Hints for the existence of hexaquark states in the baryon-antibaryon sector

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The discovery of some baryon-antibaryon resonances has led us to consider $3q\ 3\bar{q}$ systems as possible candidates. We predict their spectrum in the framework of a constituent model, where the chromomagnetic interaction plays the main role. The relevant parameters are fixed by the present knowledge of tetraquarks. The emerging scenario complies well with experiment; besides the description of the baryon-antibaryon resonances, we find evidence for new tetraquark states, namely, the $a_0(Y)$ in the hidden strangeness sector and the $Y(4140)$ and $X(4350)$ in the $cs\overline{cs}$ sector. A detailed account of the spectra and the decay channels is provided for future comparisons with data.

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I. INTRODUCTION

The presence in the hadron spectrum of mesons consisting of two q's and two $\bar{q}$'s [1–3] as well as baryons consisting of $4q$ and a $\bar{q}$ has been considered for many years [4].

A long time ago Jaffe proposed that the lightest scalar states f_0/σ, κ, together with the rest of their nonet, should be interpreted as $qq\bar{q}\,\bar{q}$ states [1].

The simplifying assumption [5] of considering only $2q$ pairs transforming as a $(\bar{3}_c, 1_s, \bar{3}_F)$ representation of $SU(3)_c \times SU(2)_s \times SU(3)_F$ straitens the whole spectrum to the lightest scalar nonet, namely, $f_0(600)$, $\kappa(800)$, and $f_0/a_0(980)$, as built with a pair of such a diquark and antidiquark [6]. This interpretation was recently enforced by experiments confirming the presence of hidden strangeness in both states $f_0(980)$ and $a_0(980)$ [7], promoting the tetraquarks to a more solid status.

Candidates with open or hidden charm will come from the study of nonleptonic B decays by *BABAR* and Belle, as anticipated in [8], and from BES.

In this paper we study the spectrum of the states consisting of three quarks and three antiquarks in the S wave, interacting via chromomagnetism (CM). Besides strangeness, we also include charm and assume for chromomagnetism its full content [9], treated along the lines of Ref. [10].

This hypothesis can successfully interpret some observed baryon-antibaryon negative parity states in $p\bar{p}$ [11], $\Lambda_c\bar{p}$ [12], and $\Lambda_c\bar{\Lambda}_c$ [13], assuming for the parameters (constituent masses and effective couplings) those values obtained from the tetraquark phenomenology. To study the case of broken flavor symmetry we had to resort to machine computation. A more precise approach, taking into account the spatial correlations, would introduce more free parameters and is beyond the scope of this paper, which aims to describe the general features of the spectrum and decay channels of some relevant multiquark states.

The paper is organized as follows: In Sec. I we introduce the basics of chromomagnetism with a formulation more suitable for algebraic computation. Section II deals with the formalism for the construction of the tetraquark states and the study of the open door decays. In Secs. III, IV, and V we discuss the phenomenology of tetraquark states and the parameter fixing of the model. Hexaquark states are introduced in Sec. VI along with the details entering the calculation. In Sec. VII we present the results we found for the spectrum and compare them with the relevant experimental data. Section VIII contains our conclusions. Finally, Appendix A contains a table with the full spectrum of baryon-antibaryon systems that were taken under consideration, while in Appendix B the crossing matrices required for the study of the decays of tetraquarks are reported. The matrix elements of the chromomagnetic operator are given, for all cases, in Appendix C.

II. THE CHROMOMAGNETIC INTERACTION

The hyperfine interaction arising from one-gluon exchange between constituents leads to a simple Hamiltonian involving the color and spin degrees of freedom:

$$H_{\rm CM} = \sum_i m_i - \sum_{i<j} C_{ij} O_{\rm CM}^{(i,j)}. \tag{1}$$

The index $i(j)$ refers to the ith (jth) quark, m_i its mass, and C_{ij} appropriate coupling constants. The kinetic energy is absorbed in the mass term, so it is not surprising that the quark masses depend on the system under consideration. The C_{ij}'s depend not only on the m_i's (as $1/m_im_j$) but also on the wave function at zero distance of the pair (i, j), thus depending on the system as well. CM is encoded in $O_{\rm CM}^{(i,j)}$, the two particles' chromomagnetic operator, which is given by

$$O_{\rm CM}^{(i,j)} = \frac{1}{4}\sum_{a=1}^{8}\sum_{k=1}^{3}(\lambda_a \otimes \sigma_k)^{(i)}(\lambda_a \otimes \sigma_k)^{(j)}, \tag{2}$$

where λ_a are the Gell-Mann matrices and σ_k the Pauli matrices. It is reminiscent of the well-known exchange interaction and can be expressed in terms of permutation operators for color and spin, $P_c^{(i,j)}$, $P_s^{(i,j)}$, respectively. The action on a (i, j) quark-quark (antiquark-antiquark) pair is given by

$$O_{\rm CM}^{qq} = (P_c - 1/3) \otimes (P_s - 1/2), \tag{3}$$

where $P_c^{(i,j)}$ and $P_s^{(i,j)}$ exchange the colors and spins (acting independently) of the pair (i, j). Eigenvectors of Eq. (3) are the diquark states of definite symmetry in color and spin, $(6, 3)(SS)$, $(6, 1)(SA)$, $(\bar{3}, 3)(AS)$, $(\bar{3}, 1)(AA)$, with eigenvalues $(1/3, -1, -2/3, 2)$, respectively.

To express the result for a quark-antiquark pair it is useful to define a generic T_N for the group $SU(N)$ as the object: T_N: $\Psi_A \Xi^B \rightarrow 1/N \Psi_A \Xi^B - \delta_A^B \Psi_C \Xi^C$, with Ψ_A in the representation N and Ξ^B in the c.c. representation $\bar{N}$. Making the identification $N = 3$ for T_c and $N = 2$ for T_s, we can write, quite simply,

$$O_{\rm CM}^{q\bar{q}} = T_c \otimes T_s. \tag{4}$$

The eigenvectors of T_N are the singlet representation $(\delta_A^B \Psi_c \Xi^C)$ with eigenvalue $(1/N - N)$ and the adjoint representation $(\Psi_A \Xi^B - 1/N \delta_A^B \Psi_c \Xi^C)$ with eigenvalue $1/N$. So eigenvectors and eigenvalues of the chromomagnetic operator in the present case are (8, 3), (8, 1), (1, 3), (1, 1) with eigenvalues $(1/6, -1/2, -4/3, 4)$, respectively.

By far the more bonded diquark is $(\bar{3}, 1)(AA)$, whose $SU(3)_F$ flavor content, as dictated by the Pauli principle, is $\bar{3}_F$. This is the so-called good diquark; it transforms as a scalar antiquark. If one assumes the hypothesis of Jaffe and Wilczek [5], the spectrum of the tetraquarks remains restricted to the scalar nonet suggested by Jaffe a long time ago. The vector, or bad diquark $(\bar{3}, 3)(AS)$, allows for higher spin states but, since it is a 6_F, it also introduces exotics, i.e. multiplets higher than $SU(3)_F$ nonets, and is excluded from most models. The other two 6_c states, which Jaffe [3,14] sometimes called "worse," are not, in general, taken into account either.

In the present approach of searching for the eigenstates of the chromomagnetic operator, we do not truncate the space in any way, such that, in some sense, all four possible diquarks enter the game.

It is easy to see that we have the following spin-flavor multiplets: spin 0 has four nonets and two 27_F's; spin 1 has two nonets, four octets, one 27_F, two decuplets, and two antidecuplets; finally, spin 2 has two nonets and one 27_F. Exotics, as $I = 2$ states, are not excluded *a priori*, but we think that these states are much less stable and difficult to observe.

Often, we have found a number of near threshold decays, usually considered as molecular states, that are well described by chromomagnetism. In particular, the introduction of the (6, 3) diquark encompass the dichotomy between diquark and molecular models, as clearly argued in [15]. They showed that the molecular state is not an independent state, but is a linear combination of $(\bar{3}, 1)(3, 1)$ and $(6, 3)(\bar{6}, 3)$; the latter (6, 3), by the way, is the only other diquark with negative chromomagnetic energy $(-1/3)$. Their observation indicates that a minimal diquark model should include both pairs, and interestingly enough, it would comprise all spin cases as S-wave tetraquarks lying in only $SU(3)_F$ nonets. From the point of view of $SU(6)_{cs}$, this means that a diquark should transform as the symmetric representation 21 (so as $\bar{3}_F$).

A purely phenomenological motivation to include the (6, 3) diquark is that the mass of the $\bar{3}$, $S = 0$, $(ud)_{I=0}$ pair, say μ, is related to the mass of the Λ hyperon by the relation[1] $\mu = m_\Lambda - m_s$, which, for a state consisting of two of these objects that have no mutual chromomagnetic interaction, implies about twice the mass of the $f_0(600)$. Instead, by considering the vector space consisting of both the $(\bar{3}, 1)(3, 1)$ and $(6, 3)(\bar{6}, 3)$ $S = 0$ color singlet states, the lightest state has a binding energy about 2.7 times larger than the diagonal matrix element for $(\bar{3}, 1)(3, 1)$ [10].

In the flavor symmetry limit, i.e. when the couplings C_{ij} are all equal to each other, it is well known that $O_{\rm CM}$ can be expressed as a combination of Casimirs. This fact has been extensively exploited in the pioneering works of Jaffe [3] and in many other works [4]. In the present paper we shall attack the more complicated issue of considering different masses and couplings; in most of such cases we have to rely on symbolic manipulations that we performed with FORM [16]. The expressions in Eqs. (3) and (4) are quite suitable for computer implementation.

III. "OPEN DOOR" CHANNELS FOR TETRAQUARKS

It was observed for the first time by Jaffe [1] that $qq\bar{q}\,\bar{q}$ mesons may decay into two ordinary (i.e. color singlet) mesons PP, PV, VV (P stands for a pseudoscalar and V for a vector) by simply separating from each other, as long as it is kinematically allowed. He called these channels "open door" or "OZI superallowed" decays, since they can occur without gluon exchange or quark annihilation. In open door channels, S-wave states have to decay into S-wave mesons with zero relative angular momentum.

In general, calculations are performed in the diquark-antidiquark basis; i.e. the tetraquark is represented as $q_1 q_2 \bar{q}_3 \bar{q}_4$, denoted $[\mathbf{12}, \mathbf{34}]$ in the following. Evidently, the diquark and the antidiquark cannot separate from each other, as they can never be color singlets. So, in order to access the open door channels it is convenient to pass to the meson-meson bases $[\mathbf{13}, \mathbf{24}]$ and $[\mathbf{14}, \mathbf{23}]$ which, obviously, coincide if antiquarks 3 and 4 have the same flavor.

In order to have some uniformity in the conventions, we maintain those of [10]. We denote the basis for spin 0 as follows: ϕ in $[\mathbf{12}, \mathbf{34}]$, α in $[\mathbf{13}, \mathbf{24}]$, and ϵ in $[\mathbf{14}, \mathbf{23}]$. In the same order one has ψ, β, and χ for spin 1, while those of

[1] We are indebted to Professor P. Minkowski for bringing this to our attention.

spin 2 are called ξ, γ, and δ. To characterize each basis, we only have to specify the color-spin content of the first and second pairs in the brackets, which combine to form the color singlets, i.e. the set of physical states.

$$\begin{aligned}
\textbf{Spin 0}\quad & (\phi)[\mathbf{12},\mathbf{34}]\colon [(6,3)(\bar{6},3)];[(\bar{3},1)(3,1)];\\
& \qquad [(6,1)(\bar{6},1)];[(\bar{3},3)(3,3)],\\
& (\alpha)[\mathbf{13},\mathbf{24}]\colon [(1,1)(1,1)];[(1,3)(1,3)];\\
& \qquad [(8,1)(8,1)];[(8,3)(8,3)],\\
& (\epsilon)[\mathbf{14},\mathbf{23}]\colon \text{as } [\mathbf{13},\mathbf{24}].
\end{aligned} \tag{5}$$

For α and ϵ the first components are PP and the second VV. The last two are P_8P_8 and V_8V_8, where P_8 is a colored pseudoscalar and V_8 a colored vector.

$$\begin{aligned}
\textbf{Spin 1}\quad & (\psi)[\mathbf{12},\mathbf{34}]\colon [(6,3)(\bar{6},3)];[(\bar{3},3)(3,3)];\\
& \qquad [(\bar{3},1)(3,3)];[(6,3)(\bar{6},1)];\\
& \qquad [(\bar{3},3)(3,1)];[(6,1)(\bar{6},3)],\\
& (\beta)[\mathbf{13},\mathbf{24}]\colon [(1,1)(1,3)];[(1,3)(1,1)];\\
& \qquad [(1,3)(1,3)];[(8,1)(8,3)];\\
& \qquad [(8,3)(8,1)];[(8,3)(8,3)],\\
& (\chi)[\mathbf{14},\mathbf{23}]\colon \text{as } [\mathbf{13},\mathbf{24}].
\end{aligned} \tag{6}$$

So β_1, χ_1 (β_2, χ_2) are PV (VP) and β_3, χ_3 are VV.

$$\begin{aligned}
\textbf{Spin 2}\quad & (\xi)[\mathbf{12},\mathbf{34}]\colon [(6,3)(\bar{6},3)];[(\bar{3},3)(3,3)],\\
& (\gamma)[\mathbf{13},\mathbf{24}]\colon [(1,3)(1,3)];[(8,3)(8,3)],\\
& (\delta)[\mathbf{14},\mathbf{23}]\colon \text{as } [\mathbf{13},\mathbf{24}].
\end{aligned} \tag{7}$$

The only open door channel for a tensor meson is, evidently, VV.

The relative probability for the particle decaying through a specific channel is given by the square of the corresponding component of the normalized eigenvector of the state multiplied by the phase space of the respective decay channel, which depends exclusively on the masses of the particles in question (as it is assumed that all dynamical amplitudes are the same). For convenience, we call the square of the component along the channel the probability factor (PF) for that channel. In some cases we also have to consider the non-open door channels, if, for instance, the open door channels have negligible probabilities or are kinematically forbidden, and so violations of the OZI rule would enter the game. In particular, the P_8P_8 or V_8V_8 channel can become relevant at order $O(\alpha_s)$, as the exchange of one gluon in the t channel converts this object into an ordinary PP or VV pair.

The so-called crossing matrices operating the change of one basis into another arise from the well-known Fierz identities for color and spin [3] and are available in many places; for definiteness we will refer to [10]. They are reproduced, together with a necessary completion, in Eqs. (B1)–(B5).

IV. TETRAQUARK STATES

One immediately realizes that the overall chromomagnetic contribution in Eq. (1) [let us call it $O_{\rm CM}$ and assume thoroughly that $C_{q\bar{q}'} = C_{qq'}$ for any (anti)quarks pair] greatly simplifies for 0^+ and 2^+ states made of at least three constituents with the same flavor, say of type $q\bar{q}q\bar{q}'$ (q is not necessarily a light quark and q and $q\prime$ can incidentally coincide), since the corresponding matrices depend exclusively on the combination $(C_{qq} + C_{qq'})$, which factorizes out. For 2^+ we have $O_{\rm CM} = -4/3(C_{qq} + C_{qq'})\mathrm{diag}(1,1)$, while for 0^+

$$O_{\rm CM} = -1/2(C_{qq} + C_{qq'}) \cdot \begin{pmatrix} 8 & 0 & 0 & -4\sqrt{\frac{2}{3}} \\ 0 & -\frac{8}{3} & -4\sqrt{\frac{2}{3}} & 0 \\ 0 & -4\sqrt{\frac{2}{3}} & -1 & -\frac{5}{\sqrt{3}} \\ -4\sqrt{\frac{2}{3}} & 0 & -\frac{5}{\sqrt{3}} & \frac{19}{3} \end{pmatrix}. \tag{8}$$

The eigenvalues of the above matrix are $\lambda_1 = 1/3(17 + \sqrt{241})$, $\lambda_2 = 1/3(\sqrt{241} - 1)$, $\lambda_3 = 1/3(17 - \sqrt{241})$, $\lambda_4 = -1/3(\sqrt{241} + 1)$, with corresponding eigenvectors (for briefness we give decimal approximations) $(-0.74, 0.04, -0.17, 0.65)$, $(0.64, 0.18, -0.41, 0.62)$, $(0.18, -0.64, 0.62, 0.41)$, and $(0.04, 0.74, 0.65, 0.17)$.

The spectrum is given by $M_a^{(0)} = 3m_q + m_{q'} - 1/2\lambda_a(C_{qq} + C_{qq'})$ ($a = 1, \ldots, 4$) for 0^+ and by $M_b^{(2)} = 3m_q + m_{q'} + 4/3(C_{qq} + C_{qq'})$ ($b = 1, 2$) for 2^+. These considerations also apply to the case of three light constituents within the approximation of exact isospin symmetry. It is worth stressing that this phenomenon does not happen for 1^+.

A simple consequence of the fact that the eigenvectors do not depend on the masses and couplings is that the scalar nonet presents a universal pattern of decays; the lowest state has about 55% probability to decay into PP (negligible in VV), and for the next states, in order of increasing mass, 41% probability to decay into PP, 41% into VV, and 55% into VV. Identifying the lowest state of the light nonet with $\sigma/f_0(600)$ and the third one with $f_0(1370)$, we get the mass of light quarks m_q and C_{qq}; we find $m_q \cong 351.65$ MeV and $C_{qq} \cong 74.4$ MeV. Notice that the quark mass and the coupling can be expressed in terms of the masses of σ and f_0 by

$$\begin{aligned} 4m_q &= m_\sigma + \left(1 + \frac{17}{\sqrt{241}}\right)\frac{m_{f_0} - m_\sigma}{2}, \\ C_{qq} &= \frac{3}{\sqrt{241}}\frac{m_{f_0} - m_\sigma}{2}. \end{aligned} \tag{9}$$

So one immediately realizes that, if we take for m_σ a lower value (as suggested by some authors), the change in m_q would be negligible but C_{qq} would appreciably increase.

A similar determination of the parameters concerning the s and c quarks is not feasible, because presently we dispose of only one strange scalar as a possible candidate for a tetraquark [$\kappa(800)$] and none for charm. For the s quark we choose the parameters in order to reproduce the masses of the $\kappa(800)$ as a $(qq\overline{qs})$ state, the $a_0(980)$ as a $(qs\overline{qs})$ state, and the $f_1(1420)$ as a 1^+ $(qs\overline{qs})$ state, getting $m_s \cong 455.21$ MeV, $C_{qs} \cong 58.04$ MeV, and $C_{ss} \cong 43.2$ MeV.

It is quite unexpected that we obtain almost exact agreement with the parameters of our previous calculation for the pentaquarks [17], where we found $m_q \cong 346.8$ MeV, $C_{qq} \cong 74$ MeV, $m_s \cong 480$ MeV, and for C_{qs} and C_{ss} we assumed the hyperfine prescription $\frac{C_{qs}}{C_{qq}} = \frac{C_{ss}}{C_{qs}} = \frac{m_q}{m_s}$ which, as a matter of fact, is also well satisfied by the tetraquark determinations.

The parameters related to charm have been obtained requiring agreement with the masses of the following states: $X(3872)$ as a 1^+ $(qc\overline{qc})$ state, the pair $D_s(2317)$ and $D_s(2573)$ as 0^+ $(qc\overline{qs})$ states, and finally $D_s(2460)$ as a 1^+ $(qc\overline{qs})$ state. The values obtained for the parameters are $m_c \cong 1631$ MeV, $C_{qc} = 26$ MeV, $C_{cc} = 18$ MeV, $C_{sc} = 17.6$ MeV. A direct determination from the J/ψ and η_c masses gives $m_c \simeq 1534$ MeV, $C_{cc} = 21.6$ MeV.[2] On the other hand, if we determine C_{qc} from the $D^* - D$ mass splitting, we get $C_{qc} = 26.2$ MeV, in excellent agreement with the determination via the tetraquarks spectrum.

Here it is interesting to notice that the system $Q\bar{q}$ should obey some general property since the recoil of Q can be safely neglected. So it should not depend on the mass of Q, but only on the radial and orbital quantum numbers of $\bar{q}$. Since $\bar{q}$ is very light, the system would have a spatial extension that falls in the region of dominance of the linear part of the confinement potential (a phenomenological analysis demonstrates that the $c\bar{c}$ system falls in the logarithmic dominated region) for which well-known scaling laws [19] prescribe that the wave function at the origin does not depend on the Q mass, so we should expect the product $m_Q C_{qQ}$ to be constant. A law equivalent to the constancy of the product $m_Q C_{qQ}$ was inferred some time ago in Ref. [20] and verified for a great number of states involving charm or beauty.

In the case of a "neutral" state $(qq'\overline{qq}')$, as for hidden strangeness or charm, the 1^+ CM matrix in the β basis is block diagonal, with a 2×2 block corresponding to $C = +$ and the other 4×4 block to $C = -$. So, independently of the parameters, we have two exact eigenvectors, one along the direction β_3 (a pair of color singlet vectors) and the other along β_6 (a pair of color octet vectors). On the other hand, all scalars and tensors have the same charge conjugation, $C = +$. One can now calculate the masses of the two C-even states: The first has mass $2m_q + 2m_{q'} + 4/3(C_{qq} + C_{q'q'})$ and the second $2m_q + 2m_{q'} - 1/6(C_{qq} + 18C_{qq'} + C_{q'q'})$. We can also calculate the 2^+ sector exactly, getting $2m_q + 2m_{q'} + 4/3(C_{qq} + C_{q'q'})$ for the mass, the corresponding eigenvector being along γ_1 (a pair of color singlet vectors); the value of the other mass is $2m_q + 2m_{q'} - 1/6(C_{qq} - 18C_{qq'} + C_{q'q'})$, corresponding to γ_2 (a pair of color octet vectors). A general trend for this case is that the highest 1^{++} state is degenerate with the highest 2^+ state, both decaying exclusively into $V_{q\bar{q}}V_{q'\bar{q}'}$. The other 1^{++} is below the light tensor state and has dominant decay into $P_{q'\bar{q}}V_{\bar{q}'q} + P_{\bar{q}'q}V_{q'\bar{q}}$, while the light tensor decays into $V_{q'\bar{q}}V_{\bar{q}'q}$. The states 0^{++} and 1^{+-} have to be calculated numerically, with the exception of the case $q = q'$, when the spectrum of the 1^+ becomes highly degenerate. In such a case, the C-even state β_6 is paired with a C-odd state with eigenvector $\chi_6 = 2/3(-1, 1, 0, 1/(2\sqrt{2}), -1/(2\sqrt{2}), 0)$; the other C-even state β_3 becomes degenerate with the C-odd state with eigenvector $\chi_3 = 2/3(-1/(2\sqrt{2}), 1/(2\sqrt{2}), 0, -1, 1, 0)$. As can be seen from the table below, the mass region 1100–1950 MeV could seem to be populated by some controversial peaks with no definite spin or C parity, due to states overlapping. When an object contains a pair of (anti)quarks, the Pauli principle implies the absence of some states or, if the pair is made of light quarks, restrictions on the isospin content, according to the correspondence $I = 0 \rightarrow 21_{cs}$ and $I = 1 \rightarrow 15_{cs}$. This has been taken into account in the elaboration of Tables I, II, and III, where Pauli forbidden states are indicated by a dash. The very interesting cases of hidden strangeness/charm and tetraquarks with $C = \pm S = 1$ were calculated numerically and are given in Table IV. The interest for the somewhat chimerical states with $C = -S = 1$ and $C = 2$, i.e. of kind $(cs\overline{qq})$ and $(cc\overline{qq})$, is justified by the fact that they provide a clear signature for tetraquarks. In the case of $I = 0$ the first decays into D^+K^- and $D^0\bar{K}^0$ and the second into D^+D^0. Since in both cases the objects carrying strangeness or charm are necessarily a pair of quarks and obviously cannot form color singlets by themselves, the occurrence of such states is possible only if the pair of quarks combines with at least a pair of antiquarks.

Even if no candidates have been observed, for completeness we give the spectrum of strange and charmed axials in Table III.

V. DISCUSSION ON THE RESULTS FOR TETRAQUARKS

First of all, let us recall that the information we used in the fit involves only the mass spectrum, so the pattern of decays may be considered as "predictions." Let us cite the observed dominance of $\pi\pi$ in the $f_0(600)$ decay and of $\rho\rho$

[2] The hyperfine law $\simeq 1/m_i m_j$ does not apply to the charm sector, since the wave function, due to a much higher mass, is much peaked around the origin, partially compensating the mass powers in the denominator. Actually, recent data on the η_b suggest a mass splitting with the Υ of the same order of the $\eta_c - \psi$, and not a factor $(m_c/m_b)^2$ smaller [18].

TABLE I. The 0^+ and 2^+ states with three light (strange) quarks calculated exactly according to Sec. IV. Values of the masses used in the fit are distinguished with a (*). Experimental results, when available, are displayed in the next row; numbers in square brackets give the reference to the experimental data. The Pauli principle fixes the isospins of the various states, so $qq\overline{qc}$ have the same isospins as $qq\overline{qs}$ while $qs\overline{ss}$ have $I = 1/2$ and $ss\overline{ss}$ have $I = 0$. The states forbidden by the Pauli principle are indicated by (–). Masses are given in MeV.

J^P	$qq\overline{qq}$	$qq\overline{qs}$	$qq\overline{qc}$	$qs\overline{ss}$	$ss\overline{ss}$	Decays
0^+	$600^{(*)}$ $I = 0$	$792.3^{(*)}$ $I = 1/2$	2141.7	–	–	$0.55(PP)$; $1.710^{-3}(VV)$
Experiment	$f_0(600)$	$\kappa(800)$				
0^+	1046.4 $I = 0, 1, 2$	1189.6 $I = 1/2, 3/2$	2442.9	1472.2	1611.6	$0.41(PP)$; $3.110^{-2}(VV)$
0^+	$1370^{(*)}$ $I = 0$	1477.6 $I = 1/2$	2661.2	–	–	$3.110^{-2}(PP)$; $0.41(VV)$
Experiment	$f_0(1370)$					
0^+	1816.4 $I = 0, 1, 2$	1874.9 $I = 1/2, 3/2$	2962.4	1996.1	2058.9	$1.710^{-3}(PP)$; $0.55(VV)$
2^+	1605 twice $I = 0$ and $I = 1, 2$	1686.7 $I = 1/2, 3/2$	2819.8	1852.3	1936.1	$0.5(VV)$; 0.5 (light mesons)
Experiment	$X(1600)$ $I = 2$ [21]				$f_2(2010)$? [22]	

TABLE II. Axial states made of all light (in the limit of exact isospin) or strange (anti)quarks calculated exactly, according to Sec. IV. These states have definite charge conjugation. The states forbidden by the Pauli principle are indicated by (–). Masses are given in MeV.

C	$-$	$+$	$-$	$-$	$+$	$-$
$qq\overline{qq}$	1109 $I = 0$	1158.6 $I = 1$	1158.6 $I = 1$	1406.6 $I = 0, 1, 2$	1605 $I = 1$	1605 $I = 1$
$ss\overline{ss}$	–	–	–	1820.8	–	–
Decays	PV	PV	PV	PV	VV	VV

in that of $f_0(1370)$ [23,24], the dominance of the πK channel for $\kappa(800)$ (unfortunately, by now, omitted from PDG).

For the axials we obtained the dominance of $\bar{K}K^* + cc$ ($KK\pi$ probably arising from an off-shell K^*) for the $f_1(1420)$ and, analogously, the dominance of $\bar{D}D^* + cc$ for the $X(3872)$.

Since they are pure β_6 states these channels are exclusive. In particular, for $X(3872)$ the observed decays into $\rho(\omega)J/\psi$ can be explained by one-gluon exchange in the t channel, since those rates are comparable with the process being $O(\alpha_s)$. For $D^{*\pm}_{s_0}(2317)$ the only kinematically allowed open door channel is $\pi^0 D^{\pm}_s$; it is just below the DK threshold, at 2359 MeV. The relevant components are $\alpha_1 = 0.78$, $\epsilon_1 = 0.70$, thus predicting the strong dominance of the $\pi^0 D^{\pm}_s$ decay. In the case of $D^{*\pm}_{s_2}(2573)$, that we interpreted to be 0^+ (even if it is also consistent with a 2^+), the only observed decay is $D^0 K^{\pm}$ [while $D^{0*}(2007)K^{\pm}$ is not observed] in agreement with the PP prescription arising from the scalar nature of the state. Nevertheless, in this case the components are also almost equal, $\alpha_1 = 0.60$ $(\pi^0 D^{\pm}_s)$, $\epsilon_1 = 0.68$ $(D^0 K^{\pm})$, and so we could expect the $\pi^0 D^{\pm}_s$ to be relevant as well. Experimental data neither confirm nor disprove this point. Finally, the axial state $D^{\pm}_{s_1}(2460)$, which we put at 2469.3 MeV, has a large component along $\beta_1(0.87)$, which corresponds to the dominant $\pi^0 D^{\pm *}_s$ channel. The $\omega D^{\pm}_s$ decay [notice that the state $D^{\pm}_{s_1}(2460)$ has $I = 0$] has a tiny component $\beta_2 = 0.024$ and is also kinematically inaccessible. It remains to explain the large branching fraction in $D^{\pm}_s \gamma$, suggesting that the state is very narrow, albeit the experimental upper bound is not very restrictive, $\Gamma \leq$ 3.5 MeV.

The two degenerate states, the isoscalar f_0 and the isovector a_0, at 980 MeV, can only decay into $\eta\pi$ and $K\bar{K}$, since other channels are too high. We predict $\alpha_1 = 0.75$, $\epsilon_1 = 0.74$, and if we take the corrections for the mixing $\eta_0 - \eta_8$ with a mixing angle $\theta = -16°$ (as obtained recently in $\gamma\gamma \to X$), we find $g^2_{a_0 K\bar{K}}/g^2_{a_0\eta\pi} \cong 2.48$, to be compared with the value recently obtained by the KLOE experiment [7] of $0.67 \pm 0.06 \pm 0.13$. This abnormally large coupling for $\eta\pi$ cannot be obtained by chro-

TABLE III. Charmed and strange axial mesons, calculated numerically. Masses are in MeV. The non-negligible decay channels are indicated in the row below.

$qc\overline{qq}$	2329.3	2515.6	2611.7	2727.8	2785.8	2877.73
Decays	$0.55(\pi, \eta)D^*$	$0.39(\pi, \eta)D^*$	$0.47(\omega, \rho)D$	$0.28(\omega, \rho)D$	$0.47(\omega, \rho)D^* 0.11(\omega, \rho)D$	$0.46(\omega, \rho)D^*$
$qq\overline{qs}$	1207.5	1302.7	1308.6	1513.4	1672.7	1703
Decays	$0.56(\pi, \eta)K^*$	$0.17(\omega, \rho)K 0.28(\pi, \eta)K^*$	$0.52(\omega, \rho)K$	$0.21(\omega, \rho)K 0.12(\pi, \eta)K^*$	$0.50(\omega, \rho)K^*$	$0.50(\omega, \rho)K^*$
Isospin	1/2	1/2, 3/2	1/2	1/2, 3/2	1/2	1/2, 3/2

TABLE IV. Spectrum of the tetraquarks calculated numerically. States used in the fit are marked with a (*). When experimental data are available they are displayed in the next row; reference to the sources are given in square brackets. Masses are in MeV.

J^P	$qs\overline{qs}$	$cs\overline{qq}$	$qc\overline{qs}$	$qc\overline{qc}$	$cc\overline{qq}$	$cs\overline{cs}$
0^+	$981^{(*)}$	2326.7	$2315^{(*)}$	3562.7	3643.1	3904.5
Experiment	$a_0(980)$		$D^{*\pm}_{s_0}(2317)$			
0^+	1330.3	2592.3	$2574^{(*)}$	3799.3	3870.8	4060.8
Experiment	$a_0(Y)$ [26]		$D^{\pm}_{s_1}(2573)$			
0^+	1586.1	2757.6	2773.7	3979.3	3898.6	4181
0^+	1934.9	3028	3028.4	4148.4	4144.5	4295
Experiment						$X(4350)$ [29]
1^+	1327.6	2503.7	$2469.3^{(*)}$	3682.9	3795.3	4016.41
Experiment			$D^{\pm}_{s_1}(2460)$			
1^+	$1420^{(*)}$	2674.5	2634.6	$3871.9^{(*)}$	3847.8	4109.4
Experiment	$f_1(1420)$			$X(3872)$		$Y(4140)$ [27]
1^+	1461	2692	2736.4	3924.6	3927.8	4132.1
1^+	1618.9	2822.8	2823	3980.5	3991.6	4172.5
1^+	1770.5	2857.8	2889.3	4057.5	3992.2	4225.9
1^+	1773	2959.5	2951	4088.5	4084.78	4254
2^+	1768.2	2889	2900.2	4027.9	4021.2	4215
2^+	1770.5	2906.9	2912.2	4088.5	4061.6	4254

momagnetism alone; it has been explained recently [25] by nonperturbative effects induced by instantons. Analogously for the dominant decay $f_0 \to \pi\pi$, which violates the OZI rule, we have to rely on the above solution, in association with $f_0(980) - \sigma$ mixing.

We predict a companion (which is a mixture of 8_F and 27_F) for the $a_0(980)$ at 1330.3 MeV coupled to $\eta\pi$, $\eta'\pi$, and $K\bar{K}$. It was recently observed [26] in $\gamma\gamma \to \eta\pi^0$ and named $a_0(Y)$, with an observed mass of 1316 ± 25 MeV.

In the hidden charm-strange sector ($cs\overline{cs}$) we have found two candidates for newly discovered states. The first is the pure β_6 1^{++} state at 4109.4 MeV, which we propose to identify with the narrow state $Y(4140)$ found by CDF [27] in $B^+ \to XK^+$, $X \to J/\psi\phi$, with a mass $4143 \pm 2.9 \pm 1.2$ MeV and a width of $11.7^{+8.3}_{-5.0} \pm 3.7$ MeV. As the $X(3872)$, the latter has dominant decays into $\bar{D}_s D_s^* + cc$ (threshold at 4080 MeV), but can also decay into $J/\psi\phi$ (threshold at 4116.4 MeV). The choice of spin 1[3] is strongly suggested by the fact that it was not observed in $\gamma\gamma \to X$ by Belle [29]. The second state is a 0^{++} at 4295 MeV, with predominant decays into $J/\psi\phi$ ($\alpha_2 \simeq 0.81$) and $D_s^*\bar{D}_s^*$ ($\beta_2 \simeq 0.69$), to be interpreted as the $X(4350)$, discovered by Belle in the same experiment [29], with a mass $4350.6^{+4.6}_{-5.1} \pm 0.7$ MeV and width $13.3^{+17.9}_{-9.1} \pm 4.1$ MeV. Taking into account phase space, we find the $J/\psi\phi$ channel to be twice as probable as the $D_s^*\bar{D}_s^*$ one.

Among the states that are not well established, there is a 2^+ state $X(1600)$ (with $I = 2$) [21] at 1600 ± 100 MeV that, if interpreted as $(qq\overline{qq})$, is compatible with our predictions and, according to the previous section, has to be degenerate with the highest 1^{++}, with the latter possibly being hidden by some ($L = 1$ $q\bar{q}$) state of the a_1 family.

We do not exclude the fact that we have already seen some ($ss\overline{ss}$) states; one of these could be the $f_0(2010)$ found around 2011 ± 70 MeV [22] that is identifiable with our 2^+ state at 1936 MeV. We predict a 1^+, ($qs\overline{qs}$) state, with a mass of 1327.6 MeV decaying predominantly into $\pi\phi$ ($\beta_1 \simeq 0.91$) and another one at 1773 MeV with important components along $\beta_1 \simeq 0.34$ ($\eta_s V$) and $\beta_2 \simeq 0.15$ ($\pi\phi$, $\eta\phi$); while χ_3 is also very large, the state is below threshold for $K^*\bar{K}^*$. The latter could possibly be identified with the $X(1835)$ found by BES [30] at 1834 ± 6 MeV and width $67.7 \pm 20.3 \pm 7.7$ MeV, decaying into $\pi^+\pi^-\eta'$. The spin parity of the $X(1835)$ is not known, and it was, initially, supposed to be related to a $p\bar{p}$ threshold enhancement, due to the strong dominance of the channel $\pi^+\pi^-\eta'$.

We also predict a 0^+ $ss\bar{s}\,\bar{s}$ state at 2058.9 MeV which is strongly coupled to $\phi\phi$, so it would arise as a $\phi\phi$ threshold enhancement.

VI. NEGATIVE PARITY STATES BUILT WITH THREE QUARKS AND THREE ANTIQUARKS

Today experimental evidence for the occurrence of baryon-antibaryon states seems to exist. There could be a tendency to interpret them as molecular states, but as said before, there is no clear distinction between chromomagnetism and the molecular point of view as long as we do not neglect some configurations of the diquarks. In obtaining

[3]The interpretation of $Y(4140)$ as an axial was already contemplated in Ref. [28], albeit not excluding the 0^{++} alternative.

the predictions of chromomagnetism, since the number of candidates is not enough to completely determine the parameters, we will tentatively assume that the masses and chromomagnetic couplings of the quarks in the baryon-antibaryon system are the same as for tetraquarks. As mentioned before, masses could be larger due to the fact that they are defined including the kinetic energy. On the other hand, couplings could be smaller mainly because the wave function is broader.

A complete calculation is very complex and probably not of immediate utility in view of the scarcity of these states. We treat two cases: The first is related to $p\bar{p}$ states and concerns $(qqq\overline{qqq})$ systems; the second deals with the production of a variety of states of the kind $(qqq\overline{qq}\,\bar{Q})$ or $(qqQ\overline{qq}\,\bar{Q})$, where Q denotes an s or a c quark.

It is natural to work with what we call the baryon-antibaryon basis. In the first case, since we are interested in a $p\bar{p}$ pair, it is enough to take the sub-block qqq in the 70 of $SU(6)_{cs}$ (and $\overline{qqq}$ in the $\overline{70}$). The decomposition of the 70, under $SU(3)_c \otimes SU(2)_s$, is given by $70_{cs} = (8_c, 4_s) + (8_c, 2_s) + (10_c, 2_s) + (1_c, 2_s)$. We can construct four color singlets of spin 0 and six of spin 1, which are given below:

Spin 0

$$|1\rangle = [(1_c, 2_s), (1_c, 2_s)]; \quad |2\rangle = [(8_c, 2_s), (8_c, 2_s)];$$
$$|3\rangle = [(8_c, 4_s), (8_c, 4_s)]; \quad |4\rangle = [(10_c, 2_s), (\overline{10}_c, 2_s)]. \tag{10}$$

Spin 1

$$|1\rangle = [(1_c, 2_s), (1_c, 2_s)]; \quad |2\rangle = [(8_c, 2_s), (8_c, 2_s)];$$
$$|3\rangle = [(8_c, 4_s), (8_c, 4_s)]; \quad |4\rangle = [(10_c, 2_s), (\overline{10}_c, 2_s)];$$
$$|5\rangle = [(8_c, 2_s), (8_c, 4_s)]; \quad |6\rangle = [(8_c, 4_s), (8_c, 2_s)]. \tag{11}$$

Evaluating the chromomagnetic operator of Eq. (1) between these states, we get the two matrices, describing chromomagnetism in the two sectors, given in Eqs. (C1) and (C2), where we assumed the same ordering as above.

This has been done using a computer, but since we are in fact in the symmetry limit, it can also be calculated by purely group theoretical means. It provides a valuable check of the machine's symbolic calculation. It is straightforward to obtain the expression in terms of Casimir operators:

$$\begin{aligned} O_{\rm CM} = {} & [C_6(R_{3q}) + C_6(R_{3\bar{q}}) - \tfrac{1}{2}C_3(R_{3q}) - \tfrac{1}{2}C_3(R_{3\bar{q}}) \\ & - \tfrac{1}{3}S_{3\bar{q}}(S_{3\bar{q}} + 1) - \tfrac{1}{3}S_{3\bar{q}}(S_{3\bar{q}} + 1) - 12] \\ & - [C_6(H) - C_6(R_{3q}) - C_6(R_{3\bar{q}}) + \tfrac{1}{2}C_3(R_{3q}) \\ & + \tfrac{1}{2}C_3(R_{3\bar{q}}) - \tfrac{1}{3}S_H(S_H + 1) + \tfrac{1}{3}S_{3q}(S_{3q} + 1) \\ & + \tfrac{1}{3}S_{3\bar{q}}(S_{3\bar{q}} + 1)], \end{aligned} \tag{12}$$

where H stands for the representation of the hexaquark in $SU(6)_{cs}$, with S_H being its spin (0 or 1 in the present case), and R_{3q} and $R_{3\bar{q}}$ are the representations of the three quark and three antiquark subsystems, respectively [of both groups, $SU(6)_{cs}$ and $SU(3)_c$], with S_{3q} and $S_{3\bar{q}}$ being their spins. As before, C_6 and C_3 are the quadratic Casimir operators of $SU(6)_{cs}$ and $SU(3)_c$. In the first square brackets we have isolated the contribution of the quark-quark and antiquark-antiquark interactions, while in the second we have isolated the contribution for quark-antiquark interactions. Here a severe complication arises: The Casimir operators in the second brackets are not diagonal. As the operator $O_{\rm CM}$ transforms as the 35 of $SU(6)_{cs}$, it does not leave the 70 and, thus, the Casimir operators present in the first brackets are diagonal, while for the second ones, representation mixing remains possible and it does in fact occur.

The hexaquark state $(qqq\overline{qqq})$, which we have designated by H, transforms under $SU(6)_{cs}$ as one of the irreducible representations (or mixings thereof) arising in the product $70 \otimes \overline{70} = 1 + 35_1 + 35_2 + 189 + 280 + \overline{280} + 405 + 3675$. For 0^- we have to select the blocks that contain components transforming as $(1_c, 1_s)$, and for 1^-, those transforming as $(1_c, 3_s)$. We indicate below the relevant representations and the number of components of the suitable color singlets contained in each one:

$$0^-\colon (1_c, 1_s) \subset 1; 189(1); 405(1); 3675(1),$$
$$1^-\colon (1_c, 3_s) \subset 35_1(1); 35_2(1); 280(1); \overline{280}(1); 3675(2).$$

The matrix elements were found through the determination of the appropriate Clebsch-Gordan coefficients for the above decomposition.[4]

Let us now consider states of the kind $(qqQ\overline{qq}\,\bar{Q})$ (Q being an s or a c quark), for which some experimental evidence is available. The Pauli principle implies that the pair of light (anti)quarks in the (anti)baryonic block qqQ $(\overline{qq}\,\bar{Q})$ must transform under $SU(6)_{cs}$ as a 21_{cs} $(\overline{21}_{cs})$ for $I = 0$ and as a 15_{cs} $(\overline{15}_{cs})$ in the case of $I = 1$. States such as $(qq)_{21_{cs}} Q(\overline{qq})_{(\overline{21}_{cs})}\bar{Q}$ have $I = 0$ and are relevant for the $\Lambda\bar{\Lambda}$ $(\Lambda_c\bar{\Lambda}_c)$ channels. For brevity, we shall call them the $(21, \overline{21})$ basis. The other case, namely, $(qq)_{15_{cs}} Q(\overline{qq})_{(\overline{15}_{cs})}\bar{Q}$, is the $(15, \overline{15})$ basis and comprises hexaquarks with $I = 0, 1, 2$. This basis will be used in the calculation of the $\Sigma\bar{\Sigma}$ channel.

A criterion to build the physical states, i.e. the color singlets of the six quark system, is to successively combine qq with Q (and analogously for the antiquarks) in all

[4] After the completion of this work we were informed by Ding, Pingand, and Yan of the existence of their paper, Ref. [31], where the spectrum of the $qqq\overline{qqq}$ system, in the flavor symmetry limit (i.e. all masses equal) is calculated. The overlap with the present work essentially consists in the CM matrices given in Appendix C, which, in fact, coincide with their results. However, it is worth noting that these authors found that the $p\bar{p}$, $p\bar{\Lambda}$, and $\Lambda\bar{\Lambda}$ channels are appreciably bounded and could show up as conspicuous enhancements in the experiments.

possible ways regarding the color group $SU(3)_c$, and then combine with those of the antiquarks. This can be easily done using the decompositions of $SU(6)_{cs} \to SU(3)_c \otimes SU(2)_s$: $21_{cs} = (\bar{3}_c, 1_s) + (6_c, 3_s)$ and $15_{cs} = (6_c, 1_s) + (\bar{3}_c, 3_s)$. Taking into account the genealogy of the states, we get for each basis a total of 14 color singlets. They are displayed below.[5] The convention we use is the following: The composition of the baryonic (qqQ) with antibaryonic blocks $(\overline{qq}\,\bar{Q})$ is indicated by a (*); each block is enclosed by square brackets, and within each set of brackets we placed on the left the color-spin content of (qq) followed by that of Q (and analogously for the antiquarks).

As will be seen in the next section, we are also interested in building the basis for the system $\Lambda_c\bar{p}$. We use the ordering convention $(\bar{q}_1\bar{q}_2\bar{q}_3q_4q_5c_6)$. The $\bar{p}$, as before, is put in a $\overline{70}_\beta$ (antisymmetric in 1, 2) and the Λ_c (as the Pauli antisymmetry applies only to the pair 4, 5) in a 70_α (symmetric with respect to 4 and 6) and a 56, which decomposes under $SU(3)_c \otimes SU(2)_s$ as $(10, 4) + (8, 2)$. The mandatory antisymmetrization with respect to flavor of the pair 4, 5 implies isospin 0 for Λ_c.

Basis $(21, \overline{21})$ for spin 1:

$$
\begin{aligned}
&[(\bar{\mathbf{3}}, \mathbf{1})(\mathbf{3}, \mathbf{2})] * [(\mathbf{3}, \mathbf{1})(\bar{\mathbf{3}}, \mathbf{2})] \Rrightarrow |1\rangle = (1, 2) * (1, 2) \quad |2\rangle = (8, 2) * (8, 2),\\
&[(\mathbf{6}, \mathbf{3})(\mathbf{3}, \mathbf{2})] * [(\mathbf{3}, \mathbf{1})(\bar{\mathbf{3}}, \mathbf{2})] \Rrightarrow |3\rangle = (8_{\rm sim}, 4) * (8, 2) \quad |4\rangle = (8_{\rm sim}, 2) * (8, 2),\\
&[(\bar{\mathbf{3}}, \mathbf{1})(\mathbf{3}, \mathbf{2})] * [(\bar{\mathbf{6}}, \mathbf{3})(\bar{\mathbf{3}}, \mathbf{2})] \Rrightarrow |5\rangle = (8, 2) * (8_{\rm sim}, 4) \quad |6\rangle = (8, 2) * (8_{\rm sim}, 2),\\
&[(\mathbf{6}, \mathbf{3})(\mathbf{3}, \mathbf{2})] * [(\bar{\mathbf{6}}, \mathbf{3})(\bar{\mathbf{3}}, \mathbf{2})] \Rrightarrow |7\rangle = (8_{\rm sim}, 4) * (8_{\rm sim}, 4) \quad |8\rangle = (8_{\rm sim}, 4) * (8_{\rm sim}, 2) \quad |9\rangle = (8_{\rm sim}, 2) * (8_{\rm sim}, 4)\\
&\qquad |10\rangle = (8_{\rm sim}, 2) * (8_{\rm sim}, 2) \quad |11\rangle = (10, 4) * (\overline{10}, 4) \quad |12\rangle = (10, 4) * (\overline{10}, 2)\\
&\qquad |13\rangle = (10, 2) * (\overline{10}, 4) \quad |14\rangle = (10, 2) * (\overline{10}, 2).
\end{aligned} \tag{13}
$$

Basis $(15, \overline{15})$ for spin 1:

$$
\begin{aligned}
&[(\bar{\mathbf{3}}, \mathbf{3})(\mathbf{3}, \mathbf{2})] * [(\mathbf{3}, \mathbf{3})(\bar{\mathbf{3}}, \mathbf{2})] \Rrightarrow |1\rangle = (1, 4) * (1, 4) \quad |2\rangle = (1, 4) * (1, 2) \quad |3\rangle = (1, 2) * (1, 4) \quad |4\rangle = (1, 2) * (1, 2)\\
&\qquad |5\rangle = (8, 4) * (8, 4) \quad |6\rangle = (8, 4) * (8, 2) \quad |7\rangle = (8, 2) * (8, 4) \quad |8\rangle = (8, 2) * (8, 2),\\
&[(\bar{\mathbf{3}}, \mathbf{3})(\mathbf{3}, \mathbf{2})] * [(\bar{\mathbf{6}}, \mathbf{1})(\bar{\mathbf{3}}, \mathbf{2})] \Rrightarrow |9\rangle = (8, 4) * (8_{\rm sim}, 2) \quad |10\rangle = (8, 2) * (8_{\rm sim}, 2),\\
&[(\mathbf{6}, \mathbf{1})(\mathbf{3}, \mathbf{2})] * [(\mathbf{3}, \mathbf{3})(\bar{\mathbf{3}}, \mathbf{2})] \Rrightarrow |11\rangle = (8_{\rm sim}, 2) * (8, 4) \quad |12\rangle = (8_{\rm sim}, 2) * (8, 2),\\
&[(\mathbf{6}, \mathbf{1})(\mathbf{3}, \mathbf{2})] * [(\bar{\mathbf{6}}, \mathbf{1})(\bar{\mathbf{3}}, \mathbf{2})] \Rrightarrow |13\rangle = (8_{\rm sim}, 2) * (8_{\rm sim}, 2) \quad |14\rangle = (10, 2) * (\overline{10}, 2).
\end{aligned} \tag{14}
$$

We have five states for spin 0 and nine states for spin 1; they are given below:

Spin 0

$$
\begin{aligned}
&|1\rangle = (1, 2)_\beta(1, 2)_\alpha \quad |2\rangle = (8, 2)_\beta(8, 2)_\alpha\\
&|3\rangle = (8, 4)_\beta(8, 4)_\alpha \quad |4\rangle = (\overline{10}, 2)_\beta(10, 2)_\alpha\\
&|5\rangle = (8, 2)_\beta(8, 2)_{56},
\end{aligned} \tag{15}
$$

Spin 1

$$
\begin{aligned}
&|1\rangle = (1, 2)_\beta(1, 2)_\alpha \quad |2\rangle = (8, 2)_\beta(8, 2)_\alpha\\
&|3\rangle = (8, 4)_\beta(8, 4)_\alpha \quad |4\rangle = (\overline{10}, 2)_\beta(10, 2)_\alpha\\
&|5\rangle = (8, 2)_\beta(8, 4)_\alpha \quad |6\rangle = (8, 4)_\beta(8, 2)_\alpha\\
&|7\rangle = (8, 2)_\beta(8, 2)_{56} \quad |8\rangle = (8, 4)_\beta(8, 2)_{56}\\
&|9\rangle = (\overline{10}, 2)_\beta(10, 4)_{56}.
\end{aligned} \tag{16}
$$

With the introduction of appropriate color and spin projectors, it is easy to build explicitly the above basis. Symbolic expressions for the matrix elements of the chromomagnetic operator $O_{\rm CM}$ were obtained with the help of FORM [16]. The explicit expressions for the CM matrices for the three cases mentioned are collected in Appendix C. For the CM matrices we assumed the same ordering as for the above states. The mass spectra of the most interesting baryon-antibaryon states are given in Appendix A.

VII. EXPERIMENTAL EVIDENCE FOR HEXAQUARKS

(1) We predict a 0^- state $(qqq\overline{qqq})$, strongly coupled to the $p\bar{p}$ channel (the component along $p\bar{p}$ is 0.894) located below the threshold (1876.54 MeV); it has a mass of 1874 MeV. This is in agreement with the first observation of a narrow enhancement near the $p\bar{p}$ threshold by the BES Collaboration [11] in $J/\psi \to p\bar{p}\gamma$, then named $X(1859)$. So far both the J^P assignments 0^+ or 0^- remain equally possible. The state was found at a mass $m_X = 1859 \pm^{3}_{10} \pm^{5}_{25}$ MeV having a width smaller than 30 MeV. The state we found is slightly higher, just 7 MeV above the experi-

[5]The representation $8_{\rm sim}$ is the color octet symmetric under the exchange of the colors of the light quark pair.

mental upper limit. The experimentally estimated branching ratio is $B(J/\psi \to \gamma X)B(X \to p\bar{p}) \simeq 7 \times 10^{-5}$.

(2) Also relevant for the light hexaquarks ($qqq\overline{qqq}$) may be a quite broad 1^- enhancement above $p\bar{p}$ threshold with mass 1935 ± 20 MeV and width $\Gamma = 215 \pm 30$ MeV which was proposed about 30 years ago [32]. We have a very good candidate for this state at a mass 1911.5 MeV with a large component (0.61) along the $p\bar{p}$ channel. However, here some caution is needed because the evidence is based on a partial wave analysis, and one would have to check if the analysis is compatible with the inclusion of the additional 0^- state just mentioned above.

(3) We also have a pretty good candidate for $Y(2175)$, a 1^{--} state recently seen at the *BABAR* detector [33] at a mass $2170 \pm 10 \pm 15$ MeV (with a width $\Gamma = 58 \pm 16 \pm 20$ MeV). We predict a singly hidden strangeness state ($qqs\overline{qqs}$) strongly coupled to the $\Lambda\bar{\Lambda}$ channel (with a component of 0.6 along this direction) with a mass 2184 MeV. Since this state is below the $\Lambda\bar{\Lambda}$ threshold (around 2231 MeV) it has to decay mostly into mesons. In fact, *BABAR* observed this state in the decay $Y \to f_0(980)\phi$ (through $f_0 \to \pi\pi$). $Y(2175)$ has been confirmed by the BES Collaboration [34] in $J/\psi \to \eta f_0(980)\phi$ at a mass $m = 2186 \pm 10 \pm 16$ MeV and a width $\Gamma = 65 \pm 23$ MeV.

(4) The peak in $\Lambda_c\ \bar{p}$ seen at the mass $m = 3350^{+10}_{-20} \pm 29$ MeV and width $\Gamma = 70^{+40}_{-30} \pm 40$ MeV in $B^- \to \Lambda_c \bar{p}\pi^-$ [12] may be identified with a 0^- charmed hexaquark, which we predict to be at 3339 MeV. There is also a 1^- at a lower mass, 3274 MeV, with a component of the same order (0.35). All the states strongly coupled to $\Lambda_c\bar{p}$ are below the threshold (3225 MeV); on the other hand, those above the threshold, with the exception of the two states mentioned previously, have negligible couplings. This implies that these two states are the only ones observable in the baryonic channel. It is useful to remark that the experiment privileges the spin 0 assignment.

(5) In the singly hidden charm sector ($qqc\overline{qqc}$), the heaviest states are loosely coupled to $\Lambda_c\bar{\Lambda}_c$, and the reasonably coupled states are just above or below the threshold (4573 MeV). We display these states and the value of the component along the baryonic channel in the table below:

Mass (MeV)	4533	4556	4575	4614	4642	4658	4670
Comp. in $\Lambda_c\bar{\Lambda}_c$	0.41	0.21	0.52	0.42	0.48	0.16	0.24

As a matter of fact, recently, a resonance decaying into $\Lambda_c\bar{\Lambda}_c$ has been seen by the Belle detector [13,35] at $m = 4634^{+8+5}_{-7-8}$ MeV and $\Gamma = 92^{+40+20}_{-24-21}$ MeV, which is compatible [35,36] with $Y(4660) \to \psi'\pi\pi$ [35,37]. Anyway, the fact that the component along the baryonic channel is not strongly dominant is welcome, since it is opportune to leave some room for the decay into $\psi'\pi\pi$, which in the present framework is naturally related to a $\psi' f_0$ state. Let us notice that this is consistent with the proposal made in [38] of a $\psi' f_0$ molecular state. Recently, it was proposed to interpret the above state as an excited $L = 1$ tetraquark [39].

We have also calculated the spectrum of the singly hidden strangeness states ($qqs\overline{qqs}$) relevant to the $\Sigma\bar{\Sigma}$ channel, using, along the same lines, the $(15, \overline{15})$ basis. We find only two states strongly coupled to $\Sigma\bar{\Sigma}$; both are around the threshold 2380 MeV, with one being below the threshold at a mass of 2356 MeV and the other above, at 2454 MeV. So far, there has been no experimental evidence for these states.

VIII. CONCLUSION

The full chromomagnetic Hamiltonian proved to be very effective in providing for a unified treatment of tetraquarks and hexaquarks. Besides reproducing the pattern of decays of currently accepted tetraquarks, it also predicts a companion for $a_0(980)$ at a mass around 1330 MeV, which has been confirmed by experiments, as the scalar named $a_0(Y)$ and two $cs\overline{cs}$ states, the $Y(4140)$ and the $X(4350)$. A number of candidates were compared with data for the baryon-antibaryon resonances, namely, $p\bar{p}$, $\Lambda_c\bar{\Lambda}_c$, $\Lambda_c\bar{p}$, quite successfully.

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APPENDIX A: SPECTRUM OF THE $B\bar{B}$ STATES

PF is the probability factor for the $B\bar{B}$ open door channel. Masses are given in MeV.

$B\bar{B}$ state	Threshold	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$p\bar{p}$ 0^-	1876	1263	1874	2151	2407										
PF		0.15	0.80	0.001	0.05										
$p\bar{p}$ 1^-	1876	1562	1732	1911	2060	2174	2624								
PF		0.43	0.002	0.37	0	0.19	6.10^{-4}								
$\Lambda_c\bar{p}$ 0^-	3225	2653	3028	3188	3339	3595									
PF		0.15	0.28	0.42	0.13	0.02									
$\Lambda_c\bar{p}$ 1^-	3225	2740	2949	3064.	3156	3223	3274	3465	3553	3759					
PF		0.02	0.11	0.56	0.12	0.06	0.12	0.002	10^{-4}	8.10^{-5}					
$\Lambda\bar{\Lambda}$ 1^-	2231	2105	2125	2142	2184	2231	2246.8	2247.2	2274	2297	2303	2325	2343.83	2343.95	2421
PF		0.003	0	0.01	0.36	0.13	0	0.008	0.31	$3.5.10^{-5}$	0	0.17	8.10^{-4}	0	5.10^{-4}

$B\bar{B}$ state	Threshold	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$\Lambda_c\bar{\Lambda}_c 1^-$	4573	4468	4510	4533	4556	4575	4598	4614	4642	4654	4658	4669	4685	4689	4736
PF		0.025	0	0.17	0.044	0.27	0	0.17	0.23	0	0.026	0.06	0	0	8.10^{-5}
$\Sigma\bar{\Sigma}\ 1^-$	2380	2211	2236	2270	2273	2283	2310	2334	2346	2349	2356	2415	2415.6	2434	2454
PF		2.10^{-5}	4.10^{-4}	0	0.005	4.10^{-4}	0	0.012	0.35	0	0.34	0.006	0	5.10^{-4}	0.29
$\Sigma_c\bar{\Sigma}_c\ 1^-$	4910	4581	4632	4638	4646	4662	4670	4679	4702	4708	4715	4741	4742	4761	4778
PF		0.007	0.027	0	0.029	0.029	0	0.23	0.3	0	0.069	0.038	0	0.21	0.059

APPENDIX B: CROSSING MATRICES

Spin 0:

$$R_{\phi\to\alpha} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{2} \\ -\frac{1}{\sqrt{6}} & \frac{1}{2} & \frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{3}} \\ \frac{1}{2} & -\frac{1}{\sqrt{6}} & \frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{\sqrt{6}} \end{pmatrix}, \tag{B1}$$

$$R_{\phi\to\epsilon} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{2} \\ -\frac{1}{\sqrt{6}} & \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{3}} \\ \frac{1}{2} & -\frac{1}{\sqrt{6}} & -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{1}{\sqrt{6}} \end{pmatrix}. \tag{B2}$$

Spin 1:

$$R_{\psi\to\beta} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{6}} \\ 0 & 0 & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{2\sqrt{3}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{2\sqrt{3}} \\ 0 & 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \end{pmatrix}, \tag{B3}$$

$$R_{\psi\to\chi} = \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & \frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{6}} & \frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{6}} & -\frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{6}} \\ 0 & 0 & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{2\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{2\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{2\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{2\sqrt{3}} \\ 0 & 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \end{pmatrix}, \tag{B4}$$

$$R_{\xi\to\gamma} = \frac{1}{\sqrt{3}}\begin{pmatrix} \sqrt{2} & 1 \\ 1 & -\sqrt{2} \end{pmatrix}, \qquad R_{\xi\to\delta} = \frac{1}{\sqrt{3}}\begin{pmatrix} \sqrt{2} & -1 \\ -1 & -\sqrt{2} \end{pmatrix}. \tag{B5}$$

APPENDIX C: CHROMOMAGNETIC OPERATOR FOR $qqq\overline{qqq}$ STATES

Parameters are $C_{qq} = r$, $C_{qc} = s$, $C_{c\bar{c}} = t$.

We have computed the matrices of chromomagnetism by inserting the operator Eq. (1) between the states at Eqs. (10) and (11); they are given below, where $\mathbf{A_0}$ is for 0^- and $\mathbf{A_1}$ for 1^-.

$$\mathbf{A_0} = \begin{pmatrix} -2 & -\sqrt{2} & -1 & 0 \\ -\sqrt{2} & -1 & -\frac{3}{\sqrt{2}} & -\sqrt{5} \\ -1 & -\frac{3}{\sqrt{2}} & -2 & -\sqrt{\frac{5}{2}} \\ 0 & -\sqrt{5} & -\sqrt{\frac{5}{2}} & 0 \end{pmatrix}, \tag{C1}$$

$$\mathbf{A_1} = \begin{pmatrix} 2 & -\frac{\sqrt{2}}{3} & \frac{\sqrt{5}}{3} & 0 & \frac{2\sqrt{2}}{3} & -\frac{2\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{3} & \frac{1}{3} & \sqrt{\frac{5}{2}} & -\frac{\sqrt{5}}{3} & -\frac{1}{3} & \frac{1}{3} \\ \frac{\sqrt{5}}{3} & \sqrt{\frac{5}{2}} & \frac{4}{3} & \frac{5}{3\sqrt{2}} & \frac{\sqrt{\frac{5}{2}}}{3} & -\frac{\sqrt{\frac{5}{2}}}{3} \\ 0 & -\frac{\sqrt{5}}{3} & \frac{5}{3\sqrt{2}} & -\frac{4}{3} & -\frac{2\sqrt{5}}{3} & \frac{2\sqrt{5}}{3} \\ \frac{2\sqrt{2}}{3} & -\frac{1}{3} & \frac{\sqrt{\frac{5}{2}}}{3} & -\frac{2\sqrt{5}}{3} & \frac{5}{6} & -\frac{1}{2} \\ -\frac{2\sqrt{2}}{3} & \frac{1}{3} & -\frac{\sqrt{\frac{5}{2}}}{3} & \frac{2\sqrt{5}}{3} & -\frac{1}{2} & \frac{5}{6} \end{pmatrix}. \tag{C2}$$

CM matrices for basis $(15, \overline{15})$:

$$\begin{pmatrix} A & C \\ C^T & B \end{pmatrix}$$

$$
\mathbf{A}=\begin{pmatrix}
-\frac{4}{3}(r+2s) & 0 & 0 & 0 & \frac{11}{27}\sqrt{2}(r-2s+t) & \frac{2}{27}\sqrt{10}(r+s-2t) & -\frac{2}{27}\sqrt{10}(r+s-2t) \\
0 & -\frac{4}{3}(r-s) & 0 & 0 & \frac{2}{27}\sqrt{10}(r+s-2t) & \frac{5}{27}\sqrt{2}(2r-s-t) & -\frac{1}{27}\sqrt{2}(r+4(s+t)) \\
0 & 0 & -\frac{4}{3}(r-s) & 0 & -\frac{2}{27}\sqrt{10}(r+s-2t) & -\frac{1}{27}\sqrt{2}(r+4(s+t)) & \frac{5}{27}\sqrt{2}(2r-s-t) \\
0 & 0 & 0 & -\frac{4}{3}(r-4s) & \frac{2}{27}\sqrt{5}(r+4(s+t)) & -\frac{4}{27}(2r+5s+2t) & \frac{4}{27}(2r+5s+2t) \\
\frac{11}{27}\sqrt{2}(r-2s+t) & \frac{2}{27}\sqrt{10}(r+s-2t) & -\frac{2}{27}\sqrt{10}(r+s-2t) & \frac{2}{27}\sqrt{5}(r+4(s+t)) & \frac{1}{54}(5r+62s+77t) & \frac{1}{27}\sqrt{5}(7r-2(s+7t)) & \frac{1}{27}\sqrt{5}(-7r+2(s+7t)) \\
\frac{2}{27}\sqrt{10}(r+s-2t) & \frac{5}{27}\sqrt{2}(2r-s-t) & -\frac{1}{27}\sqrt{2}(r+4(s+t)) & -\frac{4}{27}(2r+5s+2t) & \frac{1}{27}\sqrt{5}(7r-2(s+7t)) & \frac{1}{54}(-2r+s-35t) & \frac{1}{54}(-7r+8s-28t) \\
-\frac{2}{27}\sqrt{10}(r+s-2t) & -\frac{1}{27}\sqrt{2}(r+4(s+t)) & \frac{5}{27}\sqrt{2}(2r-s-t) & \frac{4}{27}(2r+5s+2t) & \frac{1}{27}\sqrt{5}(-7r+2(s+7t)) & \frac{1}{54}(-7r+8s-28t) & \frac{1}{54}(-2r+s-35t)
\end{pmatrix}
$$

$$
\mathbf{B}=\begin{pmatrix}
\frac{1}{54}(-100r-7(4s+t)) & \frac{1}{9}\sqrt{2}(r+8s) & \frac{1}{18}(2r+31s) & -\frac{1}{9}\sqrt{2}(r+8s) & \frac{1}{18}(2r+31s) & -\frac{r}{2} & -\frac{\sqrt{5}r}{3} \\
\frac{1}{9}\sqrt{2}(r+8s) & \frac{1}{18}(-30r+23s-5t) & -\frac{2}{9}\sqrt{2}(2s+t) & -\frac{r}{2} & \sqrt{2}r & -\frac{2\sqrt{2}s}{3} & \frac{2\sqrt{10}s}{3} \\
\frac{1}{18}(2r+31s) & -\frac{2}{9}\sqrt{2}(2s+t) & \frac{1}{18}(-30r-14s-t) & -\sqrt{2}r & -\frac{r}{2} & \frac{11s}{6} & -\frac{\sqrt{5}s}{3} \\
-\frac{1}{9}\sqrt{2}(r+8s) & -\frac{r}{2} & -\sqrt{2}r & \frac{1}{18}(-30r+23s-5t) & \frac{2}{9}\sqrt{2}(2s+t) & \frac{2\sqrt{2}s}{3} & -\frac{2\sqrt{10}s}{3} \\
\frac{1}{18}(2r+31s) & \sqrt{2}r & -\frac{r}{2} & \frac{2}{9}\sqrt{2}(2s+t) & \frac{1}{18}(-30r-14s-t) & \frac{11s}{6} & -\frac{\sqrt{5}s}{3} \\
-\frac{r}{2} & -\frac{2\sqrt{2}s}{3} & \frac{11s}{6} & \frac{2\sqrt{2}s}{3} & \frac{11s}{6} & \frac{1}{2}(-4r-t) & -\frac{\sqrt{5}t}{3} \\
-\frac{\sqrt{5}r}{3} & \frac{2\sqrt{10}s}{3} & -\frac{\sqrt{5}s}{3} & -\frac{2\sqrt{10}s}{3} & -\frac{\sqrt{5}s}{3} & -\frac{\sqrt{5}t}{3} & -\frac{2}{3}(3r+t)
\end{pmatrix},
$$

$$
\mathbf{C}=\begin{pmatrix}
\frac{2}{27}\sqrt{5}(r+4(s+t)) & \frac{2}{9}\sqrt{10}(-r+s) & -\frac{2}{9}\sqrt{5}(r+2s) & \frac{2}{9}\sqrt{10}(r-s) & -\frac{2}{9}\sqrt{5}(r+2s) & \frac{2\sqrt{5}r}{3} & 0 \\
-\frac{4}{27}(2r+5s+2t) & \frac{5}{9}\sqrt{2}(r-s) & -\frac{4}{9}(r+2s) & \frac{1}{9}\sqrt{2}(r+2s) & \frac{4}{9}(2r+s) & \frac{4r}{3} & 0 \\
\frac{4}{27}(2r+5s+2t) & \frac{1}{9}\sqrt{2}(r+2s) & -\frac{4}{9}(2r+s) & \frac{5}{9}\sqrt{2}(r-s) & \frac{4}{9}(r+2s) & -\frac{4r}{3} & 0 \\
-\frac{1}{27}\sqrt{2}(4r+4s+t) & -\frac{4}{9}(r+2s) & -\frac{1}{9}\sqrt{2}(2r+s) & \frac{4}{9}(r+2s) & -\frac{1}{9}\sqrt{2}(2r+s) & -\frac{\sqrt{2}r}{3} & 0 \\
\frac{1}{27}\sqrt{\frac{5}{2}}(7r-8s+28t) & \frac{1}{9}\sqrt{5}(r-4s) & \frac{1}{9}\sqrt{\frac{5}{2}}(r+8s) & -\frac{1}{9}\sqrt{5}(r-4s) & \frac{1}{9}\sqrt{\frac{5}{2}}(r+8s) & \sqrt{\frac{5}{2}}r & \frac{5\sqrt{2}r}{3} \\
-\frac{2}{27}\sqrt{2}(7r-5s+7t) & \frac{1}{18}(-5r+47s) & \frac{1}{9}\sqrt{2}(r+8s) & \frac{1}{18}(-r-8s) & -\frac{2}{9}\sqrt{2}(r+2s) & \sqrt{2}r & \frac{2\sqrt{10}r}{3} \\
\frac{2}{27}\sqrt{2}(7r-5s+7t) & \frac{1}{18}(-r-8s) & \frac{2}{9}\sqrt{2}(r+2s) & \frac{1}{18}(-5r+47s) & -\frac{1}{9}\sqrt{2}(r+8s) & -\sqrt{2}r & -\frac{2\sqrt{10}r}{3}
\end{pmatrix}.
$$

$$\begin{pmatrix} A & C \\ C^T & B \end{pmatrix}$$

CM matrices for basis $(21, \overline{21})$:

$$\begin{pmatrix} A & C \\ C^T & B \end{pmatrix}$$

$$\mathbf{A} = \begin{pmatrix}
\mathbf{4r} & -\frac{\sqrt{2}t}{3} & -\frac{4s}{3} & \frac{\sqrt{2}s}{3} & \frac{4s}{3} & \frac{\sqrt{2}s}{3} & \frac{2\sqrt{5}r}{3} \\
-\frac{\sqrt{2}t}{3} & \mathbf{4r}-\frac{7t}{6} & \frac{4\sqrt{2}s}{3} & \frac{5s}{6} & -\frac{4\sqrt{2}s}{3} & \frac{5s}{6} & \sqrt{\frac{5}{2}}\mathbf{r} \\
-\frac{4s}{3} & \frac{4\sqrt{2}s}{3} & \frac{7r}{3}+\frac{35s}{18}-\frac{5t}{18} & -\frac{10\sqrt{2}s}{9}-\frac{2\sqrt{2}t}{9} & -\frac{r}{2} & \sqrt{2}\mathbf{r} & -\frac{5\sqrt{5}r}{9}+\frac{2\sqrt{5}s}{9} \\
\frac{\sqrt{2}s}{3} & \frac{5s}{6} & -\frac{10\sqrt{2}s}{9}-\frac{2\sqrt{2}t}{9} & \frac{7r}{3}+\frac{5s}{9}-\frac{t}{18} & -\sqrt{2}\mathbf{r} & -\frac{r}{2} & -\frac{5}{9}\sqrt{\frac{5}{2}}\mathbf{r}-\frac{2\sqrt{10}s}{9} \\
\frac{4s}{3} & -\frac{4\sqrt{2}s}{3} & -\frac{r}{2} & -\sqrt{2}\mathbf{r} & \frac{7r}{3}+\frac{35s}{18}-\frac{5t}{18} & \frac{10\sqrt{2}s}{9}+\frac{2\sqrt{2}t}{9} & \frac{5\sqrt{5}r}{9}-\frac{2\sqrt{5}s}{9} \\
\frac{\sqrt{2}s}{3} & \frac{5s}{6} & \sqrt{2}\mathbf{r} & -\frac{r}{2} & \frac{10\sqrt{2}s}{9}+\frac{2\sqrt{2}t}{9} & \frac{7r}{3}+\frac{5s}{9}-\frac{t}{18} & -\frac{5}{9}\sqrt{\frac{5}{2}}\mathbf{r}-\frac{2\sqrt{10}s}{9} \\
\frac{2\sqrt{5}r}{3} & \sqrt{\frac{5}{2}}\mathbf{r} & -\frac{5\sqrt{5}r}{9}+\frac{2\sqrt{5}s}{9} & -\frac{5}{9}\sqrt{\frac{5}{2}}\mathbf{r}-\frac{2\sqrt{10}s}{9} & \frac{5\sqrt{5}r}{9}-\frac{2\sqrt{5}s}{9} & -\frac{5}{9}\sqrt{\frac{5}{2}}\mathbf{r}-\frac{2\sqrt{10}s}{9} & \frac{67r}{18}-\frac{5s}{3}+\frac{11t}{18}
\end{pmatrix},$$

$$\mathbf{B} = \begin{pmatrix}
\frac{31r}{9}+\frac{5s}{6}-\frac{5t}{18} & -\frac{5r}{18}-\frac{2t}{9} & -\frac{10\sqrt{2}r}{9}-\frac{2\sqrt{2}t}{9} & \frac{10r}{27}+\frac{10s}{27}-\frac{20t}{27} & \frac{10\sqrt{5}r}{27}-\frac{5\sqrt{5}s}{27}-\frac{5\sqrt{5}t}{27} & -\frac{\sqrt{5}r}{27}-\frac{4\sqrt{5}s}{27}-\frac{4\sqrt{5}t}{27} & -\frac{4\sqrt{10}r}{27}-\frac{10\sqrt{10}s}{27}-\frac{4\sqrt{10}t}{27} \\
-\frac{5r}{18}-\frac{2t}{9} & \frac{31r}{9}+\frac{5s}{6}-\frac{5t}{18} & \frac{10\sqrt{2}r}{9}+\frac{2\sqrt{2}t}{9} & -\frac{10r}{27}-\frac{10s}{27}+\frac{20t}{27} & -\frac{\sqrt{5}r}{27}-\frac{4\sqrt{5}s}{27}-\frac{4\sqrt{5}t}{27} & \frac{10\sqrt{5}r}{27}-\frac{5\sqrt{5}s}{27}-\frac{5\sqrt{5}t}{27} & \frac{4\sqrt{10}r}{27}+\frac{10\sqrt{10}s}{27}+\frac{4\sqrt{10}t}{27} \\
-\frac{10\sqrt{2}r}{9}-\frac{2\sqrt{2}t}{9} & \frac{10\sqrt{2}r}{9}+\frac{2\sqrt{2}t}{9} & -\frac{4r}{9}+\frac{10s}{3}-\frac{t}{18} & \frac{5\sqrt{2}r}{27}+\frac{20\sqrt{2}s}{27}+\frac{20\sqrt{2}t}{27} & -\frac{4\sqrt{10}r}{27}-\frac{10\sqrt{10}s}{27}-\frac{4\sqrt{10}t}{27} & \frac{4\sqrt{10}r}{27}+\frac{10\sqrt{10}s}{27}+\frac{4\sqrt{10}t}{27} & -\frac{4\sqrt{5}r}{27}-\frac{4\sqrt{5}s}{27}-\frac{\sqrt{5}t}{27} \\
\frac{10r}{27}+\frac{10s}{27}-\frac{20t}{27} & -\frac{10r}{27}-\frac{10s}{27}+\frac{20t}{27} & \frac{5\sqrt{2}r}{27}+\frac{20\sqrt{2}s}{27}+\frac{20\sqrt{2}t}{27} & \frac{106r}{27}+\frac{124s}{27}+\frac{22t}{27} & \frac{16\sqrt{5}r}{27}-\frac{8\sqrt{5}s}{27}-\frac{8\sqrt{5}t}{27} & -\frac{16\sqrt{5}r}{27}+\frac{8\sqrt{5}s}{27}+\frac{8\sqrt{5}t}{27} & \frac{8\sqrt{10}r}{27}-\frac{16\sqrt{10}s}{27}+\frac{8\sqrt{10}t}{27} \\
\frac{10\sqrt{5}r}{27}-\frac{5\sqrt{5}s}{27}-\frac{5\sqrt{5}t}{27} & -\frac{\sqrt{5}r}{27}-\frac{4\sqrt{5}s}{27}-\frac{4\sqrt{5}t}{27} & -\frac{4\sqrt{10}r}{27}-\frac{10\sqrt{10}s}{27}-\frac{4\sqrt{10}t}{27} & \frac{16\sqrt{5}r}{27}-\frac{8\sqrt{5}s}{27}-\frac{8\sqrt{5}t}{27} & \frac{98r}{27}+\frac{2s}{27}-\frac{10t}{27} & -\frac{8r}{27}+\frac{16s}{27}-\frac{8t}{27} & -\frac{32\sqrt{2}r}{27}+\frac{40\sqrt{2}s}{27}-\frac{8\sqrt{2}t}{27} \\
-\frac{\sqrt{5}r}{27}-\frac{4\sqrt{5}s}{27}-\frac{4\sqrt{5}t}{27} & \frac{10\sqrt{5}r}{27}-\frac{5\sqrt{5}s}{27}-\frac{5\sqrt{5}t}{27} & \frac{4\sqrt{10}r}{27}+\frac{10\sqrt{10}s}{27}+\frac{4\sqrt{10}t}{27} & -\frac{16\sqrt{5}r}{27}+\frac{8\sqrt{5}s}{27}+\frac{8\sqrt{5}t}{27} & -\frac{8r}{27}+\frac{16s}{27}-\frac{8t}{27} & \frac{98r}{27}+\frac{2s}{27}-\frac{10t}{27} & \frac{32\sqrt{2}r}{27}-\frac{40\sqrt{2}s}{27}+\frac{8\sqrt{2}t}{27} \\
-\frac{4\sqrt{10}r}{27}-\frac{10\sqrt{10}s}{27}-\frac{4\sqrt{10}t}{27} & \frac{4\sqrt{10}r}{27}+\frac{10\sqrt{10}s}{27}+\frac{4\sqrt{10}t}{27} & -\frac{4\sqrt{5}r}{27}-\frac{4\sqrt{5}s}{27}-\frac{\sqrt{5}t}{27} & \frac{8\sqrt{10}r}{27}-\frac{16\sqrt{10}s}{27}+\frac{8\sqrt{10}t}{27} & -\frac{32\sqrt{2}r}{27}+\frac{40\sqrt{2}s}{27}-\frac{8\sqrt{2}t}{27} & \frac{32\sqrt{2}r}{27}-\frac{40\sqrt{2}s}{27}+\frac{8\sqrt{2}t}{27} & -\frac{14r}{27}-\frac{56s}{27}-\frac{2t}{27}
\end{pmatrix},$$

$$\mathbf{C} = \begin{pmatrix}
\frac{4r}{3} & -\frac{4r}{3} & -\frac{\sqrt{2}r}{3} & 0 & 0 & 0 & 0 \\
\sqrt{2}\mathbf{r} & -\sqrt{2}\mathbf{r} & -\frac{r}{2} & \frac{5\sqrt{2}r}{3} & \frac{2\sqrt{10}r}{3} & -\frac{2\sqrt{10}r}{3} & -\frac{\sqrt{5}r}{3} \\
\frac{25r}{18}+\frac{17s}{18} & \frac{5r}{18}+\frac{2s}{9} & -\frac{5\sqrt{2}r}{9}-\frac{4\sqrt{2}s}{9} & \frac{10r}{9}-\frac{10s}{9} & -\frac{5\sqrt{5}r}{9}+\frac{5\sqrt{5}s}{9} & -\frac{\sqrt{5}r}{9}-\frac{2\sqrt{5}s}{9} & \frac{2\sqrt{10}r}{9}+\frac{4\sqrt{10}s}{9} \\
-\frac{5\sqrt{2}r}{9}-\frac{4\sqrt{2}s}{9} & -\frac{10\sqrt{2}r}{9}-\frac{2\sqrt{2}s}{9} & -\frac{5r}{9}+\frac{25s}{18} & \frac{5\sqrt{2}r}{9}+\frac{10\sqrt{2}s}{9} & \frac{2\sqrt{10}r}{9}+\frac{4\sqrt{10}s}{9} & \frac{4\sqrt{10}r}{9}+\frac{2\sqrt{10}s}{9} & \frac{2\sqrt{5}r}{9}+\frac{\sqrt{5}s}{9} \\
\frac{5r}{18}+\frac{2s}{9} & \frac{25r}{18}+\frac{17s}{18} & \frac{5\sqrt{2}r}{9}+\frac{4\sqrt{2}s}{9} & -\frac{10r}{9}+\frac{10s}{9} & -\frac{\sqrt{5}r}{9}-\frac{2\sqrt{5}s}{9} & -\frac{5\sqrt{5}r}{9}+\frac{5\sqrt{5}s}{9} & -\frac{2\sqrt{10}r}{9}-\frac{4\sqrt{10}s}{9} \\
\frac{10\sqrt{2}r}{9}+\frac{2\sqrt{2}s}{9} & \frac{5\sqrt{2}r}{9}+\frac{4\sqrt{2}s}{9} & -\frac{5r}{9}+\frac{25s}{18} & \frac{5\sqrt{2}r}{9}+\frac{10\sqrt{2}s}{9} & -\frac{4\sqrt{10}r}{9}-\frac{2\sqrt{10}s}{9} & -\frac{2\sqrt{10}r}{9}-\frac{4\sqrt{10}s}{9} & \frac{2\sqrt{5}r}{9}+\frac{\sqrt{5}s}{9} \\
\frac{5\sqrt{5}r}{9}-\frac{2\sqrt{5}t}{9} & -\frac{5\sqrt{5}r}{9}+\frac{2\sqrt{5}t}{9} & \frac{5}{9}\sqrt{\frac{5}{2}}r+\frac{2\sqrt{10}t}{9} & \frac{11\sqrt{5}r}{27}-\frac{22\sqrt{5}s}{27}+\frac{11\sqrt{5}t}{27} & \frac{10r}{27}+\frac{10s}{27}-\frac{20t}{27} & -\frac{10r}{27}-\frac{10s}{27}+\frac{20t}{27} & \frac{5\sqrt{2}r}{27}+\frac{20\sqrt{2}s}{27}+\frac{20\sqrt{2}t}{27}
\end{pmatrix}.$$

CM matrices for basis $\Lambda_c\bar{p}$:

Spin 0:

$$\begin{pmatrix}
16r & 4\sqrt{2}(-s+r) & -8r & 0 & 4\sqrt{2}(s+r) \\
4\sqrt{2}(-s+r) & 12(s+r) & \frac{8}{3}\sqrt{2}(-s+r) & \frac{4}{3}\sqrt{5}(-s+r) & 0 \\
-8r & \frac{8}{3}\sqrt{2}(-s+r) & 16r & \frac{4}{3}\sqrt{10}(2s+r) & -\frac{4}{3}\sqrt{2}(2s+7r) \\
0 & \frac{4}{3}\sqrt{5}(-s+r) & \frac{4}{3}\sqrt{10}(2s+r) & -8s+8r & -\frac{4}{3}\sqrt{5}(s+5r) \\
4\sqrt{2}(s+r) & 0 & -\frac{4}{3}\sqrt{2}(2s+7r) & -\frac{4}{3}\sqrt{5}(s+5r) & 8r
\end{pmatrix},$$

Spin 1:

$$\begin{pmatrix}
16r & \frac{2\sqrt{2}s}{3}-\frac{2\sqrt{2}r}{3} & -\frac{4\sqrt{5}r}{3} & 0 & -\frac{8\sqrt{2}r}{3} & \frac{4\sqrt{2}s}{3}-\frac{4\sqrt{2}r}{3} & -\frac{2\sqrt{2}s}{3}-\frac{2\sqrt{2}r}{3} & -\frac{4\sqrt{2}s}{3}-\frac{4\sqrt{2}r}{3} & 0 \\
\frac{2\sqrt{2}s}{3}-\frac{2\sqrt{2}r}{3} & \frac{76s}{9}+\frac{44r}{9} & -\frac{8\sqrt{10}s}{9}+\frac{8\sqrt{10}r}{9} & \frac{4\sqrt{5}s}{9}-\frac{4\sqrt{5}r}{9} & -\frac{8s}{9}+\frac{8r}{9} & \frac{56s}{9}+\frac{112r}{9} & \frac{40s}{9}-\frac{40r}{9} & -\frac{40s}{9}+\frac{40r}{9} & -\frac{16\sqrt{10}s}{9}-\frac{32\sqrt{10}r}{9} \\
-\frac{4\sqrt{5}r}{3} & -\frac{8\sqrt{10}s}{9}+\frac{8\sqrt{10}r}{9} & -\frac{8s}{9}+\frac{104r}{9} & \frac{40\sqrt{2}s}{9}+\frac{20\sqrt{2}r}{9} & -\frac{8\sqrt{10}s}{9}+\frac{20\sqrt{10}r}{9} & -\frac{4\sqrt{10}s}{9}+\frac{4\sqrt{10}r}{9} & -\frac{8\sqrt{10}s}{9}-\frac{28\sqrt{10}r}{9} & -\frac{4\sqrt{10}s}{9}+\frac{16\sqrt{10}r}{9} & \frac{40s}{9}-\frac{40r}{9} \\
0 & \frac{4\sqrt{5}s}{9}-\frac{4\sqrt{5}r}{9} & \frac{40\sqrt{2}s}{9}+\frac{20\sqrt{2}r}{9} & -\frac{40s}{9}-\frac{56r}{9} & -\frac{32\sqrt{5}s}{9}-\frac{16\sqrt{5}r}{9} & -\frac{8\sqrt{5}s}{9}+\frac{8\sqrt{5}r}{9} & \frac{4\sqrt{5}s}{9}+\frac{20\sqrt{5}r}{9} & -\frac{8\sqrt{5}s}{9}-\frac{40\sqrt{5}r}{9} & -\frac{32\sqrt{2}s}{9}+\frac{32\sqrt{2}r}{9} \\
-\frac{8\sqrt{2}r}{3} & -\frac{8s}{9}+\frac{8r}{9} & -\frac{8\sqrt{10}s}{9}+\frac{20\sqrt{10}r}{9} & -\frac{32\sqrt{5}s}{9}-\frac{16\sqrt{5}r}{9} & -\frac{20s}{9}+\frac{80r}{9} & \frac{8s}{9}-\frac{8r}{9} & -\frac{8s}{9}+\frac{32r}{9} & \frac{8s}{9}+\frac{28r}{9} & \frac{20\sqrt{10}s}{9}-\frac{20\sqrt{10}r}{9} \\
\frac{4\sqrt{2}s}{3}-\frac{4\sqrt{2}r}{3} & \frac{56s}{9}+\frac{112r}{9} & -\frac{4\sqrt{10}s}{9}+\frac{4\sqrt{10}r}{9} & -\frac{8\sqrt{5}s}{9}+\frac{8\sqrt{5}r}{9} & \frac{8s}{9}-\frac{8r}{9} & \frac{124s}{9}+\frac{104r}{9} & -\frac{40s}{9}+\frac{40r}{9} & -\frac{20s}{9}+\frac{20r}{9} & -\frac{4\sqrt{10}s}{9}-\frac{8\sqrt{10}r}{9} \\
-\frac{2\sqrt{2}s}{3}-\frac{2\sqrt{2}r}{3} & \frac{40s}{9}-\frac{40r}{9} & -\frac{8\sqrt{10}s}{9}-\frac{28\sqrt{10}r}{9} & \frac{4\sqrt{5}s}{9}+\frac{20\sqrt{5}r}{9} & -\frac{8s}{9}+\frac{32r}{9} & -\frac{40s}{9}+\frac{40r}{9} & -\frac{32s}{9}+\frac{56r}{9} & \frac{56s}{9}-\frac{32r}{9} & -\frac{16\sqrt{10}s}{9}+\frac{16\sqrt{10}r}{9} \\
-\frac{4\sqrt{2}s}{3}-\frac{4\sqrt{2}r}{3} & -\frac{40s}{9}+\frac{40r}{9} & -\frac{4\sqrt{10}s}{9}+\frac{16\sqrt{10}r}{9} & -\frac{8\sqrt{5}s}{9}-\frac{40\sqrt{5}r}{9} & \frac{8s}{9}+\frac{28r}{9} & -\frac{20s}{9}+\frac{20r}{9} & \frac{56s}{9}-\frac{32r}{9} & \frac{16s}{9}+\frac{44r}{9} & -\frac{4\sqrt{10}s}{9}+\frac{4\sqrt{10}r}{9} \\
0 & -\frac{16\sqrt{10}s}{9}-\frac{32\sqrt{10}r}{9} & \frac{40s}{9}-\frac{40r}{9} & -\frac{32\sqrt{2}s}{9}+\frac{32\sqrt{2}r}{9} & \frac{20\sqrt{10}s}{9}-\frac{20\sqrt{10}r}{9} & -\frac{4\sqrt{10}s}{9}-\frac{8\sqrt{10}r}{9} & -\frac{16\sqrt{10}s}{9}+\frac{16\sqrt{10}r}{9} & -\frac{4\sqrt{10}s}{9}+\frac{4\sqrt{10}r}{9} & \frac{64s}{9}+\frac{56r}{9}
\end{pmatrix}.$$

[1] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977).
[2] R. L. Jaffe, Phys. Rev. D **17**, 1444 (1978).
[3] R. L. Jaffe, Phys. Rev. D **15**, 281 (1977).
[4] H. Högaasen and P. Sorba, Nucl. Phys. **B145**, 119 (1978); M. De Crombrugghe, H. Högaasen, and P. Sorba, Nucl. Phys. **B156**, 347 (1979).
[5] R. L. Jaffe and F. Wilczek, Phys. Rev. Lett. **91**, 232003 (2003).
[6] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Phys. Rev. Lett. **92**, 042003 (2004).
[7] F. Ambrosino *et al.* (KLOE Collaboration), Phys. Lett. B **681**, 5 (2009).
[8] I. Bigi, L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Phys. Rev. D **72**, 114016 (2005).
[9] A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D **12**, 147 (1975).
[10] F. Buccella, H. Högaasen, J. M. Richard, and P. Sorba, Eur. Phys. J. C **49**, 743 (2007).
[11] J. Z. Bai, Y. Ban, and J. G. Bian (BES Collaboration), Phys. Rev. Lett. **91**, 022001 (2003).
[12] N. Gabyshev *et al.*, Phys. Rev. Lett. **97**, 242001 (2006).
[13] G. Pakhlova *et al.* (Belle Collaboration), Phys. Rev. Lett. **101**, 172001 (2008).
[14] R. L. Jaffe, Phys. Rep. **409**, 1 (2005); Nucl. Phys. B, Proc. Suppl. **142**, 343 (2005).
[15] A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D **77**, 094004 (2008).
[16] J. A. M. Vermaseren, arXiv:math-ph/0010025.
[17] M. Abud, F. Buccella, D. Falcone, G. Ricciardi, and F. Tramontano, Adv. Studies Theor. Phys. **2**, 929 (2008).
[18] B. Aubert *et al.*, Phys. Rev. Lett. **101**, 071801 (2008); see also J. Marks (*BABAR* Collaboration), arXiv:0906.0725v1.
[19] C. Quigg and J. L. Rosner, Phys. Rep. **56**, 167 (1979).
[20] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, F. Feruglio, R. Gatto, and G. Nardulli, Phys. Rep. **281**, 145 (1997).
[21] H. Albrecht *et al.*, Z. Phys. C **50**, 1 (1991).
[22] A. Etkin *et al.*, Phys. Lett. B **201**, 568 (1988).
[23] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008) and 2009 partial update for the 2010 edition.
[24] M. Gaspero, Nucl. Phys. **A562**, 407 (1993).
[25] G. 't Hooft, G. Isidori, L. Maiani, A. D. Polosa, and V. Riquer, Phys. Lett. B **662**, 424 (2008).
[26] S. Uehara *et al.* (Belle Collaboration), Phys. Rev. D **80**, 032001 (2009).
[27] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **102**, 242002 (2009).
[28] Fl. Stancu, arXiv0906.2485.
[29] C. P. Shen *et al.* (Belle Collaboration), arXiv:0912.2383v1 [Phys. Rev. Lett. (to be published)].
[30] M. Ablikim *et al.* (BES Collaboration), Phys. Rev. Lett. **95**, 262001 (2005).
[31] G-J. Ding, J-L. Pingand, and M-L. Yan, Phys. Rev. D **74**, 014029 (2006).
[32] C. Evangelista *et al.*, Nucl. Phys. **B153**, 253 (1979).
[33] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **74**, 091103 (2006).
[34] M. Ablikim *et al.* (BES Collaboration), Phys. Rev. Lett. **100**, 102003 (2008).
[35] A. Zupanc (Belle Collaboration), arXiv:0910.3404v1.
[36] G. Pakhlova *et al.* (Belle Collaboration), Phys. Rev. Lett. **101**, 172001 (2008) .
[37] X. L. Wang *et al.* (Belle Collaboration), Phys. Rev. Lett. **99**, 142002 (2007).
[38] F-K. Guo, C. Hanhart, and U-G. Meissner, Phys. Lett. B **665**, 26 (2008).
[39] G. Cotugno, R. Faccini, A. D. Polosa, and C. Sabelli, arXiv:0911.2178 [Phys. Rev. Lett. (to be published)].