Alpha-Particle Clustering in Nuclei and Four-Alpha-Particle Condensation in ^{16}O

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Abstract
Low density states near the 3α and 4α breakup threshold in ^{12}C and ^{16}O, respectively, are discussed in terms of the α-particle condensation. Calculations are performed in OCM (Orthogonality Condition Model) and THSR (Tohsaki-Horiuchi-Schuck-Röpke) approaches. The 0_2^+ state in ^{12}C and the 0_6^+ state in ^{16}O are shown to have dilute density structures and give strong enhancement of the occupation of the S-state c.o.m. orbital of the α-particles. The 0_6^+ state in ^{16}O has a large component of α+^{12}C(0_2^+) configuration, which is another reliable evidence of the state to be of 4α condensate nature. The possibility of the existence of α-particle condensed states in heavier $n\alpha$ nuclei is also discussed.

1 Introduction

It is well established that α-clustering plays a very important role for the structure of lighter nuclei [1, 2]. The importance of α-cluster formation has also been discussed in infinite nuclear matter, where α-particle type condensation is expected at low density [3], quite in analogy to the recently realised Bose-Einstein condensation of bosonic atoms in magneto-optical traps [4]. On the other hand, for trapped fermions, quartet condensation is also an emerging subject, discussed, so far, only theoretically [5]. In nuclei the bosonic constituents always are only very few in number, nevertheless possibly giving rise to clear condensation characteristics, as is well known from nuclear pairing [6]. Concerning α-particle condensation, the Hoyle state, i.e. the 0_2^+ state in ^{12}C has clearly been established. Several papers of the past [7–10] and also more recently [11–14] have by now established beyond any doubt that the Hoyle state, only having about one third of saturation density, can be described, to good approximation, as a product state of three α-particles, condensed, with their c.o.m. motion, into the lowest mean field $0S$-orbit [15, 16]. This shall be the definition of a Bose-condensed state in finite nuclei, clearly reflecting the situation found in infinite matter in Ref. 3. Occasionally, we also shall call it a gas-like state. The establishment of this novel aspect of the Hoyle state naturally leads us to the speculation about 4α-particle condensation in ^{16}O.

In the present paper, two topics are addressed. A brief review of the status of work on the Hoyle state is given first. Secondly, very recent progress of work on the investigation of 4α-particle condensate state in ^{16}O is shown.

2 Description of the Hoyle state as the 3α condensate

The α clustering nature of the nucleus ^{12}C has been studied by many authors using various approaches [2]. Among these studies, solving the fully microscopic three-body problem of α clusters gives us the most important and reliable theoretical information of α clustering in ^{12}C. First solutions of the microscopic 3α problem where the antisymmetrization of nucleons is exactly treated, have been given by Uegaki et al. [9] and by Kamimura et al. [8]. Their calculations reproduced reasonably well the observed binding energy and r.m.s. radius of the ground 0_1^+ state which is the state with normal density, while they both predicted a very large r.m.s. radius for the second 0_2^+ state which is larger than the r.m.s. radius of the ground 0_1^+ state by about 1 fm, i.e. by over 30%. The observed 0_2^+ state lies slightly above the 3α breakup threshold. The energies of the calculated 0_2^+ state reproduced reasonably well the observed value, together with the electron scattering form factors with respect to the 0_2^+ state [8, 9]. The dilute character of the 0_2^+ state can be described by a gas-like structure of 3α-particles which interact weakly among one another, predominantly in relative S waves. This S-wave dominance in the 0_2^+ state had been already suggested by Horiuchi on the basis of the 3α OCM (Orthogonality Condition Model) calculation [7].

Recently, based on the investigations of the possibility of α-particle condensation in low-density nuclear matter [3], the present authors proposed a conjecture that near the $n\alpha$ threshold in self-conjugate $4n$ nuclei there exist excited states of dilute density which are composed of a weekly interacting gas of self-bound α particles and which can be considered as an $n\alpha$ condensed state [11]. This conjecture was backed by examining the structure of ^{12}C and ^{16}O using a new α-cluster wave function of the α-cluster condensate type.

The new α-cluster wave function [11], which has been denoted as THSR (Tohsaki-Horiuchi-Schuck-Röpke) wave function, represents a condensation of α-clusters. This is clearly seen by the following expression

$$|\Psi\rangle = (C_\alpha^\dagger)^n |\mathrm{vac}\rangle, \tag{1}$$

with

$$C_\alpha^\dagger = \int d\boldsymbol{R} e^{-\boldsymbol{R}^2/R_0^2} \int d\boldsymbol{r}_1 d\boldsymbol{r}_2 d\boldsymbol{r}_3 d\boldsymbol{r}_4 \varphi_{0s}(\boldsymbol{r}_1 - \boldsymbol{R}) a_{\sigma_1\tau_1}^\dagger(\boldsymbol{r}_1) \cdots \varphi_{0s}(\boldsymbol{r}_4 - \boldsymbol{R}) a_{\sigma_4\tau_4}^\dagger(\boldsymbol{r}_4), \tag{2}$$

where $\varphi_{0s}(\boldsymbol{r}) = (1/(\pi b^2))^{3/4} e^{-\boldsymbol{r}^2/(2b^2)}$ and $a_{\sigma\tau}^\dagger(\boldsymbol{r})$ is the creation operator of a nucleon with spin-isospin $\sigma\tau$ at spatial point $\boldsymbol{r}$. The total $n\alpha$ wave function therefore can be written as

$$\Phi_{n\alpha} \propto \mathcal{A} \Big[\prod_{i=1}^{n} \exp\Big(-2\frac{X_i^2}{B^2}\Big) \phi(\alpha_i) \Big]. \tag{3}$$

Here $\phi(\alpha_i) \propto \exp[-(1/2b^2)\sum_j^4 (\boldsymbol{r}_{j(i)} - \boldsymbol{X}_i)^2]$ and $\boldsymbol{X}_i = \sum_j^4 \boldsymbol{r}_{j(i)}/4$, with $j(i) \equiv 4(i-1)+j$, are the internal wave functions and c.o.m. coordinates of the i-th alpha cluster, respectively. B is a variational parameter and the relation $B^2 = b^2 + 2R_0^2$ holds. Of cource, in Eq. (3) the c.o.m. coordinate of the whole nucleus should be eliminated. This is easily achieved by utilizing a helpful property of Gaussian functions. It should be noted that Eq. (1) and (3) contains two limits exactly: the one of a pure Slater determinant relevant at higher densities and the one of an ideal α-particle condensate in the dilute limit [11]. All intermediate scenarios are also correctly covered.

This THSR wave function was applied to study the structure of ^{12}C and ^{16}O, and actually succeeded to place a level of dilute density (about one third of saturation density) in each system of ^{12}C and ^{16}O in the vicinity of the 3 respectively 4 α breakup threshold, without using any adjustable parameter. In the case of ^{12}C, this success of the new α-cluster wave function may seem rather natural, because the microscopic 3α cluster models had predicted a gas-like structure of 3α-particles for the 0_2^+ state, as mentioned above.

The detailed structure analyses of ^{12}C [12] showed that the 0_2^+ wave function of ^{12}C which was obtained in past by solving the full three-body problem of the microscopic 3α cluster model is almost completely equivalent to the wave function of the 3α THSR state. This result gives us strong support to our opinion that the 0_2^+ state of ^{12}C has a gas-like structure of 3α clusters with “Bose-condensation”. The rms radius for this THSR state was calculated as $R(0_2^+)_{\mathrm{THSR}}$ = 4.3 fm which fits well with experimental data for the form factor of the Hoyle state, see Ref. 14, 17. It confirms the assumption of low density as a prerequisite for the formation of an α-cluster structure for which the Bose-like enhancement of the occupation of the S orbit is possible.

A very interesting analysis of the applicability of the THSR wave function can be performed by comparing with stochastic variational calculations [15] and OCM calculations [16]. The α density matrix $\rho(\boldsymbol{r}, \boldsymbol{r}')$ defined by integrating out of the total density matrix all intrinsic α-particle coordinates, is diagonalized to study the single-α orbits and occupation probabilities in ^{12}C states. Fig. 1 shows the occupation probabilities of the L-orbits with S, D and G waves belonging to the k-th largest occupation number (denoted by L_k), for the ground and Hoyle state of ^{12}C obtained by diagonalizing the density matrix $\rho(\boldsymbol{r}, \boldsymbol{r}')$. We found that in the Hoyle state the α-particle S orbit with zero node is occupied to more than 70 % by the three α-particles (see also Ref. 15 and Fig. 1). Taking into account the finite size of the nucleus, a reduction of the condensate fraction from 100 % to about 70 % is not surprising, and the remaining fraction (about 30 %) is due to higher orbits originating from antisymmetrization among nucleons. This huge percentage means that an almost ideal α-particle condensate is realized in the Hoyle state. One should remember that superfluid ^{4}He has only 8 % of the particles in the condensate, what represents a macroscopic amount of particles nonetheless. On the other hand, in the ground state of ^{12}C, the α-particle occupations are equally shared among S_1, D_1 and G_1 orbits, where they have two, one, and zero nodes, respectively, reflecting the SU(3)$(\lambda\mu) = (04)$ character of the ground state [16]. This fact thus invalidates a condensate picture for the ground state.

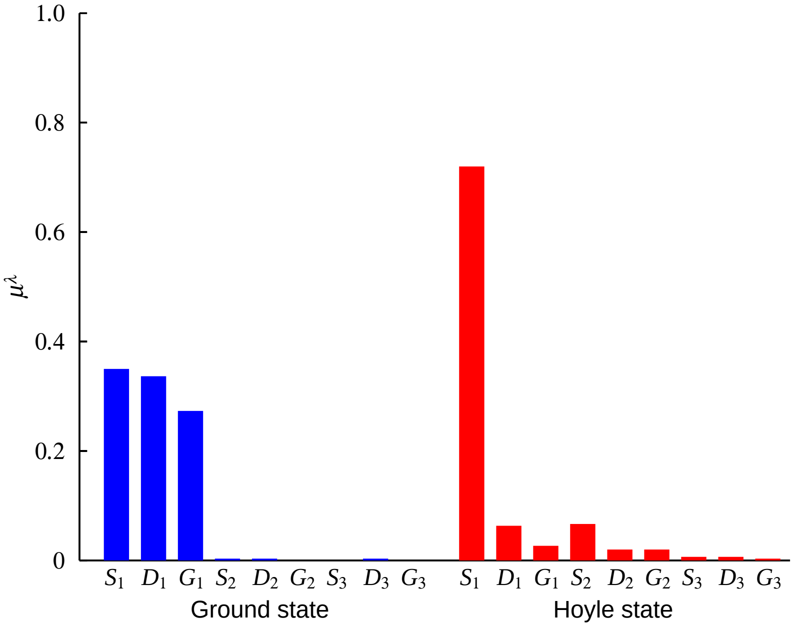

Fig. 1: (Color online) Occupation of the single-α orbitals of the ground state of ^{12}C compared with the Hoyle state [16]. For explanation see the text.

3 4α condensation in ^{16}O

The situation in ^{16}O with respect to cluster states is already much more complicated than in ^{12}C. In ^{12}C, exciting one α-particle out of the ground state necessarily leads to a dilute α-gas state, because the remaining nucleus, ^{8}Be, is itself a loosely bound two α-object. On the other hand exciting an α-particle out of the ^{16}O ground state can lead to multiple ^{12}C-α configurations, be it only for the spectrum of 0^+ states. It is well documented [21] that the 0_2^+ state at 6.06 MeV is an α-particle orbiting in a $0S$-wave around the ground state of ^{12}C. However, there are many more possibilities. The α-particle can be in higher nodal S-wave, the α can orbit in a $0D$-wave around the 2_1^+ of ^{12}C and couple to a 0^+ state. It also can orbit in an odd parity wave around the 1^- and 3^- states in ^{12}C, etc., etc. The ^{12}C can, of course, also be in the Hoyle state which then leads us to the four α gas state in ^{16}O, which is the state of our interest. Experimentally, there are six 0^+ states in ^{16}O up to around the four α disintegration threshold at 14.4 MeV: the ground state 0_1^+, 0_2^+ at 6.06 MeV, 0_3^+ at 12.05 MeV, 0_4^+ at 13.6 MeV [20], 0_5^+ at 14.01 MeV, and 0_6^+ at 15.1 MeV.

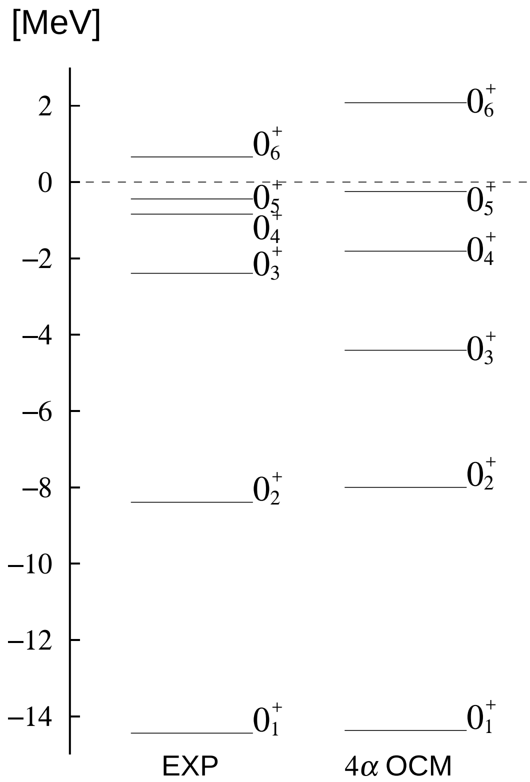

Fig. 2: Comparison of energy spectra between experiments [18] and 4α OCM calculation [19]. The 0_4^+ state in experiment is given in Ref. 20.

In analogy to the aforementioned OCM calculation for ^{12}C [16], we recently performed a quite complete OCM calculation also for ^{16}O, including many of the cluster configurations [19]. We were able to reproduce the full spectrum of 0^+ states with 0_2^+ at 6.4 MeV, 0_3^+ at 9.4 MeV, 0_4^+ at 12.6 MeV, 0_5^+ at 14.1 MeV, and 0_6^+ at 16.5 MeV. As shown in Fig. 2, we tentatively make a one to one correspondence of those states with the six lowest 0^+ states of the experimental spectrum. In view of the complexity of the situation, the agreement can be considered as very satisfactory. Considering the properties of the various states in more detail, a certain number of rather big surprises came up. The analysis of the diagonalisation of the α-particle density matrix $\rho(\boldsymbol{r}, \boldsymbol{r}')$ (as was done in [15,16,22,23]) showed that the newly discovered 0^+ state at 13.6 MeV [20], as well as the well known 0^+ state at 14.01 MeV, corresponding to our states at 12.6 MeV and 14.1 MeV, respectively, have, contrary to what we assumed previously [24], very little condensate occupancy of the $0S$-orbit (about 20 %). On the other hand, the sixth 0^+ state at 16.5 MeV calculated energy, to be identified with the experimental state at 15.1 MeV, has 61 % of the α-particles being in the $0S$-orbit. The corresponding single-α $0S$ orbit is shown in Fig. 3. It has a strong spatially extended behaviour without any node $(0S)$. This indicates that α particles are condensed into the very dilute $0S$ single-α orbit, see also Ref. 25. Thus, the 0_6^+ state clearly has 4α condensate character. We should note that the orbit is very similar to the single-α orbit of the Hoyle state [15, 16]. We also show in Fig. 3 the single-α orbit for the ground state. It has maximum amplitude at around 3 fm and oscillations in the interior with two nodal $(2S)$ behaviour, due to the Pauli principle and reflecting the shell-model configuration. The 0_6^+ state also has a very large radius of 5.6 fm, though this value may be somewhat over estimated because of the 10 % too high energy of the 0_6^+ state.

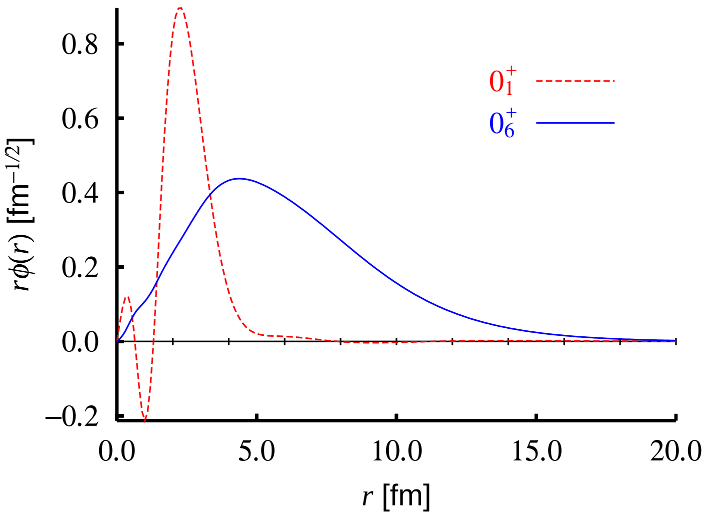

Fig. 3: (Colors online) Single-α orbits with $L = 0$ belonging to the largest occupation number, for the ground and 0_6^+ states. The radial part of the orbits are shown.

The condensate nature for the 0_6^+ state also can clearly be seen by the following analysis. We calculate an overlap amplitude, which is defined as follows:

$$\mathcal{Y}(r) = \left\langle \left[\frac{\delta(r'-r)}{r'^2} Y_L(\hat{\boldsymbol{r}}') \Phi_L(^{12}\mathrm{C}) \right]_0 \middle| \Psi(0_6^+) \right\rangle. \quad (4)$$

Here, $\Phi_L(^{12}\mathrm{C})$ is the wave function of ^{12}C, given by the 3α OCM calculation [16], and r is the relative distance between the center-of-mass of ^{12}C and the α particle. From this quantity we can see how large is the component in a certain α+^{12}C channel which is contained in the 0_6^+ state. The amplitudes for the 0_6^+ state are shown in Fig. 4. It only has a large amplitude in the α+$^{12}\mathrm{C}(0_2^+)$ channel, whereas the amplitudes in other channels are much suppressed. The amplitude in the Hoyle-state channel has no oscillations and a long tail stretches out to ~ 20 fm.

The reason why we previously tried to identify one of the two 0^+ states at 13.6 MeV or at 14.01 MeV with the analog of the Hoyle state, was that the calculation with our condensate wave function THSR [11] gave a third 0^+ state at 16.1 MeV as the highest 0^+ state of our calculation. A new analysis, however, shows that in Ref. 11 a fourth 0^+ state at 18.1 MeV with a very large radius of 6.0 fm was missed. We now identify this state with the four α-particle condensate state, corresponding to the one at 16.5 MeV obtained with the OCM calculation. This was the second surprise. We also will publish details of this calculation in Ref. 26. Since our THSR wave function cannot describe ^{12}C-α configurations, the two intermediate states at 6.06 MeV and 13.6 MeV published already in Ref. 11 must be considered as trying to mock up such ^{12}C-α states in an insufficient, average way. Only the first 0^+ state and the newly discovered highest lying 0^+ state at 18.1 MeV, obtained with the THSR wave function have a clear physical interpretation as being the ground state and the analogue to the Hoyle state, respectively. We, thus, have produced two independent results, both confirming that the α-particle condensate state in ^{16}O lies, as in ^{12}C, above the α-particle disintegration threshold, giving us good confidence for our interpretation.

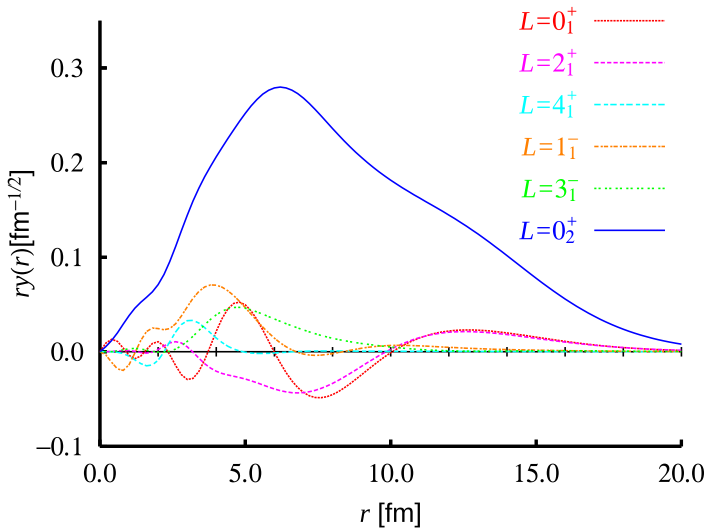

Fig. 4: (Colors online) $r\mathcal{Y}(r)$ for the 0_6^+ state is shown, where $\mathcal{Y}(r)$ is the α-particle overlap amplitude defined by Eq. (4). L denotes the orbital angular momentum of the remaining α particle coupling to ^{12}C.

It, therefore, seems that the situation with respect to cluster states in ^{16}O has become more complex and at the same time more rich and exciting. We have the feeling to have opened a window for an understanding of the whole 0^+ spectrum up to around the α disintegration threshold, giving free the way for more exotic α cluster structures to be detected at even higher energies. It would now be of greatest importance that our results be confirmed by independent calculations. Also on the experimental side more information is very much needed. Of great interest would be, as in the past for ^{12}C, the measurement of inelastic form factors, as they give strong hints to the spatial extension of the various cluster states.

4 Conclusion and remarks

Multiple successful theoretical investigations, concerning the Hoyle state in ^{12}C, have established, beyond any doubt, that it is a dilute gas-like state of three α-particles, held together only by the Coulomb barrier, and describable to first approximation by a wave function of the form $(C_\alpha^\dagger)^3|\mathrm{vac}\rangle$ where the three bosons $(C_\alpha^\dagger)$ are condensed into the $0S$-orbital. There is no objective reason, why in ^{16}O,^{20}Ne,$\cdots$ there should not exist similar "Hoyle"-like states. At least the calculations with THSR and OCM approaches show this to be the case, systematically. In this work, we give preliminary results of a complete OCM calculation which reproduces the six first 0^+ states of ^{16}O to rather good accuracy . In view of the complexity of the situation, this is to be considered as quite an achievement. In that calculation the 0_6^+ state at 16.5 MeV, to be identified with the experimental 0^+ state at 15.1 MeV, shows the characteristics typical for the Hoyle state. This is also confirmed with an extended (with respect to Ref. 11) calculation of the THSR-type, where an additional very extended fourth 0^+ state has been put forward.

Further topics to be investigated in the future in the context of α-particle condensation are numerous. An interesting question is how many α's can maximally be in a self bound α-gas state. In this respect, a schematic investigation using an effective α-α interaction in an α-gas mean field calculation of the Gross-Pitaevskii type was performed [27]. Because of the increasing Coulomb repulsion, the Coulomb barrier fades away and our estimate yields a maximum of about eight α-particles that can be held together in a condensate. However, a few extra neutrons can have a strong additional binding effect (see ^{9}Be and ^{10}Be [28, 29]) and may stabilize larger condensates. Another exciting possibility is to observe expanding α-particle condensate states. Imagine that one excites ^{40}Ca, via a heavy-ion collision, to about 60 MeV, i.e. to the total α disintegration threshold. The α condensate, being formed with a certain probability, will start expanding, since there no longer exists any Coulomb barrier to confine it. With multiparticle detectors such as INDRA or CHIMERA, all decaying α-particles could be detected in coincidence, and the coherent state could be identified by its very low energy in the c.o.m. system. This would then be analogous to an expanding atomic condensate after switching off the confining trap potential [4]. Experiments in this direction are being analysed at IPN-Orsay [30]. Another interesting idea concerning α-particle condensates was put forward by von Oertzen and collaborators [31,32]. α-particles outside a strongly bound core (e.g. ^{40}Ca) can form a condensate at the multi-α-particle threshold [31]. For the condensate with a fixed particle number, the emission of two α's and three α's must be enhanced. In fact the observation of the emission of ^{12}C in the state from the compound nucleus ^{52}Fe has been observed [33] and a very strong deviation from statistical model predictions is observed. Similar ideas have been advanced by Ogloblin [34], who hypothesizes a three α-particle cluster state on top of ^{100}Sn, and earlier by Brenner et al. [35] who reports evidence of a gaseous α-particles in ^{28}Si and ^{32}S on top of an inert ^{16}O core. Also, very interesting recent experimental work on loosely bound α-structures in light nuclei has been performed by T. Kawabata et al. [36] and M. Freer et al. [37].

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