Systematics of local pion optical model parameters

Dipika Rani Sarker[1], Fahmida Khanam[2], H. M. Sengupta[2] and R. J. Peterson[3]

1. Department of Physics, Jagannath University, Dhaka, Bangladesh
2. Physics Department, University of Dhaka, Dhaka, Bangladesh
3. Department of Physics, University of Colorado, Boulder CO 80309-0390 USA

Abstract

A simple six-parameter local optical potential has been used to fit the global supply of pion-nucleus elastic scattering data, with good success. The resulting real and imaginary well-depths show a striking resonant structure, derived from the underlying pion-nucleon resonances.

The pi meson or pion is a both a particle available as beams and the principal ingredient of the strong interaction within nuclei. In principle, the propagation of the pion field within a nucleus can be quite complex, for instance encountering structured radial dependences in the sensed potential. [1] In practice, the DWIA, proportional to only the nucleon density, may be used to describe elastic scattering for pion beam energies from about 100 to 300 MeV where the prominent 3-3 resonance requires a nonlocal p-wave formalism. [2] At lower energies, theory-based optical potentials have been created, involving higher powers of the nucleon density. [3] Above 300 MeV, an eikonal optical model has been used with success. [4] This range of models (and their respective computer codes) makes it awkward to include pion-nucleus reactions within major reaction codes such as Fluka, although some parameterizations have been presented. [5]

There is however a simple scheme to enable use of a standard local optical model, as commonly used for nucleons and light ions. Satchler showed us how to transform the Klein-Gordon equation appropriate for pions at a lab kinetic energy K_L to an equivalent Schroedinger equation, with a local potential, enabling use of a local optical model. [6] This requires transforming the true pion mass m_π and beam energy K_L to surrogate vales E_L and M_π, using

$M_\pi = \gamma_\pi m_\pi$ with m_π the free charged pion mass of 0.1499 amu or 139.6 MeV and

$$\gamma_\pi=(x+\gamma_L) / (1 + x^2 +2 x \gamma_L) \qquad x = m_\pi/m_T \qquad \gamma_L=1 + (K_L/m_\pi c^2) .$$

Here, m_T=target mass= A times the amu.
It is convenient to use the center of mass kinetic energy $E_{cm}=20.901 k^2 (M_\pi+m_T) / M_\pi m_T$ MeV, with m_π and m_A in amu, with $k=(m_\pi c/ \hbar)\ \mathrm{sqrt}(\gamma_\pi^2 -1)$, giving

$$E_L=E_{cm} (Mp + m_T) / m_T .$$

As an example, for a K_L=180 MeV charged pion incident upon ^{48}Ca, one replaces the true pion mass into the code with M_π=0.3412 amu = 318 MeV, and the actual beam energy with E_L=130.08 MeV. [6] A parameter-free version of the local potential method has been applied with good success to a limited range of pion-nucleus data. [7]

Satchler used this system to demonstrate good fits to several examples of pion-nucleus elastic scattering, from 116 to 291 MeV. However, the strong absorption at these resonance energies creates a continuous ambiguity between geometrical and well depth parameters, as familiar for alpha-particle scattering for instance, making this system difficult to specify.

The solution to this ambiguity was found by requiring the optical model parameters within a local DWBA calculation also to give the correct shape and magnitude of inelastic scattering data to collective nuclear levels. [8] Essentially, this simultaneous fitting involves both the potential and its derivative, constraining the well depth and geometrical parameter ranges.

Using the radial parameter ranges specified by the work of [8], we have used a standard six-parameter local optical potential to fit the world supply of pion-nucleus elastic scattering data, from 100 to 790 MeV. This extends the results of [9], which covered a wide energy range of data on a single nucleus, carbon. Our analyses included isotopic strings, such as ^{12,13,14}C and ^{40,42,44,48}Ca, where the sensitivities of π^+ and π^- beams is strongly influenced by the neutron excesses in the 3-3 resonance region near 160 MeV. Nonetheless, the fitting, restrained only within the range given by [8], provided good fits with real and imaginary radius parameters that may be averaged to be r_R=1.5 fm, r_I=1.1 fm, a_R=0.25+0.4((K_L-150)/500) fm, a_I=0.55 fm, with the usual radius R= r A $^{1/3}$ and diffuseness parameters a of the Woods-Saxon potential, and K_L the pion lab kinetic energy. The energy-dependent real diffuseness parameter may be the result of the varying pion mean free path within nuclei, shortest at resonant energies.

It is the real V_0 and imaginary W_0 well depth parameters that are of greatest variability, since the geometrical parameters are near those of nuclear matter or charge distributions, and not unexpected. Figures 1 and 2 show these well depth parameters, with nuclei of the mass of ^{12}C to lead, and energies K_L from 80 to 790 MeV. All six parameters were varied for the well depths shown in these figures, not using the average geometrical values cited above. Lower beam energies have not yet been included in our fitting scheme, since it is not obvious that a local potential proportional to the first-order distribution of nuclear potential should be adequate.

The scatter among points in these figures at each energy represents fitting variations, but are dominated by pion sign and neutron excess effects. These are treated in more detail for a limited range of nuclei in [8], and will be covered in a more complete presentation of the present work. Even with these scatters, a clear pattern is seen, with potential parameters that look much like the resonant structure expected for the prominent π-nucleon 3-3 or Δ resonance, with the real part vanishing near the free-space pion lab frame resonance energy of 190 MeV and the imaginary potential peaking there. The earlier work on carbon alone showed the same energy behavior [9].

These well depths, looking so resonant, may be fit by the old single-resonance-plus-hard-sphere amplitude equation. [10] To represent, but not fit, these trends, we use

$$V_0 = -22\,\Gamma_R\,(K_L-E_{0R}) / [(K_L-E_{0R}{}^2 + \Gamma_R{}^2] + 5 \text{ MeV}$$

$$W_0=33\,\Gamma_I{}^2 / [(K_L-E_{0I})^2 + \Gamma_I{}^2 /4] +50 \text{ MeV},$$

with Γ_R=Γ_I=200 MeV (larger than the free-space Δ width of 120 MeV) as the resonance widths, and E_{0R}=220, E_{0I}=180 MeV, near the free space value of 190 MeV for the pion kinetic energy K_L. Curves with these parameters are compared to the fitted well depths in the figures.

These parameters and the equivalent local optical model parameters also reproduce well the shape and magnitude of many examples of pion-nucleus inelastic scattering to collective states, as was the case for the more restricted examples of [8]. These parameters also reproduce the measured reaction cross sections for a range of nuclear masses at pion beam energies near the 3-3 resonance, and the wide energy range of reaction cross section measurements for carbon, from 100 to 1000 MeV.

Yet to be completed in this effort is the inclusion of pion beam energy data below 100 MeV. Although the potential in principle will be much more complex in its radial shape, the lack of structure in the angular distributions may enable adequate fits, even with an unsophisticated potential. It is also possible to focus on the isovector part of the simple potential by fitting strings of isotopic data, quite thoroughly done in [8], or by relating the sparse data on pion charge exchange.

This will be of little interest to overall generalized reaction codes, but of interest for more basic nuclear reaction studies.

Even without the information needed to treat pion beam energies below 100 MeV, the present work should provide a great simplification to those including optical model methods within large codes to treat complex situations. With the kinematic shift of pion mass and energy and the overall set of parameters within this work, the same local optical may be used reliably for pions incident on any nucleus (of mass 12 or greater) at any energy from 100 to 1000 MeV, equivalent to the usages for nucleons and light ions.

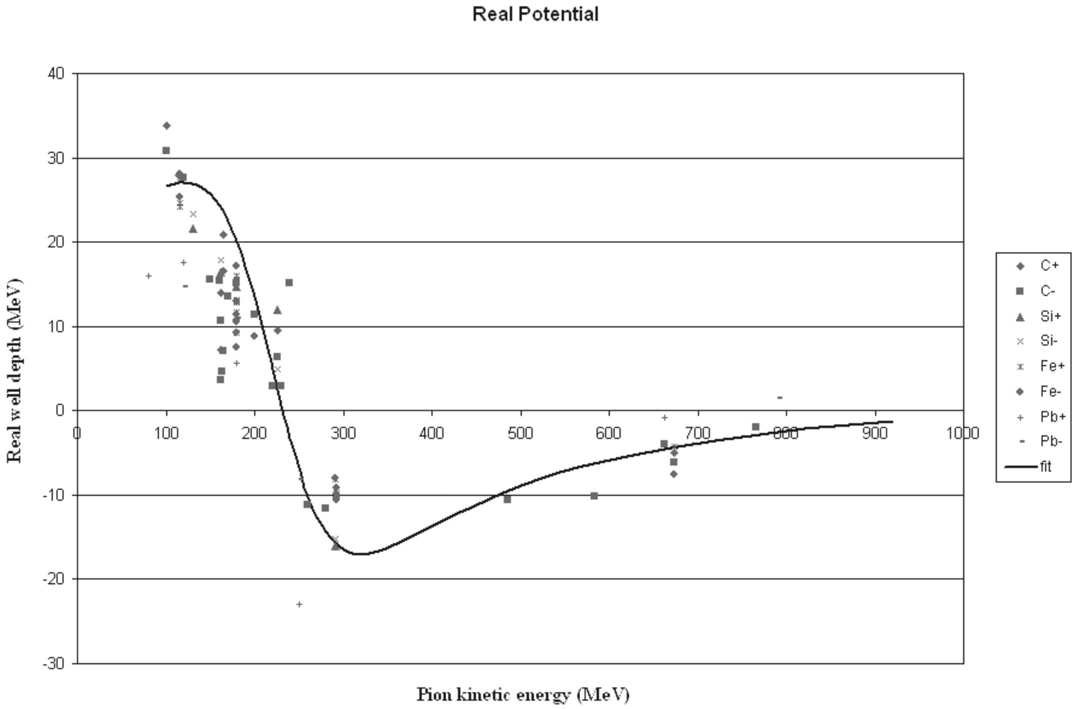

Fig. 1: The real well depths arising from six-parameter fits to pion-nucleus elastic scattering using the local optical potential method described in the text are shown. A positive potential is attractive. Note the resonant structure, with a zero crossing near 220 MeV. The solid curve is from the resonant equation in the text. Target nuclei are labeled by the pion sign, with 'C' representing samples of C, N or O, and 'Fe' for samples of Ca,Fe and Ni isotopes.

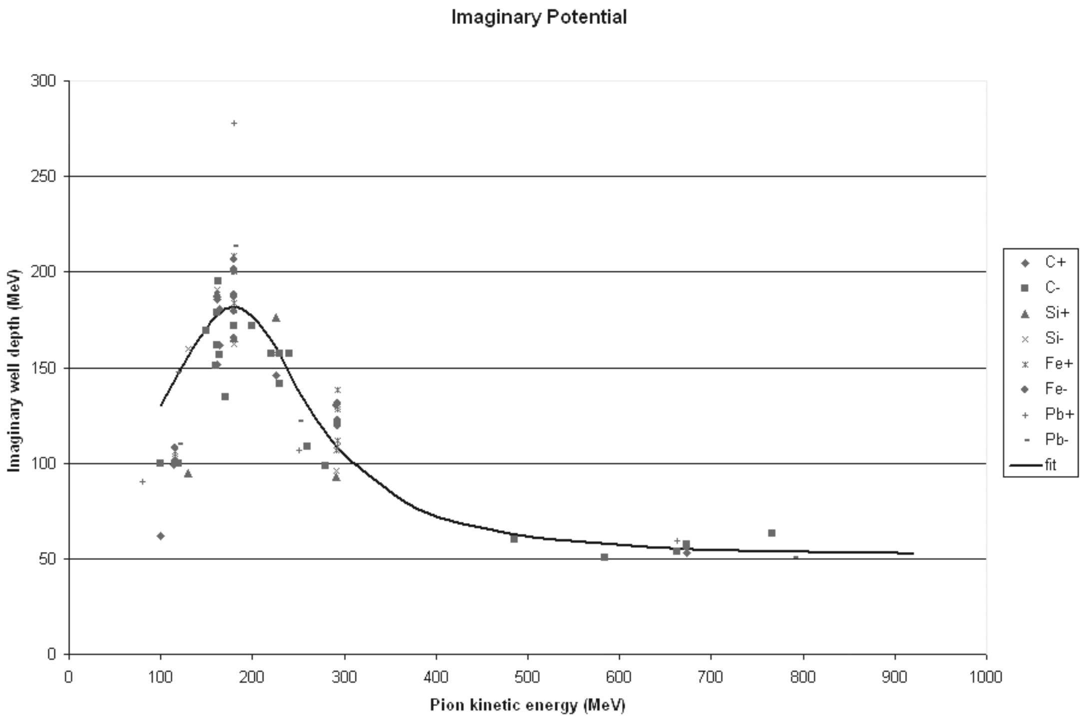

Fig. 2: As figure 1, but for the absorptive imaginary local optical potential well depths.

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