Symmetries of Quadrupole-Collective Vibrational Motion in Transitional Even-Even $^{124-134}$Xenon Nuclei

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Abstract
Projectile-Coulomb excitation of Xe isotopes has been performed at ANL using the Gammasphere array for the detection of γ-rays. The one-quadrupole phonon $2^+_{1,\mathrm{ms}}$ mixed-symmetry state (MSS) has been traced in the stable N=80 isotones down to ^{134}Xe. First, the data on absolute $E2$ and $M1$ transition rates quantify the amount of F-spin symmetry in these nuclei and provide a new local measure for the pn-QQ interaction. Second, the evolution of the $2^+_{1,\mathrm{ms}}$ state has been studied along the sequence of stable even-even $^{124-134}$Xe isotopes that are considered to form a shape transition path from vibrational nuclei with vibrational U(5) symmetry near N=82 to γ-softly deformed shapes with almost O(6) symmetry. Third, our data on more than 50 absolute $E2$ transition rates between off-yrast low-spin states of ^{124,126}Xe enable us to quantitatively test O(6) symmetry in these nuclei. As a result we find that O(6) symmetry is more strongly broken in the A=130 mass region than previously thought. The data will be discussed.

1 Introduction

Proton-neutron (pn) mixed-symmetry states (MSSs) are important sources of information on the effective proton-neutron interaction in collective nuclei. Their excitation energies are directly related to the proton-neutron interaction in the nuclear valence shell. This fact is obvious in the interacting boson model where the excitation energies of MSSs determine the strength of the Majorana interaction to which pn symmetric states at the yrast line are insensitive [1, 2]. Investigation of the proton-neutron interaction in the valence shell is an important subject of contemporary nuclear structure physics. Its evolution with neutron number represents a key-issue for future studies with intense beams of neutron-rich radioactive nuclides.

Vibrational nuclei exhibit a one-quadrupole phonon excitation as the lowest-lying state of mixed pn symmetry, *i.e* the $2^+_{1,\mathrm{ms}}$ state. Its close relation to the 2^+_1 state is evident in the Q-phonon scheme [3], where the wave functions of the one-quadrupole phonon excitations are well approximated by the expressions

$$|2^+_1\rangle \simeq Q_s\,|0^+_1\rangle = [Q_\pi + Q_\nu]\;|0^+_1\rangle \tag{1}$$

$$|2^+_{1,\mathrm{ms}}\rangle \simeq Q_m\,|0^+_1\rangle = N\left[\frac{Q_\pi}{N_\pi} - \frac{Q_\nu}{N_\nu}\right]\;|0^+_1\rangle \tag{2}$$

where $Q_{\pi,\nu}$ $(N_{\pi,\nu})$ denote the proton and neutron quadrupole operators (boson numbers), N=$N_\pi + N_\nu$, and $|0^+_1\rangle$ is the (in general highly correlated) ground state of a collective even-even nucleus. Despite its fundamental role in nuclear structure, the $2^+_{1,\mathrm{ms}}$ state has only recently been studied systematically, *e.g.*, [4–8]. The dominant fragments of the one-phonon $2^+_{1,\mathrm{ms}}$ state are observed at about 2 MeV excitation energy. Due to their isovector character, MSSs decay rapidly by dipole transitions and are very short lived, typically a few tens of femtoseconds. Large $M1$ matrix elements of $\approx$ 1 μ_N are in fact the unique signatures for MSSs and, thus, lifetime information is needed for making safe assignments of mixed symmetry. A review article on the status of experimental information on mixed symmetry states in vibrational nuclei has recently been published [9].

Projectile-Coulomb excitation has been established as a powerful method for the identification and investigation of one-phonon MSSs [5]. We have recently begun a research programme on the $2^+_{1,\mathrm{ms}}$ state at ANL with the nucleus ^{138}Ce as a case study [8]. Crucial influence of sub-shell closures on mixed-symmetry structures was first observed [8, 10], which sensitively tests the effective proton-neutron interaction in microscopic valence shell models [11]. The one-phonon $2^+_{1,\mathrm{ms}}$ state of ^{136}Ce has been identified from similar Coulomb excitation experiments at Gammasphere. A short description of our experimental method for probing the mixed-symmetry character of low-lying collective states of vibrational nuclei is given in the next section. Our results concerning the $N = 80$ isotonic chain will be presented in section 3. The evolution of the one-phonon $2^+_{1,\mathrm{ms}}$ in the U(5)→O(6) transition phase, *i.e* from spherical to γ-softly deformed nuclei in the stable even-even Xe isotopic chain are presented in section 4. Section 5 deals with our unexpected finding of severe O(6) symmetry breaking in the A=130 mass region as briefly discussed for the case of ^{124}Xe.

2 Experimental method

The experiments have been performed at Argonne National Laboratory. The superconducting ATLAS accelerator provided the Xenon ion beams with energies corresponding to $\sim$ 85 % of the Coulomb barrier for a reaction on ^{12}C nuclei. The beam intensity amounted typically to $\sim$ 1pnA. The beam was impinging on a stationary carbon target of thickness 1 mg/cm^2. Light target ions were chosen in order to favor the one-step Coulomb excitation process over multi-step processes for ease of data evaluation. The γ-rays emitted by Coulomb-excited states of the beam nuclei were detected in the Gammasphere array which consisted of $\sim$ 100 high purity Compton suppressed Germanium detectors arranged in 16 rings [12, 13]. An event was defined by a γ-ray of multiplicity 1 or higher. Two corrections had to be done in order to get the total single spectra displayed on the left side of Fig. 1, namely the Doppler correction (recoiling velocity $\sim$ 6%) and the background subtraction (difference between the "in-beam" spectrum and the "off-beam" spectrum scaled to eliminate the 1461 keV ^{40}K line).

The experimental γ-ray spectra are dominated by the decays of low-spin states, such as 2^+ or 3^- states, that are predominantly populated by one-step Coulomb excitation from the ground state. For each state observed we measured the excitation cross section relative to that of the 2^+_1 state with an accuracy of 1 - 0.1 %. By calculating the Coulomb excitation cross sections for each excited state with the multiple-Coulomb excitation code CLX and fitting them to our experimental data (normalized to the 2^+_1 state), we deduced the electromagnetic matrix elements corresponding to each transition of the excited states. The crucial multipole mixing ratios of the $2^+ \rightarrow 2^+_1$ transitions were obtained from γ-ray angular distributions if sufficient statistics have been obtained. A possible large $B(M1)$ value, signature of the MSS, is then easily derived from the data. For a further description of this method, the reader is referred to Refs. [8, 9]. This experimental technique of projectile-Coulomb excitation on a light target inside the Gammasphere array at ANL has been applied by us to 12 nuclei up to now: ^{136,138}Ce, $^{124-134}$Xe, ^{148,154}Sm, ^{96}Ru and ^{94}Mo. The experimental set up was very similar for all of these experiments and the subsequent data analysis was essentially identical.

3 Results for the N=80 isotones

In the stable N=80 isotones the 2^+_1 state decreases in energy with increasing proton number. This is understood as an increase of collectivity with increasing number valence particles. In contrast, the $2^+_{1,\mathrm{ms}}$ state increases in energy (see Fig. 1, right-hand side). Thus, the separation between the two one-phonon levels becomes larger as a function of the product of valence particle pairs $N_\pi N_\nu$ [14, 15].

According to the two-state mixing scheme outlined in Ref. [16], a fit was performed to the energy

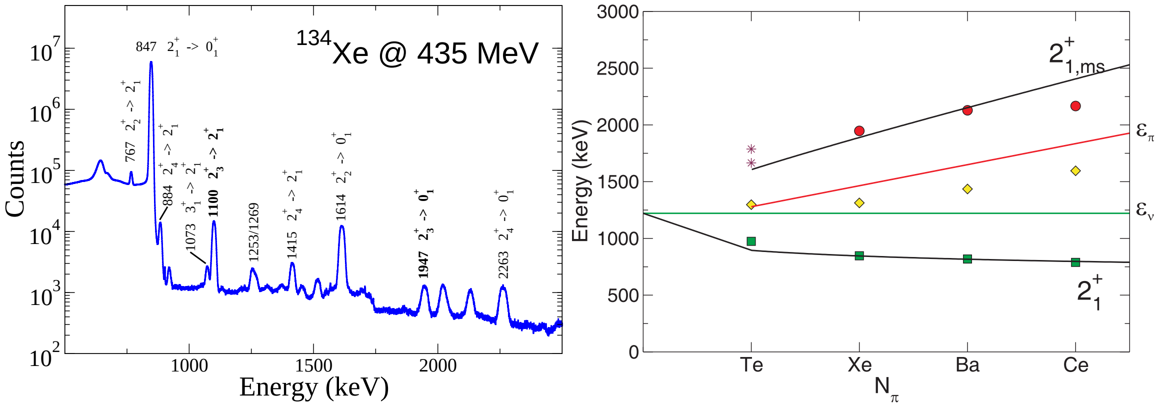

Fig. 1: *Left:* Background-subtracted and Doppler-corrected singles γ-ray spectrum summed over all Ge detectors of the Gammasphere array at ANL after Coulomb excitation of ^{134}Xe on a carbon target. *Right:* Fit of the $2^+_{1,\mathrm{ms}}$ and 2^+_1 level energies in the N=80 isotones shown as filled circles and squares, respectively. The lines labeled ϵ_π and ϵ_ν represent the unperturbed energy of the proton and neutron excitation, respectively. The experimental energies of the 2^+_1 states in the corresponding N=82 isotones are given as diamonds. Tentative 2^+ states of ^{132}Te are shown as asterisks. From Ref. [14].

splitting of the observed 2^+_1 and $2^+_{1,\mathrm{ms}}$ states. This scheme assumes that the observed 2^+_1 and $2^+_{1,\mathrm{ms}}$ states arise from a mixing of fundamental 2^+ proton (with energy ϵ_π) and neutron (with energy ϵ_ν) quadrupole excitations. In this scheme [16], the interaction between the two states results from the proton-neutron quadrupole-quadrupole interaction $V^{QQ}_{pn} = \kappa\, Q_p Q_n$ and, thus, it can serve us as a sensitive measure for this quantity. Its dependence on the number of valence-particles can be parametrized as $V^{QQ}_{pn}(N_\pi, N_\nu) = \beta\sqrt{N_\pi N_\nu}$ in leading order for predominantly spherical nuclei.

The matrix to be diagonalized in this two-state mixing scheme then becomes:

$$H = \begin{bmatrix} \epsilon_\pi & V_{pn} \\ V_{pn} & \epsilon_\nu \end{bmatrix} \Longrightarrow H_{Diag} = \begin{bmatrix} E(2^+_1) & 0 \\ 0 & E(2^+_{1,\mathrm{ms}}) \end{bmatrix}$$

with eigenvalues:

$$E(2^+_1) = \frac{\epsilon_\pi + \epsilon_\nu}{2} - \sqrt{\frac{1}{4}(\epsilon_\pi - \epsilon_\nu)^2 + \beta^2 N_\pi N_\nu} \quad \text{and} \tag{3}$$

$$E(2^+_{1,\mathrm{ms}}) = \frac{\epsilon_\pi + \epsilon_\nu}{2} + \sqrt{\frac{1}{4}(\epsilon_\pi - \epsilon_\nu)^2 + \beta^2 N_\pi N_\nu} \tag{4}$$

as shown on the right of Fig.1 by thick curves. The energy for the neutron excitation, ϵ_ν, is taken to be constant for the $N = 80$ isotones, and is fixed equal to the energy of the 2^+_1 state of ^{130}Sn, $\epsilon_\nu(N_\nu = 1) = E_{2^+_1}(^{130}_{50}\mathrm{Sn}_{80}) = 1221$ keV. The 2^+_1 states in the neighboring semi-magic N=82 isotones increase in energy almost linearly with valence proton number, as can be seen in Fig. 1. Consequently, the energy of the proton excitation, ϵ_π, was linearly parametrized for the N=80 isotones by the expression $\epsilon_\pi = a + b(N_\pi - 1)$ in a leading order approximation in N_π. Here a was again fixed to be equal to the energy of the 2^+_1 state of the neutron-closed shell nucleus ^{134}Te, $a = \epsilon_\pi(N_\pi = 1) = E_{2^+_1}(^{134}_{52}\mathrm{Te}_{82}) = 1279$ keV. The free parameters b and β were then fitted to the data on the energies of the observed one-phonon states, 2^+_1 and $2^+_{1,\mathrm{ms}}$. The resulting values for the parameters were $b = 0.23(4)$ MeV, and $\beta = 0.35(1)$ MeV [14].

A fit to the excitation energies of the 2^+_1 states of even-even Te, Xe, Ba, Ce nuclei for neutron numbers $60 \leq N \leq 80$ in Ref. [16], indicated a value of $\beta = 0.365$ MeV, a result quite close to the value derived here locally using nuclei with $N = 80$ only, considering now the excitation energies of both, the

2^+_1 and $2^+_{1,\mathrm{ms}}$ states. Making use of the fitted value of $\beta = 0.35(1)$ MeV as derived in the present study and using the expression of β as derived from Eq. (3.4) of Ref. [16], a rather precise value of the local strength κ of the proton-neutron quadrupole-quadrupole residual interaction can be derived. Considering the limit $j \to \infty$ ($\kappa = -\frac{5}{12}\beta$), which is a good approximation for the large j values 7/2, 11/2 that are relevant in this mass region (see Fig. 2 and Table II in Ref. [17]), a value of $\kappa = 0.15(1)$ MeV results from our data which again is in agreement with the previous estimate from Ref. [16]. This agreement documents the sensitivity of the $2^+_{1,\mathrm{ms}}$ state to the local V^{QQ}_{pn} in the valence shell.

4 Evolution of the one-phonon mixed-symmetry $2^+_{1,\mathrm{ms}}$ state in even-even Xe isotopes

The Interacting Boson Model enables one to classify the nuclei according to the dynamical symmetries of the IBM Hamiltonian. Three symmetries are most relevant for the description of excited states of quadrupole-collective nuclei: U(5) for vibrational nuclei [18], SU(3) for axially deformed nuclei [19], and O(6) for deformed nuclei with soft triaxiallity [20]. In the A=130 mass region, the Xenon isotopes can exhibit excitation spectra close to the O(6) symmetry. After some theoretical investigations [21–25], it was concluded that the Xenon isotopes should lie in a transitional region from U(5)- to an O(6)-like structure as the neutron number decreases from the closed shell N =82. This was later supported by Casten and von Brentano [26] who presented evidence for an extensive region of nuclei near A =130 resembling the O(6) symmetry. The evolution of the one-quadrupole phonon $2^+_{1,ms}$ with F-spin value $F = F_{max} - 1$ in a U(5)-O(6) transition is still unknown and has been investigated via the projectile-Coulomb excitation method. Some information on our experiments on ^{124,126,128,130,132,134}Xe can be found in the previous section 2.
The data enable us to deduce the $2^+ \to 2^+_1$ $M1$ transition strength distribution up to an excitation energy of about 2.2 MeV (see Table 1 and Fig. 2. The excited 2^+ states that dominate this $M1$ strength distribution are considered as fragments of the $2^+_{1,\mathrm{ms}}$ state. We observe that the detected $B(M1)$ strength decreases while the number of valence neutrons decreases (*i.e.*, while the numbers of neutron hole pairs or neutron bosons increase). Simultaneously, the nuclear collectivity increases and the corresponding excitation energy, $E(2^+_{1,\mathrm{ms}})$, goes up. Apparently, the energy of the one-quadrupole phonon MSS increases with increasing number of valence bosons. The same behavior is clearly observed for the N=80 isotones, too [14, 29]. This increase in energy of a MSS with increasing valence boson number was already observed for the 1^+ two-quadrupole phonon excitation in Ref. [30]. A quantitative understanding of this phenomenon is still missing. The simultaneous decrease of the $B(M1; 2^+_{1,\mathrm{ms}} \to 2^+_1)$ is not understood, either. Perhaps the fragmentation of the $2^+_{1,\mathrm{ms}}$ mode increases with increasing energy and increasing valence space such that smaller shares of the total $2^+_{1,\mathrm{ms}} \to 2^+_1$ $M1$ transition strength can be detected experimentally. This hypothesis may explain the disappearance of any detectable fragments of MSSs below 2.3 MeV and 2.1 MeV for ^{124}Xe and ^{126}Xe, respectively.

Isotope	MSS	Energy [keV]	B(M1; $2^+_{i,ms} \to 2^+_1$) μ_N^2	Literature μ_N^2	Reference
^{124}Xe	no MSS below 2.3 MeV				
^{126}Xe	no MSS below 2.1 MeV				
^{128}Xe	2^+_4	2127	0.04(1)	0.07(2)	[27]
^{130}Xe	2^+_4	2150	0.16(5)		
^{132}Xe	2^+_3	1986	0.22(6)	0.29	[28]
^{134}Xe	2^+_3	1947	0.30(2)		

Table 1: Absolute strengths $B(M1; 2^+_{1,\mathrm{ms}} \to 2^+_1)$ found in the even-even Xe isotopes.

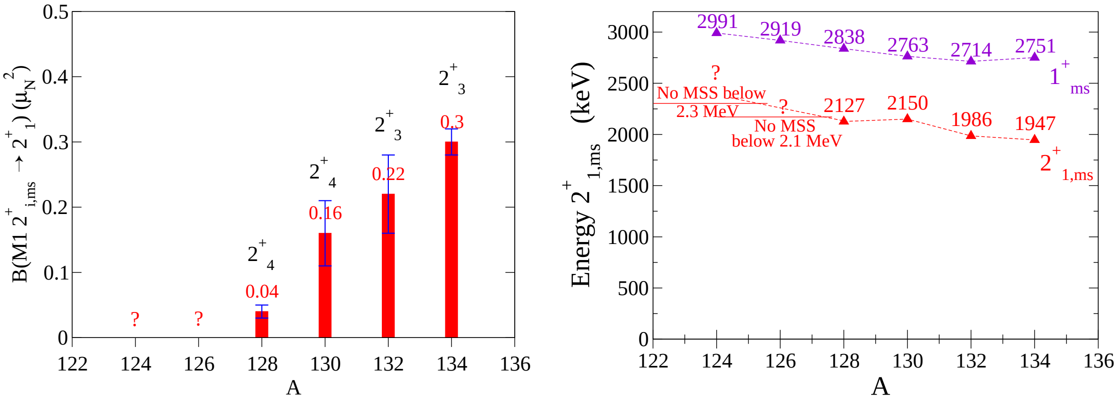

Fig. 2: *Left:* Evolution of the $B(M1; 2^+_{1,\mathrm{ms}} \rightarrow 2^+_1)$ strength in μ_N^2 for the six stable, open-shell, even-even Xe isotopes. *Right:* The corresponding energy of the first one-quadrupole phonon $2^+_{1,\mathrm{ms}}$ state (this work) in comparison to the evolution of the energy of the two-quadrupole phonon 1^+ state, investigated recently by H. von Garrel *et al.* [30].

5 O(6) symmetry breaking in ^{124}Xe

Observation of the O(6) symmetry in nuclei has first been reported in the case of ^{196}Pt [31]. This claim was based on energy level pattern and E2 decay branching ratios that closely follow the O(6) selections rules. It was later on supported by absolute $B(E2)$ values [32]. Another, even more extensive region of O(6)-candidate nuclei is found in the Xe-Ba-Ce region [26] around mass number $A = 130$. For example, it has been shown that the low-spin structures of the nuclei ^{128}Xe [33], ^{126}Xe [34] and ^{124}Xe [35] manifest O(5)-like arrangements of energy levels and $E2$ branching ratios corresponding closely to the expectations for the $\sigma = N$ family of O(6). Consequently, the nuclei from the Xe-Ba-Ce region have

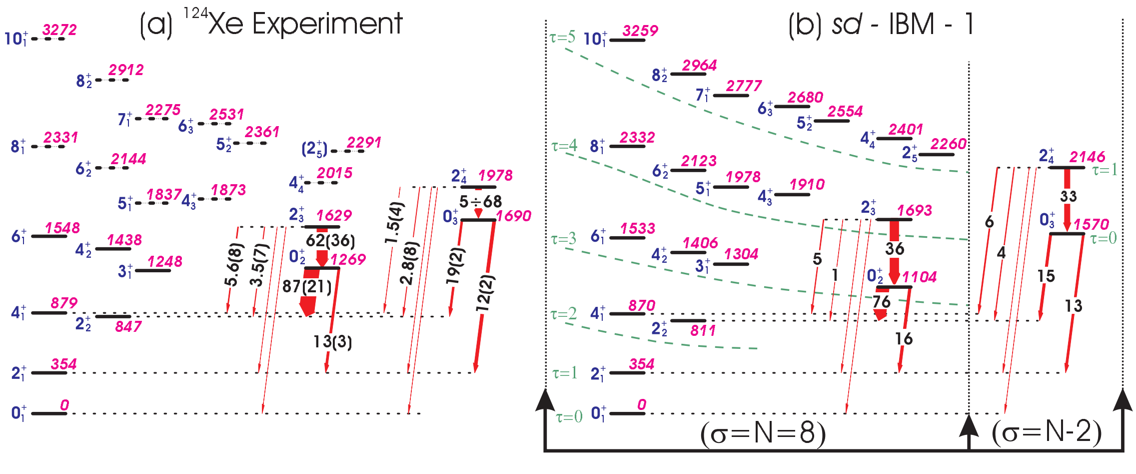

Fig. 3: *Left:* Low-energy positive-parity levels of ^{124}Xe. Levels observed in the present experiment are represented by solid lines. *Right:* *sd*-IBM-1 calculation for ^{124}Xe [35]. The eigenstates are arranged in (σ, τ) multiplets according to the O(6) dynamical symmetry as suggested in Ref. [35]. Since in Ref. [35] the σ quantum numbers have been assigned tentatively, they are presented in parentheses. The arrows represent the $E2$ transitions of off-yrast quasi-K=0 levels of particular interest. The thickness of the arrows and the numbers on them represent the absolute $B(E2)$ values in W.u. (for transitions with $B(E2) \geq 1$ W.u.). From Ref. [36].

been interpreted [26, 33–35, 37] as being located in the vicinity of the O(6) symmetry. On the other hand the nuclei from the Pt region and from the Xe-Ba-Ce region exhibit two systematical deviations from the exact O(6) limit, in particular, the smaller than expected energy staggering in the quasi-γ bands and the τ-compression effect [26]. These deviations from the exact O(6) symmetry can be accounted for by adding perturbative terms to the O(6) Hamiltonian [38]. This improves the description of the low-lying states with $\sigma = N$. It has been assumed that the small perturbations preserve the O(6) symmetry to a large extent but it was clear that $E2$ rates are needed for a stringent test of O(6) symmetry [39]. However, crucial information on absolute $E2$ transition strengths is scarce [34, 37, 40] for nuclei in the $A \approx 130$ region. It does not exist at all for transitions between off-yrast states belonging to $\tau \geq 3, 4$ and $\sigma < N$ multiplets. Thus, due to the lack of experimental data, the crucial quantitative test of the goodness of O(6) symmetry in the Xe-Ba region has not been performed to date. Thanks to the powerful detector systems at ANL available for the study of projectile-Coulomb excitation described in section 2, we have been able to measure absolute $E2$ strengths between off-yrast low-spin states of the open-shell, stable, even-even Xe-isotopes. Absolute $B(E2)$ values, particularly for the two first members of the $\sigma = N - 2$ family enable us [36] to test and quantify the presence or breaking of O(6) symmetry. We discuss here the example of ^{124}Xe. An *sd*-IBM-1 fit has been made as done in [35] and compared with the data as shown on Fig. 3.

The qualitative analysis of the selection rules shows that the new experimental data on the absolute $E2$ strengths of ^{124}Xe agree to a large extend with the $\Delta\tau = \pm 1$ selection rules but they are in severe conflict with the $\Delta\sigma = \pm 0$ selection rules. This fact leads to the hypothesis that in ^{124}Xe the O(5) symmetry is predominantly preserved while the O(6) symmetry is broken.

In order to investigate to what extent the O(6) symmetry breaks down in ^{124}Xe we used the present *sd*-IBM calculation from [35] for a symmetry analysis of the wave functions. We have projected the wave

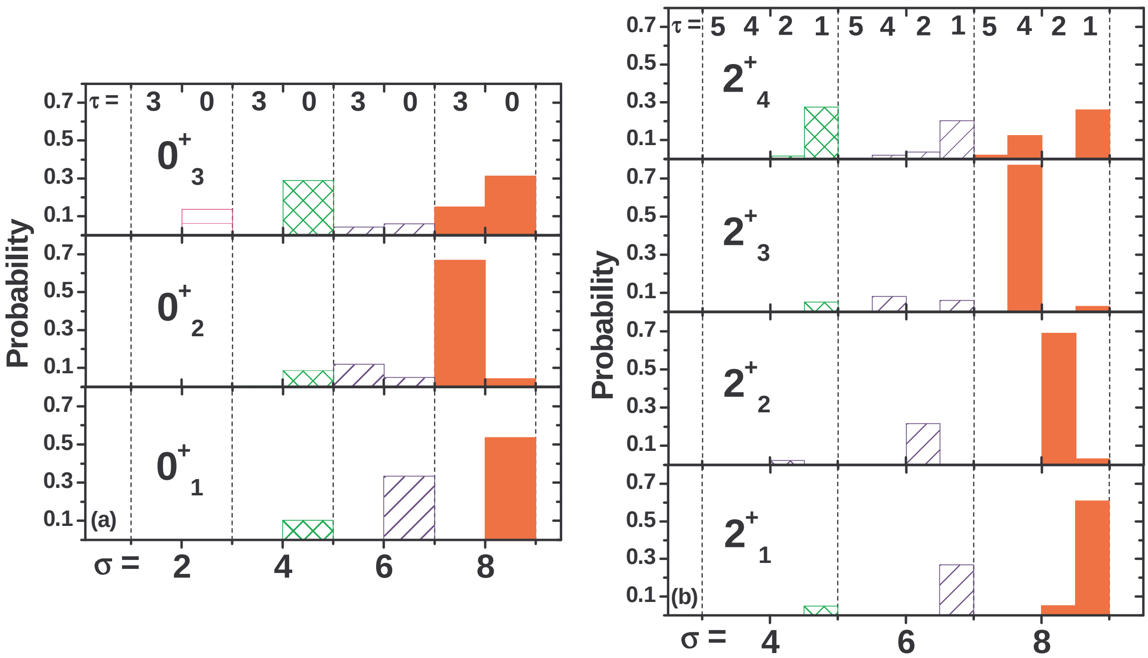

Fig. 4: Squared amplitudes of the components with different (σ, τ) values of the $0^+_{1,2,3}$ (a) and the $2^+_{1,2,3,4}$ (b) *sd*-IBM-1 wave functions. From Ref. [36].

functions of the first few 0^+ and 2^+ IBM states to the O(6) basis $|J^\pi(\sigma,\tau) >$. These results are presented in Fig. 4. Neither τ nor σ are perfect quantum numbers, of course. However, the τ quantum number is usually quite well preserved which indicates that O(5) is a valid symmetry of ^{124}Xe. The components with "correct" τ quantum number exhaust about 70% or more of the total wave functions. The small admixtures with different τs are such that the O(5) selection rules remain mostly undisturbed.

The σ quantum number is, at most, an approximate quantum number. Even the ground state contains only 54.5% of $\sigma = N = 8$. For the states which were thought to belong to the $\sigma = N - 2$ representation, the 0^+ and the 2^+ , the σ quantum number is completely diluted. In fact, the components with $\sigma = 6$ account only for 25.9 % and 10.2 % of the total wave functions of these IBM states (see Fig. 3). While the IBM is very well suited to describe the γ-soft nuclei in the $A \approx 130$ mass region, we must conclude [36] that the nucleus ^{124}Xe cannot be considered as a good example for O(6) symmetry. This conclusion is based on the new data on absolute E2 transition rates. The same analysis has been done for ^{126}Xe with a similar conclusion eventhough it appeared that the O(6) symmetry-breaking was "less" pronounced as in ^{124}Xe.

6 Conclusion

Projectile-Coulomb excitation on a light target is a very powerful tool for populating quadrupole-collective low-spin states of heavy nuclei. This experimental method is well suited to study the lowest lying mixed-symmetry state $2^+_{1,\mathrm{ms}}$ of vibrational nuclei. As a part of our research programme at ANL we have applied this technique to explore the evolution of MSSs in the $A \approx 130$ mass region in order to better understand the pn valence shell interactions in vibrational nuclei. It was possible to obtain more than 50 absolute $E2$ transition strengths in the even-even $^{124-134}$Xe chain [14, 36, 41, 42]. As a result, the strength of the quadrupole-quadrupole proton-neutron residual interaction in the N=80 even-even stable isotones has been derived ($\kappa = 0.15(1)$), the one-quadrupole phonon $2^+_{1,\mathrm{ms}}$ has been identified and tracked in the even-even $^{124-134}$Xe isotopic chain, and an unexpected O(6) symmetry-breaking was discovered (as well as quantified) in ^{124,126}Xe.

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