A neutrino-nucleon interaction generator for the FLUKA Monte Carlo code

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Abstract

Event generators that handle neutrino-nucleon interaction have been developed for the FLUKA code [1]. In earlier FLUKA versions only quasi-elastic (QEL) interactions were included, and the code relied on external event generators for the resonance (RES) and deep inelastic scattering (DIS). The new DIS+RES event generator is fully integrated in FLUKA and uses the same hadronization routines as those used for simulating hadron-nucleon interactions. Nuclear effects in neutrino-nucleus interactions are simulated within the same framework as in the FLUKA hadron-nucleus interaction model (PEANUT), thus profiting from its detailed physics modelling and longstanding benchmarking. The generators are available in the standard FLUKA distribution. They are presently under development and several improvements are planned to be implemented. The physics relevant to the neutrino-nucleon interactions and the results of comparisons with experimental data are discussed.

1 Introduction

The next generation of large-scale neutrino experiments will require substantial Monte Carlo studies that handle nuclear effects with reasonable quality. The FLUKA Monte Carlo code [1] has a well tested environment for nuclear interactions, and is therefore a good tool for these kind of studies. Recent efforts of improving the code include stand-alone event generators for neutrino-nucleon deep inelastic scattering and resonance scattering, added to the already existing package for quasi-elastic interactions. The qualities of the FLUKA nuclear environment have been extensively explained elsewhere [2, 3] and will only be briefly touched upon here, while the main focus will be on explaining the properties of the new neutrino features and plans for future improvements.

2 The FLUKA code

FLUKA is a general purpose Monte Carlo tool for calculations of particle transport and interactions with matter. The abilities of the code covers a wide range of applications including proton and electron accelerator shielding, target design, calorimetry, activation, dosimetry, detector design, Accelerator Driven Systems, cosmic rays, neutrino physics, and radiotherapy.

2.1 The PEANUT package

A powerful part of FLUKA is the PEANUT (Pre-Equilibrium Approach to NUclear Thermalization) [2, 4] nuclear interaction package, which handles interactions of nucleons, pions, kaons and γ-rays from threshold (or 20 MeV neutrons) up to a few GeV. All reaction mechanisms in this model are embedded in a generalized intranuclear cascade which smoothly overlaps with statistical preequilibrium emission

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and is followed by evaporation and gamma deexcitation. Recently, PEANUT has been extended to the high energy range, through a smooth onset of the Glauber-Gribov cascade. In this approach, hadron-hadron interactions at high energies are based on the Dual Parton Model [5], in which interactions result in the creation of QCD color strings from which hadrons are generated [1, 6]. The FLUKA nuclear interaction model has capabilities which have been successfully tested in hadron and photon induced reactions [2]. If we assume that the models for nuclear interaction, thermalization and hadronization work equally well irrespective of the incident interaction vertex, then is is reasonable to assume that the model should work well also for neutrino interactions. This is the approach we have adopted for the development of neutrino-nucleon interaction models within FLUKA.

2.2 Neutrino interactions in FLUKA

In the framework of the ICARUS experiment [7] a model for neutrino-nucleus quasi-elastic scattering (QEL), based on Ref. [8], was developed within FLUKA in 1997. For the same purpose, FLUKA was also coupled successfully to the NUX model [9, 10], which handles deep inelastic scattering (DIS) and resonance scattering (RES). The NUX-FLUKA was extensively cross-checked with data from the NOMAD experiment [11] at CERN. For these generators (QEL in FLUKA and DIS+RES from NUX) all initial and final state interactions were managed by the PEANUT model. From this experience, important knowledge was obtained about the effects of reinteractions in the nucleus [3], and was useful to produce results for the ICARUS collaboration.

NUX depends on Pythia [12] and the linking to another code package means that a number of separate parameters related to the hadronization are used, although this should be handled by the internal hadronization routines within FLUKA. This was in conflict with the ideal of consistency of models and minimization of the number of parameters for the design of the FLUKA code. The resonance region in NUX is considered on average in the sampling of DIS events, according to the duality principle, and thus does not contain a true description of the resonance region. Furthermore NUX was only available within the ICARUS collaboration, with limited possibilites for further development within FLUKA. For these reasons it was deemed necessary to have internal event generators, and now we have developed dedicated event generators for DIS and RES that are fully integrated within FLUKA.

The objective was to create event generators that handles all neutrino flavors for Neutral Current (NC) and Charged Current (CC) interactions over a wide energy range, and that would be flexible for the use of different PDF-sets (see below). The result is the packages NUNDIS and NUNRES, which are still under development, but available for use in the standard FLUKA distribution.

3 Description of the event generators

3.1 NUNDIS

NUNDIS handles deep inelastic scattering for Neutral Current (NC) and Charged Current (CC) interactions for incident neutrinos and antineutrinos of all flavours on protons and neutrons. It samples Q^2 and Bjorken-x, calculates structure functions and differential cross sections, determines whether the incident parton is valence or sea quark, samples the incident and outgoing parton flavors, calculates the invariant mass of the hadronic system, and for CC interactions it also calculates the polarization of outgoing charged leptons.

NUNDIS uses available parton distribution functions (PDFs) in order to obtain probability distributions for the kinematical variables Q^2 and Bjorken-x. PDFs are obtained from global fits to experimental data for one or more physical processes which can be calculated from perturbative QCD, and there are a number of different approaches available. For the moment NUNDIS works for neutrino energies up to 100 TeV with the GRV98 [13] PDF-set as default, but it is also possible to use GRV94 [14] and BBS [15]. Future updates will implement other PDF-sets such as MSTW [16] and CTEQ [17]. Extrapolations are performed for Q^2 and Bjorken-x whenever these variables are beyond the ranges defined by the PDF-set.

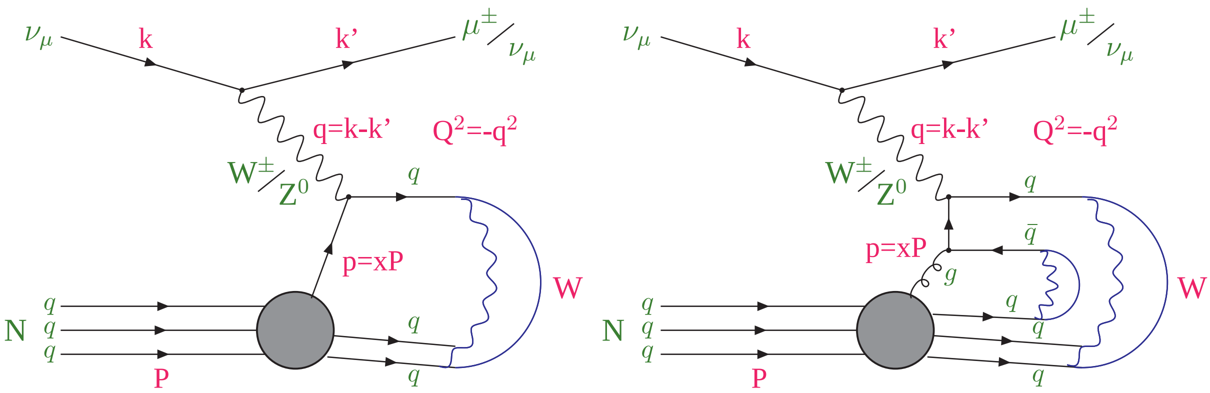

Fig. 1: Feynman diagrams of deep inelastic neutrino-nucleon scattering for CC/NC interaction with the incident neutrino interacting with a valence quark (left) and with a sea quark (right). All kinematical variables are given with red letters. The blue regions indicate the $q-\bar{q}$ and $q-qq$ hadronization strings that are considered in PEANUT.

For high Q^2 and low x extrapolations are generally straightforward and without problems, but for low Q^2 special treatment is necessary, see Sec. 3.4.

Fig. 1 shows the Feynman diagrams for neutrino–nucleon interactions in CC and NC interactions, with the kinematical variables in red letters. The lefthand plot shows the case when the incident neutrino interacts with a valence quark, and the righthand plot the case for interaction with a sea quark. All calculations in NUNDIS are done in the lab system. PEANUT handles the transformations from different reference frames before and after the interaction. For a neutrino with energy E_ν incident on a nucleon with mass M, with total energy s in the CMS system, the kinematics for the interaction is completely described by three of the following variables:

- $Q^2 = 2ME_\nu xy$: squared momentum transfer.
- $W^2 = M^2 + Q^2\frac{1-x}{x}$: squared invariant mass of the outgoing hadronic system.
- x : Bjorken-x, the fraction of the nucleon momentum carried by the struck parton in the infinite momentum frame.
- $y = \frac{W^2+Q^2-M^2}{s-M^2}$: inelasticity, the fraction of energy transferred to the hadronic system.

Each of these have kinematical constraints that need to be fulfilled. In NUNDIS the variables Q^2 and x are sampled and checked against the constraints for all four variables.

All random sampling is performed with the importance sampling method, with guess functions g that we strive to design as close as possible to the true sampling functions f, and variable substitutions are frequently used to speed up the sampling. The sampling of Q^2 is done over the allowed (Q^2,x) phase space where the sampling function is $f(Q^2) = \int f(Q^2,x)dx = \int \frac{d^2\sigma}{dxdQ2}\,dx = \frac{d\sigma}{dQ^2}$. Bjorken-x is sampled from the sampling function $f(Q^2,x) = \frac{d^2\sigma}{dxdQ^2}$. In both cases the double differential cross sections are calculated, and we follow the formulation given by Paschos and Yu [18]:

$$\frac{d^2\sigma}{dxdQ^2} = \frac{d^2\sigma}{dxdy}\cdot\frac{dy}{dQ^2} = \frac{d^2\sigma}{dxdy}\cdot\frac{1}{2ME_\nu x}\,, \tag{1}$$

$$\frac{d^2\sigma}{dxdy} = \frac{G_F^2 ME_\nu}{\pi(1+Q^2/M_{W/Z}^2)^2}\sum_{i=1}^{5}A_i(x,y,E_\nu)F_i(Q^2,x)\,, \tag{2}$$

where G_F is the Fermi constant, $M_{W/Z}$ is the vector boson mass, and the coefficients $A_i(x,y,E_\nu)$ are:

$$A_1 = y\left(xy + \frac{m_\ell^2}{2ME_\nu}\right)\,,$$

$$\begin{aligned}
A_2 &= 1 - y\left(1 + \frac{Mx}{2E_\nu}\right) - \frac{m_\ell^2}{4E_\nu^2}\,, \\
A_3 &= \pm y\left[x\left(1 - \frac{y}{2}\right) - \frac{m_\ell^2}{4ME_\nu}\right]\,, \\
A_4 &= \frac{m_\ell^2}{2ME_\nu}\left(y + \frac{m_\ell^2}{2ME_\nu x}\right)\,, \\
A_5 &= -\frac{m_\ell^2}{ME_\nu}\,.
\end{aligned} \tag{3}$$

The minus sign for A_3 is used for anti-neutrinos.

The structure functions $F_i(Q^2, x)$ in Eq. (2) follow the standard definition given by the Particle Data Group [19] and are here shown for the cases $\nu + p$ and $\bar{\nu} + p$. For neutrons the convention of isospin symmetry is taken into account. For CC interactions the structure functions are:

$$F_2^{\nu p}(Q^2, x) = 2x[d + s + \bar{u} + \bar{c}], \tag{4}$$

$$xF_3^{\nu p}(Q^2, x) = 2x[d + s - \bar{u} - \bar{c}], \tag{5}$$

$$F_2^{\bar{\nu} p}(Q^2, x) = 2x[u + c + \bar{d} + \bar{s}], \tag{6}$$

$$xF_3^{\bar{\nu} p}(Q^2, x) = 2x[u + c - \bar{d} - \bar{s}], \tag{7}$$

where u, d, s, etc. are the PDFs for each parton flavor. For NC interactions the structure functions are:

$$F_2^{(\nu p, \bar{\nu} p)}(Q^2, x) = 2x\left[\left(g_L^2 + g_R^2\right)[u + \bar{u} + c + \bar{c}] + \left(g_L'^2 + g_R'^2\right)\left[d + \bar{d} + s + \bar{s}\right]\right] \tag{8}$$

$$F_3^{(\nu p, \bar{\nu} p)}(Q^2, x) = 2x\left[\left(g_L^2 - g_R^2\right)[u - \bar{u} + c - \bar{c}] + \left(g_L'^2 - g_R'^2\right)\left[d - \bar{d} + s - \bar{s}\right]\right] \tag{9}$$

$$g_L = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W\,, \tag{10}$$

$$g_R = -\frac{2}{3}\sin^2\theta_W\,, \tag{11}$$

$$g_L' = -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\,, \tag{12}$$

$$g_R' = \frac{1}{3}\sin^2\theta_W\,, \tag{13}$$

where θ_W is the Weinberg angle. Eq. (2) is simplified in leading order due to the Callan-Gross relation [20] for F_1, and the Albright-Jarlskog relations [21] for F_4 and F_5:

$$\begin{aligned}
F_1 &= \frac{F_2}{2x}\,, \\
F_4 &= 0\,, \\
F_5 &= \frac{F_2}{x}\,.
\end{aligned}$$

If the sampled (Q^2, x) is accepted then the incident parton flavor is sampled, and sampling also determines whether the incident parton is a valence quark or a sea quark. The incident parton flavor determines the flavor of the outgoing parton, depending on whether it is CC or NC interactions. For CC interactions the relative probabilities for the transition depend on the Cabibbo favored or suppressed angles. The minimum allowed value of W^2, which initially is based on the mass for a single nucleon and a pion, may be modified depending on the outgoing parton flavor and is therefore checked again. For CC interactions the polarization of the outgoing lepton is calculated according to Ref. [22].

Total cross sections are calculated by numerical integration from the differential cross sections:

$$\sigma = \int_{Q^2_{min}}^{Q^2_{max}} \int_{x_{min}(Q^2)}^{x_{max}(Q^2)} \frac{d^2\sigma}{dx dQ^2}\, dx\, dQ^2\,. \tag{14}$$

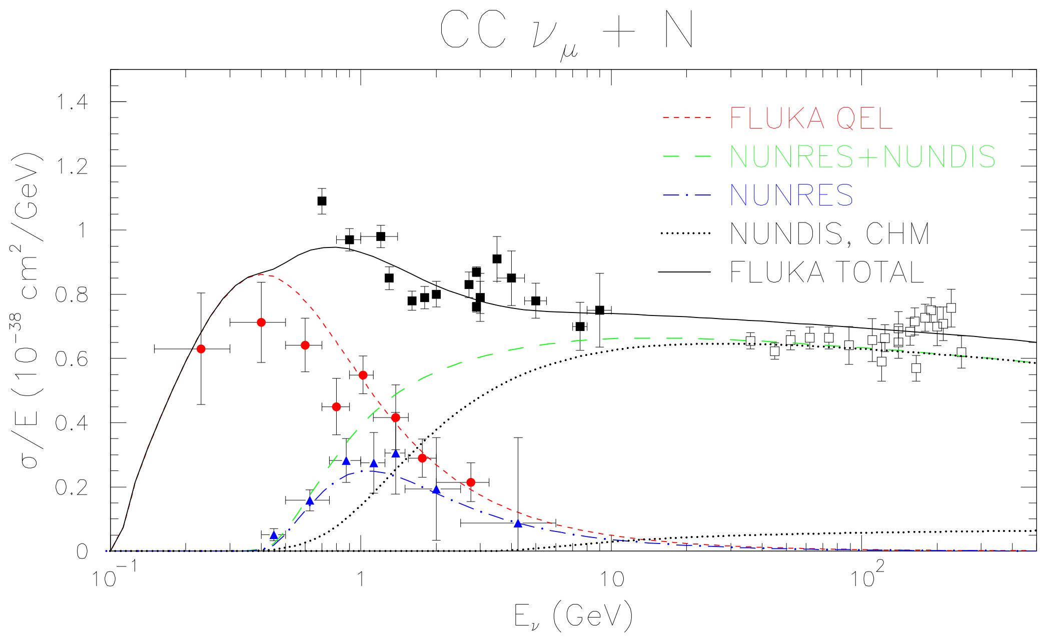

Fig. 2: Total cross sections for Charged Current ν_μ–N interactions as function of incident neutrino energy. The various contributions from the FLUKA calculation are explained in the figure. Note that charm production is displayed separately but is part of the NUNDIS event generator. Experimental data are shown for QEL [24] (red dots), RES [25] (blue triangles), DIS [26] (open squares) and total cross sections [27] (black squares).

Due to long integration times the total cross sections are tabulated for all valid neutrino-nucleon combinations and are used in the initial sampling that determines which kind of neutrino interaction that occur.

3.2 NUNRES

The NUNRES package in now under development with the purpose to generate neutrino-nucleon resonance interactions. It is built on the basis of the Rein-Sehgal formulation [23], however keeping only the contribution from Δ production. No non-resonant background term is considered, assuming that the non-resonance contribution comes from NUNDIS. For the moment, the transition from RES to DIS is performed by imposing a linear decrease of both cross sections as a function of W. With this approach the energy dependence of the cross section for DIS is automatically reduced in favor of RES interactions. As in the case of NUNDIS all calculations are performed in the lab system, with relevant transformations between reference frames handled by PEANUT.

3.3 Results

Beta versions of the NUNDIS and NUNRES generators are included in the standard FLUKA 2008 distribution and can be invoked with standard FLUKA input commands. Neutrino primary particles can be simulated by forcing the interaction at a given locations. A separate file with cross sections is available in the distribution so that the uses can weight the results. It is also possible to restrict the simulations to a subset of the interaction channels QEL, RES or DIS, and charm production can be simulated separately from the rest of the DIS contribution.

In Fig. 2 is shown the total cross section for the different contributions in the case $\nu + N$ where N is an isoscalar nucleon. Experimental data are taken from Refs. [24–27]. As seen the total cross sections agree fairly well with experimental data, but it should be noted that it is important to compare

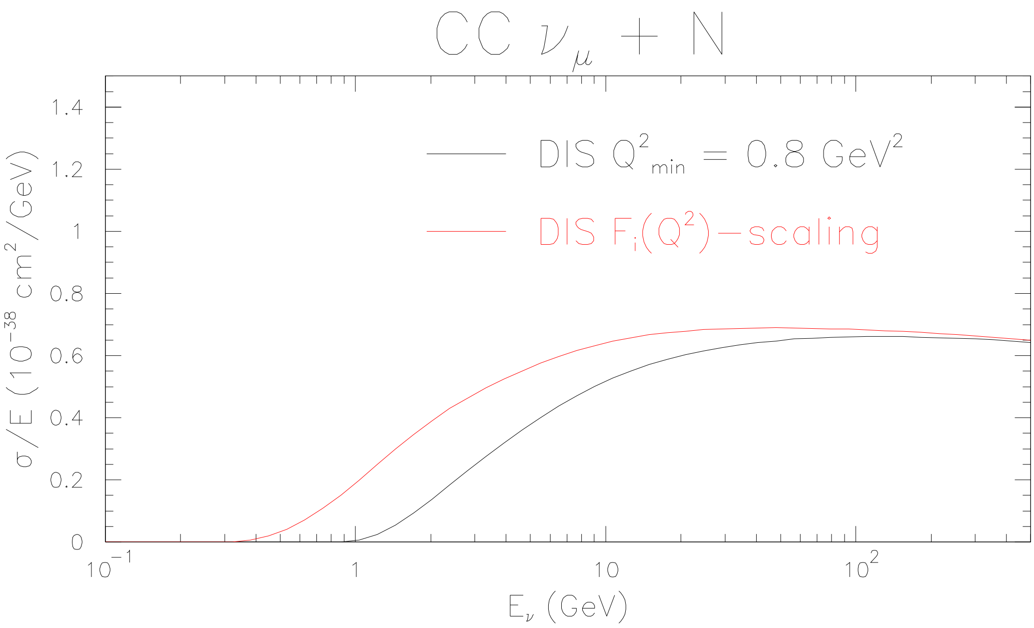

Fig. 3: Total cross sections for Charged Current ν_μ–N, DIS interactions with (red) and without (black) extrapolation below the Q^2-limit given by the GRV98 PDF-set.

with differential distributions and the resulting final states after hadronizations. Dedicated benchmarking have been initiated and will be reported elsewhere [28].

3.4 Further improvements

The first studies of the quality of the new neutrino-nucleon event generators within FLUKA are promising, but it is clear that many improvements of the code will be required. Below are a few issues that will be addressed in the near future:

- The extrapolation towards $Q^2 = 0$ will be improved. The low limit is determined by the PDF-sets and is $Q_0^2 = 0.8$ GeV^2 for the default GRV98 distribution. Different approaches have been considered, and at the moment we follow the common strategy to freeze the structure functions at $F_i(Q_0^2, x)$ and extrapolate with a rescaling function. The present extrapolation scheme is inspired by the Vector Meson Dominance two phase model [29] and uses the relation:

$$F_2(Q^2, x) = \sqrt{\frac{Q^2}{Q_0^2 + Q^2}}\, F_2(Q_0^2, x). \tag{15}$$

  This shape is very simple in comparison with a full two phase approach, and it gives unphysical effects for certain differential cross sections. Studies are ongoing to improve the situation, inspired by results with low Q^2 electron scattering from the CLAS experiment [30, 31]. Fig. 3 shows an example of how the total cross section for DIS is affected by extrapolating from Q_0^2 with Eq. (15) vs. not allowing any sampling at all below that limit.
- At present there are no incident charm quarks within NUNDIS, and charm production is limited to the transitions from $s/\bar{s}$ and $d/\bar{d}$ quarks in CC interactions. Initial attempts will be made to implement charm PDFs from other PDF-sets while retaining GRV98 as the default PDF-set. At a later step GRV98 will most likely be abandoned as default package for another PDF-set that includes charm quarks.

– Target mass corrections of Bjorken-x according to the Nachtmann variable [32] and proposals by Bodek *et al.* [33] are in principle trivial to implement. We have selected to postpone their introduction until other aspects of the code have stabilized, but they will be tested after the introduction of incident charm quarks.
– A lot theoretical and experimental progress have been made recently in the resonance region, and we should change from the Rein-Sehgal approach to something more sophisticated, though it will require continuous updates of the code in order to follow the latest developments.
– As mentioned above systematic benchmarking against experimental data has been initiated.
– The Callan-Gross relation will be replaced with the ratio of the cross section for scattering from longitudinally to transversely polarized bosons. The reason is that for lower Q^2 this ratio becomes non-negligible and then F_1 and F_2 become related through a longitudinal structure function F_L, which can be expressed as:

$$2xF_1(Q^2,x) = F_2(Q^2,x)\frac{1+4M^2x^2/Q^2}{1+R(Q^2,x)} \;, \tag{16}$$

where $R(Q^2, x)$ is the ratio of the cross sections of longitudinally to transversely polarized W-bosons. R can be parametrized in different ways, for instance as it is done in Ref. [34].
– Some of the numerical integrations in the sampling of Q^2 and Bjorken-x are very time consuming. Therefore special attention is given to improve the sampling methods.

4 Summary

The successful FLUKA Monte Carlo environment has recently been extended with event generators for neutrino–nucleon resonance scattering (NUNRES) and deep inelastic scattering (NUNDIS), to accompany the already existing model for quasi-elastic scattering. The event generators, which are included as beta versions in the standard FLUKA distribution version 2008.3 and can be used for simulating neutrino interactions in a FLUKA geometry. Nuclear effects and hadronization are handled by the internal PEANUT model, giving a consistent handling of all interactions with the same nuclear models for all processes. Initial comparisons with experimental data are promising, but a number of improvements of the event generators are deemed necessary and will be implemented for coming releases of FLUKA.

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