Detailed L3 measurements of Bose-Einstein correlations and a region of anti-correlations in hadronic Z^0 decays at LEP

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L3 preliminary data of two-particle Bose-Einstein correlations are reported for hadronic Z^0 decays in e^+e^- annihilation at LEP. The invariant relative momentum Q is identified as the eigenvariable of the measured correlation function. Significant anti-correlations are observed in the Bose-Einstein correlation function in a broad region of $0.5 - 1.6$ GeV with a minimum at $Q \approx 0.8$ GeV. Absence of Bose-Einstein correlations is demonstrated in the region above $Q \gtrsim 1.6$ GeV. The effective source size is found to decrease with increasing value of the transverse mass of the pair, similarly to hadron-hadron and heavy ion reactions. These feautes and our data are described well by the non-thermal τ-model, which is based on strong space-time momentum-correlations.

Introduction: Boson interferometry provides a powerful femtoscopic tool for the investigation of the space-time structure of particle production processes on the femtometer lengthscales. Bose-Einstein correlations (BEC) of two identical bosons reflect both geometrical and dynamical properties of the particle radiating source [1–7].

In e^+e^- annihilation BEC have been observed [8] to be maximal when the invariant momentum difference of the bosons, $Q = \sqrt{-(p_1 - p_2)^2}$, is small, even when one of the relative momentum components is large. This is not the case in hadron-hadron interactions [9] or in heavy-ion interactions [10], where BEC are found not to depend simply on Q, but to decrease even if Q is small but any of the relative momentum components is large. The size (radius) of the source in heavy-ion collisions has been found to decrease with increasing transverse momentum, p_t, or transverse mass, $m_t = \sqrt{m^2 + p_t^2}$, of the bosons, for a recent on data oriented review see ref. [6]. A similar effect has been seen in $p+p$ collisions [11], as well as in e^+e^- annihilation [12]. Such a behavior that can be described by hydrodynamical models of the source, [5], however, a simple Q dependence of Bose-Einstein correlations in e^+e^- collisions can not be explained in hydrodynamical or thermal models [13].

Event and track selection: The data used in the present analysis were collected by the L3 detector at LEP at an e^+e^- center-of-mass energy of $\sqrt{s} \approx 91.2$ GeV. In total about 0.8 million events with an average number of about 12 well-measured charged tracks are selected. This results in approximately 36 million like-sign pairs of well-measured charged tracks. Events are classified as two- or three-jet events on the basis of the Durham jet algorithm with a jet resolution

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parameter $y_{\text{cut}} = 0.006$, yielding about 0.5 million two-jet and 0.3 million three-jet events. In the present study we report only about the results for two-jet events, more detailed three-jet results will be presented elsewhere.

Bose-Einstein correlation function: The two-particle correlation function of two particles with four-momenta p_1 and p_2 is given by the ratio of the two-particle number density, $\rho_2(p_1, p_2)$, to the product of the two single-particle number densities, $\rho_1(p_1)\rho_1(p_2)$. Since we are here interested only in the correlation R_2 due to Bose-Einstein interference, the product of single-particle densities is replaced by $\rho_0(p_1, p_2)$, the two-particle density that would occur in the absence of Bose-Einstein correlations:

$$R_2(p_1, p_2) = \frac{\rho_2(p_1, p_2)}{\rho_0(p_1, p_2)} . \qquad (1)$$

An event mixing technique is used to construct ρ_0, whereby all tracks of each data event are replaced by tracks from different events having similar multiplicity to the original event.

ρ_0 is corrected for detector acceptance and efficiency in the same way as ρ_2. The mixing technique removes all correlations, for example, resonances and energy-momentum conservation, not just Bose-Einstein correlations. Hence, ρ_0 is also corrected for this by a multiplicative factor which is the ratio of the densities of events to mixed events found using events generated by JETSET [14], without BEC simulation. Thus R_2 is measured by

$$R_2 = \left(R_{2\,\text{data}} R_{2\,\text{gen}} \right) / \left(R_{2\,\text{det}} R_{2\,\text{gen-noBE}} \right) , \quad (2)$$

where data, gen, det, gen-noBE refer, respectively, to the data sample, a generator-level Monte Carlo sample, the same Monte Carlo sample passed through detector simulation and subjected to the same selection procedure as the data, and a generator-level sample of a Monte Carlo without BEC simulation.

The invariant relative momentum Q is eigenvariable of the correlation function in e^+e^- annihilation. In e^+e^- annihilation at lower energy [8] it has been observed that Q is the appropriate (eigen)variable of the Bose-Einstein correlation function, which implies an approximate spherical symmetry of particle emission in

the rest frame of the pair. A priori, one does not expect the hadron source to be so spherically symmetric in jet fragmentation. Recent investigations have, in fact, found an elongation of the source along the jet axis [12, 15–18]. While this effect is well established, the elongation is actually only about 20%, which suggests that a parametrization in terms of the single variable Q, may be a good approximation.

We have checked on L3 data, if indeed Q is an eigenvariable of the BEC or not, and confirmed [19] that this is indeed the case, both for all and for two-jet events: We observe that R_2 does not decrease when both $q^2 = (\vec{p}_1 - \vec{p}_2)^2$ and $q_0^2 = (E_1 - E_2)^2$ are large while $Q^2 = q^2 - q_0^2$ is small, but is maximal for $Q^2 = q^2 - q_0^2 = 0$, independent of the individual values of q and q_0. Furthermore, two-dimensional fits with the parametrization

$$R_2(q, q_0) = 1 + \lambda \exp\left((rq)^2 - (r_0 q_0)^2 \right) \qquad (3)$$

find r and r_0 to be equal. (Where we also note parameter λ, the intercept parameter of the correlation function.) The similar conclusion is found in a different decomposition: $Q^2 = Q_{\text{t}}^2 + Q_{L,B}^2$, where $Q_{\text{t}}^2 = (\vec{p}_{\text{t}1} - \vec{p}_{\text{t}2})^2$ is the component transverse to the thrust axis and $Q_{L,B}^2 = (p_{l1} - p_{l2})^2 - (E_1 - E_2)^2$ combines the longitudinal momentum and energy differences. Again, R_2 is maximal along the line $Q = 0$. This is a non-trivial result. For a hydrodynamical type of source, on the contrary, BEC decrease when any of the relative momentum components is large [2, 5].

In the region of $0 \leq Q \leq 0.5$ GeV, we observe as usual, a positive correlation, due to the Bose-Einstein symmetrization effect of like-sign charged identical boson pairs. In the region $0.5 \leq Q \leq 1.6$ GeV, the measured L3 correlation function R_2 decreases below unity [31], which is indicative of an anti-correlation. This is clearly seen in Fig. 1 by comparing the data in this region to an extrapolation of a linear fit, $\gamma(1 + \epsilon Q)$ that is fitted to our data in the region $Q \geq 1.6\,\text{GeV}$, where γ as an absolute normalization constant and ϵ is a measure of long-range, residual non-Bose-Einstein correlations in our measurement. The extrapolation to the low values of the invariant relative momentum Q is indicated by the dashed line on Fig. 1. Correlation functions with $1 +$ positive definit forms are by definition unable to describe this dip in R_2. This is the primary reason for the failure of several Q dependent
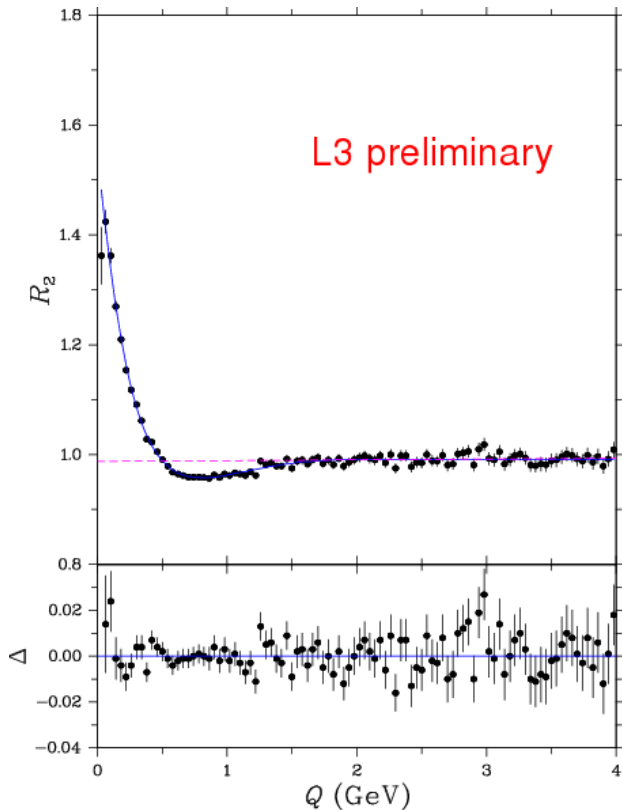
FIG. 1: L3 data on Bose-Einstein correlations of charged particles are compared to a τ model fit. The fit describes the peak structure at low $Q \leq 0.5$ GeV, as well as the region of anti-correlations in $0.5 < Q \leq 1.5$ GeV around $Q \approx 1$ GeV with a $\chi^2/NDF = 90.1/95$. Note that the background distribution in the $Q \geq 2$ GeV is within errors flat, all long-range correlations have been removed. Δ indicates theory - data.

parameterizations. We note that this dip is less apparent, if one only plots (and fits) R_2 for $Q < 2$ GeV as has usually been done in the past.

Many parametrizations discussed earlier, for example Gaussian or Lévy source distributions as well as Edgeworth expansion, have been shown before to be insufficient to describe the BEC [19–21]. These parameterizations assume a static source: the parameter R, representing the size of the source as seen in the rest frame of the pion pair, is a time independent constant. It has, however, been observed that R depends on the transverse mass, $m_t = \sqrt{m^2 + p_t^2} = \sqrt{E^2 - p_z^2}$, of the pions [12]. It has been shown [22, 23] that this dependence can be understood if the produced pions satisfy, approximately, the (generalized) Bjorken-

Gottfried condition [24, 25], whereby the four-momentum of a produced particle and the space-time position at which it is produced are linearly related. Such a correlation between space-time and momentum-energy is also a feature of the Lund string model, which, incorporated in JET-SET [14], is very successful in describing detailed features of the hadronic final states of e^+e^- annihilation.

A model which predicts such a Q-dependence while incorporating the Bjorken-Gottfried condition is the so-called τ-model, introduced in ref. [26]. In this model, it is assumed that the average production point in the overall center-of-mass system, $\overline{x} = (\overline{t}, \overline{r}_x, \overline{r}_y, \overline{r}_z)$, of particles with a given four-momentum $p = (E, p_x, p_y, p_z)$ is

$$\overline{x}^\mu(p^\mu) = a\tau p^\mu \; . \quad (4)$$

In the case of two-jet events, $a = 1/m_t$ where $m_t = \sqrt{m^2 + p_t^2} = \sqrt{E^2 - p_z^2}$ is the transverse mass and $\tau = \sqrt{\overline{t}^2 - \overline{r}_z^2}$ is the longitudinal proper time. For isotropically distributed particle production, the transverse mass is replaced by the mass in the definition of a and τ is the proper time. In the case of three-jet events the relation is more complicated. The second assumption of the τ-model is that the distribution of $x^\mu(p^\mu)$ about its average, $\delta_\Delta(x^\mu(p^\mu) - \overline{x}^\mu(p^\mu))$, is narrower than the proper-time distribution, $H(\tau)$. The emission function of the τ-model is

$$S(x, p) = \int_0^\infty \mathrm{d}\tau H(\tau) \delta_\Delta(x - a\tau p)\rho_1(p) \; , \quad (5)$$

where $H(\tau)$ is the (longitudinal) proper-time distribution, the factor $\delta_\Delta(x - a\tau p)$ describes the strength of the correlations between coordinate space and momentum space variables and $\rho_1(p)$ is the experimentally measurable single-particle spectrum.

The two-pion distribution, $\rho_2(p_1, p_2)$, is related to $S(x, p)$, in the plane-wave approximation, by the Yano-Koonin formula [27]. The resulting two-particle Bose-Einstein correlation function is indeed found to depend on the invariant relative mometum variable Q, as well as on the values of a of the two particles[28]:

$$R_2(p_1, p_2) = 1 + \mathrm{Re}\widetilde{H}\left(\frac{a_1 Q^2}{2}\right)\widetilde{H}\left(\frac{a_2 Q^2}{2}\right) \; , \quad (6)$$

3

where $\widetilde{H}(\omega) = \int \mathrm{d}\tau H(\tau) \exp(i\omega\tau)$ is the Fourier transform (characteristic function) of $H(\tau)$. Note that $H(\tau)$ is normalized to unity. This formula simplifies further if R_2 is measured with the restriction $a_1 \approx a_2 \approx \bar{a}$. In that case, for two-jet events R_2 becomes

$$R_2(p_1, p_2) = 1 + \lambda \, \mathrm{Re}\widetilde{H}^2 \left(\frac{Q^2}{2\overline{m}_\mathrm{t}} \right) . \qquad (7)$$

Thus for a given average of a of the two particles,

R_2 is found to depend only on the invariant relative momentum Q. Further, the model predicts a specific dependence on $\bar{a}$, which for two-jet events is a specific dependence on $\overline{m}_\mathrm{t}$ [32].

Since there is no particle production before the onset of the collision, $H(\tau)$ should be a one-sided distribution. We choose a one-sided Lévy distribution [28], which has the characteristic function [29] (for $\alpha \neq 1$)

$$\widetilde{H}(\omega) = \exp \left[-\frac{1}{2} \left(\Delta\tau |\omega| \right)^\alpha \left(1 - i \, \mathrm{sign}(\omega) \tan\left(\frac{\alpha\pi}{2} \right) \right) + i\,\omega\tau_0 \right] , \qquad (8)$$

where the parameter τ_0 is the proper time of the onset of particle production and $\Delta\tau$ is a measure of the width of the proper-time distribution. For the special case $\alpha = 1$, see, for example, [30].

We have tested that parameter τ_0 is within errors zero, hence we fixed it to zero. Using this simplification, the characteristic function in Eq. (6) yields

$$R_2(Q, a_1, a_2) = \gamma \left\{ 1 + \lambda \cos\left[\tan\left(\frac{\alpha\pi}{2} \right) \left(\frac{\Delta\tau Q^2}{2} \right)^\alpha \frac{a_1^\alpha + a_2^\alpha}{2} \right] \exp\left[-\left(\frac{\Delta\tau Q^2}{2} \right)^\alpha \frac{a_1^\alpha + a_2^\alpha}{2} \right] \right\} (1 + \epsilon Q) . \qquad (9)$$

The result of fitting Eq. (9) to the L3 two-jet event Bose-Einstein correlation data is presented on Fig. 1. The best values of the fit parameters and their errors are shown in Table I. Their first uncertainty is statistical, the second systematic. The confidence levels are shown in Table II for varying the transverse mass of the particles independently. The results indicate, that the τ-model is consistent with the L3 data, the fit quality is good and the model is able to describe data well not only the low relative momentum region, but also the region of $0.5 \leq Q \leq 1.6$ GeV, where anti-correlations are observed. In the large relative momentum region of $Q > 1.6$ GeV, no significant long-range correlations are found and the corresponding parameter ϵ is measured to be zero

within errors. Based on the analysis of the Bose-Einstein correlations and the single particle spectra in terms of the τ-model the space-time evolution of the particle emitting source can also be reconstructed [28]. The first L3 preliminary results on such an extremely fast movie were reported in refs. [20, 21].

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TABLE I: Results of the fit of Eq. (9) for two-jet events. The first uncertainty is statistical, the second systematic.

parameter	
λ	$0.58 \pm 0.03^{+0.08}_{-0.24}$
α	$0.47 \pm 0.01^{+0.04}_{-0.02}$
$\Delta\tau$ (fm)	$1.56 \pm 0.12^{+0.32}_{-0.45}$
ϵ (GeV^{-1})	$0.001 \pm 0.001 \pm 0.003$
γ	$0.988 \pm 0.002^{+0.006}_{-0.002}$
χ^2/DoF	90/95
confidence level	62%

TABLE II: Confidence levels and the values of λ found in fits of Eq. (9) for two-jet events in various regions of the m_{t1}-m_{t2} plane with α and $\Delta\tau$ fixed to the result of the fit to the entire plane.

m_t regions (GeV)		average m_t (GeV)		confidence level (%)
m_{t1}	m_{t2}	$Q < 0.4$	all	
0.14 – 0.26	0.14 – 0.22	0.19	0.19	10
0.14 – 0.34	0.22 – 0.30	0.27	0.27	48
0.14 – 0.46	0.30 – 0.42	0.37	0.37	74
0.14 – 0.66	0.42 – 4.14	0.52	0.52	13
0.26 – 0.42	0.14 – 0.22	0.25	0.26	22
0.34 – 0.46	0.22 – 0.30	0.32	0.33	33
0.46 – 0.58	0.30 – 0.42	0.43	0.44	34
0.66 – 0.86	0.42 – 4.14	0.65	0.65	66
0.42 – 0.62	0.14 – 0.22	0.34	0.34	17
0.46 – 0.70	0.22 – 0.30	0.41	0.41	55
0.58 – 0.82	0.30 – 0.42	0.52	0.52	59
0.86 – 1.22	0.42 – 4.14	0.80	0.81	24
0.70 – 4.14	0.22 – 0.30	0.59	0.65	4
0.82 – 4.14	0.30 – 0.42	0.71	0.76	11

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