Neutron star properties derived from relativistic mean field

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Abstract
The equation of state of asymmetric nuclear matter given by the parameterized form of the relativistic Brueckner-Hartree-Fock mean field with vector cross interaction is applied to construct spherically symmetric neutron star models. The masses and radii of the models are given as functions of the central energy density and compared with astrophysical data. The calculated radii appear to be confined to a narrow band between 12 and 13 km, nearly independent of the calculated masses. Our models are in agreement with known data and the ones with vector cross interaction cope well with the recent limitations imposed by the double pulsar J0737-3039.

1 Introduction

A wide spectrum of different equations of state (EoS) of nuclear matter and their applications to astrophysical problems has been reported in literature (see, e.g., [1–6]). Some of the EoS collections (even though not all of them are up-to-date already) give an amazingly rich general overview of the state-of-the-art, whereas the others emphasize some specific aims. All these EoS yield (nearly) the same properties close to the standard nuclear density ($\rho_N \approx$ 0.16 nucleon/fm^3 $\approx 2.7 \times 10^{14}$ g/cm^3), but when one is far off this value, s/he has to rely more on underlying principles than on possible experimental verification of predicted physical observables.

Here we concentrate our attention on a relativistic asymmetric nuclear matter where the EoS stem from an assumed form of the interaction Lagrangian. The calculations use the relativistic mean-field theory with allowance for an isospin degree of freedom [7,8]. Assuming static, homogeneous and infinite nuclear matter allows for using its invariances and symmetries. We employed the Dirac-Brueckner-Hartree-Fock mean-field approach in its parameterized form suggested by Gmuca [9] which reproduces the nuclear matter results of Huber *et al.* [10]. That has been used to calculate high-density behavior of asymmetric nuclear matter with varying neutron-to-proton ratio [11]. The proton fraction has been determined from the condition of β-equilibrium and charge neutrality, and it is density-dependent. We have extended our calculations up to $4 \times \rho_N$, what is the region typical for the interior of neutron stars.

The EoS is used to model the static, spherically symmetric neutron star in the framework of general relativity. The calculated properties i.e. its mass, radius etc. can be efficiently tested by confronting them with astrophysical data of various kind (see, e.g., [12–19]). Two recent cases were specifically selected, namely the isolated neutron star RXJ 1856.5–3754 and the double pulsar J0737–3039.

2 Asymmetric nuclear matter in relativistic mean-field approach

We follow here the Dirac-Brueckner-Hartree-Fock (DBHF) mean field (see [20–23] which easily allows to consider different neutron-proton composition of the matter, and also the inclusion of non-nucleonic degrees of freedom.

The full mean-field DBHF calculations of nuclear matter (Huber *et al.* [10, 24]) have been parameterized by Kotulič Bunta and Gmuca [25], and we employ their parameterization reproducing the calculations of Huber *et al.* [10, 24] with one-boson-exchange (OBE) potential A of Brockmann and Machleidt [26]. The model Lagrangian density includes the nucleon field ψ, isoscalar scalar meson field σ, isoscalar vector meson field $\boldsymbol{\omega}$, isovector vector meson field $\boldsymbol{\rho}$ and isovector scalar meson field δ, including also the vector cross-interaction. The Lagrangian density in the form used by Kotulič Bunta and Gmuca [25] is

$$\begin{aligned}\mathcal{L}(\psi,\sigma,\boldsymbol{\omega},\boldsymbol{\rho},\delta) &= \bar{\psi}[\gamma_\mu(i\partial^\mu - g_\omega\boldsymbol{\omega}^\mu) - (m_\mathrm{N} - g_\sigma\sigma)]\psi \\ &+\frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - {m_\sigma}^2\sigma^2) - \frac{1}{4}\boldsymbol{\omega}_{\mu\nu}\boldsymbol{\omega}^{\mu\nu} + \frac{1}{2}{m_\omega}^2\boldsymbol{\omega}_\mu\boldsymbol{\omega}^\mu \\ &-\frac{1}{3}b_\sigma m_\mathrm{N}(g_\sigma\sigma)^3 - \frac{1}{4}c_\sigma(g_\sigma\sigma)^4 + \frac{1}{4}c_\omega({g_\omega}^2\boldsymbol{\omega}_\mu\boldsymbol{\omega}^\mu)^2 \\ &+\frac{1}{2}(\partial_\mu\delta\partial^\mu\delta - {m_\delta}^2\delta^2) + \frac{1}{2}{m_\rho}^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\boldsymbol{\rho}^{\mu\nu} \\ &+\frac{1}{2}\Lambda_V({g_\rho}^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu)({g_\omega}^2\boldsymbol{\omega}_\mu\boldsymbol{\omega}^\mu) - g_\rho\boldsymbol{\rho}_\mu\bar{\psi}\gamma^\mu\tau\psi + g_\delta\delta\bar{\psi}\tau\psi, \end{aligned} \quad (1)$$

where the antisymmetric tensors are

$$\begin{aligned}\boldsymbol{\omega}_{\mu\nu} &\equiv \partial_\nu\boldsymbol{\omega}_\mu - \partial_\mu\boldsymbol{\omega}_\nu, \\ \boldsymbol{\rho}_{\mu\nu} &\equiv \partial_\nu\boldsymbol{\rho}_\mu - \partial_\mu\boldsymbol{\rho}_\nu;\end{aligned} \quad (2)$$

the strength of the interactions of isoscalar and isovector mesons with nucleons is given by (dimensionless) coupling constants g's and the self-coupling constants (also dimensionless) are b_σ (cubic), c_σ (quartic scalar) and c_ω (quartic vector). The second and the fourth lines represent non-interacting Hamiltonian for all mesons, Λ_V is the cross-coupling constant of the interaction between $\boldsymbol{\omega}$ and $\boldsymbol{\rho}$ mesons. Furthermore, m_N is the nucleon mass, $\partial^\mu \equiv \frac{\partial}{\partial x_\mu}$ and γ's are the Dirac matrices [20, 25, 27].

This Lagrangian is the starting point to obtain the nuclear matter EoS. These EoS have been used as an input to construct the models of neutron stars and their properties (mass, radius etc.) for different central parameters.

The EoS of Kotulič Bunta and Gmuca, both with and without the cross-interaction term (parameter sets A and B, see [25]), which have been found to be a good description of asymmetric nuclear matter, are easily expressed up to about $4 \times \rho_N$.

3 Neutron star models

We consider spherically symmetric models with matter being in β-equilibrium and electrically neutral.

3.1 β-equilibrium

The total energy density of *n-p-e-*μ matter is given as

$$\mathcal{E} = \mathcal{E}_\mathrm{B}(n_\mathrm{B}, x_\mathrm{p}) + \mathcal{E}_e(n_e) + \mathcal{E}_\mu(n_\mu), \quad (3)$$

where $\mathcal{E}_\mathrm{B}(n_\mathrm{B}, x_\mathrm{p})$ is the binding energy density of asymmetric nuclear matter, n_i is the number density of different particles (i = n, p, e, μ), $n_\mathrm{B} = n_\mathrm{p} + n_\mathrm{n}$ is the baryon number density and $x_\mathrm{p} = n_\mathrm{p}/n_\mathrm{B}$ is the proton fraction. The leptonic contributions $\mathcal{E}_l(n_l)$ (l = e, μ) to the total energy density are given by

$$\mathcal{E}_l(n_l) = \frac{2}{h^3}\int\limits_0^{p_{F(l)}} \left(m_l^2c^4 + p^2c^2\right)^{1/2} 4\pi p^2 \mathrm{d}p, \quad (4)$$

where $p_{F(l)}$ is the Fermi momentum of *l*-th kind of particle.

The matter in neutron stars is in β-equilibrium, i.e. in equilibrium with respect to $\mathrm{n} \leftrightarrow \mathrm{p} + \mathrm{e}^- \leftrightarrow \mathrm{p} + \mu$ [1]. The equilibrium is given by equality of chemical potentials $\mu_\mathrm{n} = \mu_\mathrm{p} + \mu_\mathrm{e} = \mu_\mathrm{p} + \mu_\mu$, where the chemical potential of each kind of particle is given by $\mu_i = \partial\mathcal{E}/\partial n_i$.

[1] The (anti)neutrinos contribution could be neglected, because the matter is assumed to be cold enough that they can freely escape.

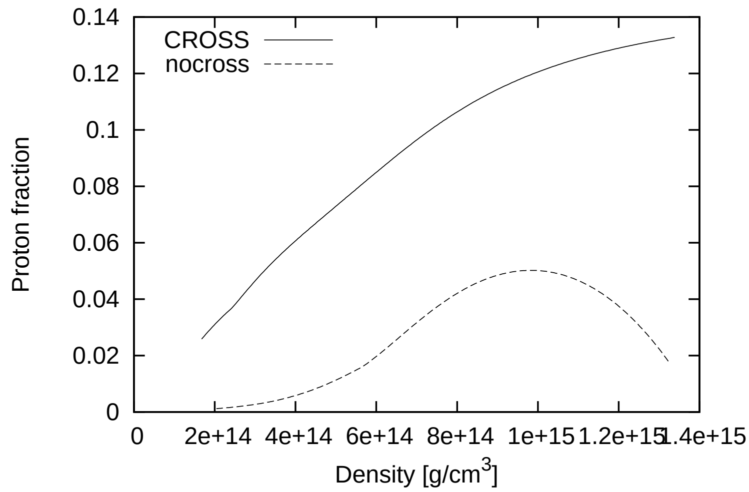

Fig. 1: Proton fraction calculated from β-equilibrium as a function of density for both considered interactions.

The binding energy density of asymmetric nuclear matter could be expressed in terms of proton fraction x_p as [28]

$$\mathcal{E}_\mathrm{B}(n_\mathrm{B}, x_\mathrm{p}) = \mathcal{E}_\mathrm{SNM}(n_\mathrm{B}) + (1 - 2x_\mathrm{p})^2 S(n_\mathrm{B}), \tag{5}$$

where $\mathcal{E}_\mathrm{SNM}$ is the energy density of symmetric nuclear matter ($x_\mathrm{p} = 0.5$) and $S(n_\mathrm{B})$ is the symmetry energy density, that corresponds to the difference of binding energy density between pure nuclear matter and symmetric nuclear matter

$$S(n_\mathrm{B}) = \mathcal{E}_\mathrm{B}(n_\mathrm{B}, x_\mathrm{p} = 0) - \mathcal{E}_\mathrm{B}(n_\mathrm{B}, x_\mathrm{p} = 0.5). \tag{6}$$

The condition of β-equilibrium then reads

$$\mu_\mathrm{e} = \mu_\mu = \mu_\mathrm{n} - \mu_\mathrm{p} = 4\frac{S(n_\mathrm{B})}{n_\mathrm{B}}\,(1 - 2x_\mathrm{p})\,. \tag{7}$$

and is solved together with condition of charge neutrality ($n_\mathrm{p} = n_\mathrm{e} + n_\mu$) to obtain the proton fraction of neutron star matter. The are depicted in Fig. 1.

3.2 EoS for low densities

The nuclear EoS have been the dominant input for the calculations in the high-density region, namely $\rho \geq 10^{14}$ $\mathrm{g/cm^3}$. For lower densities, the EoS used are the following (see also Fig. 2):

- Feynman-Metropolis-Teller EoS for $7.9\ \mathrm{g/cm^3} \leq \rho \leq 10^4\,\mathrm{g/cm^3}$ where matter consists of e^- and ${}^{56}_{26}\mathrm{Fe}$, [29];
- Baym-Pethick-Sutherland EoS for $10^4\ \mathrm{g/cm^3} \leq \rho \leq 4.3 \times 10^{11}\,\mathrm{g/cm^3}$ with Coulomb lattice energy corrections [30];
- Baym-Bethe-Pethick EoS for $4.3\ \mathrm{g/cm^3} \times 10^{11} \leq \rho \lessapprox 10^{14}\,\mathrm{g/cm^3}$: here, e^-, neutrons and equilibrated nuclei calculated using the compressible liquid drop model are considered [31].

We use the internal Schwarzschild metric with $c = G = 1$ (see, e.g., [32]). The hydrostatic equilibrium is given by the Tolman-Oppenheimer-Volkov equation (TOV) [33,34], which in this notation reads

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -(\rho + p)\frac{m(r) + 4\pi r^3 p}{r(r - 2m(r))}, \tag{8}$$

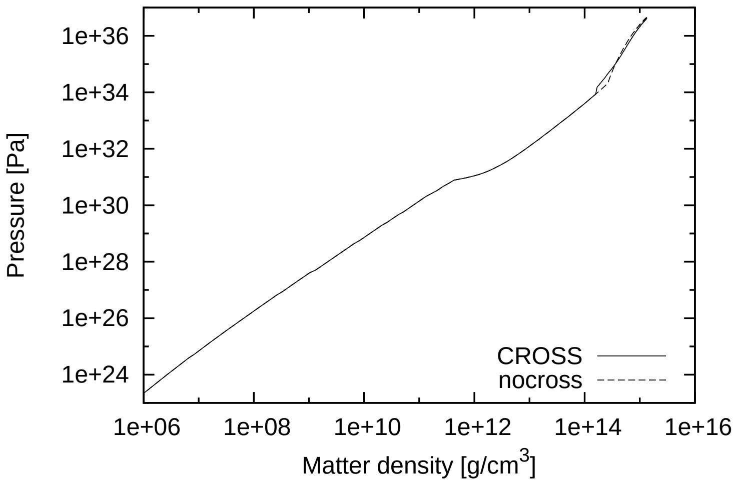

Fig. 2: Dependence of pressure in the matter density. In the nuclear region, both the calculations with and without the vector cross interaction are drawn.

where

$$m(r) = 4\pi \int_0^r \rho {r_1}^2 \mathrm{d}r_1 \tag{9}$$

is the mass inside the sphere of radius r, ρ is the energy density and p is the pressure, that are both functions of radial coordinate r. Integration of TOV starting from given central energy density ρ_c uses the EoS and finally yields the radius R, given by the boundary condition $p(R) = 0$, and the gravitational mass $M = m(R)$ of the neutron star. Another useful quantity to calculate is the so-called baryonic mass that is given

$$M_\mathrm{B} = 4\pi \mathrm{u} \int_0^R n_\mathrm{B}(r) \left[1 - \frac{2m(r)}{r}\right]^{-1/2} r^2 \mathrm{d}r, \tag{10}$$

where $n_\mathrm{B}(r)$ is the baryon number density at the radius r and u is the atomic mass unit.

4 Results

The resulting masses and the star radii are shown in Figs. 3 and 4. Whereas the mass–density dependence (Fig. 3) is monotonic for both sets (with and without the cross interaction), the mass–radius one (Fig. 4) shows S-shape typical for this kind of calculations. For a comparison, we have drawn also the relation imposed by the analysis of the isolated neutron star RX J1856.5–3754.

4.1 Recent observations — EoS tests

4.1.1 Double pulsar J0737–3039

Podsiadlowski *et al.* [15] investigated possible formation scenarios of double pulsar J0737–3039. They have shown that one can test EoS assuming the double pulsar is formed in electron-capture supernova. Such scenario enables formation of the second pulsar of very accurately measured mass $M = 1.2489 \pm 0.0007 M_\odot$ [19]. If this pulsar is born under the presented scenario, its baryonic mass M_B is in the range 1.366 to 1.375 $M_\odot$. The relation between the gravitational and the baryonic masses together with the limitations derived from the double pulsar observations are presented in Fig. 5. One can see that the interaction with cross interaction meet the requirement represented by the rectangle, while the one without cross interaction does not.

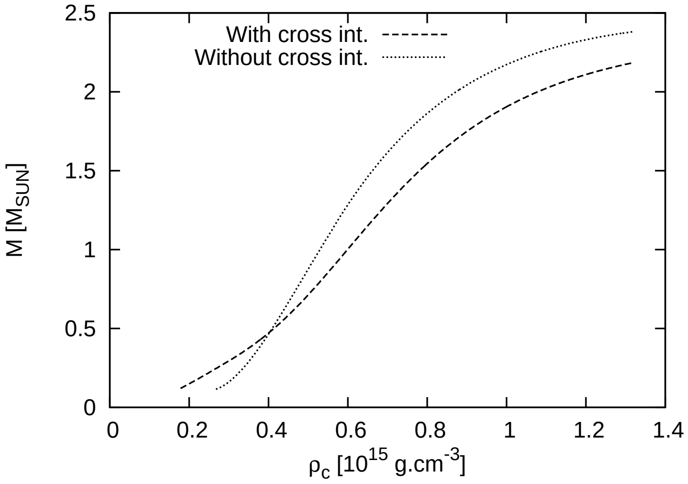

Fig. 3: Predicted mass of the neutron star as a function of the central density for two types of the EoS, namely with (dashed) and without (dotted) cross-interaction term.

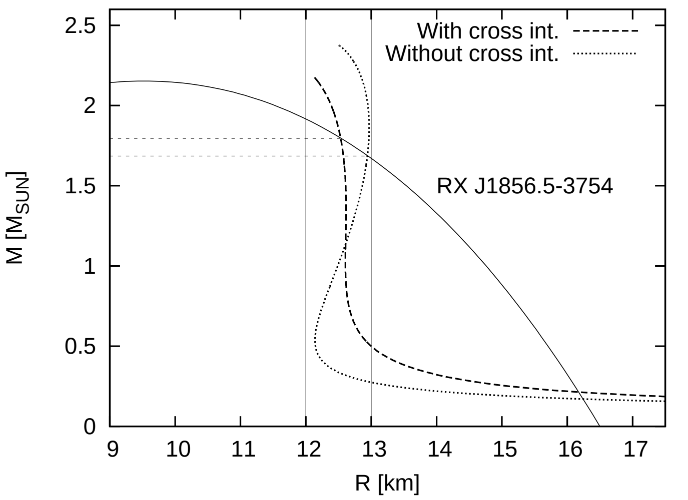

Fig. 4: The same calculations as in the preceding figure, but the radius-mass dependence. In addition, the mass-radius relation of the RX J1856.5–3754 pulsar is depicted as a full line.

4.1.2 Isolated neutron star RX J1856.5–3754

Several authors [16, 17] discussed observations of the isolated neutron star RX J1856.5–3754 and they found constraints on the mass-radius relation of this particular neutron star. They found the lower limit of the apparent radius to be $R_\infty = 16.5$ km and a simple relation between the neutron star mass and its radius, which is drawn also in Fig. 4. That could serve as a test of equation of state and also as an estimate for the mass of this neutron star. Our EoS gives mass of the isolated neutron star RX J1856.5–3754 to be $M > 1.795 M_\odot$ ($M > 1.685 M_\odot$ neglecting vector cross interaction).

5 Discussion

Our results (Figs. 3 to 6) do not contradict the recent compilations of the deduced radii of observed and analyzed compact objects. Several dozens of neutron stars and/or similar objects have their masses reported; a great majority of them is in very close vicinity of 1.4 $M_\odot$, and only very few are significantly

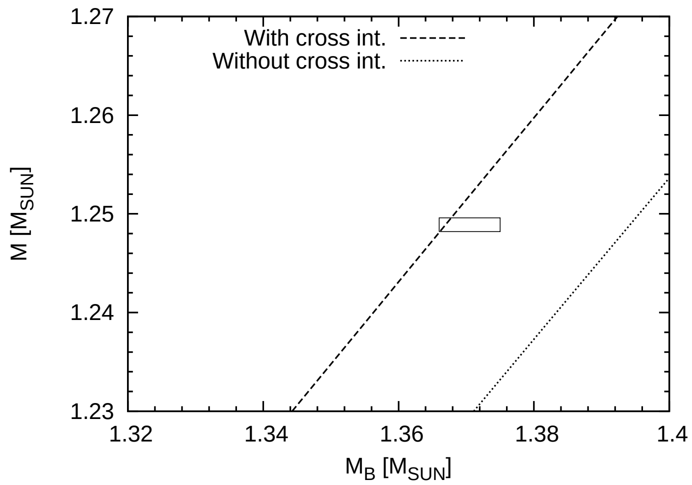

Fig. 5: Relation of calculated gravitational mass M and the baryonic one M_B with and without the cross interaction. The limitations imposed by the analysis of the J0737–3039 double pulsar are drawn as a small rectangle.

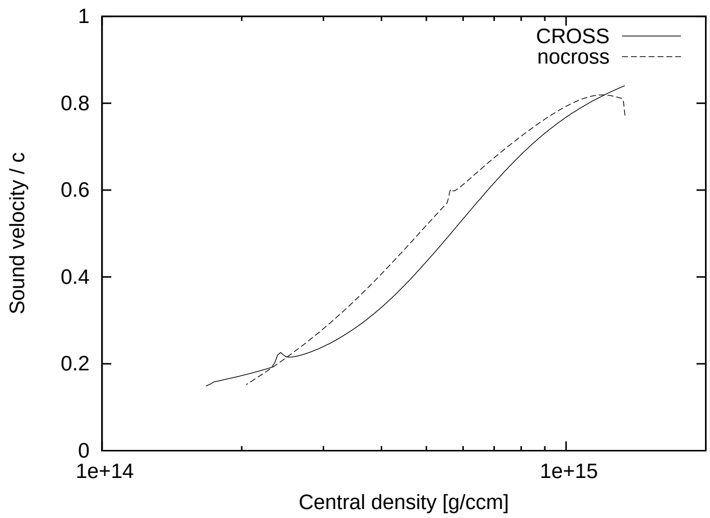

Fig. 6: Sound velocity relative to that of light, as calculated for our EoSs.

above (see, e.g., the compilations in [35, 36] and observations and analyses [5, 6, 15–19, 23, 36–42]), but the interpretation of the observations contains some model-dependent aspects. Note, however, that recent results of the data fitting of kHz quasiperiodic oscillations observed in the low-mass X-ray systems containing neutron stars indicate relatively high masses of $M > 2M_\odot$ [43–46], which have not been considered until recently. Our calculations allow for the existence of neutron stars even for so heavy masses.

We have also tested the velocity of sound in nuclear matter with respect to the velocity of light — whenever the former one exceeds c, one enters physically forbidden region (causality violation). This dependence is drawn in Fig. 6, and we are safely below the limit. (A small bumps seen in both curves are just artefacts of joining two different density regions.)

6 Conclusions

We have employed the parameterized form of the relativistic mean-field EoS for asymmetric nuclear matter with vector cross interaction. The proton fraction was varied in accord with the need of the β-equilibrium and charge neutrality. Assuming spherically symmetric geometry and using TOV equation, we constructed models of neutron stars for different central parameters. We have found the neutron star radii lie in a narrow band between 12 and 13 kms and the masses do not contradict the constraints imposed by recent neutron star analyses, including that of a double pulsar J0737–3039. They do not contradict to the earlier reported observed ones either and they are in line with the results published by other authors using different approaches. Our present calculations have been done considering only neutrons and protons in β-equilibrium with electrons and muons. We aim to continue in tests of given EoS in future. One of our plans is to include hyperons. Another is to perform more detailed tests based on the fitting of observational data of quasi-periodic oscillations in low-mass X-ray systems measurements. This necessitates to investigate the rotational effects on neutron star models based on the Hartle-Thorne metric reflecting mass, spin and the quadrupole moment of the neutron star. All these improvements could bring a new information on the validity of EoS ([14, 47]).

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