MULTI-BUNCH CALCULATIONS IN THE CLIC MAIN LINAC

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Abstract

In the main linac of the compact linear collider (CLIC [1]) , wakefield induced multi-bunch effects are important. They have a strong impact on the choice of accelerating structure design. The paper presents the limit for the wakefield that one bunch exerts on the next. It also gives estimates for the allowed level of persistent wake fields and on the resistive wall wakefield.

INTRODUCTION

Long-range wakefields impact the beam in the main linac in two ways. First, if the bunches are injected with offsets or angles long-range wakefields will drive growing oscillations of the subsequent bunches. Second, offsets of accelerating structures or other beam line elements will lead to transverse deflecting kick of leading bunches on subsequent ones. Both effects can yield a limit for the acceptable strength of the wakefields. In the following, we will first introduce some analytic estimates of the wakefield effects and then consider the impact of geometric wakefields of the accelerating structures as well as trapped modes and of resistive wall wakefields.

JITTER OF POINT-LIKE BUNCHES

Two point-like bunches with distance z in a train are considered that are injected with an offset $y_{1,0}$ for the first and $y_{2,0}$ for the second bunch into a perfectly aligned machine with constant twiss function β, energy E and wakefield W. The oscillation y_2 of the second bunch is given by

$$y_2'' + \frac{1}{\beta^2} y_2 = y_1 \frac{W(z)Ne^2}{E} \exp\left(-i\frac{s}{\beta}\right)$$

which can be easily solved using the ansatz

$$y_2 = \left(y_{2,0} + i\frac{y_1 W(z)Ne^2\beta}{2E}s\right)\exp\left(-i\frac{s}{\beta}\right)$$

If W, β or E depend on s, the solution is less straightforward. The problem can however be simplified by assuming that $\beta(s)$, $E(s)$ and $W(s)$ do not change over one betatron oscillation, i.e. treating phase and amplitude seperately. The excitation induced by the driving particle is thus approximated with a stair-case function. For a large number of oscillations and continuously increasing driving function this is a good approximation. In this case the problem is greatly simplified to solving

$$y_2(s) \approx \left(y_{2,0} + \int_0^s i\frac{y_1 W(z,s')Ne^2\beta(s')}{2E(s')}ds'\right)$$

$$\exp\left(-i\int_0^s \frac{1}{\beta(s')}ds'\right) \quad (1)$$

In the following, we will ingore the phase factor and only consider the complex oscillation amplitudes y_i defined with respect to the phase at $s = 0$.

To expand to cases with more than two bunches we define a_{j-k} to be the direkt change of the final amplitude $y_{j,f}$ of bunch j that is induced by the initial offset y_k of bunch k. a_{j-k} can be calculated by integrating to the end of the main linac $\hat{s}$

$$a_{j-k} = i\int_0^{\hat{s}} \frac{W(z_j - z_k, s)Ne^2\beta(s)}{2E(s)}ds \quad (2)$$

Hence, $y_{j,f} = a_{j-k}y_k$.

For n bunches one can define a matrix a with elements $a_{jk} = a_{j-k}$ for $j > k$ and $a_{jk} = 0$ otherwise. This matrix describes the direct impact of the initial offset of each bunch on the final offset of each other bunch. However, bunch k can modify the oscillation of bunch j also indirectly by altering the oscillation of bunches in between those two. We thus define the matrix A to determine the final bunch offsets including the indirect wakefield effects. The final offset of all bunches $\mathbf{y_f}$ is then related to the initial $\mathbf{y_0}$ by $\mathbf{y_f} = A\mathbf{y_0}$. A can then be easily calculated as

$$A = \lim_{m\to\infty}\left(1 + \frac{a}{m}\right)^m = \exp(a) = \sum_{k=0}^{\infty}\frac{a^k}{k!} = \sum_{k=0}^{n-1}\frac{a^k}{k!}$$

Here, we use $a^n = 0$ since $a_{jk} = 0$ for $j \leq k$.

GEOMETRIC WAKEFIELDS

Beam Jitter

To estimate the impact of the long-range geometric wakefields of the accelerating structures one can replace the integral in equation 2 with the sum

$$a_k = \sum_i \frac{L_i\beta_i}{2E_i}W(z_k)Ne^2 \approx 380\,\mathrm{m^2GeV^{-1}}W(z_k)Ne^2$$

We use the largest wakefield value allowed during the CLIC parameter optimisation: the long-range wake field of each bunch applies a kick only to the next following bunch and the field amplitude is $6.6\,\mathrm{kV/pCm^2}$. For this case the direct wakefield parameter can be trivially calculated to be $a_1 \approx 1.5$ and $a_{k\neq1} = 0$ and for $j \geq k$ one finds

$$A_{jk} = \frac{(ia_1)^{(j-k)}}{(j-k)!} \quad (3)$$

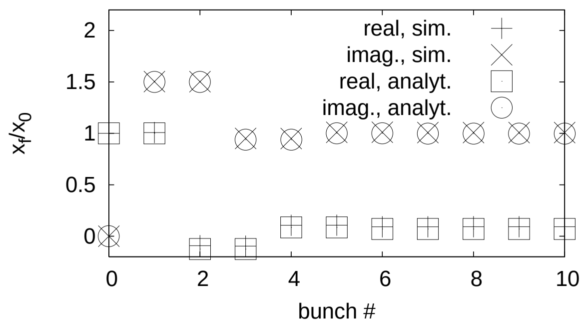

Figure 1: The normalised amplitudes of the bunches at the end of the CLIC main linac for an offset incoming train. Point-like bunches are assumed in the calculation and simulation.

The impact of an initial offset of the whole train is shown in Fig. 1. As one can easily calculate, the bunches approach along the train a phase shift of $\exp(ia_1)$, with good convergence already after only a few. The agreement between the simulation and the simple analytic model is very good.

Different variables can be used to describe the impact of the longrange wakefields on the beam:

- Coherent jitter of all bunches of the incoming beam leads to scattering of the final bunches. This can be easily calculated

$$F_c = \frac{1}{n}\sum_k \left|\sum_j A_{kj}\right|^2$$

  In our case $F_c \approx 1$.
- Random bunch-to-bunch jitter of the incoming beam also leads to scattering of the final bunches.

$$F_{rms} = \frac{\sum_{k=0}^{n-1}\sum_{j=1}^{k} A_{k,j}A^*_{k,j}}{n}$$

  From equation 3 one can also conclude that $F_{rms} \approx \left(1+\sum_{i=0}^{n-1} A_{0i}^2\right)\sigma_{b0}^2 \approx 4.9$. Hence, the fraction of the incoming beam emittance that is due to white noise bunch-to-bunch offsets is amplified by about a factor 5, which seems to be the limit of acceptability.
- Specific combinations of offsets of the incoming bunches can be more harmful than a coherent offset of all bunches. The worst combination can be found via a singular value analysis of A. The square of the largest singular value then defines F_{worst}. We find $F_{worst} \approx 20$.

If the bunches are not point-like the beam stability will be altered. Beam energy spread leads to decoherence of the motion and damps the wakefield effects but single bunch effects driven by the oscillations due to the multi-bunch wakefields can be important. Figure 2 shows the multi-pulse multi-bunch emittance growth normalised to

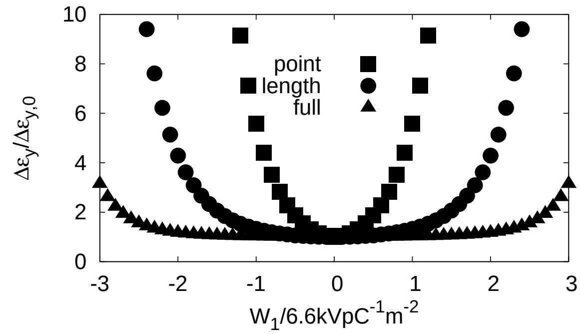

Figure 2: F_{rms} as a function of the wakefield at the second bunch, for point-like bunches, realistic bunch with no initial incoherent energy spread and bunches with initial energy spread.

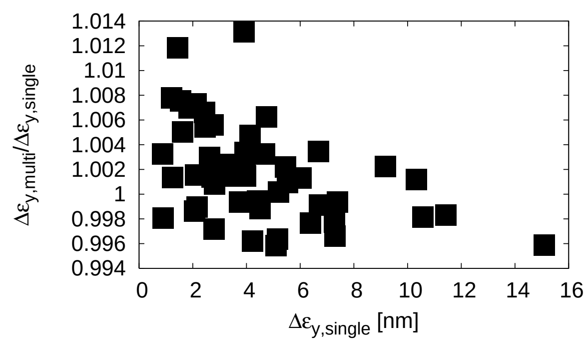

Figure 3: The emittance growth for a static misalignment of the accelerating structures after one-to-one steering for a single bunch and a bunch train.

the single bunch case for different beam models. Point-like bunches, bunches without initial energy spread and bunches with an initial uncorrelated energy spread of 2% are shown. Even the worst, the point-like case is acceptable.

Misalignment of Accelerating Structures

Misalignment of the accelerating structures leads to emittance growth. The effect is simulated with a linac that is perfectly aligned except for the accelerating structures which have an RMS offset of $10\,\mu$m. The beam is sent through the linac and one-to-one steering is performed to centre it in all BPMs. Figure 3 shows the emittance growth for the case of a single bunch and the ratio for the single and multi-bunch case. As can be seen, the difference is quite small.

The effective centres of the structures for single and multi-bunch wakefields could be slightly different. This case is studied by simulating perfect main linacs with only the centre of the multi-bunch wakefields being misaligned within each structure. For an RMS misalignment amplitude of $10\,\mu$m an emittance growth of $0.13\,$nm is found after one-to-one steering. For rigid bunches one finds only

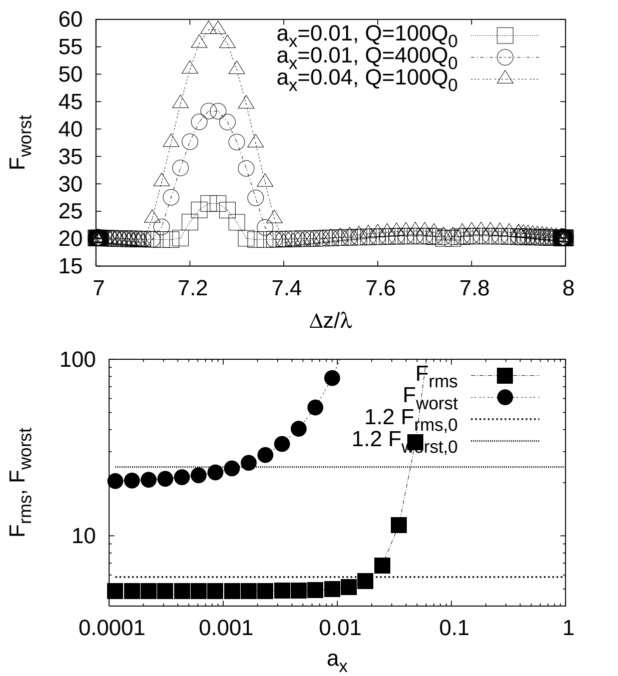

Figure 4: Upper plot shows F_{worst} as function of frequency. Lower plot shows maximum of F_{worst} and F_{rms} of all frequencies for an undamped mode.

$0.037\,\mathrm{nm}$. Misaligned long-range wakefield centres lead to increased single bunch effects, since the trajectories of the beams are modified.

TRAPPED MODES

We use a simple model, $W_\perp = W_0 \sin(2\pi z/\lambda)\exp(-\pi z/\lambda Q)$ and define a_x

$$a_x = \int_0^{\hat{s}} \frac{W_0 \beta(s')}{2E(s')} ds' \quad (4)$$

Following the previously introduced formalism, one can simply describe the combined effect of geometric wakefield and trapped mode on the beam jitter as $A_{both} = A_{geom} A_{mode} = \exp(a_{geom} + a_{mode})$, independent of the distribution of the two wakefield sources along the linac. We further assume that the trapped mode is part of the geometric wakefield, i.e. $a_{1,both} = a_{1,geom}$.

The change of F_{worst} and F_{rms} is frequency dependent, figure 4 shows this for F_{worst}. The dependence of the worst case for any frequency is shown in the same plot for both values.

RESISTIVE WALL WAKEFIELDS

The wakefield is given by

$$W(z) = \frac{cZ_0}{\pi b^3} \sqrt{\frac{1}{Z_0 \sigma_r \pi z}} \quad (5)$$

Here, Z_0 is the impedance of vacuum, b the beam pipe radius and the conductivitiy of copper $\sigma_r = 5.8 \cdot 10^7\,\Omega^{-1}\mathrm{m}^{-1}$ is assumed. We treat the resistive wall wakefield independently since it mainly induces a varation of the flat top and does not much interact with the geometric wake.

Beam Jitter

For the resistive wall wakefields in the accelerating structures we use as a simple model the average radius of the irises and weigh the length with the average iris thickness divided by the cell length. The accelerating structures alone would lead to $y_{N,f} \approx (1 + 0.02i)y_{all}$. If we require that the effect of the beam pipe in the quadrupoles, drifts and flanges is smaller one needs $b \geq 3.6\,\mathrm{mm}$, we choose $b = 4\,\mathrm{mm}$. Summing all sources, for a coherent beam jitter y_{all} the last bunch has a final amplitude of $y_{N,f} \approx (1 + 0.035i)y_{all}$.

Element Misalignments

We consider only the quadrupole beam pipe misalignment with respect to the beam. The other contributions should be small, since the structures including the flanges should be well aligned to the beam with the help of the wakefield monitors. The calculation is similar to the one for beam jitter, except that the kicks of the elements add in quadrature. The RMS bunch position scatter F_s at the end of the linac normalised to the beam size can be approximated as

$$F_s = \sum_{j=1}^{n} \frac{1}{n} \sum_i \frac{L_i^2 \beta_i \Delta_i^2}{2E_i} \frac{1}{mc^2\epsilon} \left((W_{j,sum} - \langle W_{sum} \rangle) N e^2 \right)^2 \quad (6)$$

Here, $W_{j,sum}$ is the wakefield at bunch j produced by a coherent offset of all leading bunches and $\langle W_{sum} \rangle$ is the average wakefield. For $\Delta = 100\,\mu\mathrm{m}$ this yields $F_{stat} \approx 0.012$. Simulations with point-like bunches yield the same value. If the geometric wakefields are added the average value remains the same.

CONCLUSION

With a simple analytic estimation the impact of multi-bunch wakefields on beam jitter can be quickly calculated. With this estimation and with full beam dynamics simulations it is found that the multi-bunch geometric wakefields of the accelerating structure lead to large but acceptable effects in the CLIC main linac. The analytic method also allows to quickly determine the impact of trapped modes in combination with the geometric wakefield.

REFERENCES

[1] H. Braun et al. “CLIC 2008 Parameters”, CLIC Note 764.

[2] A. Latina et al., “Recent Improvements in the tracking code PLACET”, EPAC 2008.