Eccentricity Fluctuations Make Flow Measurable in High Multiplicity *p-p* Collisions

Jorge Casalderrey-Solana and Urs Achim Wiedemann

Physics Department, Theory Unit, CERN, CH-1211 Genève 23, Switzerland

(Received 30 November 2009; published 12 March 2010)

Elliptic flow is a hallmark of collectivity in hadronic collisions. Its measurement relies on analysis techniques which require high event multiplicity and so far can only be applied to heavy ion collisions. Here, we delineate the conditions under which elliptic flow becomes measurable in the samples of high-multiplicity ($dN_{\rm ch}/dy \geq 50$) *p-p* collisions, which will soon be collected at the LHC. We observe that fluctuations in the *p-p* interaction region can result in a sizable spatial eccentricity even for the most central *p-p* collisions. Under relatively mild assumptions on the nature of such fluctuations and on the eccentricity scaling of elliptic flow, we find that the resulting elliptic flow signal in high-multiplicity *p-p* collisions at the LHC becomes measurable with standard techniques.

DOI: 10.1103/PhysRevLett.104.102301 PACS numbers: 25.75.Ag

In high-energy hadronic collisions, all projectiles, be they protons or nuclei, have a finite spatial extension and thus collide generically at a finite impact parameter. This finite impact parameter, as well as event-by-event fluctuations, both result in an azimuthally asymmetric shape of the interaction region in the plane transverse to the beam direction. If particle production is not governed solely by independent local processes, it can be modified collectively by the azimuthally asymmetric spatial gradients present in the collision geometry. In this case, the spatial eccentricity ϵ of the interaction region results in a momentum anisotropy of single inclusive particle momentum distributions dN in the azimuthal angle ϕ, which can be characterized by the harmonic flow coefficients v_n

$$\frac{dN}{dy d\mathbf{p_T}} = \frac{1}{2\pi}\frac{dN}{p_T dp_T dy}\left[1 + 2\sum_n v_n \cos n(\phi - \phi_R)\right].$$
$$v_n(p_T, y) \equiv \langle \cos n(\phi - \phi_R)\rangle|_{p_T,y}. \quad (1)$$

In general, the v_n's depend on transverse momentum p_T and rapidity y. The measurement of finite harmonic flow coefficients v_n is widely regarded as one of the most direct dynamical manifestations of collective behavior in high-energy hadronic collisions [1,2].

The measurement of flow coefficients v_n is complicated by the fact that it requires information about the azimuthal orientation ϕ_R of the reaction plane. As we recall below, whether this reaction plane is experimentally accessible depends on both, the strength of the flow signals v_n and the multiplicity $n_{\rm mult}$ in the phase space window in which v_n is determined. This is so since microscopic dynamics such as dijet production is a source of azimuthal asymmetry uncorrelated to ϕ_R and needs to be disentangled from the collectively preferred macroscopic motion of all particles, characterized by v_n. Up until now, a sufficiently large signal strength at sufficiently high event multiplicity has been observed only in heavy ion collisions, and measurements of flow coefficients v_n have not been reported for *p-p* collisions. It is the main purpose of this Letter to provide generic arguments for why at the LHC collective flow coefficients v_n may become experimentally accessible for the first time in proton-proton collisions and to discuss how this measurement would impact the understanding of soft physics both in *p-p* and in heavy ion collisions.

There are several, mutually compatible standard techniques to discriminate collective flow coefficients from nonflow effects [2–4]. Here, we focus on the cumulant analysis [4] which is based on the observation that azimuthal correlations between arbitrary particle pairs (i, j) in an event,

$$\langle e^{in(\phi_i - \phi_j)}\rangle = v_n v_n + \delta, \quad (2)$$

do not arise solely from global correlations with ϕ_R (which give rise to v_n), but that they can also have other, microscopic origins δ. Since the nonflow effects parametrized by δ are, by definition, correlated at most to a small subset of all particles in the event, they are parametrically suppressed by one power of $n_{\rm mult}$. Hence, the elliptic flow coefficient v_2 can be reliably extracted from two-particle correlations if $v_2\{2\} > 1/\sqrt{n_{\rm mult}}$. The assumption $v_n^2 \gg \delta$ that measured azimuthal two-particle correlations are dominated by a global correlation with ϕ_R can then be tested systematically by studying higher-order particle correlations. For instance, one can analyze four-particle correlations (4th order cumulants) which require only a signal size $v_2\{4\} > 1/(n_{\rm mult})^{3/4}$. By going to higher cumulants, one achieves at best a sensitivity $v_2 > 1/n_{\rm mult}$. In general, the cumulant analysis allows one to disentangle the collective flows v_n from nonflow corrections by systematically exploiting the different multiplicity scaling of flow and nonflow contributions in different n_p-particle correlations.

With such standard techniques, the lowest order collective flow coefficients, in particular v_1, v_2 and v_4, have been characterized in heavy ion collisions unambiguously and over a wide range of impact parameter at center of mass energies $\sqrt{s_{\rm NN}}$ between 2 and 200 GeV [5–9]. In the following, we focus mainly on elliptic flow v_2 at midrapidity. Odd harmonic coefficients vanish in this case by symmetry and v_2 is known to parametrize the dominant momentum space asymmetry. The size of v_2 measured in experiments at the Relativistic Heavy Ion Collider RHIC are very large. For instance, for transverse momenta in the range $p_T \simeq 2$ GeV, semiperipheral Au-Au collisions at RHIC result in more than twice as many hadrons produced in the direction of the reaction plane, than orthogonal to it ($v_2(p_T \sim 2\text{ GeV}) > 0.2$). The transverse momentum integrated v_2 reaches $v_2 \simeq O(0.1)$. Remarkably, the experimental praxis of v_2 measurements indicates that the parametric bounds on $v_2\{2\}, v_2\{4\}, \ldots, v_2\{n\}$ given above provide realistic numerical estimates for the feasibility of v_2 measurements if one uses for n_{mult} values of order of the charge multiplicity per unit rapidity dN_{ch}/dy. We summarize this information in Table I for $n_{\text{mult}} = 30$, 50, and 80. These values of n_{mult} are smaller than dN_{ch}/dy in sufficiently central heavy (Au) collisions, but they are comparable to the values in semiperipheral collisions of lighter (Cu) nuclei.

The interpretation of elliptic flow measurements in heavy ion collisions relies on the observation that v_2 is correlated with the initial spatial eccentricity ϵ of the transverse overlap region of the two projectiles [1],

$$\epsilon = \frac{\langle y'^2\rangle - \langle x'^2\rangle}{\langle y'^2\rangle + \langle x'^2\rangle}. \tag{3}$$

Here, averages are performed with respect to the matter distribution right after the collision, and x' and y' denote the lengths along the main axis of an ellipsoid describing this distribution. For sufficiently central heavy (Au) and lighter (Cu) ion collisions, it is found that $v_2 \propto \epsilon$ [10,11]. Remarkably, this is a generic expectation of fluid dynamic simulations of heavy ion collisions. Models of ideal dissipation-free hydrodynamics, which by construction describe collision scenarios of maximal collective flow, can account quantitatively for the size of the elliptic flow measured at RHIC [12–14] and the expected dissipative corrections are anomalously small [15–18]. In conjuction with this interpretation, the observation of very large v_2 signals [5–7] is arguably one of the most far reaching discoveries of the RHIC heavy ion program.

We now turn to the question whether elliptic flow may be measurable in p-p collisions at the LHC. In p-p collisions studied so far, one may explain the apparent absence of an elliptic flow signal by pointing to the fact that the

TABLE I. Estimates of the minimal signal strength $v_2\{n_p\}$, which can be discriminated from nonflow effects in an n_p-cumulant analysis based on n_{mult} particles.

	$n_p = 2$	$n_p = 4$
$v_2(n_{\text{mult}} = 30)$	>0.18	>0.09
$v_2(n_{\text{mult}} = 50)$	>0.14	>0.05
$v_2(n_{\text{mult}} = 80)$	>0.11	>0.04

dN_{ch}/dy in these collisions is too low to make v_2 measurable (see Table I). However, while Monte Carlo simulations for minimum bias dN_{ch}/dy distributions in $\sqrt{s} = 14$ TeV p-p collisions peak at low values <10 for the nondiffractive contribution, they show a pronounced high-multiplicity tail, typically reaching values as high as $dN_{\text{ch}}/dy \sim 60$. Despite their model dependence, these simulations strongly indicate that abundant samples of high-multiplicity p-p events with $dN_{\text{ch}}/dy \geq 50$ will be measured at the LHC. Such a multiplicity is comparable to that reached in semiperipheral (centrality class 40%–60%) Cu-Cu collisions at $\sqrt{s_{\text{NN}}} = 62.4$ GeV at RHIC, and for these latter collisions elliptic flow has been measured. Whether elliptic flow is also measurable in a high-multiplicity p-p event sample at the LHC then depends on the signal strength v_2 and on the relative strength of nonflow corrections in p-p collisions.

To estimate the strength of the elliptic flow signal v_2, we now discuss the initial spatial eccentricity ϵ of hadronic collisions. In the collisions of heavy (Au or Pb) ions, this eccentricity is determined solely by the transverse spatial overlap. More precisely, in nucleus-nucleus collisions dN_{ch}/dy scales approximately with the average number N_{part} of participant nucleons, which scales with the area of the nuclear overlap. Therefore, selecting a multiplicity class in A-A amounts to selecting on impact parameter and determines the shape of the nuclear overlap region. For large N_{part}, it is reasonable to make the smoothness assumption that the interactions between the N_{part} nucleons result in a homogeneous density distribution within the area of the nuclear overlap (for illustration, see the right-hand side of Fig. 1). If this assumption would carry over to p-p collisions, then the highest multiplicity p-p collisions would be the most central ones, their spatial eccentricity would be close to zero, and so would be the flow signal $v_2 \propto \epsilon$. Previous estimates of the magnitude of v_2 were based on this smoothness assumption [19,20] or on other methods [21] and reported small, nonmeasurable values.

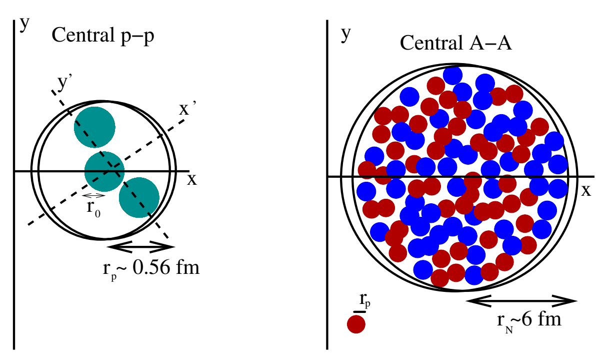

FIG. 1 (color online). Schematic view of region of hadron production may be located in the transverse overlap region of a central proton-proton and central nucleus-nucleus collision, respectively. Depending on the number and size of hadronically active regions, large eccentricities can result even in central collisions.

However, sizable deviations from the smoothness assumption have been found in modeling lighter (Cu) ion collisions [10,11]. In these systems, the relatively small number of nucleon-nucleon interactions results in event-by-event fluctuations of the density distribution which can increase the initial spatial eccentricity significantly above its geometric estimate. The predicted eccentricity scaling of v_2 is only confirmed in Cu-Cu collisions, if these fluctuations are taken into account [10,11].

For *p-p* collisions, the $N_{\rm part}$-scaling of event multiplicity used in *A-A* does not apply. It remains *a priori* unclear to what extent a cut on event multiplicity amounts to a cut on impact parameter. MC simulations of the underlying event in *p-p* do assume indeed a correlation between impact parameter and the number of independent partonic interactions $N_{\rm MPI}$, which determines event multiplicity and, depending on model assumptions, may be as large as $N_{\rm MPI}^{\rm max} \le 30$–$80$ [22]. However, the spatial distribution of hadronic activity does not enter the dynamics of these simulations. It is conceivable that all hadronic activity, even if emerging from a large number $N_{\rm MPI}$ of partonic interactions, is located in few "hot spots" in the transverse plane. Such a picture has been advocated previously in several contexts. For instance, models which view the proton as a collection of three black disks of diameter d result in a total *p-p* cross section, which matches experimental data for a surprisingly small diameter of $d \simeq 0.2$ fm. Such simple constituent quark models account well for gross features of hadronic collisions, such as the ratios of hadronic cross sections (e.g., $\sigma_{\pi p}/\sigma_{pp} \simeq 2/3$) over a wide range of $\sqrt{s}$. A more field theoretic motivation of a picture of hot spots may be based on the fact that a large value of $N_{\rm MPI}$ requires high parton densities in the proton. In QCD evolution equations such densities can arise from multiple branching of a few partonic components of high momentum fraction—and to the extent to which these branchings are collinear, partons within the proton wave function will be located indeed in few hot spots.

Irrespective of a specific dynamical picture, these considerations prompt us to consider scenarios for which the entire hadronic activity in a proton-proton collision is localized in N_s interactions regions of radius r_0. Within each interaction region, density is distributed homogenous with Gaussian profile, and the regions are distributed randomly according to the density profile of the proton (see left panel of Fig. 1 for $N_s = 3$). To calculate the eccentricity for this class of models, we take into account that for finite N_s the relevant eccentricity is the "participant eccentricity" [11]

$$\epsilon = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2}. \qquad (4)$$

Here $\sigma_x^2 = \{x^2\} - \{x\}^2$, $\sigma_y^2 = \{y^2\} - \{y\}^2$, $\sigma_{xy} = \{xy\} - \{x\}\{y\}$ and the event-by-event average $\{\ldots\}$ is taken over the distribution of interaction. This coincides with the definitions Eq. (3) in the limit of a homogeneous density. The resulting probability distribution for different sizes and different numbers of interaction regions is shown in Fig. 2. While the eccentricity vanishes indeed in the limit of a homogeneous distribution ($N_s \to \infty$), we find that it is sizable for a wide parameter range.

Elliptic flow develops on an event-by-event basis. However, the experimental determination of v_2 demands averaging over events and thus over distributions of geometric shapes. This averaging procedure is subtle; the relevant moment of the distribution $P(\epsilon)$ is different for different methods of extracting v_2 [23]. We will focus on extracting v_2 from four-particle cumulants, $v_2\{4\}$, for which ϵ scales with [23]

$$\epsilon\{4\} \equiv (2\langle\epsilon^2\rangle^2 - \langle\epsilon^4\rangle)^{1/4}. \qquad (5)$$

Here, the average is performed over the $P(\epsilon)$ shown in Fig. 2. For $N_s = 3$ our model yields the values of $\epsilon\{4\} = 0.69, 0.54, 0.40$ for the different values of the spot size in a zero impact parameter collision.

The precise transfer of spatial anisotropy $\epsilon\{4\}$ to momentum anisotropy $v_2\{4\}$ requires a full dynamical calculation, but its magnitude can be estimated on general grounds [24]. For dilute systems, there is experimental evidence [8] that the ratio v_2/ϵ depends linearly on the transverse matter density, $v_2/\epsilon \propto (dN_{\rm ch}/dy)/\langle S\rangle$ [25]. Here, $\langle S\rangle = 4\pi\sqrt{\sigma_x^2\sigma_y^2 - \sigma_{xy}^2}$ denotes the mean transverse area. For denser systems, the hydrodynamic limit is reached and the ratio becomes independent of the system

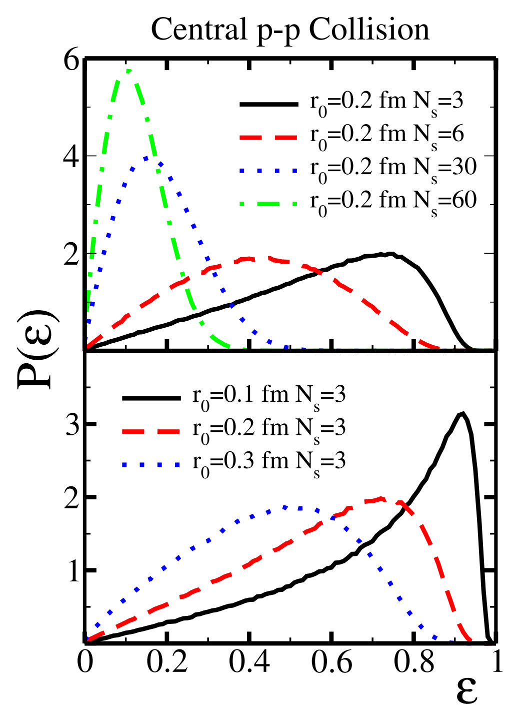

FIG. 2 (color online). Eccentricity distribution of central high-multiplicity *p-p* collisions (central) for different number of interaction regions N_s (top) of different size r_0 (bottom).

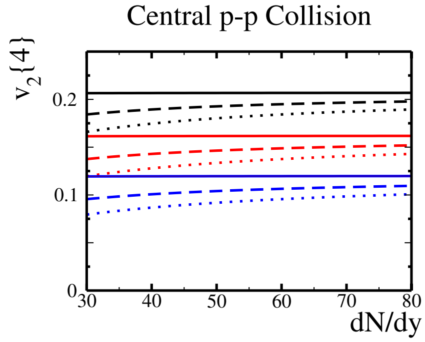

FIG. 3 (color online). The flow signal $v_2\{4\}$ as a function of multiplicity in most central p-p collisions, for models of $N_s = 3$ interaction regions of radius $r_0 = 0.1$ (top 3 curves), $r_0 = 0.2$ (middle curves), and $r_0 = 0.3$ fm (bottom curves). Signals calculated for $\bar{\lambda}/K_0 = 0$, 5.8, and 11.6 fm^{-2} are displayed by solid, dashed, and dotted lines, respectively.

size. In [24] the following interpolation formula was suggested

$$v_2\{4\} = \epsilon\{4\}\left(\frac{v_2}{\epsilon}\right)^{\text{hydro}} \frac{1}{1 + \frac{\bar{\lambda}}{K_0}\frac{\langle S\rangle}{\frac{dN}{dy}}}, \tag{6}$$

where $(v_2/\epsilon)^{\text{hydro}}$ denotes the hydrodynamic limit, and K_0 and $\bar{\lambda}$ are constants which depend on the microscopic dynamics. The combination $S\bar{\lambda}/dN/dy$ can be understood as the Knudsen number of the system [24].

To estimate v_2 for p-p at the LHC, we assume that one can use Eq. (6) with the input $(v_2/\epsilon)^{\text{hydro}} = 0.3$ and $\bar{\lambda}/K_0 = 5.8$ fm^{-2} extracted from the analysis of heavy ion collisions at RHIC [25]. This may be motivated by observing that many collective features depend mainly on event multiplicity and that the multiplicity of the p-p collisions considered here is comparable to that in peripheral lighter (Cu) ion collisions at RHIC, for which (6) applies. Under this assumption, we find for $N_s = 3$ hot spots the large elliptic flow values $v_2\{4\}$ displayed in Fig. 3. Also for $N_s = 6$ sufficiently small hot spots, the resulting $v_2\{4\}$ is found to be measurable (data not shown). To illustrate effects of theoretical uncertainties on deviations from equilibrium, we have varied in Fig. 3 the value of $\bar{\lambda}/K_0$ between its hydrodynamic limit $\bar{\lambda}/K_0 \to 0$ and twice the fitted value quoted above.

Not measuring v_2 in p-p collisions can have several reasons, such as a small spatial eccentricity of the interaction region, or viscous effects larger than expected from Eq. (6). However, combining the results shown in Fig. 3 and Table I, we conclude that strong but physically conceivable density fluctuations in the proton can lead to large values of the elliptic flow parameter v_2 in high-multiplicity p-p collisions. Measuring elliptic flow in these collisions would provide a novel constraint on the number and distribution of multiple parton interactions within a p-p collision, which is a key input in the modeling of the underlying event. At the same time, measuring flow in the smallest experimentally accessible system would show that collectivity develops on a sub-Fermi time scale; since any fluid picture demands local equilibration, this measurement would provide a tight constraint on the equilibration and dissipation mechanisms of QCD.

We would like to acknowledge P. Skands and G. Milhano for useful discussion. J. C. S. has been supported by a Marie Curie Intra-European Fellowship (PIEF-GA-2008-220207).

Note added.—While preparing this letter, Ref. [26] appeared. By studying the eccentricity for a model of two flux tubes in a p-p collision, this work arrives at similar conclusions about the measurability of v_2.

[1] J. Y. Ollitrault, Phys. Rev. D **46**, 229 (1992).
[2] A. M. Poskanzer and S. A. Voloshin, Phys. Rev. C **58**, 1671 (1998).
[3] J. Y. Ollitrault, Phys. Rev. D **48**, 1132 (1993).
[4] N. Borghini, P. M. Dinh, and J. Y. Ollitrault, Phys. Rev. C **64**, 054901 (2001).
[5] J. Adams *et al.* (STAR Collaboration), Phys. Rev. Lett. **92**, 052302 (2004).
[6] S. S. Adler *et al.* (PHENIX Collaboration), Phys. Rev. Lett. **91**, 182301 (2003).
[7] B. B. Back *et al.* (PHOBOS Collaboration), Phys. Rev. C **72**, 051901 (2005).
[8] C. Alt *et al.* (NA49 Collaboration), Phys. Rev. C **68**, 034903 (2003).
[9] J. Y. Ollitrault, Nucl. Phys. **A638**, 195c (1998).
[10] B. Alver *et al.* (PHOBOS Collaboration), Phys. Rev. Lett. **98**, 242302 (2007).
[11] B. Alver *et al.*, Phys. Rev. C **77**, 014906 (2008).
[12] D. Teaney, J. Lauret, and E. V. Shuryak, arXiv:nucl-th/0110037.
[13] P. Huovinen, P. F. Kolb, U. W. Heinz, P. V. Ruuskanen, and S. A. Voloshin, Phys. Lett. B **503**, 58 (2001).
[14] T. Hirano and K. Tsuda, Phys. Rev. C **66**, 054905 (2002).
[15] D. Teaney, Phys. Rev. C **68**, 034913 (2003).
[16] M. Luzum and P. Romatschke, Phys. Rev. C **78**, 034915 (2008) **79**, 039903(E) (2009).
[17] H. Song and U. W. Heinz, Phys. Lett. B **658**, 279 (2008).
[18] K. Dusling and D. Teaney, Phys. Rev. C **77**, 034905 (2008).
[19] D. d'Enterria, G. K. Eyyubova, V. L. Korotkikh, I. P. Lokhtin, S. V. Petrushanko, L. I. Sarycheva, and A. M. Snigirev, arXiv:0910.3029.
[20] M. Luzum and P. Romatschke, Phys. Rev. Lett. **103**, 262302 (2009).
[21] L. Cunqueiro, J. Dias de Deus, and C. Pajares, Eur. Phys. J. C **65** 423 (2010).
[22] T. Sjostrand and P. Z. Skands, J. High Energy Phys. 03 (2004) 053.
[23] R. S. Bhalerao and J. Y. Ollitrault, Phys. Lett. B **641**, 260 (2006).
[24] R. S. Bhalerao, J. P. Blaizot, N. Borghini, and J. Y. Ollitrault, Phys. Lett. B **627**, 49 (2005).
[25] H. J. Drescher, A. Dumitru, C. Gombeaud, and J. Y. Ollitrault, Phys. Rev. C **76**, 024905 (2007).
[26] P. Bozek, arXiv:0911.2397.