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# EFFECTS OF INTRABEAM SCATTERING AND SYNCHROTRON RADIATION DAMPING WHEN REDUCING TRANSVERSE EMITTANCES TO AUGMENT THE LHC LUMINOSITY

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*Abstract*

The effects of intrabeam scattering (IBS) and synchrotron radiation on the expected evolution of the LHC and SLHC beam emittances during physics coasts at 7 TeV are examined for the nominal beam and beams with reduced emittances.

## INTRODUCTION

The luminosity can be written as [1]:

$$L \approx \frac{f_{rev} n_b N_b \gamma}{4\pi\beta^*}\frac{N_b}{\varepsilon_N}\frac{1}{\sqrt{1+(\theta^2\sigma_L^2\gamma)/(4\beta^*\varepsilon_N)}} = \frac{f_{rev} n_b \gamma}{2r_p}\frac{N_b}{\beta^*}|\Delta Q_{bb}| \quad (1)$$

$$(\theta \ll 1,\ \sigma_L \ll \beta^*,\ \sigma_L \ll l_{det},\ \sigma_L \gg \sigma^* = \sqrt{\beta^*\varepsilon_N/\gamma})$$

where $f_{rev}$ is the revolution frequency, $n_b$ the number of bunches, $N_b$ the number of protons per bunch, $\varepsilon_N$ the rms normalized transverse emittance, and $\Delta Q_{bb}$ the total head-on beam-beam tune shift.

Round Gaussian bunches are considered, with a small crossing angle $\theta$, and with an rms bunch length $\sigma_L$ that is much shorter than the interaction point (IP) beta function $\beta^*$ (negligible hourglass effect) and the detector length $l_{det}$, but much larger than the IP rms beam size $\sigma^*$. Equation 1 shows that the luminosity at constant beam-beam tune shift $\Delta Q_{bb}$ is independent of the emittance and grows linearly with the bunch intensity.

## FIRST IR UPGRADE AND SLHC

Among the cases studied in [2] the nominal beam and LHC parameters and those with reduced emittances have been selected to study the effect of intrabeam scattering on emittance evolution in a coast.

**Case 1** (*Table 1, 2nd column*): The nominal beam and LHC parameters at top energy give the nominal luminosity of $10^{34}$ cm$^{-2}$s$^{-1}$ [3, Chap. 2]. A crossing angle of 285 μrad is assumed as used in the interaction regions IR1 and IR5 [3, Chap. 2-3].

**Case 2** (*Table 1, 3rd column*): The new optics foreseen for the first IR upgrade reduced $\beta^*$ from 0.55 m down to 0.30 m. The crossing angle will rise from 285 to 410 μrad as $\theta \propto \sqrt{\varepsilon_N/\beta^*}$, in the further limit that $\theta^2\sigma_L^2\gamma \gg 4\beta^*\varepsilon_N$, resulting in a luminosity increased up to 1.36×10$^{34}$cm$^{-2}$s$^{-1}$. Reducing at the same time the emittance from 3.75 to 2.54 μm diminishes the crossing angle to 337 μrad, which is enough to offset for the lower $\beta^*$ of 0.30 m. It then results in a higher luminosity of 2.00×10$^{34}$ cm$^{-2}$s$^{-1}$, and the full benefit of reduced $\beta^*$ can be gained.

**Case 3** (*Table 1, 4th column*): Considering the ultimate beam intensity with $\beta^*$=0.25 m and a reduced emittance of 2.65 μm raises the head-on beam-beam tune shift to 1.43 and the luminosity reaches 4.65×10$^{34}$ cm$^{-2}$s$^{-1}$. The resulting brightness is amply consistent with the capacity of the future SPL, PS2 and SPS injectors.

**Case 4** (*Table 1, 5th column*): A top luminosity of $10^{35}$ cm$^{-2}$s$^{-1}$ with the same bunch number can be attained reducing $\beta^*$ to 0.15 m and rising the bunch intensity to 2.36×10$^{11}$ protons within an emittance of 2.6 μm, which is the design brightness value of the future injectors.

Table 1: LHC Luminosity with nominal beam intensity and SLHC Luminosity.

| | **LHC Luminosity with nominal beam intensity** | | **SLHC Luminosity** | |
|---|---|---|---|---|
| | **Case 1** Initial IR triplet | **Case 2*** IR phase 1 triplet with $\beta^*$=0.30 m and reduced emittance | **Case 3** Ultimate $N_b$ with $\beta^*$=0.25 m and reduced emittance | **Case 4** >Ultimate $N_b$ with $\beta^*$=0.15 m and reduced emittance |
| $N_b$ (× $10^{11}$) | 1.15 | 1.15 | 1.70 | 2.36 |
| $\varepsilon_{N,H,V} = \varepsilon_N = \gamma\varepsilon$ rms (μm) | 3.75 | **2.54** | **2.65** | **2.60** |
| $\beta^*$ (m) | 0.55 | 0.30 | 0.25 | 0.15 |
| $\sigma^*_{H,V} = \sigma^*$ (μm) | 16.58 | 10.11 | 9.40 | 7.21 |
| $\sigma_L$ (mm) | 75.50 | 75.50 | 75.50 | 75.50 |
| $\sigma_{\Delta p/p}$ (×$10^{-4}$) | 1.129 | 1.129 | 1.129 | 1.129 |
| $\varepsilon_L$ rms (eVs) | 0.62 | 0.62 | 0.62 | 0.62 |
| Crossing angle $\theta$ (μrad) | 285 | 337 | 355 | 454 |
| $\Delta Q_{bb}$ head-on** | 1.00 | **1.09** | **1.43** | **1.37** |
| Luminosity (×$10^{34}$ cm$^{-2}$s$^{-1}$) | 1.00 | **2.00** | **4.65** | **10.29** |

* Case 2 adapted from Table 1 [2] to suit with a $\beta^*$ of 0.30 m instead of 0.25 m (S. Fartoukh, R. Garoby, private communication)

** $\Delta Q_{bb}$ is normalized to the value of the nominal beam

## IBS EFFECTS IN SLHC

Two lattices were used to assess the IBS growth times (calculations with the conventional Bjorken-Mtingwa theory) for the beam parameters of Table 1. Nominal LHC optics ($\beta$*=0.55 m) was used for case 1. For the other cases, the smallest $\beta$*=0.30 m at present designed (Fig. 1) has been used for all computations [4]. The small disparity about the $\beta$* in Table 1 has a negligible effect.

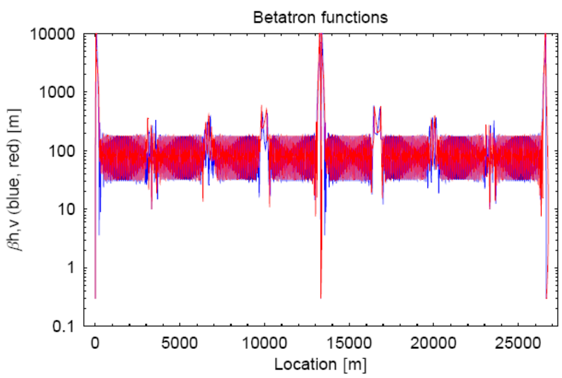


Fig. 1: SLHC betatron functions for $\beta$*=0.30 m (at IP1 and IP5).

For bunched beams, the intrabeam scattering growth rates can be written as [5]:

$$\frac{1}{\tau_{L,H,V}} = \frac{d\ln\varepsilon_{L,H,V}}{dt} = \frac{c r_p^2 (\log) N_b}{8\pi\beta^3\beta\gamma^4\varepsilon_H\varepsilon_V\sigma_{\Delta p/p}\sigma_L}\langle H_{L,H,V}\rangle \quad (2)$$

where (log) is the Coulomb logarithm, the functions $H_{L,H,V}$ depend on the optics parameters, the transverse emittances $\varepsilon_{H,V}$, the rms relative momentum spread $\sigma_{\Delta p/p}$, the rms bunch length $\sigma_L$ and $\gamma$. $H_{L,H,V}$ are averaged over the lattice. The rms longitudinal emittance (eVs) is defined as $\varepsilon_L = \pi p \sigma_{\Delta p/p}\sigma_L \beta^{-1} c^{-1}$(matched beams).

Figure 2 shows the initial IBS growth-times computed by the Bjorken-Mtingwa theory for the 4 cases of Table 1 ($\tau_{L,H}$ in hours, $\tau_V$ not shown as they are negative and of the order of 100 years). For the nominal LHC parameters (case 1), $\tau_L$ = 58 h, $\tau_H$ = 103 h, $\tau_V$ = -359 years.

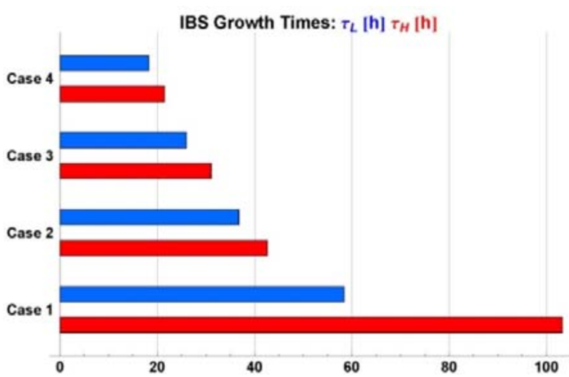


Fig. 2: Initial IBS growth-times for the 4 cases of Table 1.

The following computations assume a constant beam intensity for the duration of the beam storage period and are therefore pessimistic.

Figures 3 and 4 show the evolution of the emittances over the 10 hours period of beam coast. IBS growth-rates $\tau_{L,H,V}$ were calculated iteratively (equation 2) by step Δt of 5 minutes, updating the emittances at each iteration *i*:

$$\epsilon_{L,H,V}(i) = \epsilon_{L,H,V}(i-1)\exp\left[\Delta t/\tau_{L,H,V}(i-1)\right] \quad (3)$$

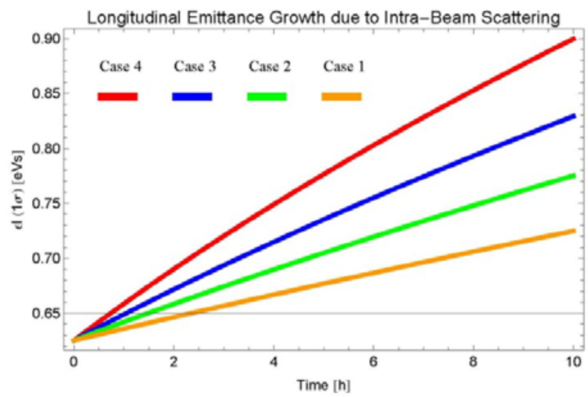


Fig. 3: IBS $\varepsilon_L$ progression for the 4 cases of Table 1.

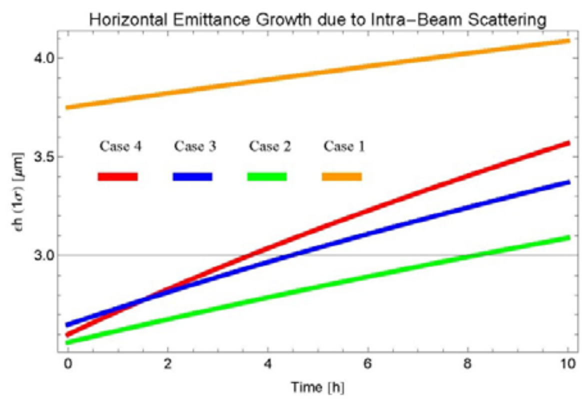


Fig. 4: IBS $\varepsilon_{N,H}$ progression for the 4 cases of Table 1.

Table 2 displays the emittance increase due to IBS for a proton beam at 7 TeV at the end of a storage period of 10 hours in the LHC/SLHC.

Table 2: IBS emittance growth after a 10 hours beam coast.

| | $\Delta\varepsilon_L/\varepsilon_L$ | $\Delta\varepsilon_h/\varepsilon_h$ | $\Delta\varepsilon_v/\varepsilon_v$ |
|---|---|---|---|
| **Initial IR triplet (case 1)** | 16% | 9% | $-10^{-4}$% |
| **IR phase1 triplet, $\beta$*=0.25 m and reduced emittance (case 2)** | 24% | 21% | $-10^{-3}$% |
| **Ultimate $N_b$, $\beta$*=0.25 m and reduced emittance (case 3)** | 32% | 27% | $-10^{-3}$% |
| **>Ultimate $N_b$, $\beta$*=0.15 m and reduced emittance (case 4)** | 44% | 37% | $-10^{-3}$% |

Computations of IBS emittance growth by the Bjorken-Mtingwa theory [5] were benchmarked with the new simulation code SIRE (Software for Intrabeam scattering and Radiation Effects) [7]. The writing of SIRE was inspired by the simulation code MOCAC (MOnte CArlo Code) [6]. Starting with a Gaussian beam, SIRE iteratively computes intrabeam collisions between pairs of macro-particles, from which the particle momentum changes are derived, and evaluates the effects of synchrotron radiation damping and quantum excitation if requested. Then, the beam distribution is updated and the rms beam emittances recomputed.

Figures 5 and 6 compare the evolution of the emittances for the first IR upgrade with reduced emittances (Table 1, case 2) between SIRE simulation and the straight IBS computation (equations 2-3) when only the IBS effect is considered. Unlike the conventional IBS formalisms which assume Gaussian beam distributions throughout the calculations, SIRE reshapes the beam distributions after each collisional process. This most likely explains the small difference observed in the emittance evolution computed with the two methods.

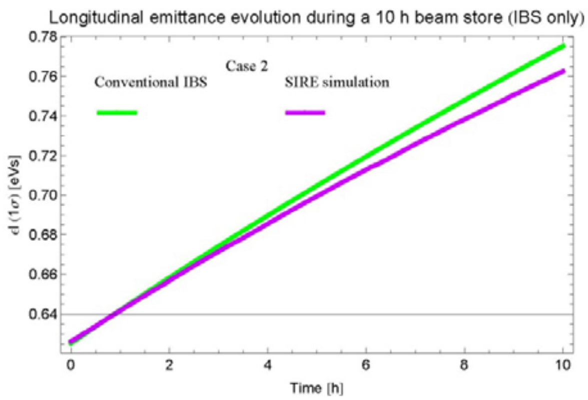


Fig. 5: Comparison of $\varepsilon_L$ between SIRE and straight IBS computations (case 2, Table 1): difference $\delta_{\max}(\Delta\varepsilon_L/\varepsilon_L) \sim 2\%$.

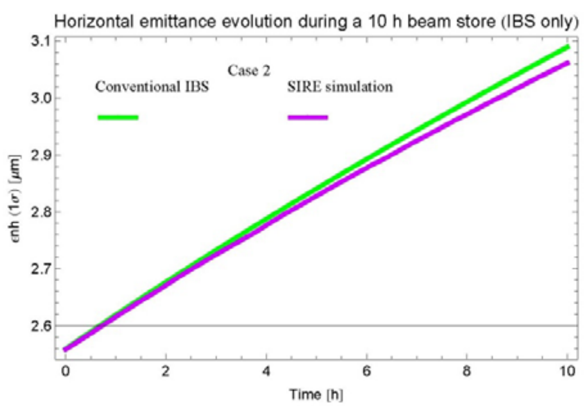


Fig. 6: Comparison of $\varepsilon_{N,H}$ between SIRE and straight IBS computations (case 2, Table 1): difference $\delta_{\max}(\Delta\varepsilon_H/\varepsilon_H) \sim 1\%$.

## IBS AND RADIATION DAMPING IN SLHC

For the LHC/SLHC proton beams at collision energy, synchrotron radiation turns into a perceptible effect. It continuously shrinks the emittances with damping times of 12.9 h in the longitudinal plane and of 26.0 h in the two transverse planes [3, Chap. 5].

The expected development of the emittances under the effect of IBS (equations 2 and 3) and radiation damping for the 4 cases during a 10 hours beam coast period is displayed in figures 7 to 9.

The synchrotron radiation damping (SRD) dominates the IBS growth in the longitudinal and vertical planes for all cases.

In the horizontal plane, excepting the nominal case 1 where the emittance damps continuously over the storage period, the others cases (2 to 4) reveal that the emittance expands at some point in time during the coast (the emittance grows all the time during the coast for case 4).

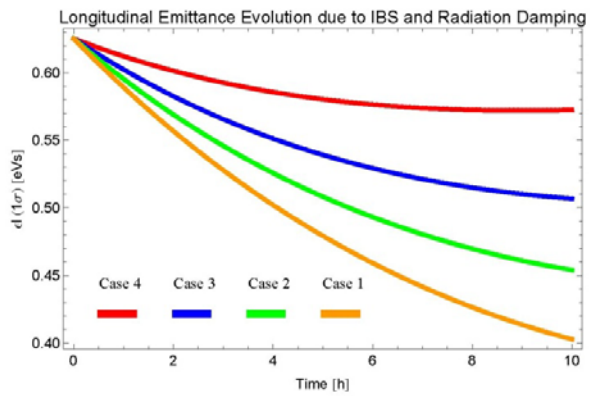


Fig. 7: IBS & SRD $\varepsilon_L$ evolution for the 4 cases of Table 1.

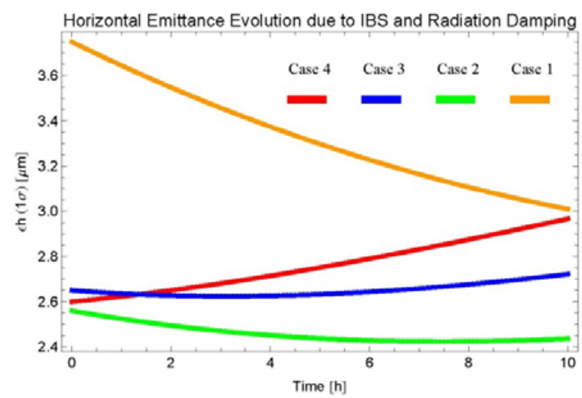


Fig. 8: IBS & SRD $\varepsilon_{N,H}$ evolution for the 4 cases of Table 1.

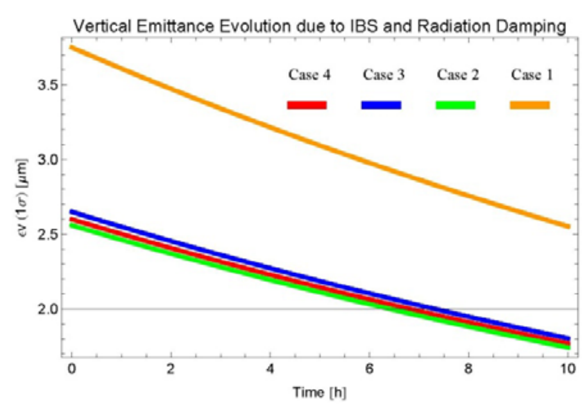


Fig. 9: IBS & SRD $\varepsilon_{N,V}$ evolution for the 4 cases of Table 1.

Table 3 shows the emittance changes caused by the combined effects of the IBS and radiation damping reached at the end of a 10 hours period of a stored proton beam at 7 TeV.

Table 3: Emittance changes after a 10 hours beam coast as a result of the effects of IBS and synchrotron radiation damping.

| | $\Delta\varepsilon_L/\varepsilon_L$ | $\Delta\varepsilon_h/\varepsilon_h$ | $\Delta\varepsilon_v/\varepsilon_v$ |
|---|---|---|---|
| **Initial IR triplet (case 1)** | -36% | -20% | -32% |
| **IR phase 1 triplet, $\beta$*=0.25 m and reduced emittance (case 2)** | -27% | -5% | -32% |
| **Ultimate $N_b$, $\beta$*=0.25 m and reduced emittance (case 3)** | -19% | 3% | -32% |
| **>Ultimate $N_b$, $\beta$*=0.15 m and reduced emittance (case 4)** | -8% | 14% | -32% |

Again, figures 10 to 12 compare the evolution of the emittances for the first IR upgrade with reduced emittances (Table 1, case 2) between SIRE simulation and the straight IBS computation (equations 2-3) taking into account the joint effects of IBS and radiation damping.

Including quantum excitation effect in Bjorken-Mtingwa calculations would yield negligible change of the LHC proton beam equilibrium emittances (estimated equal to $\varepsilon_L=10^{-4}$ eVs, $\varepsilon^N{}_H=7\times10^{-4}$ μm, $\varepsilon^N{}_V=7\times10^{-6}$ μm assuming 1% coupling between the horizontal and vertical planes).

Examination of the joint intrabeam and synchrotron radiation damping phenomena during a 10 hours physics beam store at 7 TeV in the first IR upgrade of LHC shows that over the full coast duration the evolution of emittances is kept inside the design values, as the IBS growth is largely balanced by the synchrotron radiation damping.

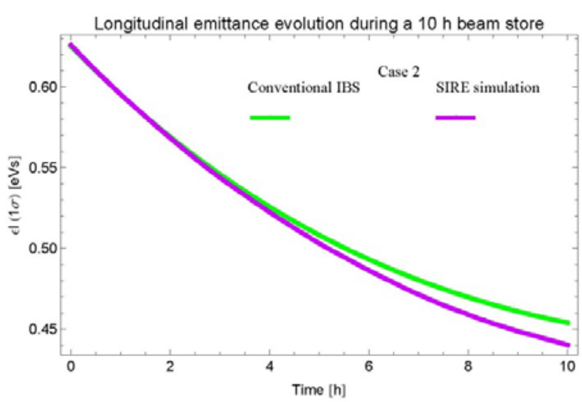


Fig. 10: Comparison of $\varepsilon_L$ between SIRE and straight IBS computations (case 2, Table 1): difference $\delta_{max}(\Delta\varepsilon_L/\varepsilon_L)$ ~ 2%.

SIRE code calculates the IBS effect by reiterative simulation of scattering events between pairs of macro-particles. So, an accurate estimate of the evolution of the particle beam density and emittance growth is done at every simulation time step. SIRE works out also the effect of radiation damping and quantum excitation (together with IBS) for each macro-particle.

Even though SIRE simulation algorithm and Bjorken-Mtingwa analytical formalism make use of distinct approaches to tackle the IBS issue, both techniques agree rather well as shown in figures 5-6 and 10-12.

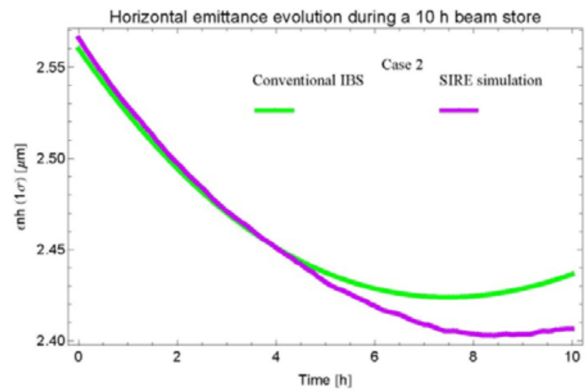


Fig. 11: Comparison of $\varepsilon^N{}_H$ between SIRE and straight IBS computations (case 2, Table 1): difference $\delta_{max}(\Delta\varepsilon_H/\varepsilon_H)$ ~ 1%.

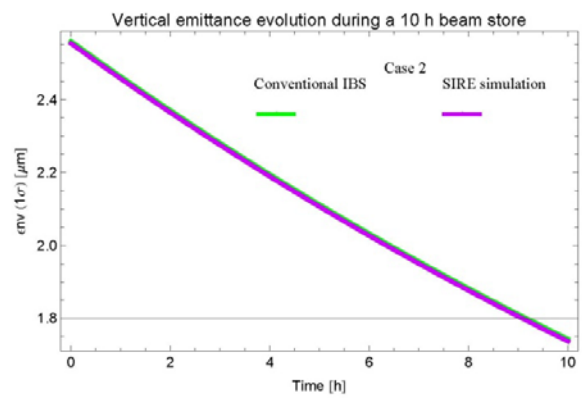


Fig. 12: Comparison of $\varepsilon^N{}_V$ between SIRE and straight IBS computations (case 2, Table 1): difference $\delta_{max}(\Delta\varepsilon_V/\varepsilon_V)$ ~ 0.1%.

## CONCLUSION

Smaller emittance is able to increase the luminosity of the LHC as discussed in [2]. Examinations of the joint IBS and synchrotron radiation damping effects during the 10 hours physics beam store in LHC at 7 TeV show that over the full coast duration:

- Longitudinal and vertical planes (figures 7, 9, 10, 12): the emittances of all the luminosity scenarios are kept within their target specifications.

- Horizontal plane (figures 8, 11): the emittances remain within their requirements for the cases 1 and 2 (nominal and first IR upgrade luminosity scenarios), a small emittance blow-up of ~3% is predictable for case 3 (ultimate beam intensity scenario); while a superior blow-up of ~14% is anticipated for case 4 (peak luminosity scenario).

Globally for most scenarios the evolution of emittances during the 10 hours coast is kept inside the design values. Unlike the other cases, controlled blow-up of the longitudinal emittance might be envisaged for the $10^{35}$ $cm^{-2}s^{-1}$ peak luminosity scheme to minimize the adverse effect of intrabeam scattering.

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