Dineutron correlations in nuclear surface

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Abstract
Two-neutron correlation in quasi two-dimensional (2D) neutron matter is studied by means of the BCS theory to understand formation of nn pairs in nuclear surface of neutron-rich nuclei. The spin-zero nn pair correlation in low density neutron systems confined in an infinite slab is investigated in a simplified model that neutron motion of one direction is frozen. It is found that, when the slab is thin enough, the nn pairing gap enhances and the size shrinking of nn Cooper pair occurs at finite low-density region in the quasi-2D system.

1 Introduction

Two-neutron correlation in neutron-rich nuclei is presently one of the fore-front subjects in the physics of unstable nuclei. In two-neutron halo nuclei, the dineutron correlation was extensively investigated with a three-body picture of a core and two valence neutrons [1–10] . It was discussed also in light neutron-rich nuclei such as in ^{8}He [11–13] and in medium-heavy neutron-rich nuclei [14, 15] as well as asymmetric nuclear matter (for example, Refs. [16–23] and references therein). The neutron-neutron interaction is attractive in the ^1S channel, and therefore, it is natural to expect that spin-zero nn pair correlations may enhance in nuclear systems though the interaction is not so strong as to form a two-neutron bound state in a free space. We here call the spin-zero nn pair with strong spatial correlations "dineutron". Originally the possible existence of a dineutron near the surface of nuclei was predicted by Migdal in 1972 [24]. His idea describes the binding mechanism of a Borromean system and is extended to the dineutron cluster picture in two-neutron halo nuclei.

Recently, dineutron correlations have also been discussed from the point of view of Bose-Einstein condensation (BEC), which is considered along similar lines as deuteron and α-particle condensation suggested in dilute nuclear matter [17,25–28] as well as α condensate states in excited states of $Z = N =$ even nuclei [29–31]. From the analysis of the spatial structure of the nn Cooper pair in infinite neutron matter, it was found that an enhancement of pairing gap and a size shrinking of Cooper pairs occur at finite low density [18, 21, 22]. Similar shrinking of the pair size were predicted also at the surface of medium-heavy nuclei [14, 15].

Thus, spatially correlated neutron pairs are expected to appear in the environment of dilute neutron matter, and therefore, the spatial structure of neutron pairs at the surface of neutron-rich nuclei attracts presently great interest. In the recent studies of neutron-rich nuclei with HFB calculations [14,15], it was found that the extension of a nn-Cooper pair rapidly decreases when going from inside to the surface before expanding again when leaving the nucleus. The nn Cooper pair attains a minimum size of about 2-3fm that is very small size reaching the dimension of the deuteron. This is also highlighted by the fact that in ^{11}Li even the single Cooper pair is very small in the surface [32].

Our aim in this paper is to understand mechanism of the formation of spatially correlated nn pairs and the shrinking of the pair size in nuclear surface. For this aim, we consider a simplified model of quasi two-dimensional (2D) neutron matter which mimics a neutron layer in the surface where distribution of valence neutrons concentrates. By ‘quasi two-dimensional (2D)’, we mean that the ‘bound’ Cooper pairs are confined within a surface layer of about $1-2$ fm and that the degree of freedom in radial direction is approximately frozen. Such a scenario could for example be realized by the fact that the density distribution of valence neutrons in very neutron rich and heavy nuclei are in radial, say z-direction, concentrated in a surface layer and that the density distribution in z-direction can be approximated by a frozen Gaussian packet, whereas the motion of the neutrons within the layer is free. Pursuing the picture to its extreme, one could imagine a slab of low density neutrons where in the transverse (z) direction only a single $0s$ level below the Fermi energy is active.

In the present work, we investigate properties of neutron pairs in the quasi-2D neutron matter by using finite-range effective nuclear forces. Concerning the degree of freedom in the transverse z direction perpendicular to the 2D plane, we for simplicity assume a Gaussian packet as mentioned above. We investigate pairing properties of the quasi-2D neutron matter based on the BCS theory. Pairing gap and size of the Cooper pair in quasi-2D neutron systems are analyzed, and BCS-BEC crossover phenomena are discussed. We also discuss the dependence of pairing features on the thickness of the neutron slab.

This paper is organized as follows. In the next section, we explain our simplified model of quasi-2D neutron matter and formulation of the present work. In section 3, the results obtained for neutron matter are shown and pairing properties are discussed. Finally, we give a summary and outlook in section 4.

2 Formulation of Nuclear 2D Pairing

We here explain our framework of the simplified model of quasi-2D neutron system confined in a slab with a certain thickness. We first describe the wave function and the Hamiltonian for a quasi-2D neutron system. We then explain the formalism for the quasi-2D infinite neutron matter within the BCS theory. More details of the formulation are described in Ref. [33].

2.1 Quasi-2D neutron systems

According to the outline of the Introduction, we propose a model for a system of neutrons confined in a slab with a certain thickness. We assume that the neutron motion in the transverse (z) direction is frozen and is represented by a simple Gaussian packet of the width parameter a. As already mentioned, this thickness a of the slab may mimic the concentration of the amplitudes of single particle wave functions in a surface layer, where z corresponds to the radial direction in finite neutron-rich nuclei.

The wave function for a single neutron is written as

$$\Phi(\mathbf{r}) = \Phi^{2D}(\mathbf{r}_\perp, \chi) \otimes \phi^{0s}(z), \quad \phi^{0s}(z) = \left(\frac{1}{\pi a^2}\right)^{1/4} \exp\left[-\frac{z^2}{2a^2}\right], \tag{1}$$

where $\mathbf{r}_\perp$ indicates the coordinates in the slab, (x, y), and χ is the intrinsic spin. The thickness, i.e., the rms width, of the slab is $2\sqrt{\langle z^2 \rangle} = \sqrt{2}a$. In a similar way, a N-neutron wave function is given as

$$\Phi(\mathbf{r_1}, \cdots, \mathbf{r_N}, \chi_1, \cdots, \chi_N) = \Phi^{2D}(\mathbf{r}_{1\perp}, \cdots, \mathbf{r}_{N\perp}, \chi_1, \cdots, \chi_N) \otimes \phi^{0s}(z_1) \cdots \phi^{0s}(z_N). \tag{2}$$

Here Φ and Φ^{2D} are the antisymmetrized many-body wave functions.

Neutrons are interacting with each other via two-body nuclear forces. The interaction $V(r_{ij})$ between two neutrons is the two-body effective nuclear force in the $T=1$ channel. The central part of the $T=1$ channel in the Gogny [35, 36] and Minnesota [37] forces are represented by a superposition of two Gaussians, $V(r) = \sum_{m=1}^{2} (W_m + B_m P_\sigma - H_m P_\tau - M_m P_\sigma P_\tau) \exp\left[-\frac{r^2}{b_m^2}\right]$.

We freeze the transverse (z) motion as mentioned before. By integrating over z_i coordinates, we reduce the three-dimensional Schrödinger equation for Φ to the two-dimensional equation for Φ^{2D} with respect to $\mathbf{r}_{i\perp}$, and get the following equation for quasi-2D neutron systems,

$$H^{2D}\Phi^{2D}(\mathbf{r}_{1\perp},\cdots,\mathbf{r}_{N\perp},\chi_1,\cdots,\chi_N) = E^{2D}\Phi^{2D}(\mathbf{r}_{1\perp},\cdots,\mathbf{r}_{N\perp},\chi_1,\cdots,\chi_N), \tag{3}$$

$$H^{2D} = \sum_i t_{i\perp} + \sum_{i<j} V^{2D}(r_{ij\perp}), \tag{4}$$

where $t_{i\perp}$ is the 2D-kinetic term and $V^{2D}(r_{ij\perp})$ is the 2D-interaction term,

$$\begin{aligned} V^{2D}(r_\perp) &= \left\langle \phi^{0s}(z_1)\phi^{0s}(z_2)\right| V(r) \left|\phi^{0s}(z_1)\phi^{0s}(z_2)\right\rangle \\ &= \sum_{m=1}^{2} Z_{b_m}(a)\left(W_m + B_m P_\sigma - H_m P_\tau - M_m P_\sigma P_\tau\right)\exp\left[-\frac{r_\perp^2}{b_m^2}\right], \end{aligned} \tag{5}$$

$$Z_{b_m}(a) \equiv \left(\frac{b_m^2}{b_m^2 + 2a^2}\right)^{1/2}. \tag{6}$$

It should be pointed that the strength of the quasi-2D two-body potential $V^{2D}(r_\perp)$ depends on the thickness of the slab with the factor $Z_{b_m}(a)$ as seen from (5) and (6). The strength decreases with the increase of the width parameter a. It means that the effective nn force in quasi 2D is stronger in thinner slab than in thicker one.

Of course, the present assumption of frozen z motion with a simple Gaussian form might be too simple to describe the detailed behavior of the radial motion of valence neutrons. However, it is expected that such effects may effectively be taken into account by a modification of the width parameter a. In the present work, we analyze properties of neutron pairs in quasi-2D neutron systems by taking various values of the slab thickness a.

2.2 Quasi two-dimensional neutron matter within BCS theory

We here apply the BCS theory, which is equivalent to the HFB approximation in a homogeneous case, to the quasi-2D infinite neutron matter in the slab in order to investigate the behavior of neutron pairing.

Let us explain the equations for ^1S pairing in the BCS theory applied to the 2D Schrödinger equation (3). The S-wave pairing gap depends on $|\mathbf{p}_\perp|$ and it is written as

$$\Delta(p) = -\frac{1}{2}\int \frac{d^2k_\perp}{(2\pi)^2} v_{\rm pp}(\mathbf{p}_\perp - \mathbf{k}_\perp)\frac{\Delta(k)}{\sqrt{(e(k)-\mu)^2 + \Delta(k)^2}}, \quad e(p) = \frac{\mathbf{p}_\perp^2}{2m} + V^{HF}(p). \tag{7}$$

Here p and k denote $|\mathbf{p}_\perp|$ and $|\mathbf{k}_\perp|$, respectively, and $v_{\rm pp}$ and $V^{HF}(p)$ is the pairing force and the Hartree-Fock potential. The occupation probability is

$$v_k^2 = \frac{1}{2}\left(1 - \frac{e(k)-\mu}{\sqrt{(e(k)-\mu)^2 + \Delta(k)^2}}\right), \tag{8}$$

and the chemical potential μ is determined so as to satisfy the number equation, $\frac{\rho}{2} = \frac{k_F^2}{4\pi} = \int \frac{d^2k_\perp}{(2\pi)^2} v_k^2$, where ρ and k_F are the density and the Fermi momentum in 2D, respectively. In this paper, the k_F dependence of pairing properties in neutron matter is discussed as a function of the ratio k_F/k_0, where $k_0 = 1.36$ fm^{-1} is the Fermi momentum at the normal density of 3D symmetric nuclear matter.

To analyze the spatial structure of a nn-Cooper pair it is useful to study the pair wave function in the coordinate space. It is defined by the Fourier transform of the anomalous density $\kappa(k)$,

$$\Psi_{\rm pair}(r_\perp) \equiv n_0\langle\Phi_{\rm BCS}|a^\dagger(\mathbf{r}'_\perp\uparrow)a^\dagger(\mathbf{r}''_\perp\downarrow)|\Phi_{\rm BCS}\rangle = n_0\int\frac{d^2k_\perp}{(2\pi)^2}\kappa(k)e^{i\mathbf{k}_\perp\cdot(\mathbf{r}'_\perp - \mathbf{r}''_\perp)}. \tag{9}$$

Here $r_\perp = |\mathbf{r}'_\perp - \mathbf{r}''_\perp|$, and n_0 is the normalization factor so that $\int d^2 r_\perp |\Psi_{\rm pair}(r_\perp)|^2 = 1$. The pair size (coherence length) $\xi_\perp$ is calculated from the root-mean-square distance of the pair wave function

$$\xi_\perp \equiv \sqrt{\langle r_\perp^2 \rangle}, \quad \text{where} \quad \langle r_\perp^2 \rangle = \int d^2 r_\perp r_\perp^2 |\Psi_{\rm pair}(r_\perp)|^2. \tag{10}$$

In the low-density limit $k_f \to 0$, the pair wave function $\Psi_{\rm pair}$ and the energy of the BCS state approach the wave function and the half of the binding energy of the bound state of an isolate two neutron system.

3 Results

In this section, two-neutron correlations in quasi-2D neutron systems are investigated by performing BCS calculations for 2D neutron matter with the Gogny D1S and Minnesota forces, which are often-used effective nuclear forces. The results for various values of a of the confinement in the frozen direction (z) are analyzed, and the behavior of Cooper pairs in quasi-2D neutron matter is discussed as a function of Fermi momentum(k_F/k_0).

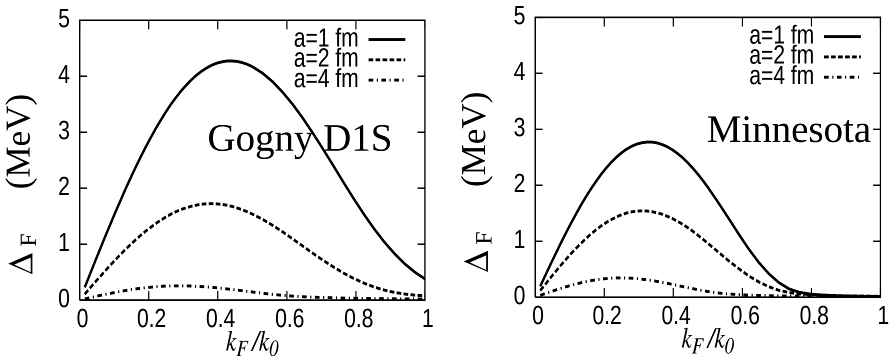

Fig. 1: Pairing gap at the Fermi momentum $\Delta_F = \Delta(k = k_F)$ in quasi-2D neutron matter as a function of k_F/k_0 ($k_0 = 1.36$ fm^{-1}). Left and right panels show the results with the Gogny D1S force and Minnesota force, respectively.

We first show the pairing gap at the Fermi momentum $\Delta_F \equiv \Delta(k = k_F)$ as a function of k_F/k_0 in Fig. 1. The Gogny D1S and Minnesota forces give qualitatively similar results in the low k_F region. At $k_F/k_0 \geq 0.3$, the pairing gap is more suppressed in case of the Minnesota force, because the short-range repulsion is larger than with the Gogny force. For $a = 1$ fm, the pairing gap obtained with the Gogny D1S force has a peak about 4 MeV high at $k_F/k_0 \sim 0.4$, while that calculated with the Minnesota force has a peak $\sim$ 3 MeV high at $k_F/k_0 \sim 0.3$. With the increase of the width a of the slab, the pairing gap Δ_F is quenched because the attraction of the pairing force is weaker in a wider slab.

We next discuss the pair size $\xi_\perp$ in quasi-2D neutron matter plotted as a function of k_F/k_0 in Fig. 2. In the $k_F \to 0$ limit, the pair size $\xi_\perp$ equals the size of the nn bound state in the quasi-2D two-neutron system. With the increase of k_F, the pair size reduces first before it becomes large again with further increase of k_F. Thus the minimum pair size $\xi_\perp$ is found to be smaller than the size $\xi_\perp^{nn}$ of the isolated two-neutron bound state in the quasi-2D system. This size reduction of the nn-Cooper pair indicates the enhancement of the dineutron correlation at finite low k_F, i.e. due to the existence of a Fermi sea. It should be pointed out that the pair size significantly depends also on the width a of the slab.

A striking difference of the pair size between the quasi-2D and 3D neutron matter is that the pair size at the $k_F \to 0$ limit is finite in the quasi-2D neutron matter while it becomes infinite in the 3D neutron matter. In the 3D neutron matter, the Cooper pair size ξ has a minimum around $k_F/k_0 = 0.5$, and it rapidly increases as k_F goes 0. Interestingly, the size shrinking of the Cooper pair in quasi-2D neutron

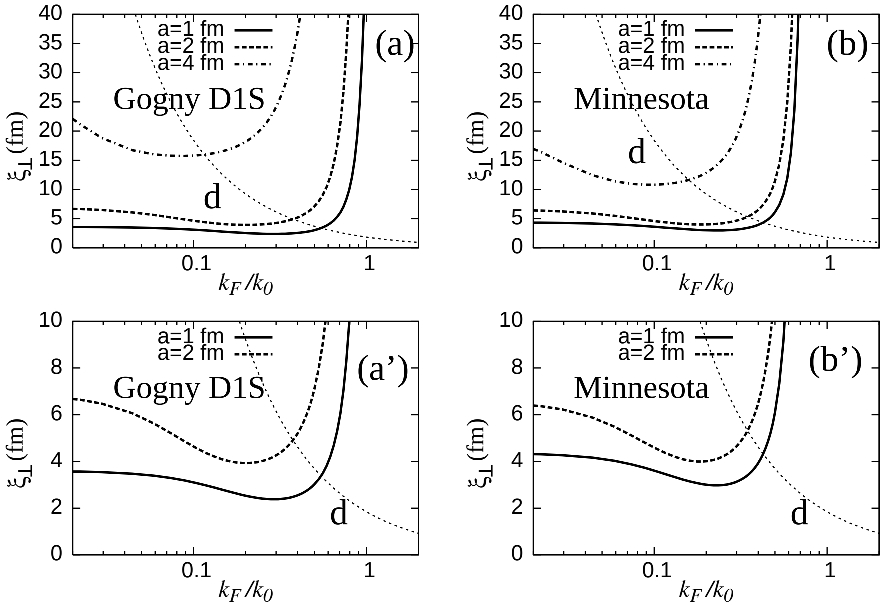

Fig. 2: (a)(b) Size $\xi_\perp$ of a Cooper pair $\Psi_{\rm pair}$ in quasi-2D neutron matter obtained with the Gogny D1S and Minnesota forces. The calculated values are plotted as a function of k_F/k_0 ($k_0 = 1.36$ fm^{-1}). The average inter-neutron distance $d = \rho^{-1/2}$ is plotted with the dotted lines. (a')(b') Same as (a)(b) but scaled up for the vertical axis.

matter has a close analogy with that of the deuteron in 3D symmetric matter predicted by Lombardo and Schuck [34].

As mentioned above, the spatial correlation of Cooper pairs enhances at finite low k_F in quasi-2D neutron matter with a small width a, where the BCS-BEC crossover phenomena is expected. To discuss pairing properties from the point of view of the BCS-BEC crossover, it is useful to compare the pair size $\xi_\perp$ with the average inter-neutron distance $d \equiv \rho^{-1/2}$ as discussed in the works for 3D neutron matter [21, 22]. In the quasi-2D neutron matter, the ratio $\xi_\perp/d$ monotonically deceases as k_F becomes small, and finally the system goes to the strong coupling BEC limit. This is one of the interesting differences from 3D neutron matter where the ratio decreases once in finite low-density region and it goes to infinite again in the $k_F \to 0$ limit.

To reveal the features of BCS-BEC crossover it is also useful to analyze the spatial structure of the nn-Cooper pair wave function[1] as done in Refs. [21,22]. Let us discuss the pair wave function $\Psi_{\rm pair}(r_\perp)$ in the quasi-2D neutron matter with $a = 2$ fm. At low k_F such as $k_F = k_0/8$ and $k_F = k_0/4$, the pair wave function shows a single peak structure at $r_\perp \sim 0$. It indicates the strong spatial correlation of the Cooper pair, namely, a feature of a BEC-like dineutron pair. With increase of k_F from $k_F = k_0/8$ to $k_F = k_0/4$, the first peak becomes narrow due to the Pauli principle from the other Cooper pairs, and the size shrinking of the dineutron occurs as mentioned before. With further increase of k_F, the nodal structure appears at $k_F = k_0/2$ which indicates the transition from the BCS-BEC crossover to the BCS regime, and at $k_F = k_0$ the short-range correlation disappears as in the weak coupling BCS phase.

[1]We are aware that the notion 'Cooper pair wave function' has, recently, become a subject of debate (see, e.g., G.G. Dussel, S. Pittel, J. Dukelsky, P. Sarriguren, Phys. Rev. C **76**, 011302 (2007) and references in there). We do not want to enter this discussion here and stay with the traditional jargon.

4 Summary and outlook

In order to understand mechanism of the formation of spatially correlated nn pairs and the shrinking of the pair size in nuclear surface, we considered a simplified model of quasi two-dimensional neutron matter confined in an infinite slab which mimics a neutron layer in the surface where distribution of valence neutrons concentrates. Spin-zero nn pair correlations in low density neutron systems in an infinite slab with a certain thickness were investigated by means of the BCS theory with a simple assumption that neutron motion of one direction is frozen.

It is found that, when the slab is thin enough, the nn pairing gap enhances and the size shrinking of nn Cooper pair occurs at finite low-density region in the quasi-2D system. We find that for reasonable slab thicknesses of $a \sim 1$ fm, the pair radius is of the order of $2-3$ fm in agreement with realistic 3D HFB calculations [15]. We also show that the transition region of the BCS-BEC crossover appears in the quasi-2D neutron matter at $k_F/k_0 < 0.4 \sim 0.5$ for $a \sim 1$ fm.

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