0.00100000
  7,  28,   29,   21,  0.00100000
  8,  29,   22,   21,  0.00100000
  9,  29,   30,   22,  0.00100000
 10,  26,   20,   27,  0.00100000
 11,  26,   21,   20,  0.00100000
 12,  20,   21,   14,  0.00100000
 13,  21,   22,   14,  0.00100000
 14,  27,   20,   25,  0.00100000
 15,  25,   20,   19,  0.00100000
 16,  19,   20,   13,  0.00100000
 17,  20,   14,   13,  0.00100000
 18,  13,   14,    7,  0.00100000
 19,  19,   13,   12,  0.00100000
 20,  13,    7,   12,  0.00100000
 21,  12,    7,    6,  0.00100000
 22,   6,    7,    2,  0.00100000
 23,   6,    2,    3,  0.00100000
 24,   2,    1,    3,  0.00100000
 25,   7,    8,    2,  0.00100000
 26,   8,    9,    2,  0.00100000
```

```
27,   2,   9,   4,  0.00100000
28,   2,   4,   1,  0.00100000
29,   9,  10,   4,  0.00100000
30,   4,  10,   5,  0.00100000
31,   4,   5,   1,  0.00100000
32,   8,  15,   9,  0.00100000
33,  15,  16,   9,  0.00100000
34,   9,  16,  10,  0.00100000
35,  10,  11,   5,  0.00100000
36,  10,  17,  11,  0.00100000
37,  16,  17,  10,  0.00100000
38,  15,  22,  16,  0.00100000
39,  22,  23,  16,  0.00100000
40,  17,  18,  11,  0.00100000
41,  17,  24,  18,  0.00100000
42,  16,  24,  17,  0.00100000
43,  16,  23,  24,  0.00100000
44,  22,  30,  23,  0.00100000
 9.81000
 0.0000000000
 0.0001000000
 0.0000000000,   0.0000000000,   0.0000000000,   0.0000000000
        3
        1,        5
        5,       11
       15,        8
        5
        1,  -0.9300000072,    0.0000000000,  -1.7999999523,   0.0000000000
        5,  -1.2100000381,    0.0000000000,  -1.6000000238,   0.0000000000
       11,  -1.5299999714,    0.0000000000,  -1.2699999809,   0.0000000000
        8,  -0.6899999976,    0.0000000000,  -0.7400000095,   0.0000000000
       15,  -0.6899999976,    0.0000000000,  -0.7400000095,   0.0000000000
        4
       31,   0.0000000000,    0.0000000000
       32,   0.0000000000,    0.0000000000
       33,   0.0000000000,    0.0000000000
       34,   0.0000000000,    0.0000000000
```

Appendix 2.2

```
ccccccccccccccccccccccccccccccc
cc    program for automatic renumbering
cc    it will give the following input variables
cc    for program tea :    alphid,bcv
cc                   nmel  nmnp
cc                   node  x-coord    y-coord
cc                   elt  icon1 icon2 icon3 depth
ccccccccccccccccccccccccccccccc
        dimension next(600),xord(600),yord(600),dep(600),icon(600,3),j
     &nt(600),nint(600),fric(600)
      character*10 alphid,bcv
      read(5,1000) alphid
      read(5,1000) bcv
 1000 format(a10)
      write(6,1001) alphid
      write(6,1001) bcv
 1001 format(1x,a10)
      read(5,*) nmel,nmnp
      write (6,2001) nmel,nmnp
 2001 format (1x,i3,1h,,i3)
      do 10 i=1,nmnp
      read(5,*) next(i),xcrd(i),yord(i),dep(i)
      nint(next(i))=i
   10 continue
      read(5,*) (n,(icon(n,j),j=1,3),fric(n),n=1,nmel)
cc
      call renum(nmnp,nmel,icon,jnt,idiff,ndiff)
cc
      maxbwh=ndiff+1
      do 13 ii=1,nmnp
      do 11 i=1,nmnp
      j=jnt(i)
      if (ii.ne.j) go to 11
      write(6,3001) next(ii),xord(i),yord(i),dep(i)
  11  continue
  13  continue
      do 12 i=1,nmel
      i1=icon(i,1)
      i2=icon(i,2)
      i3=icon(i,3)
      j1=jnt(i1)
      j2=jnt(i2)
      j3=jnt(i3)
      write (6,4001) i,j1,j2,j3,fric(i)
   12 continue
 3001 format(1x,i4,1h,,f10.2,1h,,f10.2,1h,f10.4)
 4001 format(1x,4(i4,1h,),f12.8) !
      read (5,*) g
      write(6,5001) g
 5001 format(1x,f10.5)
      read(5,*) omega
      write(6,5003) omega
 5003 format(1x,f15.10)
      read(5,*) corfac
      write(6,5003) corfac
      read(5,*) win1,win2,win3,win4
      write(6,5005) win1,win2,win3,win4
 5005 format(1x,3(f15.10,','),f15.10)
      read(5,*) nbsf
      write(6,5006) nbsf
 5006 format(1x,i10)          !
      if(nbsf.eq.0) goto 5100
      do 5040 i=1,nbsf
      read(5,*) n1,n2
      jn1=jnt(n1)
      jn2=jnt(n2)
```

```
      write(6,5030) jn1,jn2
 5030 format(1x,i10,',',i10)
 5040 continue
      read(5,*) nbnf
      write(6,5006) nbnf
      do 5070 i=1,nbnf
      read(5,*) node,sv1,sv2,sv3,sv4
      jnode=jnt(node)
      write(6,5065) jnode,sv1,sv2,sv3,sv4
 5065 format(1x,i10,4(',',f15.10))
 5070 continue
 5100 read(5,*) nbl
      write(6,5006) nbl
      if(nbl.eq.0) goto 9999
      do 5300 i=1,nbl
      read(5,*) node,htpr,phase
      jnode=jnt(node)
      write(6,5250) jnode,htpr,phase
 5250 format(1x,i10,2(',',f15.10))
 5300 continue
cc
 9999 stop
      end
ccccc
      subroutine renum(nodes,lments,icon,jnt,idiff,ndiff)
ccccc
      dimension icon(600,3),jnt(600),jt(2400),memjt(4800)
      dimension jmem(600),ix(5),jt1(5),jt2(5),jt3(5),jt4(5)
      dimension jx(5),jntx(5)
      write (8,19)
      do 100 i=1,lments
      i1=i
      i2=i+600
      i3= i+600*2
      i4=i+600*3
      jt(i1)=icon(i,1)
      jt(i2)=icon(i,2)
      jt(i3)=icon(i,3)
      jt(i4)=0
  100 continue
  19  format(1h0,'input data'/1h0,'no.',3x,'1.',3x,'2.',3x,'3.'/1h0)
      lx=lments/4+1
      do 150 i=1,lx
      lp=0
      do 160 ii=1,4
      ix(ii)=i+lp
      jt1(ii)= jt(i+lp)
      jt2(ii)= jt(i+lp+600)
      jt3(ii)= jt(i+lp+1200)
      jt4(ii)=jt(i+lp+1800)
      lp=lp+lx
  160 continue
      write (8,2) (ix(ii),jt1(ii),jt2(ii),jt3(ii),jt4(ii),ii=1,4)
  150 continue
  2   format(1h,4(5i5,3x))
ccc
      call setup(nodes,lments,jt,memjt,jmem,jnt,idiff,ndiff)
      write(8,20) idiff
   20 format(1h0,' maximum difference idiff=',i5)
      call optnum(nodes,lments,jt,memjt,jmem,jnt,idiff,ndiff)
      write(8,29)
  29  format(1h0,' results od nodes after renumbering')
      write(8,30)
  30  format(1h0,9x,'j',2x,'jnt(j)')
      nx=nodes/5+1
      do 200 j=1,nx
```

```
      np=0
      do 210 jj=1,5
      jx(jj)=j+np
      jntx(jj)=jnt(j+np)
      np=np+nx
  210 continue
      write(8,31) (jx(jj),jntx(jj),jj=1,5)
  200 continue
  31  format(1h,5(5x,2i5))
      write (8,32)
      do 300 j=1,lx
      lp=0
      do 310 jj=1,4
      ix(jj)=j+lp
      jt1(jj)=jt(j+lp)
      jt2(jj)=jt(j+lp+600)
      jt3(jj)=jt(j+lp+1200)
      lp=lp+lx
  310 continue
      write (8,33) (ix(jj),jnt(jt1(jj)),jnt(jt2(jj)),jnt(jt3(jj)),
     vjj=1,4)
  300 continue
  32  format(1h0,9x,'j',3x,'i1',3x,'i2',3x,'i3'/1h0)
  33  format(1h,4(5x,4i5))
      return
      end
cccc
      subroutine setup(nodes,lments,jt,memjt,jmem,jnt,idiff,ndiff)
cccc
      dimension jt(2400),memjt(4800),jmem(600),jnt(600)
      idiff=0
      do 10 j=1,nodes
   10 jmem(j)=0
      do 60 j=1,lments
      do 50 i=1,4
      jnti=jt(600*(i-1)+j)
      if (jnti.eq.0) go to 60
      jsub=(jnti-1)*8
      do 40 ii=1,4
      if(ii.eq.i) go to 40
      jjt=jt(600*(ii-1)+j)
      if (jjt.eq.0) go to 50
      mem1=jmem(jnti)
      if (mem1.eq.0) go to 30
      do 20 iii=1,mem1
      if( memjt(jsub+iii).eq.jjt) go to 40
  20  continue
  30  jmem(jnti)=jmem(jnti)+1
      memjt(jsub+jmem(jnti))=jjt
      if(iabs(jnti-jjt).gt.idiff) idiff=iabs(jnti-jjt)
  40  continue
  50  continue
  60  continue
      return
      end
ccccc
      subroutine optnum(nodes,lments,jt,memjt,jmem,jnt,idiff,ndiff)
ccccc
      dimension jt(2400),memjt(4800),jmem(600),jnt(600)
      dimension newjt(600),joint(600)
      njts=nodes
      minmax=idiff
      do 60 ik=1,nodes
      do 20 j=1,nodes
      joint(j)=0
 20   newjt(j)=0
```

```
      max=0
      i=1
      newjt(1)=ik
      joint(ik)=1
      k=1
      k4=jmem(newjt(i))
      if(k4.eq.0) go to 45
      jsub=(newjt(i)-1)*8
      do 40 jj=1,k4
      k5=memjt(jsub+jj)
      if(joint(k5).gt.0) go to 40
      k=k+1
      newjt(k)=k5
      joint(k5)=k
      ndiff=iabs(i-k)
      if(ndiff.ge.minmax) go to 60
      if(ndiff.gt.max) max=ndiff
40    continue
      if(k.eq.njts) go to 50
45    i=i+1
      go to 30
50    minmax=max
      do 55 j=1,nodes
55    jnt(j)=joint(j)
60    continue
      nmdiff=ndiff+1
      write(8,100) ndiff,nmdiff
100   format(1h0,'maximum difference after renumbering ndiff',i5,'new ma
     &xbwh',i5)
      return
      end
```

Appendix 3: Massachusetts Bay Example

Appendix 3.1

MASEXAC
B-01:F3V10
224, 140

1,	-21654.0000,	68040.0000,	5.0000
2,	-17685.0000,	70549.0000,	5.0000
3,	-19558.0000,	64643.0000,	10.0000
4,	-13462.0000,	76581.0000,	5.0000
5,	-10763.0000,	71215.0000,	34.0000
6,	-14224.0000,	66326.0000,	34.0000
7,	-14001.0000,	61786.0000,	30.0000
8,	-18955.0000,	59531.0000,	15.0000
9,	-24416.0000,	60233.0000,	5.0000
10,	-24829.0000,	65024.0000,	5.0000
11,	-3065.0000,	80677.0000,	10.0000
12,	-5568.0000,	74517.0000,	46.0000
13,	-7420.0000,	66834.0000,	40.0000
14,	-10160.0000,	62548.0000,	30.0000
15,	-11462.0000,	59055.0000,	25.0000
16,	-14288.0000,	57817.0000,	25.0000
17,	-17621.0000,	53023.0000,	5.0000
18,	-23876.0000,	55817.0000,	15.0000
19,	-1365.0000,	82804.0000,	15.0000
20,	1016.0000,	75184.0000,	62.0000
21,	-1334.0000,	66358.0000,	50.0000
22,	-1715.0000,	62167.0000,	60.0000
23,	-6255.0000,	60738.0000,	40.0000
24,	-8604.0000,	59055.0000,	25.0000
25,	-11239.0000,	56198.0000,	30.0000
26,	-13335.0000,	53721.0000,	15.0000
27,	-23304.0000,	51435.0000,	10.0000
28,	-14446.0000,	49371.0000,	10.0000
29,	3905.0000,	86265.0000,	20.0000
30,	5556.0000,	80137.0000,	55.0000
31,	10065.0000,	73660.0000,	65.0000
32,	5334.0000,	68898.0000,	75.0000
33,	2953.0000,	62421.0000,	76.0000
34,	1588.0000,	55817.0000,	60.0000
35,	-3016.0000,	57245.0000,	45.0000
36,	-5874.0000,	57023.0000,	35.0000
37,	-8541.0000,	56293.0000,	23.0000
38,	-9366.0000,	52737.0000,	20.0000
39,	-9906.0000,	48419.0000,	15.0000
40,	8033.0000,	90297.0000,	15.0000
41,	13018.0000,	81820.0000,	75.0000
42,	17621.0000,	73724.0000,	80.0000
43,	13335.0000,	67247.0000,	55.0000
44,	9610.0000,	61055.0000,	85.0000
45,	6255.0000,	55372.0000,	71.0000
46,	1683.0000,	50102.0000,	42.0000
47,	-2604.0000,	52292.0000,	30.0000
48,	-5884.0000,	52959.0000,	34.0000
49,	-6223.0000,	48768.0000,	25.0000
50,	-6191.0000,	44006.0000,	10.0000
51,	21971.0000,	66104.0000,	50.0000
52,	16955.0000,	59436.0000,	80.0000
53,	12922.0000,	53435.0000,	78.0000
54,	7874.0000,	47625.0000,	50.0000
55,	2477.0000,	42228.0000,	25.0000
56,	-1969.0000,	46228.0000,	25.0000
57,	-3302.0000,	38068.0000,	10.0000
58,	26384.0000,	58452.0000,	30.0000
59,	22606.0000,	51562.0000,	85.0000
60,	15875.0000,	45800.0000,	70.0000
61,	9938.0000,	41656.0000,	37.0000
62,	4191.0000,	36862.0000,	22.0000
63,	-254.0000,	32226.0000,	10.0000

64,	30480.0000,	51245.0000,	30.0000
[illegible]	26226.0000,	44037.0000,	50.0000
[illegible]	18352.0000,	40577.0000,	62.0000
[illegible]	12446.0000,	36195.0000,	50.0000
[illegible]	7049.0000,	31560.0000,	25.0000
[illegible]	3239.0000,	25654.0000,	10.0000
[illegible]	34544.0000,	44323.0000,	22.0000
[illegible]	30798.0000,	37052.0000,	21.0000
72,	24892.0000,	36639.0000,	50.0000
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74,	14224.0000,	29749.0000,	47 0000
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76,	5207.0000,	19653.0000,	5.0000
77,	37433.0000,	39465.0000,	32.0000
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80,	20066.0000,	26289.0000,	55.0000
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82,	10065.0000,	18225.0000,	30.0000
83,	6096.0000,	16764.0000,	10.0000
84,	1475.0000,	16462.0000,	5.0000
85,	40799.0000,	33814.0000,	55.0000
86,	36735.0000,	27464.0000,	30.0000
87,	33941.0000,	25527.0000,	55.0000
88,	26988.0000,	23463.0000,	53.0000
89,	21146.0000,	20574.0000,	48.0000
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94,	5969.0000,	13430.0000,	10.0000
95,	40545.0000,	28067.0000,	30.0000
96,	44577.0000,	27464.0000,	30.0000
97,	33150.0000,	19431.0000,	46.0000
98,	36576.0000,	22416.0000,	40.0000
99,	27877.0000,	16510.0000,	45.0000
100,	23146.0000,	14288.0000,	43.0000
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102,	14542.0000,	9081.0000,	25.0000
103,	10287.0000,	10097.0000,	5.0000
104,	7938.0000,	12033.0000,	5.0000
105,	39053.0000,	19558.0000,	38.0000
106,	35433.0000,	13875.0000,	35.0000
107,	29655.0000,	10573.0000,	35.0000
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109,	20003.0000,	7366.0000,	33.0000
110,	15399.0000,	3239.0000,	22.0000
111,	11557.0000,	4953.0000,	5.0000
112,	44037.0000,	23844.0000,	20.0000
113,	45942.0000,	19050.0000,	15.0000
114,	41815.0000,	14034.0000,	27.0000
115,	36703.0000,	8954.0000,	31.0000
116,	31845.0000,	4699.0000,	29.0000
117,	26321.0000,	381.0000,	20.0000
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121,	48000.0000,	12954.0000,	5.0000
122,	42736.0000,	8192.0000,	18.0000
123,	38132.0000,	3620.0000,	22.0000
124,	32925.0000,	-1016.0000,	22.0000
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126,	20479.0000,	-5017.0000,	20.0000
127,	16764.0000,	-3526.0000,	17.0000
128,	13399.0000,	-5080.0000,	5.0000
129,	11239.0000,	-2953.0000,	5.0000

130,	49086.0000,	6921.0000,	5.0000
131,	44154.0000,	2159.0000,	5.0000
132,	39592.0000,	-2032.0000,	10.0000
133,	33909.0000,	-5429.0000,	15.0000
134,	27432.0000,	-10160.0000,	5.0000
135,	21939.0000,	-9335.0000,	10.0000
136,	15939.0000,	-6985.0000,	5.0000
137,	49879.0000,	-762.0000,	5.0000
138,	44925.0000,	-5525.0000,	5.0000
139,	39656.0000,	-8446.0000,	5.0000
140,	33560.0000,	-10668.0000,	5.0000

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6,	19,	11,	12,	0.005000
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9,	4,	2,	5,	0.005000
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11,	2,	3,	6,	0.005000
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17,	20,	21,	32,	0.005000
18,	20,	12,	21,	0.005000
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35,	21,	13,	22,	0.005000
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40,	14,	7,	15,	0.005000
41,	7,	16,	15,	0.005000
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44,	8,	18,	17,	0.005000
45,	8,	9,	18,	0.005000
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47,	51,	43,	52,	0.005000
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49,	44,	53,	52,	0.005000
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60,	24,	15,	25,	0.005000
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63,	16,	17,	26,	0.005000
64,	51,	52,	58,	0.005000
65,	52,	59,	58,	0.005000
66,	52,	53,	59,	0.005000
67,	53,	45,	54,	0.005000
68,	45,	46,	54,	0.005000
69,	34,	46,	45,	0.005000
70,	34,	47,	46,	0.005000
71,	35,	47,	34,	0.005000
72,	35,	48,	47,	0.005000
73,	36,	48,	35,	0.005000
74,	37,	48,	36,	0.005000
75,	37,	38,	48,	0.005000
76,	25,	38,	37,	0.005000
77,	25,	26,	38,	0.005000
78,	26,	28,	38,	0.005000
79,	26,	17,	28,	0.005000
80,	58,	59,	64,	0.005000
81,	59,	65,	64,	0.005000
82,	59,	60,	65,	0.005000
83,	60,	66,	65,	0.005000
84,	53,	60,	59,	0.005000
85,	53,	54,	60,	0.005000
86,	60,	61,	66,	0.005000
87,	60,	54,	61,	0.005000
88,	54,	55,	61,	0.005000
89,	46,	55,	54,	0.005000
90,	46,	56,	55,	0.005000
91,	47,	56,	46,	0.005000
92,	47,	49,	56,	0.005000
93,	48,	49,	47,	0.005000
94,	48,	38,	49,	0.005000
95,	38,	39,	49,	0.005000
96,	38,	28,	39,	0.005000
97,	64,	65,	70,	0.005000
98,	65,	71,	70,	0.005000
99,	70,	71,	77,	0.005000
100,	65,	72,	71,	0.005000
101,	65,	66,	72,	0.005000
102,	66,	73,	72,	0.005000
103,	66,	67,	73,	0.005000
104,	61,	67,	66,	0.005000
105,	61,	62,	67,	0.005000
106,	55,	62,	61,	0.005000
107,	55,	57,	62,	0.005000
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111,	39,	50,	49,	0.005000
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113,	77,	71,	78,	0.005000
114,	71,	79,	78,	0.005000
115,	71,	72,	79,	0.005000
116,	72,	73,	79,	0.005000
117,	73,	80,	79,	0.005000
118,	73,	74,	80,	0.005000
119,	73,	67,	74,	0.005000
120,	67,	68,	74,	0.005000
121,	67,	62,	68,	0.005000

122,	62,	63,	68,	0.005000
123,	62,	57,	63,	0.005000
124,	85,	95,	96,	0.005000
125,	85,	86,	95,	0.005000
126,	85,	78,	86,	0.005000
127,	78,	87,	86,	0.005000
128,	78,	79,	87,	0.005000
129,	79,	88,	87,	0.005000
130,	79,	80,	88,	0.005000
131,	80,	89,	88,	0.005000
132,	80,	81,	89,	0.005000
133,	80,	74,	81,	0.005000
134,	74,	75,	81,	0.005000
135,	68,	75,	74,	0.005000
136,	68,	69,	75,	0.005000
137,	68,	63,	69,	0.005000
138,	87,	97,	98,	0.005000
139,	87,	88,	97,	0.005000
140,	88,	99,	97,	0.005000
141,	88,	89,	99,	0.005000
142,	89,	100,	99,	0.005000
143,	89,	90,	100,	0.005000
144,	81,	90,	89,	0.005000
145,	81,	82,	90,	0.005000
146,	81,	75,	82,	0.005000
147,	75,	76,	82,	0.005000
148,	75,	69,	76,	0.005000
149,	112,	105,	113,	0.005000
150,	98,	97,	105,	0.005000
151,	105,	114,	113,	0.005000
152,	105,	106,	114,	0.005000
153,	105,	97,	106,	0.005000
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159,	100,	101,	109,	0.005000
160,	100,	90,	101,	0.005000
161,	90,	91,	101,	0.005000
162,	90,	82,	91,	0.005000
163,	82,	92,	91,	0.005000
164,	82,	83,	92,	0.005000
165,	76,	83,	82,	0.005000
166,	76,	84,	83,	0.005000
167,	91,	102,	101,	0.005000
168,	83,	84,	93,	0.005000
169,	83,	93,	94,	0.005000
170,	83,	94,	92,	0.005000
171,	94,	104,	92,	0.005000
172,	113,	114,	121,	0.005000
173,	114,	122,	121,	0.005000
174,	114,	115,	122,	0.005000
175,	106,	115,	114,	0.005000
176,	106,	107,	115,	0.005000
177,	107,	116,	115,	0.005000
178,	107,	108,	116,	0.005000
179,	108,	117,	116,	0.005000
180,	108,	118,	117,	0.005000
181,	109,	118,	108,	0.005000
182,	109,	110,	118,	0.005000
183,	102,	110,	109,	0.005000
184,	102,	111,	110,	0.005000
185,	103,	111,	102,	0.005000
186,	91,	103,	102,	0.005000
187,	104,	103,	91,	0.005000

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188,   92,  104,   91,           0.005000
189,  121,  122,  130,           0.005000
190,  122,  131,  130,           0.005000
191,  122,  123,  131,           0.005000
192,  115,  123,  122,           0.005000
193,  115,  116,  123,           0.005000
194,  116,  124,  123,           0.005000
195,  116,  117,  124,           0.005000
196,  117,  125,  124,           0.005000
197,  117,  126,  125,           0.005000
198,  118,  126,  117,           0.005000
199,  118,  127,  126,           0.005000
200,  118,  119,  127,           0.005000
201,  110,  119,  118,           0.005000
202,  110,  120,  119,           0.005000
203,  111,  120,  110,           0.005000
204,  130,  131,  137,           0.005000
205,  131,  138,  137,           0.005000
206,  131,  132,  138,           0.005000
207,  123,  132,  131,           0.005000
208,  123,  124,  132,           0.005000
209,  124,  133,  132,           0.005000
210,  124,  125,  133,           0.005000
211,  125,  134,  133,           0.005000
212,  125,  135,  134,           0.005000
213,  125,  126,  135,           0.005000
214,  126,  136,  135,           0.005000
215,  127,  136,  126,           0.005000
216,  127,  128,  136,           0.005000
217,  119,  128,  127,           0.005000
218,  119,  129,  128,           0.005000
219,  119,  120,  129,           0.005000
220,  132,  139,  138,           0.005000
221,  133,  140,  139,           0.005000
222,  133,  134,  140,           0.005000
223,  132,  133,  139,           0.005000
224,  101,  102,  109,           0.005000
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        0.00013963
        0.00010000
     0.0000000,      0.0000000,      0.0000000,      0.0000000
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 41,      1.24460,      0.00000
 42,      1.23950,      0.00000
 51,      1.23460,      0.00000
 58,      1.22980,      0.00000
 64,      1.22520,      0.00000
 70,      1.22080,      0.00000
 77,      1.21770,      0.00000
 85,      1.21410,      0.00000
 96,      1.21000,      0.00000
```

Appendix 3.2

MASEXAC
B-02:F3V10
224, 140

1,	-21654.0000,	68040.0000,	5.0000
2,	-17685.0000,	70549.0000,	5.0000
3,	-19558.0000,	64643.0000,	10.0000
4,	-13462.0000,	76581.0000,	5.0000
5,	-10763.0000,	71215.0000,	34.0000
6,	-14224.0000,	65326.0000,	34.0000
7,	-14001.0000,	61786.0000,	30.0000
8,	-18955.0000,	59531.0000,	15.0000
9,	-24416.0000,	60833.0000,	5.0000
10,	-24829.0000,	65024.0000,	5.0000
11,	-8065.0000,	80677.0000,	10.0000
12,	-5588.0000,	74517.0000,	46.0000
13,	-7420.0000,	66834.0000,	40.0000
14,	-10160.0000,	62548.0000,	30.0000
15,	-11462.0000,	59055.0000,	25.0000
16,	-14288.0000,	57817.0000,	25.0000
17,	-17621.0000,	53023.0000,	5.0000
18,	-23876.0000,	55817.0000,	15.0000
19,	-1365.0000,	82804.0000,	15.0000
20,	1016.0000,	75184.0000,	62.0000
21,	-1334.0000,	68358.0000,	50.0000
22,	-1715.0000,	62167.0000,	60.0000
23,	-6255.0000,	60738.0000,	40.0000
24,	-8604.0000,	59055.0000,	25.0000
25,	-11239.0000,	56198.0000,	30.0000
26,	-13335.0000,	53721.0000,	15.0000
27,	-23304.0000,	51435.0000,	10.0000
28,	-14446.0000,	49371.0000,	10.0000
29,	3905.0000,	86265.0000,	20.0000
30,	5556.0000,	80137.0000,	55.0000
31,	10065.0000,	73660.0000,	65.0000
32,	5334.0000,	68898.0000,	75.0000
33,	2953.0000,	62421.0000,	76.0000
34,	1588.0000,	55817.0000,	60.0000
35,	-3016.0000,	57245.0000,	45.0000
36,	-5874.0000,	57023.0000,	35.0000
37,	-8541.0000,	56293.0000,	23.0000
38,	-9366.0000,	52737.0000,	20.0000
39,	-9906.0000,	48419.0000,	15.0000
40,	8033.0000,	90297.0000,	15.0000
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Appendix 3.3

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PROGRAM TEA
DEPT OF CIVIL ENGINEERING, M. I. T.
2-D LINEAR FINITE ELEMENT FREQUENCY DOMAIN ANALYSIS OF TIDAL WAVES FOR  SMALL SCALE GEOMETRY.

ENTER 10 DIGIT F.E. GRID GEOMETRY IDENTIFICATION

ENTER 10 DIGIT I.D. CODE FOR B.C. VERSION

INPUT NUMBER OF ELEMENTS AND NUMBER OF NODE POINTS

INPUT THE NODE NO.,X-COORD.,Y-COORD., AND NODAL M.S.L. DEPTH, ONE NODE/LINE

INPUT ELEMENT NUMBER, THREE NODE NUMBERS FOR THE ELEMENTAND THE ELEMENT LINEAR FRICTION FACTOR

MAX. NODAL POINT DIFFERENCE =    13

INPUT FREQUENCY IN RADIANS/SEC.

INPUT CORIOLIS PARAMETER

INPUT AMPLITUDE OF WIND SPEED, WIND PHASE SHIFT, ANGLE OF WIND DIRECTION  (IN DEGREES)  AND WIND DRAG COEFFICIENT

INPUT HOW MANY BOUNDARY SEGMENTS HAVE NON-ZERO NORMAL FLUX PRESCRIBED

INPUT NO. OF NODES WHERE ELEVATION IS PRESCRIBED
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
```

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PROGRAM TEA
DEPT OF CIVIL ENGINEERING, M. I. T.
2-D LINEAR FINITE ELEMENT FREQUENCY DOMAIN ANALYSIS OF TIDAL WAVES FOR  SMALL SCALE GEOMETRY.

F.E. GRID GEOMETRY IDENTIFICATION :  MASEXAC
BOUNDARY CONDITION IDENTIFICATION  :  B-01:F3V1

FREQUENCY = 0.00013963
CORIOLIS PARAMETER = 0.00010000

WIND SPEED AT 10 METERS ABOVE SURFACE =      0.0000
WIND PHASE SHIFT =     0.00000 RADIANS
WIND DIRECTION =    0.00000 DEGREES
WIND DRAG COEFFICIENT =    0.00000000

NO. OF NODE POINTS =140
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NO. OF ELEMENTS =224

MAX. NODAL POINT DIFFERENCE = 13

************************************ NODAL COORDINATES AND DEPTHS ****************************

NODE	X-COORD	Y-COORD.	DEPTH
1	-21654.00	68040.00	5.00
2	-17566.00	70349.00	5.00
3	-19566.00	64543.00	10.00
4	-13462.00	76681.00	5.00
5	-10763.00	71215.00	34.00
6	-14224.00	66326.00	34.00
7	-14001.00	61786.00	30.00
8	-18955.00	59531.00	15.00
9	-24416.00	60833.00	5.00
10	-24829.00	65024.00	5.00
11	-8065.00	80577.00	10.00
12	-5568.00	74517.00	45.00
13	-7420.00	68834.00	40.00
14	-10160.00	62543.00	30.00
15	-11462.00	59035.00	25.00
16	-14258.00	57817.00	25.00
17	-17621.00	53023.00	5.00
18	-23676.00	55817.00	15.00
19	-1365.00	82804.00	15.00
20	1016.00	75184.00	62.00
21	-1334.00	68358.00	50.00
22	-1716.00	62167.00	60.00
23	-6255.00	60738.00	40.00
24	-8604.00	59055.00	25.00
25	-11239.00	56198.00	30.00
26	-13335.00	53721.00	15.00
27	-23304.00	51435.00	10.00
28	-14445.00	49371.00	10.00
29	3905.00	85255.00	20.00
30	6556.00	80137.00	55.00
31	10065.00	73660.00	65.00
32	5334.00	68898.00	75.00
33	2963.00	62421.00	76.00
34	1588.00	65817.00	60.00
35	-3016.00	57245.00	45.00
36	-5674.00	57023.00	35.00
37	-8541.00	56293.00	23.00
38	-9366.00	52737.00	20.00
39	-9906.00	48419.00	15.00
40	8033.00	90297.00	15.00
41	13018.00	81820.00	75.00
42	17621.00	73724.00	80.00
43	13335.00	67247.00	55.00
44	9610.00	61055.00	65.00
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46	1663.00	50102.00	42.00
47	-2604.00	52292.00	30.00
48	-5894.00	52959.00	34.00
49	-6223.00	45768.00	25.00
50	-6191.00	44006.00	10.00
51	21971.00	65104.00	50.00
52	16955.00	59435.00	80.00
53	12922.00	53435.00	78.00
54	7874.00	47525.00	50.00
55	2477.00	42273.00	25.00
56	-1969.00	46226.00	25.00

57	-3302.00	38058.00	10.00
58	25284.00	55462.00	30.00
59	22506.00	51552.00	65.00
60	15875.00	45800.00	70.00
61	9938.00	41555.00	57.00
62	4131.00	30862.00	22.00
63	-254.00	32226.00	10.00
64	30480.00	51245.00	30.00
65	26225.00	44037.00	60.00
66	18352.00	40577.00	62.00
67	12446.00	35196.00	50.00
68	7049.00	31550.00	25.00
69	3239.00	25654.00	10.00
70	34544.00	44328.00	22.00
71	30795.00	37052.00	21.00
72	24932.00	36503.00	50.00
73	20267.00	33557.00	58.00
74	14224.00	29749.00	47.00
75	9303.00	24733.00	33.00
76	5207.00	19553.00	5.00
77	37433.00	39465.00	32.00
78	33719.00	31052.00	60.00
79	27496.00	23337.00	60.00
80	20056.00	25239.00	55.00
81	14732.00	22479.00	40.00
82	10065.00	18225.00	30.00
83	6035.00	15764.00	10.00
84	1475.00	16452.00	5.00
85	40799.00	33814.00	55.00
86	35735.00	27454.00	30.00
87	33941.00	25527.00	55.00
88	25968.00	23453.00	53.00
89	21146.00	20574.00	45.00
90	16129.00	16732.00	40.00
91	11970.00	13357.00	30.00
92	8255.00	14542.00	15.00
93	3250.00	13012.00	5.00
94	5959.00	13430.00	10.00
95	40545.00	28067.00	30.00
96	44577.00	27454.00	30.00
97	33150.00	19431.00	45.00
98	30576.00	22416.00	40.00
99	27877.00	15510.00	45.00
100	23146.00	14288.00	43.00
101	17612.00	12503.00	37.00
102	14542.00	9061.00	25.00
103	10287.00	10097.00	5.00
104	7932.00	12033.00	5.00
105	39053.00	19558.00	38.00
106	35433.00	13575.00	35.00
107	29655.00	10573.00	35.00
108	25718.00	6287.00	32.00
109	20003.00	7356.00	33.00
110	15399.00	3233.00	22.00
111	11557.00	4953.00	6.00
112	44037.00	23844.00	20.00
113	45942.00	19050.00	15.00
114	41815.00	14034.00	27.00
115	35703.00	8954.00	31.00
116	31845.00	4699.00	29.00
117	25321.00	381.00	20.00
118	20013.00	794.00	25.00
119	14478.00	-1048.00	15.00
120	10795.00	567.00	5.00
121	48000.00	12954.00	5.00
122	42735.00	8192.00	18.00

123	38130 00	3020 00	22.00
124	32925 00	-1016 00	22 00
125	27178 00	-4599 00	21 00
126	20479.00	-6017 00	20 00
127	15764 00	-3588.00	17.00
128	13399.00	-5050.00	5 00
129	11239 00	-2953 00	5 00
130	49086 00	6921 00	5 00
131	44164 00	2159 00	5 00
132	39592 00	-2032 00	10 00
133	33909 00	-6429 00	15 00
134	27432 00	-10100 00	5 00
135	21939 00	-9335 00	10 00
136	15939 00	-6335 00	5 00
137	49579 00	-762 00	5 00
138	44926 00	-5525 00	5 00
139	39555 00	-8446 00	5 00
140	33560 00	-10668 00	5 00

************************************ ELEMENT ARRAY **

ELEMENT	I	J	K	ELEMENT FRICTION FACTOR
1	40	29	41	0.0050000
2	29	30	41	0.0050000
3	29	19	30	0.0050000
4	19	20	30	0.0050000
5	19	12	20	0.0050000
6	19	11	12	0.0050000
7	11	4	12	0.0050000
8	4	5	12	0.0050000
9	4	2	5	0.0050000
10	2	6	5	0.0050000
11	2	3	6	0.0050000
12	2	1	3	0.0050000
13	41	31	42	0.0050000
14	41	30	31	0.0050000
15	30	20	31	0.0050000
16	20	32	31	0.0050000
17	20	21	32	0.0050000
18	20	12	21	0.0050000
19	12	13	21	0.0050000
20	12	5	13	0.0050000
21	5	6	13	0.0050000
22	6	14	13	0.0050000
23	6	7	14	0.0050000
24	3	7	6	0.0050000
25	3	8	7	0.0050000
26	3	9	8	0.0050000
27	3	10	9	0.0050000
28	42	43	51	0.0050000
29	31	43	42	0.0050000
30	31	32	43	0.0050000
31	32	44	43	0.0050000
32	32	33	44	0.0050000
33	32	21	33	0.0050000
34	21	22	33	0.0050000
35	21	13	22	0.0050000
36	13	23	22	0.0050000
37	13	14	23	0.0050000
38	14	24	23	0.0050000

39	14	15	24	0.0050000
40	14	7	15	0.0050000
41	7	16	15	0.0050000
42	7	8	16	0.0050000
43	8	17	16	0.0050000
44	8	18	17	0.0050000
45	8	9	18	0.0050000
46	18	27	17	0.0050000
47	51	43	52	0.0050000
48	43	44	52	0.0050000
49	44	53	52	0.0050000
50	44	45	53	0.0050000
51	44	33	45	0.0050000
52	33	34	45	0.0050000
53	22	34	33	0.0050000
54	22	35	34	0.0050000
55	22	23	35	0.0050000
56	23	36	35	0.0050000
57	23	24	35	0.0050000
58	24	37	36	0.0050000
59	24	25	37	0.0050000
60	24	15	25	0.0050000
61	15	16	25	0.0050000
62	16	26	25	0.0050000
63	16	17	26	0.0050000
64	51	52	58	0.0050000
65	52	59	58	0.0050000
66	52	53	59	0.0050000
67	53	45	54	0.0050000
68	45	46	54	0.0050000
69	34	46	45	0.0050000
70	34	47	46	0.0050000
71	35	47	34	0.0050000
72	35	48	47	0.0050000
73	36	48	35	0.0050000
74	37	48	36	0.0050000
75	37	38	48	0.0050000
76	25	38	37	0.0050000
77	25	26	39	0.0050000
78	26	28	38	0.0050000
79	26	17	28	0.0050000
80	58	59	64	0.0050000
81	59	65	64	0.0050000
82	59	60	65	0.0050000
83	60	66	65	0.0050000
84	53	60	59	0.0050000
85	53	54	60	0.0050000
86	60	61	66	0.0050000
87	60	54	61	0.0050000
88	54	55	61	0.0050000
89	46	55	54	0.0050000
90	46	56	55	0.0050000
91	47	56	46	0.0050000
92	47	49	56	0.0050000
93	48	49	47	0.0050000
94	48	38	49	0.0050000
95	38	39	49	0.0050000
96	38	28	39	0.0050000
97	64	65	70	0.0050000
98	65	71	70	0.0050000
99	70	71	77	0.0050000
100	65	72	71	0.0050000
101	65	66	72	0.0050000
102	66	73	72	0.0050000
103	66	67	73	0.0050000
104	61	67	66	0.0050000

105	61	52	67	0.0050000
106	55	62	61	0.0050000
107	55	57	62	0.0050000
108	55	57	55	0.0050000
109	56	50	57	0.0050000
110	49	50	56	0.0050000
111	39	50	49	0.0050000
112	77	78	85	0.0050000
113	77	71	78	0.0050000
114	71	79	78	0.0050000
115	71	72	79	0.0050000
116	72	73	79	0.0050000
117	73	80	79	0.0050000
118	73	74	80	0.0050000
119	73	67	74	0.0050000
120	67	68	74	0.0050000
121	67	62	68	0.0050000
122	62	63	68	0.0050000
123	62	57	63	0.0050000
124	85	95	95	0.0050000
125	85	85	95	0.0050000
126	85	78	85	0.0050000
127	78	87	95	0.0050000
128	78	79	87	0.0050000
129	79	88	87	0.0050000
130	79	80	88	0.0050000
131	80	89	88	0.0050000
132	80	81	89	0.0050000
133	80	74	81	0.0050000
134	74	75	81	0.0050000
135	68	75	74	0.0050000
136	68	69	75	0.0050000
137	68	63	69	0.0050000
138	87	97	98	0.0050000
139	87	88	97	0.0050000
140	88	99	97	0.0050000
141	88	89	99	0.0050000
142	89	100	99	0.0050000
143	89	90	100	0.0050000
144	81	90	89	0.0050000
145	81	82	90	0.0050000
146	81	75	82	0.0050000
147	76	76	82	0.0050000
148	75	69	76	0.0050000
149	112	105	113	0.0050000
150	98	97	105	0.0050000
151	105	114	113	0.0050000
152	105	106	114	0.0050000
153	105	97	106	0.0050000
154	97	99	106	0.0050000
155	99	107	106	0.0050000
156	99	100	107	0.0050000
157	100	108	107	0.0050000
158	100	109	108	0.0050000
159	100	101	109	0.0050000
160	100	90	101	0.0050000
161	90	91	101	0.0050000
162	90	82	91	0.0050000
163	82	92	91	0.0050000
164	82	83	92	0.0050000
165	76	83	82	0.0050000
166	75	84	83	0.0050000
167	91	102	101	0.0050000
168	83	84	93	0.0050000
169	83	93	94	0.0050000
170	83	94	92	0.0050000

171	94	104	92	0.0050000
172	113	114	121	0.0050000
173	114	122	121	0.0050000
174	114	113	122	0.0050000
175	105	115	114	0.0050000
176	106	107	115	0.0050000
177	107	116	115	0.0050000
178	107	108	115	0.0050000
179	108	117	115	0.0050000
180	108	118	117	0.0050000
181	109	118	108	0.0050000
182	109	110	118	0.0050000
183	102	110	109	0.0050000
184	102	111	110	0.0050000
185	103	111	102	0.0050000
186	91	103	102	0.0050000
187	104	103	91	0.0050000
188	92	104	91	0.0050000
189	121	122	130	0.0050000
190	122	131	130	0.0050000
191	122	123	131	0.0050000
192	115	123	122	0.0050000
193	115	116	123	0.0050000
194	116	124	123	0.0050000
195	116	117	124	0.0050000
196	117	125	124	0.0050000
197	117	126	125	0.0050000
198	118	126	117	0.0050000
199	118	127	126	0.0050000
200	118	119	127	0.0050000
201	110	119	118	0.0050000
202	110	120	119	0.0050000
203	111	120	110	0.0050000
204	130	131	137	0.0050000
205	131	138	137	0.0050000
206	131	132	138	0.0050000
207	123	132	131	0.0050000
208	123	124	132	0.0050000
209	124	133	132	0.0050000
210	124	125	133	0.0050000
211	125	134	133	0.0050000
212	125	135	134	0.0050000
213	125	126	135	0.0050000
214	126	136	135	0.0050000
215	127	136	126	0.0050000
216	127	128	136	0.0050000
217	119	128	127	0.0050000
218	119	129	128	0.0050000
219	119	120	129	0.0050000
220	132	139	138	0.0050000
221	133	140	139	0.0050000
222	133	134	140	0.0050000
223	132	133	139	0.0050000
224	101	102	109	0.0050000

******************************   BOUNDARY CONDITIONS   ******************************

NO. OF BOUNDARY SEGMENTS WHICH HAVE NON-ZERO NORMAL FLUX PRESCRIBED =   0

NO. OF NODES WHERE ELEVATION IS PRESCRIBED = 10

NODE	PRESCRIBED ELEVATION	PHASE
40	1.2500	0.00000
41	1.2446	0.00000
42	1.2395	0.00000
51	1.2346	0.00000
58	1.2298	0.00000
64	1.2252	0.00000
70	1.2208	0.00000
77	1.2177	0.00000
85	1.2141	0.00000
95	1.2100	0.00000

****************************** RESULTS OF COMPUTATIONS ******************************

------------------------------ NODAL ELEVATIONS ------------------------------

NODE	MODULUS	PHASE
1	1.32107698	-0.04420
2	1.31532657	-0.04051
3	1.31131938	-0.04507
4	1.29951988	-0.02411
5	1.28024875	-0.01894
6	1.28550871	-0.02718
7	1.28885076	-0.03216
8	1.30879955	-0.05523
9	1.33326414	-0.07721
10	1.33591558	-0.08136
11	1.28928822	-0.01576
12	1.27037581	-0.01339
13	1.27769090	-0.02221
14	1.28508052	-0.03016
15	1.29177553	-0.04281
16	1.29127607	-0.03418
17	1.32976766	-0.07972
18	1.30733879	-0.05576
19	1.27052267	-0.00522
20	1.25107945	-0.00792
21	1.26595393	-0.01435
22	1.26789476	-0.01852
23	1.27656987	-0.02481
24	1.28522968	-0.02512
25	1.28846252	-0.03369
26	1.30069917	-0.04694
27	1.31914924	-0.05556
28	1.30794358	-0.04988
29	1.25540751	0.01023
30	1.25438317	-0.00421
31	1.25011315	-0.00480
32	1.25717380	-0.01210
33	1.26107974	-0.01536
34	1.25557572	-0.02270
35	1.27342194	-0.02297
36	1.28096143	-0.03435

37	1.28745102	-0.03593
38	1.29294899	-0.04244
39	1.29734111	-0.04907
40	1.25000000	0.00000
41	1.24450000	0.00000
42	1.23950000	0.00000
43	1.25025103	-0.00929
44	1.25331212	-0.01248
45	1.25146311	-0.02075
46	1.27139008	-0.02263
47	1.28007918	-0.03545
48	1.25008204	-0.02740
49	1.25646509	-0.03855
50	1.30017476	-0.04342
51	1.23460000	0.00000
52	1.24758895	-0.01333
53	1.25243391	-0.01716
54	1.26692531	-0.02991
55	1.28493103	-0.04127
56	1.28502079	-0.03773
57	1.30205493	-0.05527
58	1.22980000	0.00000
59	1.24311614	-0.01761
60	1.25521657	-0.02176
61	1.27424443	-0.03500
62	1.29525989	-0.04681
63	1.31135454	-0.05253
64	1.22520000	0.00000
65	1.24193807	-0.03358
66	1.25350972	-0.03541
67	1.27627791	-0.04387
68	1.29992519	-0.05558
69	1.32378908	-0.07408
70	1.22080000	0.00000
71	1.25044471	-0.04223
72	1.25726037	-0.04706
73	1.27368435	-0.04532
74	1.28710305	-0.05226
75	1.30739111	-0.06450
76	1.35705158	-0.09783
77	1.21770000	0.00000
78	1.24310745	-0.03816
79	1.26942710	-0.06280
80	1.28969108	-0.06517
81	1.30599871	-0.07019
82	1.32105490	-0.07862
83	1.35402221	-0.10515
84	1.39035475	-0.14339
85	1.21410000	0.00000
86	1.22320394	-0.06504
87	1.27705258	-0.09774
88	1.28843605	-0.05926
89	1.30331129	-0.07850
90	1.31905651	-0.08132
91	1.33017206	-0.09652
92	1.34430078	-0.09818
93	1.37588943	-0.12890
94	1.35884596	-0.10947
95	1.20658247	-0.02055
96	1.21000000	0.00000
97	1.31059076	-0.10315
98	1.31959093	-0.12568
99	1.31739204	-0.10581
100	1.31992335	-0.09153
101	1.32905168	-0.09681
102	1.34625677	-0.10040

103	1.33197595	-0.12121
104	1.37532166	-0.12254
105	1.33031201	-0.12835
106	1.33044726	-0.11879
107	1.33757830	-0.11274
108	1.34647592	-0.12097
109	1.34500144	-0.10931
110	1.35280597	-0.12335
111	1.41650277	-0.16038
112	1.35393374	-0.16307
113	1.35628555	-0.15990
114	1.35203150	-0.14800
115	1.34989175	-0.14096
116	1.35555517	-0.13573
117	1.37508247	-0.14334
118	1.36436098	-0.12900
119	1.38401993	-0.14557
120	1.42941929	-0.17584
121	1.41485080	-0.21401
122	1.37049959	-0.16015
123	1.35833510	-0.14990
124	1.37450700	-0.15843
125	1.37985312	-0.15171
126	1.37774030	-0.14852
127	1.38356551	-0.14738
128	1.43515103	-0.18721
129	1.44026634	-0.18821
130	1.44586527	-0.24797
131	1.44820139	-0.25080
132	1.41737555	-0.20259
133	1.39515710	-0.17854
134	1.44545913	-0.20732
135	1.41151389	-0.17288
136	1.43887703	-0.19058
137	1.44335649	-0.27295
138	1.46217139	-0.26637
139	1.45704040	-0.23999
140	1.46085534	-0.23006

---------------------------- NODAL VELOCITIES ----------------------------

NODE	X-DIRECTION MODULUS	X-DIRECTION PHASE	Y-DIRECTION MODULUS	Y-DIRECTION PHASE
1	0.00690895	0.83647	0.04942572	-1.25297
2	0.06445719	-1.40589	0.03351592	-1.27762
3	0.07306741	-1.64850	0.04192369	-1.50592
4	0.05056778	-1.42768	0.02998963	-1.16517
5	0.06801092	-1.59387	0.03024153	-1.47684
6	0.07401651	-1.66679	0.02759089	-1.65126
7	0.07494970	-1.70494	0.03733711	-1.80144
8	0.08235728	-1.71214	0.02999027	-1.66485
9	0.05571792	-1.65986	0.00652031	-1.47248
10	0.08051321	-1.59937	0.01899510	1.65934
11	0.06946384	-1.27546	0.02100246	-1.17412
12	0.07575073	-1.58156	0.02107980	-1.13256
13	0.08721866	-1.69749	0.02665721	-1.36574
14	0.09938410	-1.66302	0.02019958	-1.39238
15	0.07833697	-1.66710	0.01570135	-1.50353
16	0.08013930	-1.65800	0.02258045	-1.80374
17	0.07354738	-1.55613	0.03072230	-1.74402
18	0.00331815	1.22432	0.01107259	-2.33987
19	0.08730815	-1.29659	0.02847558	-1.17211

20	0.08090078	-1.87585	0.02231702	-1.23278
21	0.08315531	-1.82110	0.02161203	-1.13297
22	0.08058182	-1.88259	0.01751365	-1.12003
23	0.05412805	-1.67416	0.01488140	-0.98372
24	0.11169185	-1.89104	0.02337989	-1.42513
25	0.08881905	-1.88339	0.01256390	-1.81783
26	0.08880282	-1.54377	0.02289257	-2.37804
27	0.02216489	0.94187	0.01413338	-1.48862
28	0.03401032	-1.28823	0.01784733	-1.87835
29	0.07045370	-1.21009	0.03773890	-0.91320
30	0.05425246	-1.40849	0.02357414	-0.88112
31	0.10392707	-1.89822	0.02559417	-0.88089
32	0.08883939	-1.51821	0.02204794	-0.91923
33	0.08155016	-1.72140	0.02510348	-1.19100
34	0.07807209	-1.88539	0.02257951	-0.97289
35	0.02344916	-1.72442	0.03281518	-1.85070
36	0.08895114	-1.72238	0.01748430	-0.77804
37	0.08291417	-1.83886	0.02554710	-1.87151
38	0.07707926	-1.80383	0.00508107	-1.88519
39	0.03322358	-1.44937	0.00799878	2.65132
40	0.04920949	-0.66641	0.02235253	0.03335
41	0.08092592	-1.42273	0.12510288	-0.03985
42	0.12895274	-1.88709	0.03921453	-0.00513
43	0.12708333	-1.53759	0.02599520	-0.62117
44	0.08754728	-1.53955	0.02322455	-0.81120
45	0.08571376	-1.71011	0.03549852	-1.34242
46	0.05172081	-1.53353	0.03573580	-1.40978
47	0.02252547	-1.52544	0.02955018	-1.38203
48	0.08854000	-1.57883	0.01016955	-1.38209
49	0.04315803	-1.50090	0.01479407	-1.53330
50	0.01243531	0.88057	0.00631105	-1.95499
51	0.19015545	-1.52565	0.05783355	-0.21090
52	0.13246458	-1.74466	0.04481350	-0.57589
53	0.09870139	-1.70400	0.04008906	-1.10930
54	0.08302044	-1.67474	0.05717254	-1.38238
55	0.05329394	-1.57040	0.05859011	-1.47939
56	0.05288558	-1.63436	0.03147573	-1.44980
57	0.01027515	-2.49839	0.04882721	-1.44742
58	0.25935377	-1.71761	0.08133718	-0.55323
59	0.14983440	-1.84028	0.05197948	-1.25119
60	0.09438987	-1.66103	0.07708670	-1.80525
61	0.08255127	-1.72078	0.09859652	-1.53028
62	0.03424341	-1.41600	0.08704146	-1.45208
63	0.01907834	2.81497	0.06615467	-1.40274
64	0.32191943	-1.84403	0.09475362	-0.91522
65	0.19537508	-1.81324	0.11080753	-1.31242
66	0.09465426	-1.58538	0.11910534	-1.70258
67	0.05454795	-1.75672	0.11531285	-1.59194
68	0.02752704	-1.35410	0.10127911	-1.49122
69	0.03411658	2.10217	0.10080381	-1.35180
70	0.35909077	-1.59650	0.12543713	-0.74121
71	0.33681092	-1.74383	0.12281290	-0.95755
72	0.14197189	-1.72114	0.14328064	-1.44832
73	0.07899317	-1.58338	0.14561378	-1.72450
74	0.04950359	-1.86583	0.15084650	-1.65899
75	0.02463991	-1.76744	0.13164306	-1.55931
76	0.00384235	-2.63099	0.13038814	-1.38837
77	0.43098891	-1.65753	0.09055113	-0.49475
78	0.45919369	-1.93495	0.22235127	-1.59260
79	0.15454337	-1.92942	0.22472316	-1.70470
80	0.08871182	-1.75314	0.15945271	-1.70482
81	0.01848967	-1.71221	0.15718577	-1.71636
82	0.01837556	-2.09549	0.12794550	-1.59175
83	0.04027281	-1.92229	0.05777072	-1.43422
84	0.07947520	-1.54829	0.05400715	-1.34455
85	0.44139143	-2.05751	0.11433536	-1.37302

86	0.65341778	-1.65190	0.21451581	-1.90291
87	0.09327137	-1.90370	0.40520455	-1.76508
88	0.01509753	-0.69272	0.24765172	-1.76680
89	0.02647134	-2.07893	0.20254080	-1.75929
90	0.00853251	-2.41277	0.15652409	-1.72570
91	0.03243844	1.56578	0.10820521	-1.64351
92	0.03033784	-2.20787	0.08025255	-1.36525
93	0.05261491	-1.81784	0.02309108	1.85134
94	0.01920526	-2.72399	0.02537520	-1.21509
95	0.55005577	-2.14276	0.11222527	-0.40132
96	0.40542552	-2.44573	0.13460574	-1.32495
97	0.13006434	1.35581	0.24522768	-1.75946
98	0.35553029	1.41243	0.24411180	-1.51528
99	0.00501071	2.10575	0.22377515	-1.75221
100	0.02522141	1.14206	0.16230102	-1.76586
101	0.01009809	1.91253	0.17305355	-1.75962
102	0.02883153	1.39119	0.15807025	-1.65157
103	0.05891536	2.04139	0.12917045	-1.38491
104	0.04412494	2.37390	0.04400055	-1.08500
105	0.14866004	1.35501	0.05022538	-1.85952
106	0.09477653	1.41577	0.15785505	-1.77553
107	0.06039100	1.30743	0.16615326	-1.80239
108	0.02101338	1.79225	0.17000524	-1.80575
109	0.01309023	1.65353	0.16273522	-1.75920
110	0.02393348	-2.13239	0.13705234	-1.70819
111	0.03558550	-1.91384	0.16547788	-1.48258
112	0.07392800	1.35892	0.11595961	1.41422
113	0.02563966	2.04597	0.04551253	-1.57517
114	0.07706535	1.50126	0.10127037	-1.73953
115	0.05058544	1.57949	0.13586683	-1.75530
116	0.04501552	1.34471	0.12993812	-1.80221
117	0.04433249	1.21497	0.11045988	-1.75743
118	0.01600257	1.76413	0.12097254	-1.79579
119	0.01855828	-2.05847	0.08435533	-1.63947
120	0.01676499	-1.68593	0.08171937	-1.53252
121	0.05591482	1.18446	0.09167173	-1.53021
122	0.07089177	1.24152	0.10399338	-1.76259
123	0.07198302	1.23184	0.12353957	-1.78326
124	0.04593830	1.31094	0.11040188	-1.80486
125	0.04562007	1.27926	0.07488442	-1.80770
126	0.01704299	1.62245	0.07659858	-1.78044
127	0.00701074	2.17371	0.08821250	-1.73739
128	0.01287425	2.22357	0.02865504	-1.79960
129	0.01090795	2.12477	0.03953948	-1.60923
130	0.04996831	1.42319	0.11399253	-1.55919
131	0.10912470	1.27737	0.10749524	-1.69764
132	0.09885924	1.21603	0.07891162	-1.74948
133	0.05560509	1.21691	0.05529105	-1.82378
134	0.06665079	1.39186	0.04671839	-1.80017
135	0.04501656	1.45780	0.01820587	-1.55513
136	0.02492268	1.35531	0.01973553	-1.63929
137	0.01407806	0.91704	0.02906213	-1.65870
138	0.06835311	1.07543	0.01763277	-1.35572
139	0.05656334	1.16127	0.03176735	-1.87021
140	0.05635456	1.39418	0.01859831	-1.85382

Appendix 3.4

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PROGRAM TEA
DEPT OF CIVIL ENGINEERING,  M. I. T.
2-D LINEAR FINITE ELEMENT FREQUENCY DOMAIN ANALYSIS OF TIDAL WAVES FOR  SMALL SCALE GEOMETRY.

ENTER 10 DIGIT F.E. GRID GEOMETRY IDENTIFICATION

ENTER 10 DIGIT I.D. CODE FOR B.C. VERSION

INPUT NUMBER OF ELEMENTS AND NUMBER OF NODE POINTS

INPUT THE NODE NO.,X-COORD.,Y-COORD., AND NODAL M.S.L. DEPTH, ONE NODE/LINE

INPUT ELEMENT NUMBER, THREE NODE NUMBERS FOR THE ELEMENTAND THE ELEMENT LINEAR FRICTION FACTOR

MAX. NODAL POINT DIFFERENCE =    13

INPUT FREQUENCY IN RADIANS/SEC.

INPUT CORIOLIS PARAMETER

INPUT AMPLITUDE OF WIND SPEED, WIND PHASE SHIFT, ANGLE OF WIND DIRECTION  (IN DEGREES)  AND WIND DRAG COEFFICIENT

INPUT HOW MANY BOUNDARY SEGMENTS HAVE NON-ZERO NORMAL FLUX PRESCRIBED

INPUT NO. OF NODES WHERE ELEVATION IS PRESCRIBED
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE
INPUT THE NODE NUMBER AND VALUES FOR THE MODULUS AND PHASE (IN RAD) OF THE PRESCRIBED ELEVATION AT NODE

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PROGRAM TEA
DEPT OF CIVIL ENGINEERING,  M. I. T.
2-D LINEAR FINITE ELEMENT FREQUENCY DOMAIN ANALYSIS OF TIDAL WAVES FOR  SMALL SCALE GEOMETRY.

F.E. GRID GEOMETRY IDENTIFICATION :  MASEXAC
BOUNDARY CONDITION IDENTIFICATION  :  B-02:F3V1

FREQUENCY = 0.00000000
CORIOLIS PARAMETER = 0.00010000

WIND SPEED AT 10 METERS ABOVE SURFACE =      0.0000
WIND PHASE SHIFT =     0.00000 RADIANS
WIND DIRECTION =    0.00000 DEGREES
WIND DRAG COEFFICIENT =    0.00000000

NO. OF NODE POINTS =140
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NO. OF ELEMENTS =224

NODAL POINT DIFFERENCE = 13

********************************** NODAL COORDINATES AND DEPTHS *******************************

NODE	X-COORD.	Y-COORD.	DEPTH
1	-21654.00	62040.00	5.00
2	-17685.00	70549.00	5.00
3	-19558.00	64643.00	10.00
4	-13452.00	75581.00	5.00
5	-10763.00	71215.00	34.00
6	-14224.00	65326.00	34.00
7	-14001.00	61785.00	30.00
8	-18955.00	59531.00	15.00
9	-24416.00	60833.00	5.00
10	-24329.00	65024.00	5.00
11	-8055.00	80677.00	10.00
12	-5588.00	74517.00	46.00
13	-7420.00	66634.00	40.00
14	-10160.00	62548.00	30.00
15	-11462.00	59055.00	25.00
16	-14288.00	57817.00	25.00
17	-17621.00	53023.00	5.00
18	-23576.00	55817.00	15.00
19	-1365.00	82804.00	15.00
20	1016.00	75184.00	62.00
21	-1334.00	68358.00	50.00
22	-1715.00	62167.00	60.00
23	-6255.00	60738.00	40.00
24	-8604.00	59055.00	25.00
25	-11239.00	56198.00	30.00
26	-13335.00	53721.00	15.00
27	-23304.00	51435.00	10.00
28	-14446.00	49371.00	10.00
29	3905.00	85265.00	20.00
30	5556.00	80137.00	55.00
31	10065.00	73660.00	65.00
32	6334.00	68898.00	75.00
33	2953.00	62421.00	76.00
34	1688.00	55817.00	60.00
35	-3016.00	57245.00	45.00
36	-5874.00	57023.00	35.00
37	-8541.00	56293.00	23.00
38	-9366.00	52737.00	20.00
39	-9906.00	48419.00	15.00
40	8033.00	90297.00	15.00
41	13018.00	81820.00	75.00
42	17621.00	73724.00	80.00
43	13335.00	67247.00	65.00
44	9510.00	61055.00	85.00
45	6255.00	55372.00	71.00
46	1683.00	50102.00	42.00
47	-2604.00	52292.00	30.00
48	-5884.00	52959.00	34.00
49	-6223.00	48768.00	25.00
50	-6191.00	44006.00	10.00
51	21971.00	66104.00	50.00
52	16955.00	59436.00	80.00
53	12922.00	53435.00	78.00
54	7674.00	47625.00	50.00
55	2477.00	42228.00	25.00
56	-1969.00	46228.00	25.00

57	-3302.00	38068.00	10.00
58	25364.00	58452.00	30.00
59	22606.00	51562.00	85.00
60	15875.00	46800.00	70.00
61	9938.00	41655.00	37.00
62	4191.00	36852.00	22.00
63	-254.00	32226.00	10.00
64	30480.00	51245.00	30.00
65	26226.00	44037.00	50.00
66	18352.00	40577.00	62.00
67	12446.00	35195.00	50.00
68	7049.00	31560.00	25.00
69	3239.00	25654.00	10.00
70	34544.00	44323.00	22.00
71	30798.00	37052.00	21.00
72	24892.00	36339.00	50.00
73	20257.00	33657.00	58.00
74	14224.00	29749.00	47.00
75	9303.00	24733.00	33.00
76	5207.00	19553.00	5.00
77	37433.00	39455.00	32.00
78	33719.00	31052.00	60.00
79	27495.00	28337.00	60.00
80	20056.00	26289.00	55.00
81	14732.00	22479.00	40.00
82	10065.00	18225.00	30.00
83	6096.00	16764.00	10.00
84	1475.00	16462.00	5.00
85	40799.00	33814.00	55.00
86	36735.00	27464.00	30.00
87	33941.00	25527.00	55.00
88	26988.00	23463.00	53.00
89	21146.00	20574.00	48.00
90	16129.00	16732.00	40.00
91	11970.00	13367.00	30.00
92	8255.00	14542.00	15.00
93	3250.00	13012.00	5.00
94	6969.00	13430.00	10.00
95	40545.00	28067.00	30.00
96	44577.00	27464.00	30.00
97	33150.00	19431.00	45.00
98	36576.00	22416.00	40.00
99	27877.00	16510.00	45.00
100	23146.00	14288.00	43.00
101	17812.00	12509.00	37.00
102	14542.00	9081.00	25.00
103	10267.00	10097.00	5.00
104	7938.00	12033.00	5.00
105	39053.00	19558.00	38.00
106	35433.00	13675.00	35.00
107	29655.00	10573.00	35.00
108	25718.00	6287.00	32.00
109	20003.00	7366.00	33.00
110	15399.00	3239.00	22.00
111	11557.00	4953.00	5.00
112	44037.00	23844.00	20.00
113	46942.00	19050.00	15.00
114	41815.00	14034.00	27.00
115	35703.00	8954.00	31.00
116	31845.00	4639.00	29.00
117	26321.00	381.00	20.00
118	20013.00	794.00	25.00
119	14478.00	-1048.00	15.00
120	10795.00	657.00	5.00
121	48000.00	12954.00	5.00
122	42736.00	8192.00	18.00

123	33132.00	3620.00	22.00
124	32925.00	-1016.00	22.00
125	27178.00	-4699.00	21.00
126	20479.00	-5017.00	20.00
127	16764.00	-3588.00	17.00
128	13399.00	-5060.00	5.00
129	11239.00	-2953.00	5.00
130	49086.00	6921.00	5.00
131	44164.00	2159.00	5.00
132	39592.00	-2032.00	10.00
133	33909.00	-5429.00	15.00
134	27432.00	-10160.00	5.00
135	21939.00	-9335.00	10.00
136	15939.00	-6985.00	5.00
137	49879.00	-762.00	5.00
138	44926.00	-5525.00	5.00
139	39556.00	-8446.00	5.00
140	33560.00	-10668.00	5.00

************************************** ELEMENT ARRAY **

ELEMENT	I	J	K	ELEMENT FRICTION FACTOR
1	40	29	41	0.0050000
2	29	30	41	0.0050000
3	29	19	30	0.0050000
4	19	20	30	0.0050000
5	19	12	20	0.0050000
6	19	11	12	0.0050000
7	11	4	12	0.0050000
8	4	5	12	0.0050000
9	4	2	5	0.0050000
10	2	6	5	0.0050000
11	2	3	6	0.0050000
12	2	1	3	0.0050000
13	41	31	42	0.0050000
14	41	30	31	0.0050000
15	30	20	31	0.0050000
16	20	32	31	0.0050000
17	20	21	32	0.0050000
18	20	12	21	0.0050000
19	12	13	21	0.0050000
20	12	5	13	0.0050000
21	5	6	13	0.0050000
22	6	14	13	0.0050000
23	6	7	14	0.0050000
24	3	7	6	0.0050000
25	3	8	7	0.0050000
26	3	9	8	0.0050000
27	3	10	9	0.0050000
28	42	43	51	0.0050000
29	31	43	42	0.0050000
30	31	32	43	0.0050000
31	32	44	43	0.0050000
32	32	33	44	0.0050000
33	32	21	33	0.0050000
34	21	22	33	0.0050000
35	21	13	22	0.0050000
36	13	23	22	0.0050000
37	13	14	23	0.0050000
38	14	24	23	0.0050000

39	14	15	24	0.0050000
40	14	7	15	0.0050000
41	7	16	15	0.0050000
42	7	8	16	0.0050000
43	8	17	16	0.0050000
44	8	18	17	0.0050000
45	8	9	18	0.0050000
46	18	27	17	0.0050000
47	51	43	52	0.0050000
48	43	44	52	0.0050000
49	44	53	52	0.0050000
50	44	45	53	0.0050000
51	44	33	45	0.0050000
52	33	34	45	0.0050000
53	22	34	33	0.0050000
54	22	35	34	0.0050000
55	22	23	35	0.0050000
56	23	36	35	0.0050000
57	23	24	36	0.0050000
58	24	37	36	0.0050000
59	24	25	37	0.0050000
60	24	15	25	0.0050000
61	15	16	25	0.0050000
62	16	26	25	0.0050000
63	16	17	26	0.0050000
64	51	52	58	0.0050000
65	52	59	58	0.0050000
66	52	53	59	0.0050000
67	53	45	64	0.0050000
68	45	46	64	0.0050000
69	34	46	45	0.0050000
70	34	47	46	0.0050000
71	35	47	34	0.0050000
72	35	48	47	0.0050000
73	36	48	35	0.0050000
74	37	48	36	0.0050000
75	37	38	48	0.0050000
76	25	38	37	0.0050000
77	25	26	38	0.0050000
78	26	28	38	0.0050000
79	26	17	28	0.0050000
80	58	59	64	0.0050000
81	59	65	64	0.0050000
82	59	60	65	0.0050000
83	60	66	65	0.0050000
84	53	60	59	0.0050000
85	53	54	60	0.0050000
86	60	61	66	0.0050000
87	60	54	61	0.0050000
88	54	55	61	0.0050000
89	46	55	54	0.0050000
90	46	56	55	0.0050000
91	47	56	46	0.0050000
92	47	49	56	0.0050000
93	48	49	47	0.0050000
94	48	38	49	0.0050000
95	38	39	49	0.0050000
96	38	28	39	0.0050000
97	64	65	70	0.0050000
98	65	71	70	0.0050000
99	70	71	77	0.0050000
100	65	72	71	0.0050000
101	65	66	72	0.0050000
102	66	73	72	0.0050000
103	66	67	73	0.0050000
104	61	67	66	0.0050000

105	61	62	67	0.0050000
106	55	62	61	0.0050000
107	55	57	62	0.0050000
108	56	57	55	0.0050000
109	56	50	57	0.0050000
110	49	50	56	0.0050000
111	39	50	49	0.0050000
112	77	78	85	0.0050000
113	77	71	78	0.0050000
114	71	79	78	0.0050000
115	71	72	79	0.0050000
116	72	73	79	0.0050000
117	73	80	79	0.0050000
118	73	74	80	0.0050000
119	73	67	74	0.0050000
120	67	68	74	0.0050000
121	67	62	68	0.0050000
122	62	63	68	0.0050000
123	62	57	63	0.0050000
124	85	95	96	0.0050000
125	85	86	95	0.0050000
126	85	78	86	0.0050000
127	78	87	86	0.0050000
128	78	79	87	0.0050000
129	79	88	87	0.0050000
130	79	80	88	0.0050000
131	80	89	88	0.0050000
132	80	81	89	0.0050000
133	80	74	81	0.0050000
134	74	75	81	0.0050000
135	68	75	74	0.0050000
136	68	69	75	0.0050000
137	68	63	69	0.0050000
138	87	97	98	0.0050000
139	87	88	97	0.0050000
140	88	99	97	0.0050000
141	88	89	99	0.0050000
142	89	100	99	0.0050000
143	89	90	100	0.0050000
144	81	90	89	0.0050000
145	81	82	90	0.0050000
146	81	75	82	0.0050000
147	75	76	82	0.0050000
148	75	69	76	0.0050000
149	112	105	113	0.0050000
150	98	97	105	0.0050000
151	105	114	113	0.0050000
152	105	106	114	0.0050000
153	105	97	106	0.0050000
154	97	99	106	0.0050000
155	99	107	106	0.0050000
156	99	100	107	0.0050000
157	100	108	107	0.0050000
158	100	109	108	0.0050000
159	100	101	109	0.0050000
160	100	90	101	0.0050000
161	90	91	101	0.0050000
162	90	82	91	0.0050000
163	82	92	91	0.0050000
164	82	83	92	0.0050000
165	76	83	82	0.0050000
166	76	84	83	0.0050000
167	91	102	101	0.0050000
168	83	84	93	0.0050000
169	83	93	94	0.0050000
170	83	94	92	0.0050000

171	94	104	92	0.0050000
172	113	114	121	0.0050000
173	114	122	121	0.0050000
174	114	115	122	0.0050000
175	106	115	114	0.0050000
176	106	107	115	0.0050000
177	107	116	115	0.0050000
178	107	108	116	0.0050000
179	108	117	116	0.0050000
180	108	118	117	0.0050000
181	109	118	108	0.0050000
182	109	110	118	0.0050000
183	102	110	109	0.0050000
184	102	111	110	0.0050000
185	103	111	102	0.0050000
186	91	103	102	0.0050000
187	104	103	91	0.0050000
188	92	104	91	0.0050000
189	121	122	130	0.0050000
190	122	131	130	0.0050000
191	122	123	131	0.0050000
192	115	123	122	0.0050000
193	115	116	123	0.0050000
194	116	124	123	0.0050000
195	116	117	124	0.0050000
196	117	125	124	0.0050000
197	117	126	125	0.0050000
198	118	126	117	0.0050000
199	118	127	126	0.0050000
200	118	119	127	0.0050000
201	110	119	118	0.0050000
202	110	120	119	0.0050000
203	111	120	110	0.0050000
204	130	131	137	0.0050000
205	131	138	137	0.0050000
206	131	132	138	0.0050000
207	123	132	131	0.0050000
208	123	124	132	0.0050000
209	124	133	132	0.0050000
210	124	125	133	0.0050000
211	125	134	133	0.0050000
212	125	135	134	0.0050000
213	125	126	135	0.0050000
214	126	136	135	0.0050000
215	127	136	126	0.0050000
216	127	128	136	0.0050000
217	119	128	127	0.0050000
218	119	129	128	0.0050000
219	119	120	129	0.0050000
220	132	139	138	0.0050000
221	133	140	139	0.0050000
222	133	134	140	0.0050000
223	132	133	139	0.0050000
224	101	102	109	0.0050000

****************************** BOUNDARY CONDITIONS ******************************

NO OF BOUNDARY SEGMENTS WHICH HAVE NON-ZERO NORMAL FLUX PRESCRIBED = 0

OF NODES WHERE ELEVATION IS PRESCRIBED = 10

NODE	PRESCRIBED ELEVATION	PHASE
40	0.0400	0.00000
41	0.0346	0.00000
42	0.0295	0.00000
51	0.0246	0.00000
58	0.0198	0.00000
64	0.0152	0.00000
70	0.0108	0.00000
77	0.0077	0.00000
85	0.0041	0.00000
96	0.0000	0.00000

****************************** RESULTS OF COMPUTATIONS ******************************

------------------------------- NODAL ELEVATIONS -------------------------------

NODE	MODULUS	PHASE
1	0.04239742	0.00000
2	0.04146851	0.00000
3	0.03871197	0.00000
4	0.04140596	0.00000
5	0.03409668	0.00000
6	0.03367290	0.00000
7	0.03430751	0.00000
8	0.03695301	0.00000
9	0.04199020	0.00000
10	0.04231058	0.00000
11	0.04012124	0.00000
12	0.03313332	0.00000
13	0.03328799	0.00000
14	0.03411663	0.00000
15	0.03449610	0.00000
16	0.03425399	0.00000
17	0.04144649	0.00000
18	0.03667599	0.00000
19	0.03895312	0.00000
20	0.03295935	0.00000
21	0.03205407	0.00000
22	0.03066650	0.00000
23	0.03254232	0.00000
24	0.03329435	0.00000
25	0.03403015	0.00000
26	0.03629130	0.00000
27	0.03893867	0.00000
28	0.03818117	0.00000
29	0.03938028	0.00000
30	0.03370254	0.00000
31	0.03071956	0.00000
32	0.02975796	0.00000
33	0.02913253	0.00000
34	0.02996507	0.00000
35	0.03114561	0.00000
36	0.03291786	0.00000

37	0.03425405	0.00000
38	0.03437238	0.00000
39	0.03580912	0.00000
40	0.04000000	0.00000
41	0.03450000	0.00000
42	0.02950000	0.00000
43	0.02774080	0.00000
44	0.02706090	0.00000
45	0.02759821	0.00000
46	0.02993071	0.00000
47	0.03225334	0.00000
48	0.03222310	0.00000
49	0.03346445	0.00000
50	0.03555364	0.00000
51	0.02450000	0.00000
52	0.02338679	0.00000
53	0.02555646	0.00000
54	0.02787447	0.00000
55	0.03070406	0.00000
56	0.03135355	0.00000
57	0.03361690	0.00000
58	0.01980000	0.00000
59	0.02017039	0.00000
60	0.02415353	0.00000
61	0.02634127	0.00000
62	0.02784290	0.00000
63	0.03047233	0.00000
64	0.01520000	0.00000
65	0.02052242	0.00000
66	0.02164907	0.00000
67	0.02522183	0.00000
68	0.02595268	0.00000
69	0.02774690	0.00000
70	0.01080000	0.00000
71	0.01113698	0.00000
72	0.01871591	0.00000
73	0.02052861	0.00000
74	0.02284622	0.00000
75	0.02432832	0.00000
76	0.02581006	0.00000
77	0.00770000	0.00000
78	0.01250209	0.00000
79	0.01976190	0.00000
80	0.02123672	0.00000
81	0.02301749	0.00000
82	0.02367440	0.00000
83	0.02541543	0.00000
84	0.02579009	0.00000
85	0.00410000	0.00000
86	0.01634657	0.00000
87	0.02160755	0.00000
88	0.01913685	0.00000
89	0.02242700	0.00000
90	0.02213530	0.00000
91	0.02325523	0.00000
92	0.02439890	0.00000
93	0.02620263	0.00000
94	0.02535208	0.00000
95	0.00677536	0.00000
96	0.00000000	0.00000
97	0.02124819	0.00000
98	0.02296447	0.00000
99	0.02228786	0.00000
100	0.02144485	0.00000
101	0.02334156	0.00000
102	0.02267252	0.00000

103	0.02524565	0.00000
104	0.02567436	0.00000
105	0.02100811	0.00000
105	0.02174995	0.00000
107	0.02155953	0.00000
108	0.02293981	0.00000
109	0.02248442	0.00000
110	0.02323389	0.00000
111	0.02507854	0.00000
112	0.02244596	0.00000
113	0.02227215	0.00000
114	0.02244031	0.00000
115	0.02214914	0.00000
116	0.02240808	0.00000
117	0.02259472	0.00000
118	0.02263812	0.00000
119	0.02340451	0.00000
120	0.02438837	0.00000
121	0.02347312	0.00000
122	0.02207911	0.00000
123	0.02257496	0.00000
124	0.02257196	0.00000
125	0.02299174	0.00000
126	0.02312905	0.00000
127	0.02331226	0.00000
128	0.02470194	0.00000
129	0.02513803	0.00000
130	0.02448409	0.00000
131	0.02438530	0.00000
132	0.02331891	0.00000
133	0.02308339	0.00000
134	0.02444104	0.00000
135	0.02373517	0.00000
136	0.02463100	0.00000
137	0.02377207	0.00000
138	0.02428318	0.00000
139	0.02451650	0.00000
140	0.02454542	0.00000

------------------------------ NODAL VELOCITIES ------------------------------

NODE	X-DIRECTION MODULUS	X-DIRECTION PHASE	Y-DIRECTION MODULUS	Y-DIRECTION PHASE
1	0.00175050	0.00000	0.00191328	0.00000
2	0.00089564	3.14159	0.00150522	3.14159
3	0.00076546	3.14159	0.00009580	3.14159
4	0.00245184	3.14159	0.00233961	3.14159
5	0.00193925	3.14159	0.00503779	3.14159
6	0.00057356	3.14159	0.00334094	3.14159
7	0.00004535	3.14159	0.00362769	3.14159
8	0.00030854	3.14159	0.00122589	3.14159
9	0.00013587	3.14159	0.00062505	3.14159
10	0.00115912	3.14159	0.00089568	3.14159
11	0.00134495	3.14159	0.00072811	3.14159
12	0.00612395	3.14159	0.00457581	3.14159
13	0.00017548	3.14159	0.00553301	3.14159
14	0.00034812	3.14159	0.00507954	3.14159
15	0.00137569	0.00000	0.00494394	3.14159
16	0.00052958	0.00000	0.00246354	3.14159
17	0.00178535	0.00000	0.00019001	3.14159
18	0.00007070	3.14159	0.00103313	3.14159
19	0.00654348	3.14159	0.00554703	3.14159

20	0.00852800	3.14159	0.01521603	3.14159
21	0.00416815	3.14159	0.01463559	3.14159
22	0.00165025	0.00000	0.01232933	3.14159
23	0.00216996	0.00000	0.01008998	3.14159
24	0.00143317	0.00000	0.00478452	3.14159
25	0.00260572	0.00000	0.00427507	3.14159
26	0.00196755	0.00000	0.00355405	3.14159
27	0.00112903	0.00000	0.00034819	3.14159
28	0.00321039	0.00000	0.00104311	3.14159
29	0.01476024	3.14159	0.00259954	3.14159
30	0.01848597	3.14159	0.01327950	3.14159
31	0.02146579	3.14159	0.02599517	3.14159
32	0.00454109	3.14159	0.02526832	3.14159
33	0.00013235	0.00000	0.02077943	3.14159
34	0.00699908	0.00000	0.01577448	3.14159
35	0.00311071	0.00000	0.00935985	3.14159
36	0.00342715	0.00000	0.00850010	3.14159
37	0.00205989	0.00000	0.00653517	3.14159
38	0.00353534	0.00000	0.00397352	3.14159
39	0.00229908	0.00000	0.00306331	3.14159
40	0.00068157	3.14159	0.01200100	0.00000
41	0.02542014	3.14159	0.01318256	3.14159
42	0.02736788	3.14159	0.03399335	3.14159
43	0.00807370	3.14159	0.03578452	3.14159
44	0.00641599	0.00000	0.02885954	3.14159
45	0.00852358	0.00000	0.02146475	3.14159
46	0.00722885	0.00000	0.01505735	3.14159
47	0.00426536	0.00000	0.01131097	3.14159
48	0.00312188	0.00000	0.00679231	3.14159
49	0.00589803	0.00000	0.00774021	3.14159
50	0.00577702	0.00000	0.00867886	3.14159
51	0.00834701	3.14159	0.04210852	3.14159
52	0.00971926	0.00000	0.03521359	3.14159
53	0.01803253	0.00000	0.03139546	3.14159
54	0.01108586	0.00000	0.02153943	3.14159
55	0.00747050	0.00000	0.01613010	3.14159
56	0.00451749	0.00000	0.00917026	3.14159
57	0.00607950	0.00000	0.01058583	3.14159
58	0.01244333	0.00000	0.03906020	3.14159
59	0.02456351	0.00000	0.03061842	3.14159
60	0.01480311	0.00000	0.02914454	3.14159
61	0.01260251	0.00000	0.02494676	3.14159
62	0.00553506	0.00000	0.01485448	3.14159
63	0.00806158	0.00000	0.00886291	3.14159
64	0.04258078	0.00000	0.02884662	3.14159
65	0.02537910	0.00000	0.03382655	3.14159
66	0.01057695	0.00000	0.02980215	3.14159
67	0.01334340	0.00000	0.02212571	3.14159
68	0.00538957	0.00000	0.01351915	3.14159
69	0.00505535	0.00000	0.00865477	3.14159
70	0.04215877	0.00000	0.03833579	3.14159
71	0.05098547	0.00000	0.03208422	3.14159
72	0.02784564	0.00000	0.02980213	3.14159
73	0.01865297	0.00000	0.01855527	3.14159
74	0.01120297	0.00000	0.01785550	3.14159
75	0.00579239	0.00000	0.00867762	3.14159
76	0.00017347	0.00000	0.00527287	3.14159
77	0.04219230	0.00000	0.03500457	3.14159
78	0.07596456	0.00000	0.00357210	0.00000
79	0.03869250	0.00000	0.00480311	3.14159
80	0.01615997	0.00000	0.00875328	3.14159
81	0.00761589	0.00000	0.00591184	3.14159
82	0.00359449	0.00000	0.00673667	3.14159
83	0.00040526	0.00000	0.00035006	3.14159
84	0.00006542	0.00000	0.00071536	0.00000
85	0.09794519	0.00000	0.01777024	3.14159

86	0.11622079	0.00000	0.05796856	0.00000
87	0.02550812	0.00000	0.02977408	0.00000
88	0.00911381	0.00000	0.00485003	0.00000
89	0.01230833	0.00000	0.00215404	3.14159
90	0.00423225	0.00000	0.00394934	3.14159
91	0.00252005	0.00000	0.00149100	3.14159
92	0.00201497	0.00000	0.00281829	3.14159
93	0.00056456	3.14159	0.00131442	3.14159
94	0.00113570	0.00000	0.00125250	3.14159
95	0.14779993	0.00000	0.02564557	3.14159
96	0.14817184	0.00000	0.02594122	3.14159
97	0.00210415	3.14159	0.00787749	0.00000
98	0.02291919	3.14159	0.00577943	3.14159
99	0.00521180	0.00000	0.00348577	0.00000
100	0.00241660	0.00000	0.00200599	3.14159
101	0.00628864	0.00000	0.00157973	3.14159
102	0.00125243	3.14159	0.00332979	3.14159
103	0.00363435	0.00000	0.00278559	3.14159
104	0.00103848	0.00000	0.00178592	3.14159
105	0.00119358	3.14159	0.00024522	0.00000
106	0.00237921	0.00000	0.00156125	0.00000
107	0.00168940	0.00000	0.00042744	0.00000
108	0.00327197	0.00000	0.00154609	0.00000
109	0.00187653	0.00000	0.00105971	3.14159
110	0.00148176	0.00000	0.00058501	3.14159
111	0.00186985	0.00000	0.00004848	3.14159
112	0.00564487	3.14159	0.00254638	3.14159
113	0.00054186	3.14159	0.00294539	0.00000
114	0.00038178	0.00000	0.00262091	0.00000
115	0.00047221	0.00000	0.00165542	0.00000
116	0.00188215	0.00000	0.00083069	0.00000
117	0.00051836	0.00000	0.00117572	3.14159
118	0.00022968	0.00000	0.00135159	3.14159
119	0.00039673	0.00000	0.00060835	3.14159
120	0.00176357	3.14159	0.00065756	3.14159
121	0.00068461	0.00000	0.00029952	0.00000
122	0.00078819	0.00000	0.00019324	0.00000
123	0.00111919	0.00000	0.00091625	0.00000
124	0.00029475	0.00000	0.00033234	0.00000
125	0.00077863	0.00000	0.00073324	0.00000
126	0.00100347	0.00000	0.00075037	0.00000
127	0.00053032	0.00000	0.00058711	0.00000
128	0.00007171	0.00000	0.00075537	3.14159
129	0.00026041	0.00000	0.00082569	0.00000
130	0.00102921	3.14159	0.00122001	0.00000
131	0.00006522	3.14159	0.00077237	0.00000
132	0.00011941	0.00000	0.00010616	3.14159
133	0.00057442	0.00000	0.00009338	0.00000
134	0.00025808	0.00000	0.00043097	3.14159
135	0.00027081	3.14159	0.00109775	3.14159
136	0.00038648	3.14159	0.00102708	3.14159
137	0.00078059	0.00000	0.00051044	3.14159
138	0.00039875	0.00000	0.00003586	0.00000
139	0.00030053	0.00000	0.00060997	0.00000
140	0.00008951	3.14159	0.00004408	3.14159