Evaporation of Microscopic Black Holes in String Theory and the Bound on Species

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Abstract

We address the question how string compactifications with D-branes are consistent with the black hole bound, which arises in any theory with number of particle species to which the black holes can evaporate. For the Kaluza-Klein particles, both longitudinal and transversal to the D-branes, it is relatively easy to see that the black hole bound is saturated, and the geometric relations can be understood in the language of species-counting. We next address the question of the black hole evaporation into the higher string states and discover, that contrary to the naive intuition, the exponentially growing number of Regge states does not preclude the existence of semi-classical black holes of sub-stringy size. Our analysis indicates that the effective number of string resonances to which such micro black holes evaporate is not exponentially large but is bounded by $N = 1/g_s^2$, which suggests the interpretation of the well-known relation between the Planck and string scales as the saturation of the black hole bound on the species number. In addition, we also discuss some other issues in D-brane compactifications with a low string scale of order TeV, such as the masses of light moduli fields.

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Contents

1 Introduction

String theory, being the theory of one-dimensional extended objects, contains an infinite number of elementary particles, when seen from the field theory point of view. Specifically, the excitation spectrum of a relativistic string contains a tower of infinitely many higher spin excitations, whose number grows exponentially with the masses of the excited string modes. In fact, these states follow the well-known Regge excitation pattern with masses

$$M_n^2 = nM_s^2 \,, \tag{1}$$

where M_s is the intrinsic string scale ($M_s^{-2} = \alpha'$), and n counts the n$^{th.}$ excitation level.

In string compactifications from ten to four space-time dimensions there are several additional generic mass scales and particles, which are related to the internal geometry of the compact space. These are Kaluza-Klein (KK) particles, whose masses scale inversely with some internal radii, possible winding states, as well as moduli fields that determine the parameters of the underlying geometry.

In particular, in intersecting D-brane compactifications, there are several distinct mass scales and moduli fields, which could be also of experimental relevance. Basically, each of these mass scales is related to the Compton wave-lengths of some particles, which generically arise during the compactification process.

A particularly interesting scenario is given by D-brane compactifications with a string scale M_s being near the scale of the Standard Model (SM), i.e. $M_s = \mathcal{O}(\text{TeV})$ [1], which implies that some of the extra dimensions must be much larger than M_s^{-1}. Moreover, since M_s sets the fundamental scale of gravity in the higher dimensional space, the gravitational effects will start to play an important role already at low energies, and possibly be accessible experimentally. In fact, in this scenario there is an explosion of string states at the TeV scale, and some spectacular model-independent stringy signatures are expected at the LHC in case the string scale is low [2, 3, 4]. D-brane compactifications with a low string scale are indeed possible, because the open string gauge interactions are confined to the lower dimensional world volumes of the D-branes. On the other hand, the gravitational force is enormously weakened by the fact, that the closed string gravitons can leak out into the large transversal bulk space. Of course, one has to investigate, if low strings scale D-brane compactifications can be dynamically realized by a certain moduli potential, a question, which is not a focus of our paper, but which we shall only briefly touch later.

Low string scale D-brane models are a concrete, microscopic realization of the large extra-dimension scenario of ADD, which was originally introduced as a phenomenologically-motivated solution to the hierarchy problem [5]. Of course, in string theory compactifications, some new phenomenological questions compared to ADD arise, such as the appearance of several new mass scales, the existence of additional states and in particular the problem of light moduli at tree level, which typically arise in case of a large internal string manifold. We will discuss the light moduli problem and radiative mass corrections to the moduli fields later on.

The main focus of the present paper is to investigate how the string theory compactifications, viewed as the theories of many particle species, cope with the consistency requirements of the semi-classical black hole (BH) physics [6, 7, 8, 9].

This question emerges already at the level of high-dimensional field theories. Indeed, from the point of view of a four-dimensional observer, any high-dimensional theory is a theory of large number of particle species, since every high-dimensional field (e.g., a graviton) produces a tower of KK excitations. An each member of this tower represents a four-dimensional particle with a definite mass and other quantum numbers. The intrinsic geometric property of large extra dimensional scenarios is the hierarchy between the four-dimensional (M_4) and high-dimensional (M_{4+d}) Planck scales produced by the large compactification volume

$$M_4^2 = M_{4+d}^2 (M_{4+d}^d V^{(d)}) , \qquad (2)$$

where the quantity $M_{4+d}^d V^{(d)}$ is the volume of the compact manifold measured in the units of the high-dimensional Planck length $l_{4+d} \equiv M_{4+d}^{-1}$.

Recently [6, 7, 8, 9], it was understood that the above simple geometric relation, is in fact a particular realization of a much more general bound, according to which in any theory with N particle species, there is an inevitable hierarchy between the Planck mass and the fundamental gravitational cutoff of the theory, which we shall denote by M_*. Among several possible equivalent definitions of M_*, we shall use the one in which $l_* \equiv M_*^{-1}$ is the size of the smallest possible semi-classical neutral non-rotating BH. In four-dimensions, the bound then reads [6, 8, 9]:[3]

$$M_* = \frac{M_4}{\sqrt{N_4}} \, . \tag{3}$$

Correspondingly in $4 + d$-dimensions we have

$$M_* = \frac{M_{4+d}}{N_{4+d}^{\frac{1}{2+d}}} \, , \tag{4}$$

where both M_{4+d} and N_{4+d} must be understood from the point of view of $4+d$-dimensional theory. That is, N_{4+d} counts the number of $4+d$-dimensional species. Notice, that since the scale M_* is unique for any given theory, the saturation of the bound in a high-dimensional theory automatically implies its saturation in a dimensionally-reduced theory. This implies, that the geometric relation between the Planck masses of high and low dimensional theories is controlled by the number of KK species.

As said above, the scale l_* marks the size below which BH can no longer maintain their semi-classical properties, such as, the small relative rate of the temperature change,

$$\frac{1}{T^2} \frac{dT}{dt} \ll 1 \, , \tag{5}$$

where the temperature is the inverse of the gravitational radius $r_g = T^{-1}$. The latter relation must hold for any static, neutral and non-rotating semi-classical BH. The bounds (3) and (4) follow from the fact that the semi-classicality condition (5) is violated for any BH smaller than l_*, due to an unsustainably high rate of Hawking evaporation into the N particle species [6].

Alternatively [8], the bound follows from the fact that at length scales shorter than l_*, the resolution of species is impossible *in principle*, since any detector capable of differentiating among the species at such a short length-scale would itself collapse into a BH.

It is obvious, that the geometric relation (2) is a particular form of (3). For seeing this, it is enough to notice that the factor $M_{4+d}^d V^{(d)} \equiv N_{KK}$ simply counts the number of KK species, whereas the high-dimensional Planck length is an obvious cutoff of the high-dimensional gravity theory.

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[3]This bound has been generalized for de Sitter spaces and cosmological backgrounds in [7].

As it follows from (3), in case the mass M_* is of order TeV, one expects $N = 10^{32}$ different species at or below this scale, and the strength of the gravitational interactions is diluted to its observed value due to the existence of this large number of particles. Therefore, ADD model can be viewed as a particular representative of much more general class of the large species models that address the hierarchy problem between the Planck scale and the TeV scale.

The purpose of the present work is to explore the physical meaning of (3) and (4) in string theory. As explained above, from the effective field theory perspective, string theory is a theory of infinite number of particle species. Since, the bound relies on the BH evaporation, it is thus crucial to understand which are the relevant string species participating in this process. The question we would like to address now is, which particles play the role of the different species in D-brane string compactifications, and how the black hole bound eqs.(3) and (4) is satisfied in these string models.

In other words, we wish to understand how the string compactifications fit into the large species scenario. Thus, the following questions will be addressed:

- What is relation of the different mass scales in D-brane compactifications to the different particle species participating in BH evaporation?

- How is the black hole bound satisfied by the stringy excitations?

As we shall see these questions can be relatively easily answered for the KK modes, both for KK modes corresponding to the open strings on the wrapped D-branes and also for the closed string KK modes in the bulk space, transversal to the D-branes. However, the most interesting and more subtle question is, how the stringy excitation modes (Regge states) respect the bound (4), because naively the exponentially growing number of these states seems to violate it for any semi-classical BH smaller than the string length.

There are the two regimes that we shall explore.

1.1 Super-Stringy Black Holes

The first regime explores the BHs with the gravitational radius larger than the string length $r_g > l_s$, where $l_s \equiv M_s^{-1}$. Such BHs can only evaporate into the lowest closed and open strings modes and their KK excitations. An obvious requirement, that such BHs must be semi-classical, implies the absolute upper bound on the value of the string scale

$$M_s \leqslant \frac{M_{4+d}}{N_{4+d}^{\frac{1}{2+d}}(m < M_s)} , \qquad (6)$$

where $N_{4+d}(m < M_s)$ counts the number of all $4+d$-dimensional elementary particle species with masses below the string scale. In a consistent compactification, the bound must be satisfied for any d, and as we shall see, it indeed is.

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1.2 Sub-Stringy Black Holes

The second regime, in which BHs horizon shrink below the string length, $r_g < l_s$, is much more profound. The naive impression is, that such BHs simply cannot exist as semi-classical objects with the well-defined Hawking temperature, since for $T > M_s$ the number of thermally-accessible species diverges exponentially and one is bound by the Hagedorn effect. The result would be an exponential increase of BH evaporation channels and the semi-classicality condition (5) would be violated almost immediately. We shall see, that this naive argument is false, and it alone cannot prevent the semi-classical BHs from continuing existence in sub-l_s domain. The reason is, that although number of states is indeed exponentially large, only few of them are effectively produced in the BH evaporation. Our findings indicate that the effective number of emitted string resonances is maximum

$$N_{eff} \sim 1/g_s^2, \tag{7}$$

where g_s is the string coupling constant[4].

In order to illustrate this suppression, we shall develop an effective theory of BH evaporation into the string resonances. The emission of a given string resonance is suppressed by a power of T/M_{4+d} defined by the oscillator number. A crude intuitive understanding of this suppression can come from thinking of higher string resonances as of vibrating long strings. Production of such a string by a tiny BH is only possible for certain patterns of the string vibration, namely if during its vibration the strings contracts down to a BH size. For the other vibration patterns, the production must be suppressed.

This simple qualitative picture is also supported by a toy model, which allows for a microscopic field-theoretic description of the BHs evaporation into stringy states. The latter model is an $SU(n)$-QCD coupled to Einsteinian gravity. The advantage of this system is, that it allows for the complementary descriptions of the micro BH evaporation both in the language of closed strings (glueballs), as well as in the language of elementary gluons. The latter description reproduces (7).

Remarkably, the relation (7) when substituted in (4), reproduces the known relation between the string and the Planck scales

$$M_{10}^8 = \frac{1}{g_s^2} M_s^8 . \tag{8}$$

Although this remarkable connection is indicative of intrinsic consistency of micro BH physics in string theory, it has to be interpreted with care. In particular, it cannot be regarded as a derivation of (8) from the species bound (4). From the way we arrived to it, this relation by no means indicates that the BH semi-classicality scale is bounded by l_s. This would be the case if all N_{eff} modes

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[4]The alternative evidence [11] for this bound comes from applying the arguments of [8] that are restricting the information-storage by use of species

were opening up at or below the scale M_s. However, they open up gradually at higher energies and for $g_s \ll 1$ most of them are crowded up at the temperatures $T \gg M_s$. This is why, the BHs smaller than l_s are not inconsistent with semiclassical BH physics.

Our results have some fundamental as well as phenomenological implications. On one hand they indicate, that BH semi-classicality properties combined with the exponentially large number of string resonances do not *a priory* prevent the small BHs from probing distances shorter than the string length l_s. In this respect, micro BHs seem to share some properties of D-branes, which are known to probe the scales parametrically shorter than the string length [12].

Secondly, our findings shed some light at the expected properties of the micro BH that may be produced in particle collisions at super-stringy energies, e.g., in the high energy cosmic rays or at LHC. Since even in the most optimistic scenarios, the largest produced BHs at LHC will still be much smaller than the size of the extra dimensions, their evaporation will not be democratic in species [13]. Instead, such BHs will predominantly evaporate into the species whose wave-function profiles in the extra dimensions have maximal overlaps with the BH localization site. In this respect, democracy of the evaporation products will be a function of the BH size (and thus of the center of mass energy in LHC collisions). Observation of the mass-dependent democracy will be most easy within the Standard Model species, e.g., for the different quark and lepton families, since these species cannot be too far displaced in the extra space, due to their gauge quantum numbers. For example, by gauge invariance, all the electrically-charged states must have the same overlap with the photon wave-function. Thus, in this case, variation in BH democracy may be in principle noticeable even for a relatively small variations of the BH mass. For example, heavier BHs produced in the collision of the first generation quarks, must evaporate more democratically into other flavors, than the lighter BHs produced in the same collision.

Finally, after shrinking to the size l_s BHs must evaporate into $N_{eff} \sim 1/g_s^2$ string resonances, some of which which may then be identified via their decay channels into the SM particles.

2 Preliminaries

2.1 The three fundamental length scales in brane compactifications:

In the following we will consider type II orientifold compactifications with D-branes (for reviews see [14, 15]). Let us recall that there are three basic, dimensional parameters in type II string compactifications with wrapped and/or intersecting D-branes.

Of course, first there is the fundamental string scale, given in terms of the slope

parameter α' as:

$$(1): \quad M_s = \frac{1}{\sqrt{\alpha'}} \, . \tag{9}$$

The string scale is related to the fundamental 10-dimensional Planck mass M_{10} through a dimensionless string coupling constant g_s via the relation (8)

Second, we have to compactify from ten to four dimensions on an internal 6-dimensional space $\mathcal{M}_6$. So as the second basic string mass scale, we introduce the scale defined by the volume $V^{(6)}$ of the internal space:

$$(2): \quad m_6 \equiv \frac{1}{(V^{(6)})^{1/6}} \, . \tag{10}$$

When measuring masses in terms of Planck units, it is useful to introduce a dimensionless overall volume, given in 10-dimensional Planck mass units, as

$$V_6' = V^{(6)} M_{10}^6 = \frac{M_4^2}{g_s^{1/2} M_s^2} \, . \tag{11}$$

We are considering scenarios, where the SM is located at some stacks of D(3+p)-branes, which are wrapped around some internal p-cycles inside $\mathcal{M}_6$: $\Sigma_p \subset \mathcal{M}_6$. So, the third basic string mass scale is related to the volume $V_p^{\parallel}$ of the sub-space Σ_p, around which the D-branes are wrapped:

$$(3): \quad m_p^{\parallel} = \frac{1}{(V_p^{\parallel})^{1/p}} \, . \tag{12}$$

Alternatively we can introduce the volume $V_{6-p}^{\perp}$ of the space transversal to the D-branes,

$$(3'): \quad m_{6-p}^{\perp} = \frac{1}{(V_{6-p}^{\perp})^{1/(6-p)}} \, , \tag{13}$$

where the following relation holds:

$$V^{(6)} = V_p^{\parallel} V_{6-p}^{\perp} \, . \tag{14}$$

These three fundamental dimensional parameters of D-brane models are linked to two 4D physical observables in the following way:

(A) the strength of gravitational interactions, as determined by the 4D Planck mass:

$$(A): \quad M_4^2 = g_s^{-2} \, M_s^8 \, V^{(6)} \, . \tag{15}$$

(B) the strength of the gauge interactions, as determined by the 4D gauge coupling:

$$(B): \quad g_{Dp}^{-2} = M_s^p \, g_s^{-1} \, V_p^{\parallel} \, . \tag{16}$$

Hence, for fixed string coupling g_s, knowing M_4 and g_{Dp}^{-2} leaves one of the three dimensionful string parameters undetermined (unlike for the heterotic string). Since the 4D gauge coupling must not be too small, we take it of order unity. This fixes $(V_p^{\parallel})^{-1/p}$ to be of order M_s, i.e. $(V_p^{\parallel})^{-1/p} \simeq M_s$. This leaves only M_s (or $V^{(6)}$) as free parameter. Then, the transversal volume $V_{6-p}^{\perp}$ is of the oder

$$V_{6-p}^{\perp} = V^{(6)} M_s^p \,. \tag{17}$$

From the model-building perspective there are several "natural choices" for M_s which scan the mass scales starting from just below $M_4 \sim 10^{19}$ GeV all the way to TeV scale. The later choice being motivated by the solution to the hierarchy problem, as well as by the prospect of the experimental discovery of string theory in particle physics and in table-top experiments.

2.2   Choices of M_s and low string scale compactifications

Although in our discussion M_s will be kept as a free parameter, we wish to briefly display some phenomenologically-motivated choices for it.

We shall consider cases of $(V^{(6)})^{-1/6} \sim M_s$ and $(V^{(6)})^{-1/6} \ll M_s$ separately. The choice $(V^{(6)})^{-1/6} \sim M_s$ is special, since in this case the super-l_s-size ($r_g \gg l_s$) BHs evaporate only into the zero modes of the lowest string excitations, and never in their KK or Regge resonances. In the other words, for this choice the semi-classical BHs directly jump from four-dimensional Einsteinian regime to the stringy one in which they start evaporation into the Regge excitations. Notice, that in every other case, there is an intermediate regime, when semi-classical BHs become high-dimensional and evaporate in KK resonances but not in the Regge ones.

2.2.1   String Length Compactifications: High String Scale

We shall first focus on the choice $(V^{(6)})^{-1/6} \sim M_s$. Notice, that for any given g_s this choice corresponds to the maximal proximity between the string and four-dimensional Planck masses. This choice is also rather special from the point of view of species counting. As said above, in this case the larger than the l_s BHs are automatically four-dimensional Einstein BHs, with the temperature $T < M_s, m_6$. Thus, none of the KK or Regge excitations are thermally accessible. Thus, the bound on species automatically translates as the bound on the possible number of the four-dimensional zero modes. We shall now express this bound as the relation between the number of zero modes and the string coupling.

The relation between M_s and M_4 is given by

$$M_s = \frac{M_4}{g_s} \,, \tag{18}$$

which follows from the relation (15) and (2) and from setting the compactification radius equal to l_s.

The realistic numerical value of M_s can be estimated by using (16) and assuming the standard UV boundary condition for the running gauge coupling g_{Dp}. We thus get the estimate for the highest phenomenologically-consistent choice of M_s in the weakly-coupled description (which is similar to the value in heterotic string compactifications):

$$M_s \sim 10^{17-18} \text{ GeV} . \tag{19}$$

The masses of all the KK excitations as well as of the string Regge modes are at or above M_s and are irrelevant for the dynamics of semi-classical BHs of size $r_g \gg l_s$. Since such BHs are larger than both the string length as well as the the compactification radius, they are well within the domain of semiclassical description and for all practical purposes represent the four-dimensional Schwarzschild BHs. Their semi-classicality immediately implies the bound (3), which dictates that the number of all possible zero modes in such compactifications is automatically restricted by

$$N_{zero} \lesssim \frac{1}{g_s^2} . \tag{20}$$

This bound is absolute, since, by default, at distances $\gg l_s$ there are no new gravitational modes available that could modify the universal thermal properties of the Einsteinian BHs. The only gravitational degree of freedom at large distances is a four-dimensional massless graviton. Because of this, the classical metric of neutral BHs, by standard no-hair theorems [23], is exactly Schwarzschild. Thus, the energy loss of such a BH is

$$\frac{dM_{BH}}{dt} = -T^2 N_{zero} , \tag{21}$$

which implies the breakdown of the semi-classicality condition (5) at the temperature

$$T_* = M_* = \frac{M_4}{\sqrt{N_{zero}}} . \tag{22}$$

Since by consistency T_* cannot be below M_s (below string scale there are no new modes available for changing the semi-classical regime), taking into the account (18), we prove (20).

2.2.2 Large Volume Compactifications: Low String Scale

Choices of M_s with values lower than (18), imply at least some compactification radii larger than the string length l_s. In this case the evaporating semi-classical BHs cross over to a high-dimensional regime before reaching the string size, and evaporate not only into the zero modes but also in their KK excitations. The evaporation of such BHs will be considered in details in the next section. Here we just list the few interesting choices of M_s values.

Motivated by gauge coupling unification, one often assumes the existence of a GUT gauge group and makes the identification:

$$M_s \equiv M_{GUT} \simeq 10^{16} \text{ GeV} . \tag{23}$$

In this scenario, $V^{(6)}$ is of order

$$(V^{(6)})^{1/6} \simeq 10^{-15} \text{ GeV}^{-1} . \tag{24}$$

Another choice of lower string scale compactifications is to identify M_s with the scale of spontaneous supersymmetry breaking:

$$M_s \equiv M_{SUSY} \simeq 10^{11} \text{ GeV} . \tag{25}$$

The rational for this choice is that one does not have to worry about lowering the scale of supersymmetry breaking below the cutoff, since the gravity-mediated contribution to the Standard Model masses are automatically of the right order of magnitude. However, one has to be careful in avoiding $1/M_s$-suppressed interaction between the Standard Model and the supersymmetry-breaking sectors, which would destabilize the hierarchy.

This scenario implies that $V^{(6)}$ is of order

$$(V^{(6)})^{1/6} \simeq 10^{-(6-7)} \text{ GeV}^{-1} . \tag{26}$$

The most radical choice (motivated by the hierarchy problem) is to identify M_s right away with the scale of the SM, i.e. with the TeV scale:

$$M_s \equiv M_{SM} \simeq 10^3 \text{ GeV} . \tag{27}$$

Therefore, in this scenario, $V^{(6)}$ is very large, and is of order

$$(V^{(6)})^{1/6} \simeq 10^{14/6} \text{ GeV}^{-1} , \tag{28}$$

and hence this scenario can also be called *very large extra dimension scenario.*

Let us summarize these four possibilities, and the corresponding energy and length scales in the table 1 (we use that 1 GeV $\sim (10^{-16} \text{ m})^{-1}$, we have set $g_s \simeq 1$, $p = 4$, and we neglect factors of 2π). V_6' takes the following values:

$$V_6' \sim 1, 10^6, 10^{16}, 10^{32} . \tag{29}$$

2.3 Dynamical realization of low string scale compactifications

In our analysis, we shall make no specific assumption on how the moduli are stabilized. Our results will apply to any consistent static or a nearly static compactification. What is important is, that the possible time-dependence of the

Table 1: The different mass scales in D-brane models

	M_s (GeV)	L_s (m)	$m_6 = (V^{(6)})^{-1/6}$ (GeV)	$(V^{(6)})^{1/6}$ (m)	$m_2^\perp = (V_2^\perp)^{-1/2}$ (GeV)	$(V_2^\perp)^{1/2}$ (m)
(o)	10^{18}	10^{-35}	10^{18}	10^{-35}	10^{18}	10^{-35}
(i)	10^{16}	10^{-32}	10^{15}	10^{-31}	10^{13}	10^{-29}
(ii)	10^{11}	10^{-27}	10^{6-7}	$10^{-(22-23)}$	10^3	10^{-19}
(iii)	10^3	10^{-19}	$10^{-14/6}$	10^{-14}	10^{-13}	10^{-3}

background is slower than the evaporation process of the *smallest* semi-classical BHs existing on the background of interest [7].

For example, for the bounds obtained via the four-dimensional semi-classical BHs this will imply, that there must exist such BHs for which the evaporation time is shorter than the effective four-dimensional Hubble time. Needless to say, for any realistic compactification this condition must be automatically satisfied.

As long as the above condition is satisfied, the precise mechanism of compactification and moduli stabilization is unimportant for our analysis. On the backgrounds of our interest, the moduli fields may be: Stable (positive masses-squares), neutrally-static (massless) or even slowly evolving in time (either tachyonic or runaway) . As long as the background supports the existence of semi-classical BHs, which evaporate faster than the characteristic time of the background-evolution, our bounds are applicable.

As for the high-dimensional or sub-l_s BHs, the issue of compactification plays no role, since their evaporation can be considered directly in ten-dimensional theory. As long as throughout their half-life they stay smaller than the size (or the curvature radius) of the extra space, the time-dependence of the background is unimportant.

We thus see, that derivation of our bounds is rather insensitive to the stabilization of the radii of the compactified space. The converse, however, is not true. The BH bound on number of species puts severe constraints on the possible static (or slowly-changing) backgrounds in string compactifications. For example, any background that violates (20), must be *clasically-unstable* with the instability time less than the lifetime of any Schwarszchild BH of size larger than the compactification radius.

Of course, as the necessary model-building aspect, one has to address the issue of how moduli stabilization might lead to such big internal volumes, as required in scenarios (ii) and (iii). This issue will not concern our work. However, our results can be used as an useful consistency checks of such compactifications, in terms of number of species.

Some plausible mechanisms in the direction of moduli stabilization in large volume compactification were developed in [16], where in was argued that for the case of the SUSY breaking scenario (ii), loop effect and non perturbative effects in the effective moduli potential can easily and naturally lead to such big,

exponential hierarchies. The relevant geometries are called Swiss cheese Calabi-Yau space, where the D-branes of SM model are wrapped around small cycles (holes) inside a huge internal space.

In order to be a little bit more specific, let us recall the main points in the construction of large volume D-brane comapctifications. The effective field theory potential $\mathcal{V}$ for the moduli fields is based on the following Kähler potential and superpotential [16, 17]

$$
\begin{aligned}
\frac{\mathcal{K}}{M_4^2} &= -2\log\left(V_6' + \frac{\xi}{2}\right) - \log\left(-i(\tau - \bar{\tau})\right) + \mathcal{K}_{cs}\,, \\
W &= \frac{g_s^{\frac{3}{2}} M_4^3}{\sqrt{4\pi}} \sum_i A_i e^{-\frac{2\pi T_i}{g_s}} + W_{cs}\,,
\end{aligned}
\tag{30}
$$

where τ denotes the axion-dilaton field, the T_i correspond to the volumes of the 4-cycles, being wrapped by D7-branes, the ξ-terms is a higher order loop correction to the Kähler potential, and $\mathcal{K}_{cs}$, W_{cs} are functions, which depend on the complex structure moduli fields. Using $\mathcal{K}$ and W it is straightforward to compute the standard supergravity scalar potential given as

$$
\mathcal{V} = e^{\mathcal{K}}\left(|DW|^2 - 3|W|^2\right).
\tag{31}
$$

As it was shown in [16, 17] it is possible to obtain minima of $\mathcal{V}$ with hierarchically large values for $V_6' \sim 10^{16}$ and correspondingly low values of the string scale M_s. This depends crucially on the loop correction ξ to the Kähler potential as well as on the exponential dependence of W on the moduli T_i, as it was demonstrated in [16, 17] for the SUSY breaking scenario (ii). However it is equally possible by a small variation of the parameters in $\mathcal{K}$ and W to obtain minima of $\mathcal{V}$, where V_6' takes even much larger values of the order 10^{32}.[5] However, the masses of some moduli fields obtained from this scalar potentiual can be unacceptably small in case the volume V_6' is very large (see [17] for a table with masses computed from this potential). We will discuss this potential problem in the next section for the overall modulus field T, and we will argue that generic radiative mass corrections to the overall T-field modulus will lift its mass to a phenomenologically acceptable value.

3 Several generic particles in brane compactifications and their associated length scales

Now let us recall several generic particles that are related to the different mass scales in D-brane compactifications, as well as their masses, measured in the Einstein frame, and expressed in terms of Planck units.

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[5]We like to thank F. Quevedo for discussion on this point.

(i) String Regge excitations:

The most generic and model independent stringy excitations are the so-called Regge modes. Their masses are proportional to the string scale M_s, and in terms of Planck units given by:

$$M_{\text{Regge}} = \sqrt{n} M_s = \frac{\sqrt{n} M_4}{g_s^{1/4} \sqrt{V_6'}} \, . \tag{32}$$

Here n denotes the oscillator number of these states. In case they are open strings, the stringy Regge excitations can carry the SM quantum numbers, i.e. they can be produced by pp-collisions via gauge interaction. Their discovery in LHC experiments is then possible in the TeV scale string scenario [2, 3].

(ii) Transversal direction KK particles:

For the masses of the KK particles related to the large volume cycles one obtains:

$$M_{KK}^{\perp} = \frac{m}{(V_{6-p}^{\perp})^{1/(6-p)}} = m \, M_4 (V_6')^{\frac{8-p}{2p-12}} \, , \tag{33}$$

where m counts the internal closed string KK momentum of each state. In the four scenarios discussed above $M_{KK}^{\perp}$ takes the following values (here for $m = 1$, $p = 4$):

$$M_{KK}^{\perp} \sim 10^{18}, 10^{13}, 10^{3}, 10^{-13} \text{ GeV} \, . \tag{34}$$

These KK particles are closed strings, and they are neutral under the SM gauge group, and hence interact only gravitationally with SM particles.

(iii) Small (longitudinal) cycle KK particles:

For the masses of the KK particles related to the small volume cycles one obtains:

$$M_{KK}^{\parallel} = \frac{m'}{(V_p^{\parallel})^{1/p}} \tag{35}$$

Here m' is the open string KK momentum along the wrapped $(3+p)$-branes. For the compactification on the parallel directions being l_s, in the four scenarios discussed above $M_{KK}^{\parallel}$ takes the following values ($m' = 1$):

$$M_{KK}^{\parallel} = 10^{19}, 10^{16}, 10^{11}, 10^{3} \text{ GeV} \, . \tag{36}$$

These KK particles are open strings, and hence they carry SM quantum numbers, i.e. they are the KK excitations of the SM fields. Hence they can be directly produced at the LHC.

(iv) Overall volume modulus:

Related to the overall six-dimensional volume V_6, there is a generic closed string modulus field T in any compactification. In the low string scale compactifications,

the tree-level mass of this modulus may be unacceptably small, (e.g., its typical tree level mass was calculated using the moduli potential calculated in [17]: $M_T = \frac{M_{\text{Planck}}}{(V_6')^{3/2}}$).

However, the point we would like to stress is, that there is a model-independent lower bound on the typical mass that the volume modulus should get from the quantum correction, which is given by

$$\Delta M_T^2 \sim \frac{\langle T_\mu^\mu T_\mu^\mu \rangle}{M_4^2} \gtrsim \frac{\Lambda_{SM}^4}{M_4^2} , \qquad (37)$$

where Λ_{SM} is the scale that cuts-off the contribution to the $\langle T_\mu^\mu T_\mu^\mu \rangle$ correlator from the loops of the standard model particles. The source of this contribution is in the fact that the volume modulus is coupled to trace of the energy-momentum tensor, T_μ^μ, gravitationally by (at most) $1/M_4$-suppressed interactions. Thus, the Standard Model loops inevitably contribute to the generation of its mass. The relevant diagrams are loops with the external T-legs, and with the Standard Model particles circulating in the loop. Notice, that Λ_{SM} is entirely determined by the scale at which the momenta of the Standard Model particles flowing in the loop get regulated, and is independent of the momenta flowing in the external T-legs. Due to this, the result is insensitive to the UV properties of the effective coupling between the standard Model species and the volume modulus, which makes (37) very robust. An obvious lower experimental bound on Λ_{SM} is around TeV, which taking into the account the multiplicity of the contributing species (and ignoring the accidental cancellations) translates as the expected lowest value of the volume modulus mass around

$$\Delta M_T \sim 10^{-3} \text{ eV} , \qquad (38)$$

which corresponds to a length scale in sub-millimeter range. This is the range where the existing table-top fifth force experiments [18] start to penetrate. The resulting gravity-competing force should share similarity with the ones considered previously from light moduli in gauge-mediated supersymmetric theories [19] and from volume-modulus in orbifolds string compactifications with Scherk-Schwarz supersymmetry breaking [20].

Notice, however, that in many cases, the expected mass in reality must be somewhat higher, either due to multiple sectors contributing in the $\langle T_\mu^\mu T_\mu^\mu \rangle$-correlator, or due to higher cutoff (or both).

(v) Small (longitudinal) cycle volume moduli:

Related to the small cycles of longitudinal volume $V_p^{\parallel}$, there are a moduli fields t with tree level masses (here for $p = 4$)

$$M_t = \frac{M_4}{V_6'} . \qquad (39)$$

In the four scenarios discussed above M_t takes the following values:

$$M_t = 10^{19}, 10^{13}, 10^3, 10^{-13} \text{ GeV}. \tag{40}$$

Again, these t-moduli are neutral and interact only gravitationally with SM fields.

4 Black Hole evaporation in String D-brane compactifications and the bound on species-number

We now wish to discuss the question, in what sense string D-brane comapctifiactions are a concrete realization of the large species-number scenario of [6]. Namely we want to analyze how the different particles species in D-brane models respect the BH bound eq.(3).

Since the number entering in (3) is the effective number of species to which semi-classical BHs can evaporate, we thus focus our analysis on the study of micro-BH evaporation process in string compactifications. We shall consider the regimes $r_g \gg l_s$ and $r_g < l_s$ separately. In the first case the string Regge states are unaccessible and the only relevant degrees of freedom are zero modes and their KK excitations. We start with this case first.

4.1 Evaporation into KK particles

In this section we shall work under the choice $g_s = 1$. This choice allows us to avoid possible production of the higher string Regge excitations in any semi-classical BH evaporation, since the latter by default must have a temperature below M_{10}. The particles contributing in the evaporation of such BHs are only the string zero modes and their KK resonances. Hence, these will be the states contributing into the BH bounds (3) and (4), and our goal in this section will be to observe how the consistency of these bounds is maintained.

We shall consider sub-l_s BH regime in the next section, where we shall allow for the hierarchy between the M_s and M_{10} scales.

In any $4+d$-dimensional scenario, with the coordinates in extra d-dimensional space denoted by $y_1, ... y_d$, the BH evaporation rate is given by

$$\frac{dM_{BH}}{dt} = -T^2 N_{eff}, \tag{41}$$

where, it is assumed that the BH is a perfect quantum emitter, with the temperature given by $T = r_g^{-1}$. The effective number of particle species N_{eff} is given by

$$N_{eff} \simeq \sum_m e^{-\frac{m}{T}} \int_{|y| < r_g} |\psi^{(m)}(y)|^2 \tag{42}$$

where the exponent stands for the Boltzmann suppression, and the d-dimensional integral evaluates the overlap of each particle wave-function profile with the BH horizon.

To be most precise, the integral has to be taken over a background metric in the presence of a *classically-static* BH. However, for the micro BHs with short evaporation time, this subtlety can be ignored, and in the leading approximation the deformations of the particle wave-function profiles due to the BH metric can be disregarded.

Of course, for the BHs with the size r_g smaller than the size of an extra dimension, the corresponding high-dimensional relation between the mass and the Hawking temperature must be assumed.

The key point then is, that the bound (4) between the gravitational cutoff of the theory M_* and M_{4+d} is reproduced by $N_{d+4} = N_{eff}$. Moreover, since the scale M_* is the universal cutoff, the saturation of the bound in one particular dimensionality, automatically saturates it in all. E.g., the four-dimensional bound (3) is simultaneously reproduced by N_4 being the total number of four-dimensional species, participating in the sum in (42).

We shall now illustrate this point on some examples.

(i) Isotropic compact space

We first discuss the simplest case of an isotropic, d-dimensional compact space of volume $V_d = R^d$. (This includes the case of no (wrapped) branes) Then the masses M_{KK}^i of the bulk KK-particles are given as

$$M_{KK}^i = \frac{\sqrt{\sum_i^d m_i^2}}{R}, \quad (i = 1, \ldots, d), \tag{43}$$

where the m_i are the KK quantum numbers in the $i^{th.}$ direction of the internal space. From above it follows that the total number of KK particle with the masses up to the scale M_{4+d}, for each $4 + d$-dimensional species is given by

$$N_{KK} = (M_{4+d}R)^d. \tag{44}$$

Applying this counting to a theory with a single $4 + d$-dimensional graviton, and using (3) with identification $M_* = M_{4+d}$, we obtain the expression for the effective Planck scale in four dimensions in terms of number of KK-species,

$$M_4^2 = M_{4+d}^2 N_{KK} = M_{4+d}^2 (M_{4+d}R)^d, \tag{45}$$

which indeed agrees with the geometric expression for the Planck mass in four dimensions after compactification [5].

On the other hand the BH bound (3) counts the total number N of four-dimensional species, which in case of N_{4+d} high-dimensional species increases as

$N = N_{4+d} N_{KK}$. Substituting this in (3) and taking into the account (45), we automatically get (4).

For $N_{4+d} \sim 1$, which is the case for example for pure gravity in $4 + d$-dimensions, the both BH bounds, (3) and (4), are automatically satisfied by $M_* = M_{4+d}$, as long as $M_{KK} < M_{4+d}$ or equivalently as long as

$$R > 1/M_{4+d} \,. \tag{46}$$

However, for $N_{4+d} \gg 1$, the bound indicates that the true gravitational cutoff, must be below M_{4+d}! This result is absolutely compatible with our understanding of the structure of string theory in ten-dimensions. Indeed, for $g_s \sim 1$, the number of ten-dimensional species with mass below M_{10} is limited by the fixed number of closed string zero modes, and cannot be arbitrarily large. That is, for $g_s \sim 1$, $N_{10} = N_{zero}$. Only possibility for increasing N_{10} arbitrarily is to take a weak string coupling $g_s \to 0$, but then the string scale M_s, which is the gravitational cutoff of the theory, inevitably comes below M_{10}.

Let us now check how the above mode-counting reproduces the correct relation between the M_{4+d} and the M_* through the relation (41). For a BH of size $r_g \ll R$, the relation between the mass and the temperature is,

$$M_{BH} = M_{4+d}(M_{4+d}T^{-1})^{1+d} \,. \tag{47}$$

Noticing, that the overlap between the KK wave-functions and the BH horizon is $(r_g/R)^d$, and the thermally available number of KK states per each $4 + d$-dimensional particle is $N_{KK}(m < T) = (TR)^d$, the value of N_{eff} in (42) becomes,

$$N_{eff} = (r_g/R)^d (TR)^d N_{4+d} = N_{4+d} \,. \tag{48}$$

Using (47) and (48), the eq(41) can be written as,

$$\frac{1}{T^2}\frac{dT}{dt} = \frac{1}{(1+d)} \left(\frac{T}{M_{4+d}}\right)^{2+d} N_{4+d} \,. \tag{49}$$

It is obvious that the semi-classicality condition (5) breaks down exactly at $T = M_*$, where M_* is given by (4). There is a complete consistency between the BH bounds obtained in four and high-dimensional theories.

(ii) Non-isotropic compact space – wrapped D-branes

Now we are ready to discuss the case of non-isotropic extra space, in which we have $(3 + p)$-branes wrapped around internal p-cycles. The longitudinal volume of the wrapped branes is $V_\parallel = R_\parallel^p$, whereas the transversal volume is $V_\perp = R_\perp^{d-p}$. Hence the total volume is $V_d = V_\parallel V_\perp = R_\parallel^p R_\perp^{d-p}$. As before, the total number of KK particles produced by each $d + 4$-dimensional degree of freedom is given by

$$N_{KK,total} = (M_{4+d})^d R_\parallel^p R_\perp^{d-p} \,, \tag{50}$$

and hence if we had a single $4+d$-dimensional particle, we would again reproduce the geometric relation for M_4 in terms of number of species [6],

$$M_4^2 \; = \; M_{4+d}^2 N_{KK,total} \; = \; (M_{4+d})^{d+2} V_d \,. \tag{51}$$

Again M_4 coincides with the Planck mass eq.(15) in string compactifications with $d = 6$.

However, we now have two different kinds of KK species. The first kind are the KK excitations along the wrapped p-branes. Their masses and numbers are given as:

$$M_{KK}^{\parallel} = \frac{\sqrt{\sum_{i=1}^{p}(m_i')^2}}{R_{\parallel}} \,, \quad N_{KK,\parallel} = (M_{4+d}R_{\parallel})^p \,. \tag{52}$$

The second type are the KK excitations in the bulk directions, transversal to the wrapped p-branes. Their masses and numbers are given as:

$$M_{KK}^{\perp} = \frac{\sqrt{\sum_{i=p+1}^{d} m_i^2}}{R_{\perp}} \,, \quad N_{KK,\perp} = (M_{4+d}R_{\perp})^{d-p} \,. \tag{53}$$

Note that $N_{KK,total} = N_{KK,\parallel} N_{KK,\perp}$.

Thus, we have to distinguish two types of high-dimensional particles as seen by a $4+d$-dimensional observer. The first category are $4+d$ dimensional bulk species of number N_{4+d}, which poses KK excitations both in transverse as well as in longitudinal direction. The total number of the KK-excitations in both directions is given by (50). The second category are the $4+p$-dimensional particles localized on the p-brane world volume. These particles have no transverse momentum and thus only poses KK excitations in longitudinal directions. Each of these localized states deposits $N_{KK,\parallel}$ four-dimensional KK species.

We are now ready to generalize the BH evaporation analysis to the unisotropic situation. One small complication is, that the BH evaporation rate now depends on its relative location with respect to the p-brane. For instance, a small BH that is positioned away from the brane cannot evaporate into the $4 + p$-dimensional modes that are localized on the brane. It was suggested in [13], that such BHs that are decoupled from the brane species cannot be *classically-static*, and must evolve in time until they enclose the brane. However, if the BH is small enough and the initial distance to the brane it sufficiently large, the time scale of classical evolution can be much longer than the evaporation time. In this case the BH shall evaporate practically only into the bulk species. In order to avoid irrelevant details of the above complications, we shall assume that the BH is pierced by the brane to start with, and thus all the brane-localized species are readily accessible.

The BH in question will undergo the different evaporation regimes depending on its size (and thus temperature). The BHs that are larger than the size of longitudinal dimensions, but still smaller than the transverse ones $R_{\parallel} \ll r_g \ll R_{\perp}$,

will evaporate as effectively $(4 + d - p)$-dimensional. The analysis of the previous section can be directly applied to such a BH via a simple replacement $d \to d - p$. The only caveat is, that N_{eff} will also include N_{4+p} zero modes localized on the p-brane to which the BH will evaporate without any extra suppression. In this respect, each zero mode localized on the p-brane counts as a whole transverse KK tower of a single $4 + d$-dimensional bulk state.

Let us now turn to the evaporation of a small BH, $r_g \ll R_\parallel$. The mass-to-temperature relation of such a BH is the same as (47). The only novelty in comparison with the previous section is the anisotropy of the evaporation into the transverse and the longitudinal KK modes. The production rate of the individual longitudinal and transverse KK modes are suppressed by the wave-function overlap suppression factors, equal to $(r_g/R_\parallel)^p$ and $(r_g/R_\perp)^{d-p}$ respectively. However, these suppression factors are exactly compensated by the corresponding numbers of the thermally available states, which are given by $N_{KK\parallel}(m < T) = (TR_\parallel)^p$ and $N_{KK\perp}(m < T) = (TR_\perp)^{d-p}$ respectively. As a result, every $4 + d$-dimensional bulk degree of freedom as well as every $4 + p$-dimensional brane mode, each contribute as one mode into N_{eff}. That is,

$$N_{eff} \; = \; N_{4+d} \, N_{4+p} \, , \tag{54}$$

and

$$\frac{1}{T^2} \frac{dT}{dt} \; = \; \frac{1}{(1+d)} \left(\frac{T}{M_{4+d}} \right)^{2+d} N_{4+d} \, N_{4+p} \, . \tag{55}$$

The breakdown of the semi-classicality condition (5) again happens at the scale M_* given by (4) where N_{4+d} has to be replaces by $N_{4+d} \, N_{4+p}$. Thus, in creating the hierarchy between the Planck mass and the gravitational cutoff, bulk and brane modes contribute equally. Again, we see a complete consistency between the four- and the high-dimensional BH bounds.

Notice, that the relation (54) tells us that every brane mode has the same effect on decreasing gravitational cutoff of the theory as the bulk mode. So one could reduce M_* relative to M_{4+d} at the expense of increasing N_{4+p} rather than N_{4+d}. However, as was shown in [21], the limit $N_{4+p} \gg N_{4+d}$ is very subtle, since in this limit the gravitational effects of the p-branes can no loner be ignored. For instance, in case of many coincident branes, they develop a horizon, and the BH evaporation into the brane modes changes at shorter scales.

4.2 Black hole evaporation into the string Regge excitations – the effect of the string tower

It is known that in string theory the density of states grows exponentially for $M > M_s$:

$$\rho(M) \; \sim \; e^{\sqrt{bd}\frac{M}{M_s}} \, , \tag{56}$$

where M_s is the string mass scale, b is a numerical constant and the factor d accounts for the fact that in D space dimensions each string oscillator can come in $d = D - 2$ varieties.

The obvious question is what is the implication of this exponentially-growing number for the BH bound (4).

A simple quick answer to the above question would be, that, since M_s is an obvious cutoff, whereas all the higher Regge states are heavier, they simply should not be counted in eq.(4). Hence the only relevant species would be the string zero modes and their KK excitations, which we have already taken into the account in the previous section.

This answer, however, is far from being satisfactory. The reason is, that the number N_{4+d} is the number of species to which the semi-classical BH can evaporate. Can semi-classical BHs smaller than l_s exist? Naively, the answer to this question is negative, as the exponentially growing numbers seems to be in conflict with the black hole semi-classicality for temperatures above $T > M_s$, similarly to the usual Hagedorn phenomenon. Indeed, for the exponentially-growing N, the condition (5) would be violated almost immediately. We shall argue however, that this naive view is not supported by more detailed analysis. In fact, we shall discover quite the opposite, that the decay rate of small BHs into the stringy Regge states is suppressed. In other words, the effective number of string species, into which the the black hole can decay, is smaller that the exponentially growing number (56). Thus, the thermal arguments do not forbid existence of the substringy size BHs, and the BH bound is not violated by the string Regge exciaitions.

Before we start the discussion about the black hole decays, let us note that the above-mentioned suppression of the string Regge modes in the BH evaporation is qualitatively similar to the rate of scattering of light SM particles into string Regge excitations, which exhibits Veneziano-type softening. In other words, in the perturbative string regime the excitation of the Regge states is effectively suppressed in scattering proceses among the light fields. The relevant scattering amplitudes were recently computed in [2]. As an example consider the $2 \to 2$ scattering amplitudes of 4 open string gluons on the D-brane, or 2 gluons and 2 quarks with an intermediate Regge particles exchanged. This amplitude is only dominated by the exchange of string Regge excitations, no KK particles can be exchanged in this process (KK particles are exchanged in 4-point scattering of 4 quark fields). Generically, this 4-point, tree–level string amplitudes are described by the Euler Beta function depending on the kinematic invariants $s = -(k_1 + k_2)^2$, $t = -(k_1 + k_3)^2$, $u = -(k_1 + k_4)^2$, with $s + t + u = 0$ and k_i the four external momenta. The whole amplitudes $A(k_1, k_2, k_3, k_4; \alpha')$ may be understood as an infinite sum over s–channel poles with intermediate string Regge states $|k; n\rangle$ exchanged. After neglecting kinematical factors the string amplitude

$A(k_1, k_2, k_3, k_4; \alpha')$ takes the form

$$A(k_1, k_2, k_3, k_4; \alpha') \sim -\frac{\Gamma(-\alpha's)\,\Gamma(1-\alpha'u)}{\Gamma(-\alpha's-\alpha'u)} = \sum_{n=0}^{\infty} \frac{\gamma(n)}{s - M_n^2} \qquad (57)$$

as an infinite sum over s–channel poles at the masses

$$M_n^2 = M_s^2\, n \qquad (58)$$

of the string Regge excitations. In (57) the factor $\gamma(n)$ is the three–point coupling of the intermediate states $|k; n\rangle$ to the external particles and is given by

$$\gamma(n) = t\,\frac{(u\,\alpha', n)}{n!} = \frac{t}{n!}\,\prod_{j=1}^{n}[a(u) + j] \sim \frac{(\alpha'u)^n}{n!} \quad , \qquad (59)$$

with $a(u) = u\alpha' - 1$ the Regge trajectory and $n+1$ the highest possible spin of the state $|k; n\rangle$. As one can see, the growth of this coupling is not exponential. On the contrary, although the degeneracy d_n of the Regge states at the level exponentially grows,

$$d_n \sim \text{const.}\ n^{-27/4}\exp(4\pi\sqrt{n}) \sim \exp(M_n/M_s)\,, \qquad (60)$$

the coupling function $\gamma(n)$ tends to zero for large n due to the $n!$ in the denominator. Therefore the effective number of Regge states that can be excited in scattering processes among the light fields is much smaller compared to d_n.

We now wish to study how the existence of the string tower affects the evaporation of the semiclassical BHs in string theory. Of course, for the BHs with the Schwarzschild radius $r_g > l_s$, the existence of the string tower has no effect, since their production is Boltzmann suppressed. As already discussed above, such BHs evaporate into the lowest excitations and their KK tower.

The non-trivial question is, what happens when BH horizon shrinks to the string size. Can quasi-classical BHs continue to exist even at sub-stringy distances? The exact answer to this question is unknown to us. However, as we shall explain, at least from the point of view of an effective theory, there is no obvious inconsistency in continuing semi-classical BH regime beyond the string scale.

In order to provide an evidence for the above statement, let us start with a semi-classical BH of size $r_g \gg l_s$, and follow its evolution. We shall assume, that (by design) the only non-zero characteristic quantum number of our BH is its mass (M_{BH}). In the other words, by construction, BH is neutral under all possible massless gauge fields, with corresponding charges that could have been measured in form of the Gaussian fluxes at infinity. It also carries no topological or quantum charge measurable by an Aharonov-Bohm type effect [22]. Such a BH can always be prepared, for example, by collapsing a neutral dust cloud with

zero angular momentum composed of equal number of particles and anti-particles. Because the resulting BH is sufficiently large and classical, by no-hair theorem [23], all the possible short distance excitations must vanish outside the horizon.

Until the BH shrinks to the string length, the evaporation is semi-classical and emission takes place into the few lowest massless string states. At some point, the BH reaches the size $r_g \sim l_s$. Notice, that in weakly-coupled string theory (to which we limit our consideration here) the mass of such BH is still much larger than the Planck mass. This is obvious from the fact that for $r_g > l_s$ the BH is semi-classical and its mass is given by the usual Schwarzschild relation. For example, in ten dimensions

$$M_{BH} \sim M_{10}(M_{10}\, r_g)^7 . \tag{61}$$

So, for $r_g \sim l_s$ we get

$$M_{BH} \sim \frac{M_s}{g_s^2} \sim \frac{M_{10}}{g_s^{\frac{7}{4}}} . \tag{62}$$

Thus, for all the accounts, from the point of view of ten-dimensional Einstein gravity, such a BH is semi-classical.[6] Once the BH crosses over to $r_g \ll l_s$, the semi-classicality can potentially be compromised because of the following two reasons.

First, the new heavy fields can emerge outside the horizon and modify the BH metric. Since, by construction, the BH of interest only poses a non-zero $T_{\mu\nu}$ current, only the spin-2 and spin-0 fields may be directly sourced by it. For example, graviton and dilaton are the simplest examples. In a ghost-free theory, gravity mediated by the exchange of spin-2 and spin-0 fields is always attractive. Thus, it is unlikely that such exchanges could lead to either a shrinkage or a removal of the BH horizon. Of course, the role of the other spins can be enhanced by non-linearities. However, again, since string theory is weakly coupled, at least in the closed string theory, the couplings of individual degrees of freedom are suppressed by powers of M_D, so it is unclear how the non-linearities could undo the strength of gravity, at least up to the scale,

$$M_* = \frac{M_D}{N_{eff}^{\frac{1}{D-2}}} , \tag{63}$$

where M_D refers to the D dimensional Planck mass, and N_{eff} is an effective number of D-dimensional degrees of freedom participating in the gravitational exchanges at scale M_*. The whole question, thus, boils down to what is N_{eff}?

In ten dimensions the semi-classical BH parametrically smaller than l_s can exist, if N_{eff} is not exponentially large. This means that the number of effective degrees of freedom affecting the BH metric at a given distance, must grow

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[6]For weak string coupling, $g_s < 1$, the black hole mass M_{BH} is higher than the string scale M_s and therefore also higher that the mass of string Regge excitations. E.g. for $g_s = 10^{-2}$ there exist $n_* = g_s^{-2} = 10^4$ Regge states before one reaches the BH mass.

slower than the number of the available string states with the (naive) Compton wavelength less than that distance.

The second consideration also indicates that the number of effective degrees of freedom must be less than the actual number of stringy states of a given mass. The shift of the focus, however, is from the modes affecting the gravitational exchange to the modes participating in the BH evaporation.

Indeed, when $r_g \ll l_s$ i.e. when the temperature $T \sim r_g^{-1} \gg M_s$, one arrives to an apparent conflict with BH semi-classicality: The number of the energetically-available species in string theory seems to be exponentially large.

We shall now show, that the key to resolving this puzzle is, that although the number of species is large, only very few participate in the evaporation of the BHs. In the other words $N_{eff} \ll N$.

Let us quantify the argument. As already explained above, it is a well known fact in string theory that the density of states grows exponentially fast for $M > M_s$. This growth takes place according to (56). The thermodynamics consequences of this exponential growth are well known, and in particular give rise to Hagedorn type transition above the temperatures $T > M_s$.

Let us now consider an elementary process leading to the BH evaporation in which a semiclassical nonrotating BH state of temperature T emits, for example, a closed string state and decays to another nonrotating BH state. If we ignore the backreaction, the BH remains at temperature T,

$$BH(T) \rightarrow BH(T) + \text{String State}. \qquad (64)$$

This is a good approximation for semiclassical BHs, since, by default, in semiclassical regime the BH is much heavier than its temperature, and the change of temperature due to a single particle emission is a sub-leading effect. In the leading order, therefore, the temperature can be kept unchanged for an elementary emission act. One might have thought that the exponential growth of the density of states would indicate that the number of available string states for the process (64) is exponentially large, however, this is not the case.

Let us first show that in string theory there is an absolute upper pound, set by $1/g_s^2$, on the effective number of Regge states participating in the evaporation of a semiclassical BH. For this, let us first define N_{eff}. Let the one-particle production rate (integrated over all the species) in the evaporation of a semi-classical BH be $\Gamma_{total}(T)$. This quantity is related to the change of temperature as,

$$\frac{1}{T}\frac{dT}{dt} \sim \Gamma_{total}(T). \qquad (65)$$

The violation of semi-classicality simply means that $\Gamma_{total} > T$. For example, for a ten-dimensional BH evaporating into a single graviton, we have

$$\Gamma_{BH \rightarrow graviton} \sim T \left(\frac{T}{M_{10}}\right)^8, \qquad (66)$$

and the BH half-lifetime can be obtained by integrating (65). We can now define the N_{eff} as the measure of the relative increase of the evaporation rate with respect to graviton,

$$\Gamma_{total}(T) = \Gamma_{BH \rightarrow graviton} N_{eff}(T) . \tag{67}$$

Obviously, the effective number of species at the critical temperature, $\Gamma_{total}(T_c) = T_c$, is

$$N_{eff}(T_c) = \frac{M_{10}^8}{T_c^8} . \tag{68}$$

Since Regge states only participate in the BH evaporation above the temperature M_s, we have $T_c \gtrsim M_s$. This fact, after taking into the account the relation between M_{10} and M_s, automatically implies the upper bound on the number Regge states that can be emitted from a semi-classical BHs,

$$N_{eff} \lesssim \frac{1}{g_s^2} . \tag{69}$$

We shall now study the physics behind this bound. We shall rely on the effective field theory treatment that explains why the number of states produced in the process (64) is not exponentially large. This result also matches the general intuition, that in the decay of a small size object the contribution of the long strings should be suppressed. It is also in agreement with the general Veneziano-type softening of the stringy amplitudes in high-energy scattering processes, discussed earlier.

For quantifying this result, we shall use the effective field theory approach of [24], for describing the closed string emission by external sources, adopting it to the BH evaporation process. Some results of the analogous study were also briefly reported in [9].

The key point in our approach is, that from the point of view of an effective theory the process (64) can be described by an effective vertex. The emission of a closed string state $|a^{\mu_1 \cdots \mu_l \mu_{l+1} \cdots \mu_{l+r}} p\rangle$ of momentum p by a black hole of temperature T (and size $r_g = T^{-1}$) has the following form

$$\langle 0|a_{BH}(T') a_{BH}^\dagger(T) \, \alpha_{m_1}^{\mu_1 \dagger} ... \alpha_{m_l}^{\mu_l \dagger} \tilde{\alpha}_{\widetilde{m}_1}^{\mu_{l+1} \dagger} ... \tilde{\alpha}_{\widetilde{m}_r}^{\mu_{l+r} \dagger} |0\rangle, \tag{70}$$

where $a_{BH}(T)^\dagger$ ($a_{BH}(T)$) is the creation (annihilation) operator for a BH of temperature T, $\alpha_{m_l}^{\mu_l \dagger}$ ($\alpha_{m_l}^{\mu_l}$) and $\tilde{\alpha}_{\widetilde{m}_r}^{\mu_r \dagger}$ and ($\tilde{\alpha}_{\widetilde{m}_r}^{\mu_r \dagger}$) are the usual string creation (annihilation) operators for the left and right oscillators, respectively. The μ-s are the Lorentz indexes, and m_l, m_r label the oscillator levels. Because of the obvious Lorentz structure, the effective vertex is suppressed by $l + r + 2$ powers of M_{10},

$$\frac{1}{M_{10}^{l+r+2}} \, p^{\mu_1} ... p^{\mu_l} p^{\mu_{l+1}} ... p^{\mu_{l+r}} . \tag{71}$$

The suppression scale is M_{10} and not M_s. This is also obvious from the fact that the closed string states are partners of graviton and couple with gravitational strength. As a consistency check, this correctly reproduces the graviton vertex, for $l = r = 1$, as it should.

Since in BH evaporation, the typical momenta involved are $\sim T$, the emission rate of a given string state is suppressed by the factor

$$\Gamma_{\text{single}} \sim T \left(\frac{T}{M_{10}} \right)^{2(l+r)+4} . \tag{72}$$

Before proceeding we shall perform a consistency check, noticing that the above effective vertex correctly accounts for the decay width of an usual semiclassical BH due to graviton emission. Indeed, the graviton emission rate is (66). This is exactly the rate of the temperature-charge of a semi-classical BH due to evaporation into a single graviton,

$$\frac{1}{T} \frac{dT}{dt} \sim \frac{T^9}{M_{10}^8} . \tag{73}$$

The half life-time of such a BH can be estimated by integrating the above equation, which gives

$$\tau_{BH} \sim \frac{M_{10}^8}{T^9} . \tag{74}$$

This matches the lifetime obtained by the usual consideration of a BH as of a perfect quantum emitter. In the absence of any species other than graviton, the change of BH mass would be given by,

$$\frac{dM_{BH}}{dt} \sim -r_g^8 T^{10} \sim -T^2 . \tag{75}$$

Integrating this equation, we get exactly (74).

In order to obtain the total one particle emission rate we have to sum (72) over all the states with masses up to T, or equivalently up to the level $n = T^2/M_s^2$. Although the number of states is exponentially large, what matters is the number of states with *fixed* oscillator number $l + r$. Finding this number reduces to a combinatorics problem of partitions $p_{l+r}(n)$ of an integer n, which for $l + r \ll n$ scales as

$$p_{l+r}(N) \sim \frac{(n - (l+r))^{l+r-1}}{(l+r)!(l+r-1)!} . \tag{76}$$

Thus, the total number of Regge states up to the level n that are created by l left and r right oscillators grows with n as

$$p_{l+r}^{(d)}(N) \sim n^{l+r-2} d^{l+r} . \tag{77}$$

Thus, the total one particle emission rate into closed string states from a BH of temperature T is,

$$\Gamma_{\text{total}} \sim \sum_{r+l} \sum_{n}^{n_{max}=T^2/M_s^2} T \left(\frac{T}{M_{10}}\right)^{2(l+r)+4} n^{l+r-2} d^{l+r} . \tag{78}$$

Performing summation over n we get

$$\Gamma_{\text{total}} \sim T \left(\frac{T}{M_{10}}\right)^6 \sum_{r+l} \left(\sqrt{d}\frac{T^2}{M_{10}M_s}\right)^{2(l+r)-2} \tag{79}$$

This expression clearly indicates the physical reason behind the suppression of the effective number of species. For example, till the temperature $T_{(2)} \equiv \sqrt{M_{10}M_s} = M_s/g_s^{1/8}$. the rate is dominated by $l + r = 2$. Therefore, we have

$$\Gamma_{total}(T < T_{(2)}) \sim \Gamma_{(2)} \equiv T \left(\frac{T}{M_{10}}\right)^8 \left(\frac{T}{M_s}\right)^2 . \tag{80}$$

Thus, up to the temperature $T \sim M_s/g_s^{1/8}$, the only states that participate in the process (64) are the two-oscillator states. These include the spin-0 and spin-2 states only. Thus, the effective number of species contributing to the evaporation of a neutral nonrotating BHs at temperatures $T \ll T_{(2)}$ is

$$N_{eff} \sim \left(\frac{T_{(2)}}{M_s}\right)^2 \sim g_s^{-1/4} , \tag{81}$$

despite the fact that the total number of thermally-available species is exponentially large.

Although, the (80) breaks down above $T_{(2)}$, above which series have to be resumed, it serves the purpose illustrating that the number of string species participating in the BH evaporation at temperatures $\gg M_s$ is *not* exponentially large, in accordance with (69). In the next section we shall provide another example of BH evaporating in the string states, in which relation (69) is explicit.

When extending the above analysis to the open string states, the following subtlety appears.

First, the perturbative open string states live on a lower dimensional sub-manifolds, so for them we have to take into the account the non-democracy factor in BH evaporation.

Secondly, the open string states may inter-couple without any M_{10}-suppressed interactions. Such unsuppressed couplings to the semi-classical BHs are impossible, because the evaporation strength has to be controlled by gravity. Consider, for example, a single photon emission from a neutral BH. The fact that emission must be controlled by the gravitational strength implies, that the effective vertex

through which the emission of photon from a neutral BH takes place must be of a dipole type,

$$F_{\mu\nu} J_{BH}^{\mu\nu},\qquad(82)$$

where $J_{BH}^{\mu\nu}$ is the BH thermal current, which classically should vanish for any neutral non-rotating BH, by no-hair theorem [23]. This form of the vertex is also dictated by the gauge invariance, since the BH is electrically neutral.

The M_{10} dependence of $J_{BH}^{\mu\nu}$ must be such that to reproduce the correct species number dependence of the lifetime of a semi-classical BH that evaporates into photons and gravitons,

$$\Gamma_{BH\to\gamma} \sim T \left(\frac{T}{M_{10}} \right)^8 .\qquad(83)$$

The above is of course very different from the photon emission rate from an ordinary relativistic hot plasma composed of, say, open string zero modes (e.g., electron positron), the photon emission rate out of which can be estimated as $\sim \alpha_{EM} T$.

5 Physical Analogy: Black Hole Evaporation in QCD Strings

It is a well known idea [25] that at large distances $SU(n)$ QCD without light quarks can be described as a theory of closed strings. The strings in question are cromo-electric flux tubes, with the tension $\sim \Lambda_{QCD}$. In the presence of quarks, the QCD closed strings can break into the open ones, that connect quarks-antiquark pairs.

The suggested connection between the QCD and string theory becomes more apparent in large n limit. In this limit QCD flux tubes interact as fundamental strings with the effective string coupling $g_s \equiv \frac{1}{n}$ and the corresponding string scale $M_s \equiv \Lambda_{QCD}$. At distances $\ll \Lambda_{QCD}^{-1} \equiv l_{QCD}$ such a string theory becomes a gauge theory of $N \simeq n^2$ gluon species.

We shall now consider the $SU(n)$-QCD coupled to Einsteinian gravity, and investigate evaporation of semi-classical black holes in such a theory. Due to obvious reasons, we shall limit our analysis to four dimensions. Since we are primarily interested in the evaporation of the small quasi-classical black holes with the size $R < l_{QCD}$, we shall assume the hierarchy $\frac{M_4}{n} \gg \Lambda_{QCD}$.

Although in no way we can claim a full physical equivalence between the above system and the semiclassical BH evaporation in a "real" string theory, nevertheless, it enables to grasp in more controllable way the important general properties of the system in which BHs can evaporate into the stringy objects. The advantage of the QCD string example is, that it admits the short-distance description in terms of n^2 gluon degrees of freedom, and thus, allows for the cross-check of the results obtained within the effective string-black hole interaction picture. As

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we shall see, the above complementary (gluon) description qualitatively confirms our results obtained in the previous section. Although naively the number of thermally-available string resonances (in QCD-string case, glueballs) is exponentially large, the effective number of species to which the black hole evaporation is taking place is much less, and is given by the number of gluons, rather than glueballs.

Let us consider the evaporation of a BH in such a theory. The large BHs, with size $\gg l_{QCD}$, cannot evaporate into the QCD degrees of freedom and are uninteresting for our purposes. We shall thus focus on the black holes with size $l_* \ll R \ll l_{QCD}$. Such black holes are semi-classical and undergo the usual Hawking evaporation. They are much hotter than the QCD scale, and thus, must be able to evaporate into the QCD strings states (glueballs). The existence of the two descriptions gives us an useful tool for understanding the evaporation rate.

For this, consider the evaporation process described by the two observers. One is a microscopic observer operating at distances $\ll l_{QCD}$ and describing the black hole evaporation in terms of the gluon emission. Another is a long-distance observer that is using the effective description at distances $\gg l_{QCD}$. The latter observer cannot resolve the structure of the QCD strings, and can only describe the black hole evaporation process in terms of closed string (glueball) emission.

The black hole evaporation rates obtained by the two observers must agree, since the two descriptions are complementary. Of course, the gluons observed by the short-distance observer will undergo the process of glueballization after the QCD time $t_{QCD} \sim l_{QCD}$, but the details of this process are unimportant for the large-distance description. The macroscopic observer interested in detecting evaporation process at large distances, must be able to integrate out the microscopic physics and understand the glueballization process in terms of the effective interaction vertex between the black hole and the closed string states. This is the key point of the effective long-distance description.

Thus, if large-n QCD can indeed be approximated as a closed string theory, the general structure of the black hole - glueball interaction must be understandable in form of the effective black hole -string interactions similar to the one considered in the previous section.

The complementarity of the two descriptions then allows for the effective number of closed strings (glueballs) participating in the emission process to be understandable in terms of the number of gluons. The microscopic theory of black hole - gluon evaporation, then immediately puts the limit on the number of macroscopic closed string states produced. In microscopic description the black hole evaporates in N^2 elementary gluon states, and hence its evaporation rate satisfies the quasi-classicality condition till the temperature $\sim M_*$, despite the fact that the naive number of thermally-accessible glueball states may be much larger.

Translating in terms of the string coupling, we obtain that the effective num-

ber of species participating in black hole evaporation in QCD string theory is $N_{eff} \sim 1/g_s^2$, as opposed to being set by the number of glueballs! It is rather tempting to generalize this number to the ordinary strings. Of course, in the usual string theory no QCD type complementary description is known that would enable us to think of fundamental strings as of flux-tubes of an underlying theory with finite number of elementary degrees of freedom. However, for our analogy, the absence of such description is unimportant, since the effective black hole -string interactions must be qualitatively insensitive to such descriptions, and should only rely on the stringy nature of the macroscopic description.

For QCD strings, existence of the complementary short-distance description gives simple microscopic explanation to the fact that the long closed strings are produced very selectively. The long strings that are produced more efficiently are the ones that are the results of the glueballization process of a single gluon emission from the black hole. At temperature T the maximal length of such flux tubes is $l_{max} \sim T l_{QCD}^2$, but not all the possible tube configurations of this length are produced. Thinking in terms of the different harmonics of the macroscopic oscillating flux-tubes, the strings that can be produced efficiently are the ones that during their oscillation undergo contraction to the BH size of T^{-1}.

6   Conclusions

In this paper we have studied the question of how the string theory compactifications accommodate the BH bound (3). The bound imposes a strict condition on the gravitational cutoff of the theory in terms of number of species to which the semi-classical BHs can evaporate in any consistent theory of gravity. The issue is therefore dramatically linked to the questions, down to what critical size the semi-classical BHs can exist in string theory, and to what number of particle species they can efficiently evaporate?

We have divided our studies into the cases of BHs larger or smaller than l_s. The first category of the BHs can only evaporate into the open and closed string zero modes and their KK resonances. We have shown that the number of species to which such BHs can evaporate automatically saturates the bound (3). In fact, saturation of this bound becomes equivalent to the geometric relations obtained between the four-dimensional Planck mass and the string scale in terms of the geometry of the internal manifold.

We thus discover, that the bound allows for understanding of seemingly-complicated fundamental geometric relations in terms of simple counting of the number of species.

We next considered the question of sub-stringy BHs. The naive intuition tells us, that such BHs should not exist based on thermal arguments a la Hagedorn. Since the number of string Regge states increases exponentially above M_s, the semi-classicality condition (5) should be violated almost instantly as soon as

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the BH size crosses l_s. We showed that this intuition is false, and the thermal arguments do not prevent the semi-classical BHs from shrinking down to sub-stringy size. The key reason is, that despite the exponentially growing number of string oscillator states, only few are produced in the evaporation of the small BHs.

Our study suggest that the effective number of string species is given by $N_{eff} = 1/g_s^2$. Interestingly, the BH bound (4) applied to this number exactly reproduces the relation between the string and Planck masses. This remarkable fact suggest an yet to be understood fundamental connection between the particle species and gravity.

Our findings have obvious phenomenological implications for TeV scale string scenario, motivated by the solution of the hierarchy problem. We predict that the maximal effective number of species to which the micro BHs can decay must be given by $1/g_s^2$.

Acknowledgements

References

[1] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, "New dimensions at a millimeter to a Fermi and superstrings at a TeV," Phys. Lett. B **436**, 257 (1998) [arXiv:hep-ph/9804398].

[2] D. Lüst, S. Stieberger and T. R. Taylor, "The LHC String Hunter's Companion," Nucl. Phys. B **808**, 1 (2009) [arXiv:0807.3333 [hep-th]].

[3] L. A. Anchordoqui, H. Goldberg, D. Lüst, S. Nawata, S. Stieberger and T. R. Taylor, "Dijet signals for low mass strings at the LHC," Phys. Rev. Lett. **101**, 241803 (2008) [arXiv:0808.0497 [hep-ph]].

[4] L. A. Anchordoqui, H. Goldberg, D. Lüst, S. Nawata, S. Stieberger and T. R. Taylor, "LHC Phenomenology for String Hunters," Nucl. Phys. B **821**, 181 (2009) [arXiv:0904.3547 [hep-ph]].

[5] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, "The hierarchy problem and new dimensions at a millimeter," Phys. Lett. B **429**, 263 (1998) [arXiv:hep-ph/9803315].

[6] G. Dvali, "Black Holes and Large N Species Solution to the Hierarchy Problem," arXiv:0706.2050 [hep-th].

G. Dvali and M. Redi, "Black Hole Bound on the Number of Species and Quantum Gravity at LHC," Phys. Rev. **D77** (2008) 045027, arXiv:0710.4344 [hep-th];

G. Dvali, "Nature of Microscopic Black Holes and Gravity in Theories with Particle Species", arXiv:0806.3801 [hep-th];'

G. Dvali, M. Redi, "Phenomenology of 10^{32} Dark Sectors", Phys.Rev. **D80** (2009) 055001, arXiv:0905.1709 [hep-ph].

[7] G. Dvali and D. Lüst, "Power of Black Hole Physics: Seeing through the Vacuum Landscape," JHEP **0806**, 047 (2008), arXiv:0801.1287 [hep-th].

[8] G. Dvali and C. Gomez, "Quantum Information and Gravity Cutoff in Theories with Species", Phys. Lett. **B674** (2009) 303, arXiv:0812.1940 [hep-th]; "Strong Coupling Holography", arXiv:0907.3237 [hep-th].

[9] R. Brustein, G. Dvali and G. Veneziano, " A Bound on Effective Gravitational Coupling from Semiclassical Black Holes", JHEP 0910 (2009) 085, arXiv:0907.5516 [hep-th].

[10] G. Dvali and C. Gomez, "Quantum Information and Gravity Cutoff in Theories with Species," Phys. Lett. B **674**, 303 (2009) [arXiv:0812.1940 [hep-th]].

[11] C. Gomez, private communications.

[12] M. R. Douglas, D. N. Kabat, P. Pouliot and S. H. Shenker, "D-branes and short distances in string theory," Nucl. Phys. B **485**, 85 (1997) [arXiv:hep-th/9608024].

[13] G. Dvali and O. Pujolas, "Micro Black Holes and the Democratic Transition," Phys. Rev. D **79**, 064032 (2009) [arXiv:0812.3442 [hep-th]].

[14] D. Lüst, "Intersecting brane worlds: A path to the standard model?," Class. Quant. Grav. **21**, S1399 (2004) [arXiv:hep-th/0401156].

[15] R. Blumenhagen, B. Körs, D. Lüst and S. Stieberger, "Four-dimensional String Compactifications with D-Branes, Orientifolds and Fluxes," Phys. Rept. **445**, 1 (2007) [arXiv:hep-th/0610327].

[16] V. Balasubramanian, P. Berglund, J. P. Conlon and F. Quevedo, "Systematics of Moduli Stabilisation in Calabi-Yau Flux Compactifications," JHEP **0503**, 007 (2005) [arXiv:hep-th/0502058].

[17] J. P. Conlon, F. Quevedo and K. Suruliz, "Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking," JHEP **0508**, 007 (2005) [arXiv:hep-th/0505076].

[18] C.D. Hoyle, U. Schmidt, B.R. Heckel, E.G. Adelberger, J.H. Gundlach, D.J. Kapner and H.E. Swanson, "Sub-millimeter tests of the gravitational inverse-square law: A search for ' large' extra dimensions," hep-ph/0011014

[19] S. Dimopoulos, and G.F. Giudice, " Macroscopic forces from supersymmetry", Phys. Lett. **B379** 105 (1996), [arXiv: hep-ph/9602350]

[20] I. Antoniadis , S. Dimopoulos and G. Dvali, " Millimeter range forces in superstring theories with weak scale compactification", Nucl. Phys. **B516** , 70 (1998). [arXiv: hep-ph/9710204]

[21] G. Dvali and C. Gomez, "Strong Coupling Holography," arXiv:0907.3237 [hep-th].

[22] L.M. Krauss and F. Wilczek, *Phys. Rev. Lett.* **62** (1989) 1221.

J. Preskill and L.M. Krauss, *Nucl. Phys.* **B341** (1990) 50.

S. Coleman, J. Preskill and F. Wilczek, *Phys. Rev. Lett.* **67** (1991) 1975.

M.J. Bowick, S.B. Giddings, J.A. Harvey, G.T. Horowitz, A. Strominger, *Phys.Rev.Lett.* **61** (1988) 2823.

T.J. Allen, M.J. Bowick, A. Lahiri, *Phys.Lett.* **B237** (1990) 47

G. Dvali, *Phys. Rev.* **D74** (2006) 044013, hep-th/0605295; hep-th/0607144.

[23] W. Israel, *Phys. Rev.* **164** (1967) 1776; *Commun. Math. Phys.* **8**, (1968) 245;

B. Carter, *Phys. Rev. Lett.* **26** (1971) 331.

J. Hartle, *Phys. Rev.* **D 3** (1971) 2938.

J. Bekenstein, *Phys. Rev.* **D 5**, 1239 (1972); *Phys. Rev.* **D 5**, (1972) 2403; *Phys. Rev. Lett.* **28** (1972) 452.

C. Teitelboim, *Phys. Rev.* **D 5** (1972) 294.

[24] G. Dvali, G. Gabadadze, M. Kolanovic and F. Nitti, " Scales of Gravity", Phys. Rev. **D 65** (2002) 024031, hep-th/0106058.

[25] G.'t Hooft, Nucl. Phys. **B72** (1974) 461