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Teaching an Insurance Model: An Interactive Exercise

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Teaching an Insurance Model: An Interactive Exercise[1]

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Abstract: In this paper we present a tutorial exercise where advanced undergraduate students can solve a problem of insurance demand using Excel Solver, and explore the main features of this problem. We also present an interactive Excel file that contains the graphical analysis of different attitudes toward risk using some well-known utility functions. The interactive Excel includes scroll bars that can be used to perform both comparative static and interactive graphical analyses, with the aim to improve students´ understanding of the model.

Keywords: Attitudes toward risk, Demand for Insurance, Excel Solver.

JEL codes: A22, C65, D89

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1. Introduction

The objective of this paper is to use Microsoft Excel, in particular, Excel´s Solver to analyze an insurance demand problem where an agent maximizes his expected utility in making decisions under uncertainty. After introducing the insurance demand model, we present a tutorial exercise that solves it by creating an Excel spreadsheet and then present some comparative static analyses in order to help the students to understand the role of each parameter of the model. We also present an Excel file that contains the standard graphical analysis of different attitudes toward risk using some well-known utility functions. The Excel spreadsheet includes scroll bars that can be used to perform comparative static and interactive graphical analyses, improving students´ understanding of the demand insurance model.

Although the insurance model introduced below is sometimes presented in chapters devoted to choice under uncertainty in microeconomics textbooks,[3] we present the model in detail with the aim to connect analytical implications of the model with their corresponding simulated scenarios. This exercise allows students, for example, to improve the intuition of what happens when a company offers an actuarially fair premium (that is, when the insurance premium is equal to the probability of loss).

Theoretical textbooks on economics do not cover information technology tools in order to solve choice under uncertainty exercises. Excel tools are increasingly being used for teaching economics, helping to improve learning outcomes (Barreto, 2015). Along these lines, many authors have used Excel Solver (see, for example, Silva and Xabadia, 2013; Benninga, 2010; Barreto, 2009; Strulik, 2004; Tohamy and Mixon, 2004; Nævdal, 2003; Cahill and Kosicki, 2000 and Houston, 1997), which is a user-friendly and flexible tool for economic optimization problems (MacDonald, 1996). Excel Solver can be used to solve choice under uncertainty exercises and, in our case, to teach the insurance demand model. As far as we know, articles using Excel Solver do not address such types of exercises apart from Barreto (2009), who presents a somewhat similar problem using an endowment model instead of the expected utility model used by us.

This paper targets advanced undergraduate students, in particular those who already have a theoretical background on choice under uncertainty. For those students not familiar with the topic we include the most important tools and concepts in an Appendix.

[3] See, for example, Jehle and Reny (2011), Gravelle and Rees (2004) and Mas-Colell, Whinston, and Green (1995).

This document is structured as follows. In Section 2, we formally present the insurance demand model with its graphical interpretation and we state the effects of the parameters on the decision variables. In Section 3 we guide the student on how to build an Excel spreadsheet to solve the model and propose some comparative static analyses using, as an example, a particular utility function. In Section 4 we include some additional exercises and further questions for students. In Section 5 we present an interactive Excel file that contains the graphical analysis of different utility functions. Finally, we introduce some of the most important concepts of the topic of choice under uncertainty in the Appendix.

2. The Insurance Demand Model

In this section we present and formally analyze, mathematically and graphically, the problem of the demand for insurance in the setting defined below. We recommend teaching this section in two hours.

Let us start with the explanation of the problem. Consider a risk averse agent who has to decide whether to buy insurance against a possible robbery. This agent has an initial wealth of w_0, and π is the probability of an amount l of wealth being stolen (l referring to the loss). The uncertainty is captured by the existence of two possible situations or states of the world: there is a robbery with probability π or there is no robbery with probability $(1-\pi)$. We assume that there is an insurance company that is willing to sell insurance cover against the robbery at a premium rate of $p,\ 0 < p < 1$. This means that the agent pays $p \cdot q$ to the insurer in exchange for the insurer's payment of q if the robbery occurs. The agent's utility function u is strictly increasing and strictly concave in levels of wealth w, that is: $u'(w) > 0$ and $u''(w) < 0$ (the latter by risk aversion). Moreover, u satisfies the expected utility property.[4]

As we said, we focus on the problem of the demand for insurance. We consider the insurer's decision fixed and represented by the risk premium parameter p. Thus, the agent's decision problem is to decide the amount $q \in [0, l]$ to buy as insurance cover (the decision variable) given w_0, l, π, p as parameters of the problem, and with the objective to maximize his expected utility.[5]

The agent's insurance decision problem can be stated mathematically as an optimization problem [P1] with a single variable, $0 \leq q \leq l$, and several additional constraints:

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[4] See these definitions with more detail in the Appendix.

[5] We assume that the agent's behavior cannot affect the parameters: w_0, l, π, and p. When allowing that agent's actions affect π would generate a moral hazard problem that is not analyzed in this work (see, for instance, an advanced analysis in Chapter 19G, Gravelle and Rees, 2004).

[P1]

$$\max_{\{q\}} \pi u(w_1(q)) + (1-\pi)u(w_2(q))$$

$$w_1(q) = w_0 - l + q - p \cdot q = w_0 - l + q \cdot (1-p), \tag{1}$$

$$w_2(q) = w_0 - p \cdot q, \tag{2}$$

where $w_1(q)$ and $w_2(q)$ are the agent's contingent wealth level in each state of the world, satisfying the following conditions:

$$w_0 - l \leq w_1(q) \leq w_0 - p \cdot l\,, \tag{3}$$

$$w_0 - p \cdot l \leq w_2(q) \leq w_0\,. \tag{4}$$

The feasibility constraint (3) says that $w_1(q)$, the agent's wealth, if a robbery takes place, has a lower bound $w_0 - l$ when no insurance is taken out ($q = 0$), and an upper bound $w_0 - p \cdot l$, when full insurance is taken out ($q = l$). Similarly, the feasibility constraint (4) says that $w_2(q)$, agent's wealth with no robbery, has a lower bound $w_0 - p \cdot l$, when full insurance is taken out and an upper bound w_0, when no insurance is taken out.

In this section we will tackle the problem using the two intermediate variables: the wealth level in each state of the world. The solution and interpretation of the problem using two such variables have similarities with the standard consumer choice problem with two goods (under certainty), a basic topic in microeconomics. Excel Solver can be used to resolve the two equivalent optimization problems, with one and with two variables, respectively. We solve the insurance optimization problem [P1] (with variable q) as Practice exercise 5 in Section 4.

We can now formally state the agent's insurance problem as [P2] where the agent chooses the levels of wealth w_1 and w_2 such that his expected utility is maximized subject to the constraints faced. That is:

[P2]

$$\max_{\{w_1, w_2\}} \pi u(w_1) + (1-\pi)u(w_2)$$

subject to (3), (4) and

$$p \cdot w_1 + (1-p) \cdot w_2 = w_0 - p \cdot l \tag{5}$$

Note that [P2] is obtained from [P1] as follows: Isolate q from Equations (1) and (2) and equate them, obtaining what we call the agent budget constraint (5).[6]

To solve the optimization problem [P2], we get w_1 and w_2 by solving the system of Equations (3) to (6). That is, the feasibility constraints in Equations (3) and (4), the agent budget constraint in Equation (5), and the first order condition of [P2]:

$$\frac{\pi}{(1-\pi)} \frac{u\prime(w_1)}{u\prime(w_2)} = \frac{p}{1-p}\,. \tag{6}$$

[6] We call it a budget constraint because it can be thought of as the one in a standard consumer decision problem over bundles of goods. See, for example, Page 508 in Gravelle and Rees (2004).

The economic interpretation of this problem can be easily handled by a graphical analysis. Figure 1 shows the main elements of the decision problem [P2].[7] The indifference curve and the budget constraint (5) are depicted in red (solid) and black (dashed) lines, respectively. The feasibility constraints in Equations (3) and (4) are also depicted limiting the values of w_1 and w_2, respectively. The optimal levels of wealth w_1^* and w_2^* correspond to the unique point where the two curves cut tangently, meaning that Equation (6) holds. Additionally, the amount of insurance q can be obtained from Equation (2) in [P1]:

$$q = \frac{w_0 - w_2}{p}. \tag{7}$$

As we are going to see next, Equations (5) to (7) are crucial for comparative static analyses. These three equations simultaneously determine variables w_1, w_2, and q.

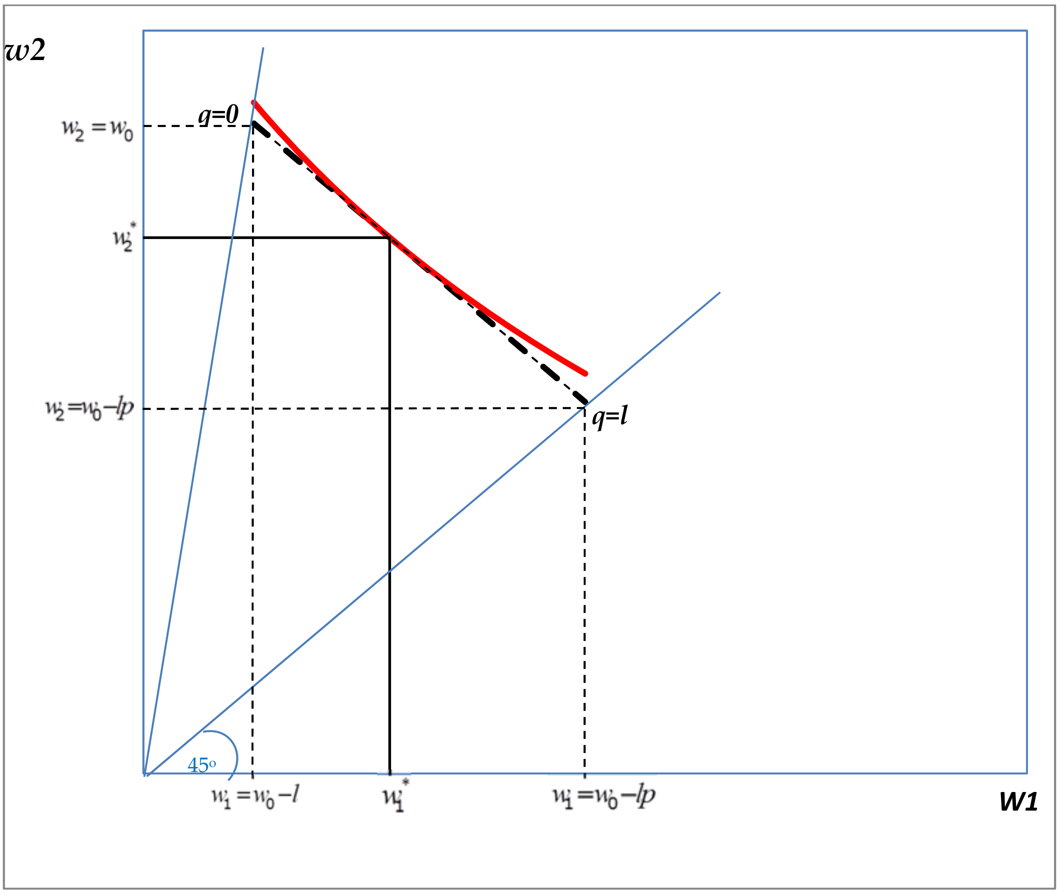

Figure 1: Graphical representation of the insurance optimization problem A risk averse agent [P2]

[7] See, for example, Appendix E (Page 675) in Gravelle and Rees (2004) for topics about uniqueness of solutions in optimization problems.

2.1 The Effect of Probability of Robbery

A first interesting relationship in the insurance model is the effect of the probability of robbery, π, on the optimal decision variables w_1 and w_2, keeping the rest of parameters constant. If π increases, $\frac{\pi}{1-\pi}$ increases too. And to restore the equality in Equation (6), $\frac{u'(w_1)}{u'(w_2)}$ must decrease. [8] Then, according to the utility functions used for risk averse agents in the next sections, a decrease in $\frac{u'(w_1)}{u'(w_2)}$ is equivalent to an increase in $\frac{w_1}{w_2}$.[9] Then, w_1 and w_2 must increase and decrease, respectively, to satisfy Equation (5).[10] Finally, according to Equation (7), the reduction in w_2 generates an increase in the amount of insurance q. The effect of the probability of robbery is analyzed in the comparative static analysis 1 in Section 3.

2.2 The Effect of Initial Wealth

A second interesting relationship in the model is the effect of initial wealth w_0. To see how initial wealth affect w_1 and w_2, consider Equations (5) and (6). Note that if w_0 increases, then to restore the equilibrium in Equations (5) and (6) both w_1 and w_2 must increase. In more detail, a higher w_0 in Equation (5) implies that either w_1 or w_2 has to increase. Then, by Equation (6), an increase in one of them implies an increase in the other since $u' > 0$ and $\frac{u'(w_1)}{u'(w_2)}$ cannot vary. Therefore, according to Equation (7), the amount of insurance will increase only if the increase in w_0 is higher than the increase in w_2. The effect of initial wealth is analyzed in the comparative static analysis 2 in Section 3.

2.3 Actuarially Fair Premium

Another interesting relationship in the insurance model is the one between the probability of robbery (π) and the insurance premium rate (p). We say that the insurance is actuarially fair if the insurance premium pq is equal to the expected losses πq. That is, the actuarially fair premium rate is $p = \pi$. It is worth mentioning that in a competitive market of insurers the premium rate would be actuarially fair. Note that under this situation, a risk averse individual fully insures against all risk. Formally, observe that from Equation (6), we get that $w_2 = w_1$. Therefore, by Equation (5), $w_1 = w_2 = w_0 - lp$ which corresponds to full insurance ($q = l$).

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[8] It is important to emphasize that the opposite relationship would hold when the parameter varies in the opposite direction. For simplicity, we only analyze one direction in all comparative static analyses below.

[9] For example, for $u(w) = w^{\alpha}$, $\frac{u'(w_1)}{u'(w_2)} = \left(\frac{w_1}{w_2}\right)^{\alpha-1}$. Thus, a decrease in this expression takes place only if $\frac{w_1}{w_2}$ increases since $0 < \alpha < 1$.

[10] Dividing the budget constraint by $w_2 > 0$, we obtain $p \cdot \frac{w_1}{w_2} + (1 - p) = \frac{w_0 - p \cdot l}{w_2}$. Then, an increase in $\frac{w_1}{w_2}$ implies that w_2 decreases.

However, insurers typically charge premiums above the expected costs of the robbery: $p > \pi$. According to Equation (6) this implies that $u'(w_1) > u'(w_2)$ and, since the agent is risk averse, $w_2 > w_1$. This scenario corresponds to a partial insurance one as, for example, the optimal point (w_1^*, w_2^*) in Figure 1. Finally, since $q \in [0, l]$, w_1 must be less than or equal to w_2. Therefore, $p < \pi$ would imply that $w_2 = w_1$ and, therefore, $q = l$. Graphically (Figure 1), this means that it is not possible to be at the right of the 45º line. The actuarially fair premium is analyzed in the comparative static analysis 3 in Section 3.

2.4 Risk Neutral Agent

The demand for insurance is traditionally explained by the assumption that insured agents are risk averse (see Kirstein, 2000). However, the model also applies for risk neutral agents. The first observation is that the indifference curves are now linear ($u''(w) = 0$), implying that natural candidates for the agent's decision are either *q=0* or *q=l*: when the slope of the indifference curve $\frac{\pi}{1-\pi}$ is higher than the slope of the budget constraint $\frac{p}{1-p}$, then the agent will choose full insurance. However, he will choose no insurance when the opposite strict inequality holds, that is, when $\frac{\pi}{1-\pi} < \frac{p}{1-p}$. The graph in Figure 2 shows, precisely, this last possibility. The risk neutral agent case is analyzed in comparative static analysis 4.

It is also worth mentioning that for the actuarially fair premium each amount of insurance q can be the agent' choice: for a risk neutral agent ($u'(w_1) = u'(w_2)$) Equation (6) holds, that is $\frac{\pi}{1-\pi} = \frac{p}{1-p}$, if and only if $p = \pi$. Graphically, this means that since the slopes of the indifference curve and the budget constraint coincide, any amount of insurance can be optimal.

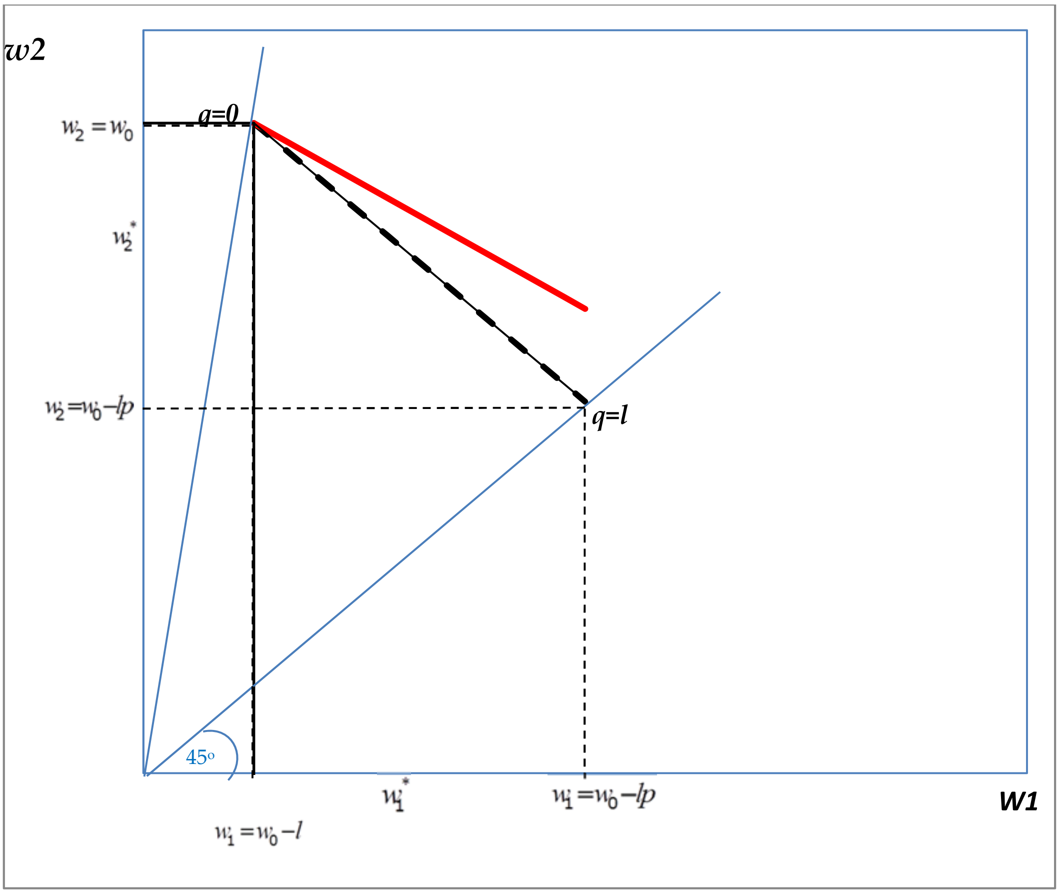

Figure 2: Graphical representation of the insurance optimization problem
A risk neutral agent [P2]

3. Solving the Insurance Decision Problem with Excel Solver from the Beginning

The insurance decision problem can also be solved numerically in a computer classroom exercise of two hours using Excel Solver. The instructor guides the students with the step-by-step narrative and accompanying screen shots as explained below.

To do this classroom exercise, we use Excel to solve the optimization problem [P2] with the agent's two contingent wealth levels w_1 and w_2 as decisions variables subject to several constraints obtained from Equations (3)-(5), and utility function $u(w) = w^{\alpha}$ with $0 < \alpha \leq 1$.[11] Additionally, the amount of insurance q can be obtained from Equation (2) in [P1]. More precisely, we solve [P3] where the constraints (3) and (4) in [P1] have been modified to introduce one inequality for each constraint.

[P3]

[11] Note that $0 < \alpha < 1$ corresponds to a risk averse agent. In the Comparative analysis 4 below we also consider the case of risk neutrality, $\alpha = 1$. Moreover, other functional forms of risk averse utility functions are considered in Section 5.

$$\max_{\{w_1,w_2\}} \pi u(w_1) + (1-\pi)u(w_2)$$
subject to:
$$p \cdot w_1 + (1-p) \cdot w_2 = w_0 - p \cdot l,$$
$$w_0 - l - w_1 \leq 0,$$
$$w_0 - p \cdot l - w_1 \geq 0,$$
$$w_0 - p \cdot l - w_2 \leq 0,$$
$$w_0 - w_2 \geq 0,$$
with
$$q = \frac{w_0 - w_2}{p}.$$

Since our objective function is smooth nonlinear,[12] we implement the Generalized Reduced Gradient Nonlinear Optimization Method (GRG Nonlinear) available in Excel Solver. Next, we present a tutorial that first builds a table with the expected utility function, variables, parameters and constraints for the initial stage of the insurance model. And then, we introduce an additional table to perform the comparative static analysis.

(a) Initial Stage Solution

Starting with an Excel worksheet, students have to build a table as Table I in Figure 3, setting up the initial optimization insurance problem of the representative agent. Rows 5 to 9 in column C include standard parameter values that we introduce. There is a probability $\pi = 0.2$ of the robbery being the amount $l = 700$ with initial wealth $w_0 =$ 1,000. The insurance company offers to cover the individual against the robbery at a premium rate of $p = 0.3$. Finally, the parameter of the expected utility function is $\alpha =$ 0.3, implying that the individual is risk adverse.

We introduce the initial values for the decision variables w_1 and w_2 in rows 12 and 13 of column C (we set them at 100 monetary units but we recommend to verify the solution using other initial values to corroborate the optimal solution). Just note that the expected utility function is included in cell C26 while the constraints are introduced in cells C16-C20 without including the equalities or inequalities that appear at the end of them.[13] Finally, the function q is included in row 23 of column C.

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[12] A smooth function is a function that has derivatives of all orders everywhere in its domain.

[13] For example, the restriction *w0-w2≥0* is introduced in C20 just as "= C7-C13". The inequality ≥0 will be included later with Solver as shown in the *Solver Parameters* window in Figure 3.

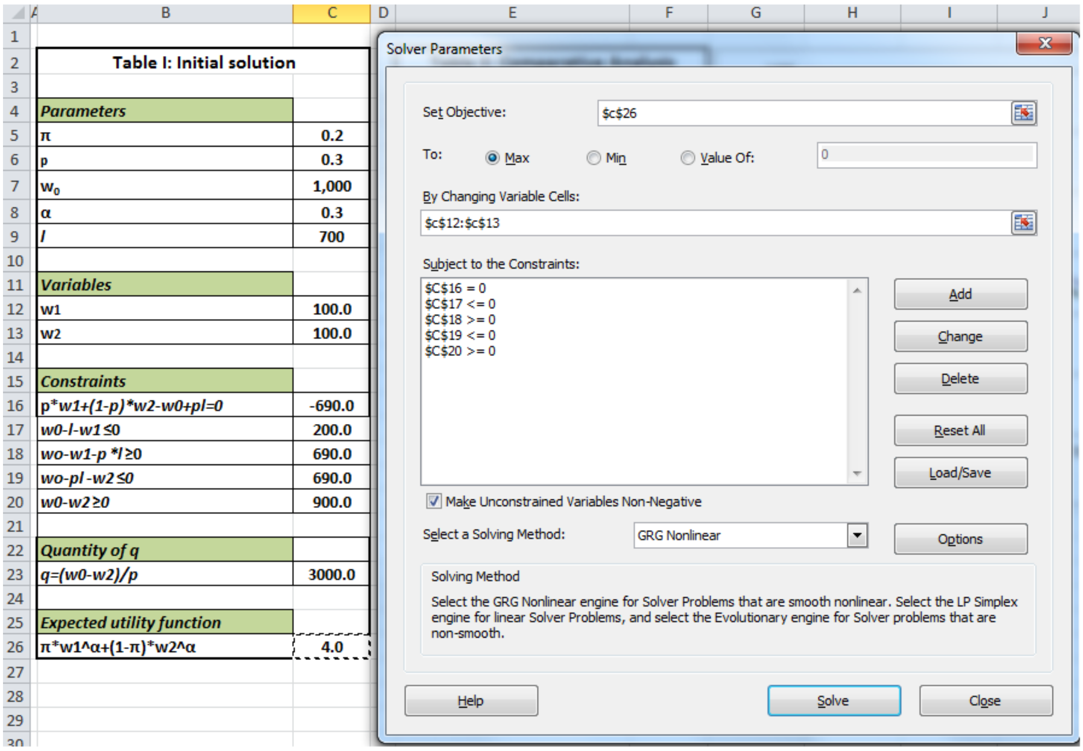

Figure 3: Setting up the problem for the initial solution

Now we can use Solver. Choose Solver from the Data menu in Excel.[14] The *Solver Parameters* window will open. Set the *Objective Cell C26* to the location of the objective function value, which is the expected utility function in our case, select *Max*, and set the *Changing Variable Cells* C12 and C13 to add the decisions variables w_1 and w_2 in Table I. To introduce the constraints in [P3], go to the *Subject to Constraints* box and select Add. The *Add Constraint* window will appear. In this window, we tell the Solver that cell C16 is " = 0", cells C17 and C19 are " ≤ 0" while cells C18 and C20 are " ≥ 0". Then select OK since there are no more constraints to add. You will return to the *Solver Parameters* window as shown in Figure 3. Select GRG Nonlinear as the *Solving Method*.

Once we have defined all the necessary components of the model, click *Solve* in the *Solver Parameters* window. A window will appear telling us that Solver has found a solution. Select Keep Solver Solution and click OK. The insurance decision problem solution is shown in Table I of Figure 4. As you can see, the individual has maximized his expected utility when the agent's wealth spent in both states of the world are $w_1 = 436.0$ and $w_2 = 941.7$. This implies an amount of insurance q of 194.3 monetary units. Also notice that the values of the cells C16-C20 show that the problem satisfies the constraints in [P3].

[14] If the command Solver does not appear in the Data, you can follow the instructions that appear in Excel help and type "Load the Solver Add-in".

	A	B	C
1			
2		**Table I: Initial solution**	
3			
4		***Parameters***	
5		π	**0.2**
6		p	**0.3**
7		w_o	**1,000**
8		α	**0.3**
9		*l*	**700**
10			
11		***Variables***	
12		w1	**436.0**
13		w2	**941.7**
14			
15		***Constraints***	
16		*p*w1+(1-p)*w2-w0+pl=0*	**0.0**
17		*w0-l-w1 ≤0*	**-136.0**
18		*wo-w1-p *l ≥0*	**354.0**
19		*wo-pl -w2 ≤0*	**-151.7**
20		*w0-w2 ≥0*	**58.3**
21			
22		***Quantity of q***	
23		*q=(w0-w2)/p*	**194.3**
24			
25		***Expected utility function***	
26		π*w1^α+(1-π)*w2^α	**7.5**

Figure 4: Finding the initial solution

(b) Comparative Static Analysis

Next, we can carry out some comparative static analyses by modifying the parameters of the model. In order to do that we duplicate all the components from Table I to a new Table II in the same worksheet as shown in Figure 5. To change the optimization problem open Solver and change the *Objective Cell* from C26 to F26 as well as the *Changing Variable Cells* from C12 and C13 to F12 and F13, respectively. Then, go to the *Subject to Constraints* box, select the constraints and click *Change*. The *Change Constraint* window will appear. In this window, we tell Solver that, in each constraint, the letter C must be changed to F. Then select OK and you will return to the Excel sheet as shown in Figure 5. Now, the setting is ready to include the new values of the parameters in the corresponding cells of Table II and compare the new solution we would obtain with the initial one in Table I.

	A	B	C	D	E	F
1						
2		**Table I: Initial solution**			**Table II: Comparative Analysis**	
3						
4		*Parameters*			*Parameters*	
5		π	0.2		π	0.2
6		p	0.3		p	0.3
7		w_0	1,000		w_0	1,000
8		α	0.3		α	0.3
9		*l*	700		*l*	700
10						
11		*Variables*			*Variables*	
12		w1	436.0		w1	436.0
13		w2	941.7		w2	941.7
14						
15		*Constraints*			*Constraints*	
16		*p*w1+(1-p)*w2-w0+pl=0*	0.0		*p*w1+(1-p)*w2-w0+pl=0*	0.0
17		*w0-l-w1≤0*	-136.0		*w0-l-w1≤0*	-136.0
18		*wo-w1-p *l≥0*	354.0		*wo-w1-p *l≥0*	354.0
19		*wo-pl -w2≤0*	-151.7		*wo-pl -w2≤0*	-151.7
20		*w0-w2≥0*	58.3		*w0-w2≥0*	58.3
21						
22		*Quantity of q*			*Quantity of q*	
23		*q=(w0-w2)/p*	194.3		*q=(w0-w2)/p*	194.3
24						
25		*Expected utility function*			*Expected utility function*	
26		π*w1^α+(1-π)*w2^α	7.5		π*w1^α+(1-π)*w2^α	7.5

Figure 5: Setting up the comparative static analysis

Comparative analysis 1: *What happens if the probability of a robbery increases from* $\pi = 0.2$ *to* $\pi = 0.4$*?*

In this case, it is necessary to change only the value of cell F5 from 0.2 to 0.4. Then call Solver again and, click *Solve* in the *Solver Parameters* window.

In Figure 6 we can see, in line with Section 2.1, how the increase in the probability of robbery from $\pi = 0.2$ to $\pi = 0.4$ increases optimal level of wealth w_1 from 436.0 to 790.0 but reduces w_2 from 941.7 to 790.0. This new solution corresponds to an amount of insurance q, which increases from 175 to its maximum level 700 corresponding to the total loss in case of robbery $l = 700$. In other words, the individual has decided to drastically increase the amount of insurance since the probability of robbery has exogenously increased considerably. Notice that the individual is worse off since the level of the utility falls from 7.5 to 7.4.

	B	C	D	E	F
1					
2	**Table I: Initial solution**			**Table II: Comparative Analysis**	
3					
4	***Parameters***			***Parameters***	
5	π	**0.2**		π	**0.4**
6	p	**0.3**		p	**0.3**
7	w_0	**1,000**		w_0	**1,000**
8	α	**0.3**		α	**0.3**
9	*l*	**700**		*l*	**700**
10					
11	***Variables***			***Variables***	
12	w1	**436.0**		w1	**790.0**
13	w2	**941.7**		w2	**790.0**
14					
15	***Constraints***			***Constraints***	
16	*p*w1+(1-p)*w2-w0+pl=0*	**0.0**		*p*w1+(1-p)*w2-w0+pl=0*	**0.0**
17	*w0-l-w1≤0*	**-136.0**		*w0-l-w1≤0*	**-490.0**
18	*wo-w1-p *l≥0*	**354.0**		*wo-w1-p *l≥0*	**0.0**
19	*wo-pl -w2≤0*	**-151.7**		*wo-pl -w2≤0*	**0.0**
20	*w0-w2≥0*	**58.3**		*w0-w2≥0*	**210.0**
21					
22	***Quantity of q***			***Quantity of q***	
23	*q=(w0-w2)/p*	**194.3**		*q=(w0-w2)/p*	**700.0**
24					
25	***Expected utility function***			***Expected utility function***	
26	π*w1^α+(1-π)*w2^α	**7.5**		π*w1^α+(1-π)*w2^α	**7.4**

Figure 6: The effect of an increase in the probability of robbery π

Comparative analysis 2: *What happens if initial wealth increases from* $w_0 = 1{,}000$ *to* $w_0 = 1{,}100$?

Proceeding as in the previous case, it is easy to see in Figure 7 that an increase in the initial wealth from $w_0 = 1{,}000$ in cell C7 to $w_0 = 1{,}100$ in F7 reduces the insurance amount from $q = 194.3$ to $q = 130.3$. In line with Section 2.2, both w_1 and w_2 go up while the amount of insurance decreases. The value of q decreases since the increase in w_0 (from 1,000 to 1,100 in Figure 7) is smaller than the increase in w_2 (from 941.7 to 1,060.9). Finally, note that in this case agent's utility increases since $u'(w) > 0$.

	B	C	D	E	F
1					
2	**Table I: Initial solution**			**Table II: Comparative Analysis**	
3					
4	*Parameters*			*Parameters*	
5	π	0.2		π	0.2
6	p	0.3		p	0.3
7	w_0	1,000		w_0	1,100
8	α	0.3		α	0.3
9	*l*	700		*l*	700
10					
11	*Variables*			*Variables*	
12	w1	436.0		w1	491.2
13	w2	941.7		w2	1060.9
14					
15	*Constraints*			*Constraints*	
16	*p*w1+(1-p)*w2-w0+pl=0*	0.0		*p*w1+(1-p)*w2-w0+pl=0*	0.0
17	*w0-l-w1≤0*	-136.0		*w0-l-w1≤0*	-91.2
18	*wo-w1-p *l≥0*	354.0		*wo-w1-p *l≥0*	398.8
19	*wo-pl -w2≤0*	-151.7		*wo-pl -w2≤0*	-170.9
20	*w0-w2≥0*	58.3		*w0-w2≥0*	39.1
21					
22	*Quantity of q*			*Quantity of q*	
23	*q=(w0-w2)/p*	194.3		*q=(w0-w2)/p*	130.3
24					
25	*Expected utility function*			*Expected utility function*	
26	π*w1^α+(1-π)*w2^α	7.5		π*w1^α+(1-π)*w2^α	7.8

Figure 7: The effect of an increase in the initial wealth w_0

Comparative analysis 3: *What happens if the government regulates the insurance premium to generate an actuarially fair premium?*

Table I: Initial solution		Table II: Comparative Analysis	
Parameters		*Parameters*	
π	0.2	π	0.2
p	0.3	p	0.2
w_0	1,000	w_0	1,000
α	0.3	α	0.3
l	700	*l*	700
Variables		*Variables*	
w1	436.0	w1	860.0
w2	941.7	w2	860.0
Constraints		*Constraints*	
*p*w1+(1-p)*w2-w0+pl=0*	0.0	*p*w1+(1-p)*w2-w0+pl=0*	0.0
w0-l-w1≤0	-136.0	*w0-l-w1≤0*	-560.0
*wo-w1-p *l ≥0*	354.0	*wo-w1-p *l ≥0*	0.0
wo-pl -w2≤0	-151.7	*wo-pl -w2≤0*	0.0
w0-w2≥0	58.3	*w0-w2≥0*	140.0
Quantity of q		*Quantity of q*	
q=(w0-w2)/p	194.3	*q=(w0-w2)/p*	700.0
Expected utility function		*Expected utility function*	
π*w1^α+(1-π)*w2^α	7.5	π*w1^α+(1-π)*w2^α	7.6

Figure 8: The effect of an introducing the actuarially fair premium

This case requires changing $p = 0.3$ in cell C6 to $p = \pi = 0.2$ in F6 and then finding the new and final solution in Table II, Figure 8. In this case, the individual chooses full insurance $q = 700$, which is consistent with the result obtained for the actuarially fair premium presented at the end of Section 2.3.

Comparative analysis 4: *What happens if the probability of robbery increases from* $\pi = 0.2$ *to* $\pi = 0.4$ *when the individual is risk neutral* $u(w) = w$*?*

In this case, we will use the same parameter values as in Comparative analysis 1 (see Figure 6) except for the parameter of the expected utility function α, which will be set equal to one in both tables to change the individual from being risk averse to be risk neutral. First, we set $\alpha = 1$ in cells C8 and F8. Next, to get the solution in Table II go to Solver and click Solve in the *Solver Parameters* window. Finally, to obtain the new initial solution in Table I we need to open the Solver again, change the letter F for C in all cells of the *Solver Parameters* window and, finally, click Solve.

Figure 9 shows the effects of an increase in the probability of robbery when the individual is risk neutral.

As we mentioned in Section 2.4, the natural candidates for the agent's decision are either *q=0* or *q=l*. The former holds in Table I because $\frac{\pi}{1-\pi} < \frac{p}{1-p}$ while the latter takes place in Table II due to the fact that $\frac{\pi}{1-\pi} > \frac{p}{1-p}$.

	B	C	D	E	F
1					
2	**Table I: Initial solution**			**Table II: Comparative Analysis**	
3					
4	*Parameters*			*Parameters*	
5	π	0.2		π	0.4
6	p	0.3		p	0.3
7	w_0	1,000		w_0	1,000
8	α	1		α	1
9	*l*	700		*l*	700
10					
11	*Variables*			*Variables*	
12	w1	300.0		w1	790.0
13	w2	1000.0		w2	790.0
14					
15	*Constraints*			*Constraints*	
16	p*w1+(1-p)*w2-w0+pl=0	0.0		p*w1+(1-p)*w2-w0+pl=0	0.0
17	w0-l-w1≤0	0.0		w0-l-w1≤0	-490.0
18	wo-w1-p *l≥0	490.0		wo-w1-p *l≥0	0.0
19	wo-pl -w2≤0	-210.0		wo-pl -w2≤0	0.0
20	w0-w2≥0	0.0		w0-w2≥0	210.0
21					
22	*Quantity of q*			*Quantity of q*	
23	q=(w0-w2)/p	0.0		q=(w0-w2)/p	700.0
24					
25	*Expected utility function*			*Expected utility function*	
26	π*w1^α+(1-π)*w2^α	860.0		π*w1^α+(1-π)*w2^α	790.0

Figure 9: The effect of an increase in the probability of robbery π under risk neutrality

4. Practice Exercises

We now present additional exercises. To solve them the students can use the Excel worksheet built in the previous section. Note that practice exercises from 1 to 4 below require parameter variations as the comparative analyses in Section 3. Exercises 5 and 6, however, require solving optimization problem [P1] and, therefore, changing the variables and constraints in the Excel worksheet. The objective of this section is twofold: to reinforce the relevance of each one of the parameters in the insurance problem, and to check the capacity of the students to introduce changes to the optimization problem using Excel Solver.

Practice exercise 1

In comparative analysis 1, we saw that when π increases from $\pi = 0.2$ to $= 0.4$, then the amount of insurance q also increases. Check that this fact holds for different values of π. And find the value of π above which the agent demands full insurance.

Practice exercise 2

In comparative analysis 2, we saw that when w_0 increases from $w_0 = 1{,}000$ to $w_0 = 1{,}100$, then the amount of insurance q decreases. Check that this fact holds for different values of initial wealth until you get null insurance. Find the value for w_0 above which null insurance holds.

Practice exercise 3

In comparative analysis 3, we saw that, after decreasing p from $p = 0.3$ to $p = 0.2$, full insurance arises. In fact, there is always full insurance if the actuarially fair premium $p = \pi$ is fixed whatever their values are (see Section 2.3). Check this fact for another $\pi = p$.

Practice exercise 4

In comparative analysis 4, we saw the presence of full insurance when the probability of robbery increases from $\pi = 0.2$ to $\pi = 0.4$ and the individual is risk neutral $u(w) = w$. In fact, it is easy to see that full insurance will hold for any $\pi > p$ and no insurance for any $\pi < p$. Now, for $p = 0.3$, check that this statement holds for one particular value of π strictly bigger than p and for one value of π strictly lower than p.

Practice exercise 5

In Section 3, we used Excel Solver to solve the optimization problem [P2] with the two agent's contingent wealth levels w_1 and w_2 as variables of decisions. With the same parameter values from the initial solution in Section 3, this exercise consists of solving the optimization problem [P1] with insurance amount q as the only decision variable. (*Hint*: To obtain the initial solution, you need to express $w_1(q)$ and $w_2(q)$ as a function of q by using equations (1) and (2). Additionally, you need to modify constraints (3) and (4) in [P1] to introduce one inequality for each restriction.)

Practice exercise 6

Once you have found the initial solution in Practice exercise 5, then, repeat the static comparative analysis carried out in Section 3 and check that the results are identical.

5. An Interactive Excel File with Different Utility Functions

In this section we present an interactive Excel file (available as supplementary material) that allows students to use the comparative static and graphical analysis of the insurance model with different well-known utility functions. We compare demand insurance problems defined by the same parameters but with utility functions that differ in terms of the attitude toward risk. Comparing the insurance decision in those problems allows the student to see the effect of the attitude toward risk on the amount of insurance.

Prior to our analyses we need to define and discuss the Arrow-Pratt coefficients to measure the degree of risk aversion of an agent with utility function u.

5.1 Degree of Risk Aversion

Let u be the utility function. The Arrow-Pratt coefficient of **absolute** risk aversion is defined as $A(w) = -\frac{u''(w)}{u'(w)}$. The Arrow-Pratt coefficient of **relative** risk aversion is defined as $R(w) = wA(w)$. The two coefficients are invariant with respect to the utility representation of agents' preferences, and $R(w)$ does not depend on the units in which income is measured. The coefficients are positive for risk averse agents, zero for risk neutral agents, and negative for risk loving agents. Notice that both coefficients are usually discussed for risk averse agents, as we do in this paper; therefore, both $A(w)$ and $R(w)$ are positive. For risk averse agents, the higher $A(w)$ and $R(w)$, the higher the corresponding degree of risk aversion.

Since $A(w)$ and $R(w)$ are local measures of risk aversion at w, it is interesting to know how wealth affects them. We say that a given utility function u is: (i) Decreasing Absolute Risk Averse (DARA) if $A(w)$ is decreasing in w; (ii) Increasing Absolute Risk Averse (IARA) if $A(w)$ is increasing in w and; (iii) Constant Absolute Risk Averse (CARA) if $A(w)$ is constant in w. In words, DARA requires that individual be less averse to taking small risks at higher level of wealth. Under IARA the greater the wealth, the more averse one become to accepting the same small risk. Finally, CARA means that the degree of risk aversion does not change with the level of wealth. Similarly, we can classify utilities using the Arrow-Pratt coefficient of relative risk aversion.[15]

[15] Formally, we say that a given utility function u is: (i) Decreasing Relative Risk Averse (DRRA) if $R(w)$ is decreasing in w; (ii) is Increasing Relative Risk Averse (IRRA) if $R(w)$ is increasing in w and; (iii) is Constant Relative Risk Averse (CRRA) if $R(w)$ is constant in w.

For example, observe that if $u(w) = w^{\frac{1}{2}}$, then $A(w) = -\frac{u''(w)}{u'(w)} = -\frac{-\frac{1}{4}w^{\frac{-3}{2}}}{\frac{1}{2}w^{\frac{-1}{2}}} = \frac{1}{2}w^{-1} > 0$ and $R(w) = wA(w) = \frac{1}{2}$. Therefore, $u(w)$ is DARA and CRRA.

5.2 The Effect of the Degree of Risk Aversion on the Insurance

To formalize the relationship between the degree of risk aversion *R(w)* and the amount of insurance, we can use Equations (5) to (7) in Section 2 and state the following result: "For agents with utility function u such that *R(w)=a* and $\frac{u\prime(w_1)}{u\prime(w_2)} = \left(\frac{w_2}{w_1}\right)^a$ with *a*>0, an increase in a implies an increase of the amount of insurance *q*."

To show this result, observe that if $\frac{u\prime(w_1)}{u\prime(w_2)} = \left(\frac{w_2}{w_1}\right)^a$ holds and *a*>0, an increase in a requires an increase in $\frac{w_1}{w_2}$ to restore the equality in Equation (6) since no other parameter changes. Following the same argument as in Section 2.1, we can see that w_1 and w_2 must increase and decrease, respectively, and therefore, the amount of insurance *q* increases with respect to a. We next check the result just stated for three types of utility functions differing in *a* that we define in Options 1 to 3. For each option, we additionally simulate one of the comparative static analyses in Section 3.

Option 1: Utility function $u(w) = w^{\alpha}$ *for 0<α<1*

In this option we consider the type of risk averse utility functions used in Section 3. For each given *α*, $u(w) = w^{\alpha}$ is DARA with $A(w) = (1-\alpha)w^{-1}$ and CRRA with *R(w)=1-α*. The above result can be applied for this option by letting *a=1-α*. Thus, we expect a positive relationship between the degree of risk aversion, $(1-\alpha)$, and the amount of insurance *q*.

As an example consider a decrease in the degree of relative risk aversion from $1-\alpha = 0.7$ ($\alpha = 0.3$) to $1-\alpha = 0.5$ $\alpha = 0.5$. Note that the absolute risk aversion coefficient also decreases for any given level of wealth *w*. The student only needs to use the scroll bar below cell H12, adjusting $\alpha = 0.3$ to $\alpha = 0.5$. Then call Solver and click *Solve* in the *Solver Parameters* window. The interactive Excel sheet also presents a graph with the optimal solution. Figure 10 compares these two cases showing that the amount of insurance decreases from $q = 194.3$ with $\alpha = 0.3$ to $q = 50.2$ with $\alpha = 0.5$.

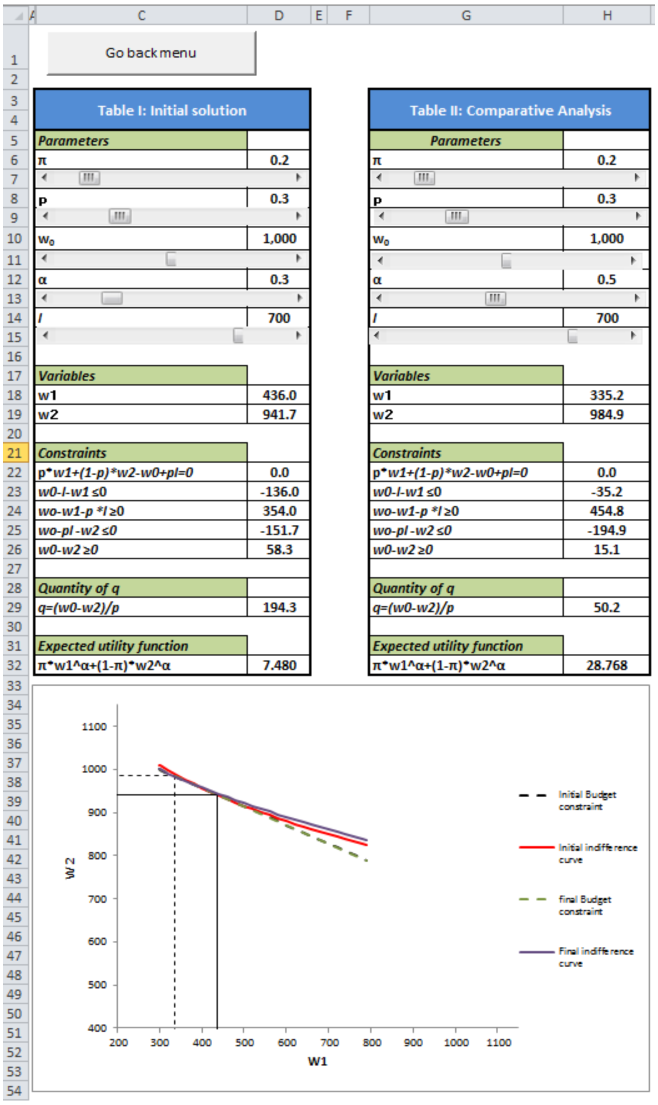

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Table I: Initial solution		Table II: Comparative Analysis	
Parameters		*Parameters*	
π	0.2	π	0.2
p	0.3	p	0.3
w_0	1,000	w_0	1,000
α	0.3	α	0.5
l	700	*l*	700
Variables		*Variables*	
w1	436.0	w1	335.2
w2	941.7	w2	984.9
Constraints		*Constraints*	
*p*w1+(1-p)*w2-w0+pl=0*	0.0	*p*w1+(1-p)*w2-w0+pl=0*	0.0
w0-l-w1 ≤0	-136.0	*w0-l-w1 ≤0*	-35.2
*wo-w1-p *l ≥0*	354.0	*wo-w1-p *l ≥0*	454.8
wo-pl -w2 ≤0	-151.7	*wo-pl -w2 ≤0*	-194.9
w0-w2 ≥0	58.3	*w0-w2 ≥0*	15.1
Quantity of q		*Quantity of q*	
q=(w0-w2)/p	194.3	*q=(w0-w2)/p*	50.2
Expected utility function		*Expected utility function*	
π*w1^α+(1-π)*w2^α	7.480	π*w1^α+(1-π)*w2^α	28.768

Figure 10: The effect of an increase in the utility parameter α

The interactive Excel file can be also used when teaching and solving the comparative static analyses in Section 3. For example, Figure 11 shows Comparative analysis 1. In this case, it is necessary to use the scroll bar below the probability of a robbery π (cell H6), adjusting it from $\pi = 0.2$ to $\pi = 0.4$. Then call Solver and click *Solve* in the *Solver Parameters* window. The interactive Excel sheet also presents a graph with the optimal solution. The initial and final indifference curves, as well as the initial and final budget constraints, appear in the graph.

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Table I: Initial solution	
Parameters	
π	0.2
p	0.3
W_0	1,000
α	0.3
l	700
Variables	
w1	436.0
w2	941.7
Constraints	
*p*w1+(1-p)*w2-w0+pl=0*	0.0
w0-l-w1 ≤0	-136.0
*wo-w1-p *l ≥0*	354.0
wo-pl -w2 ≤0	-151.7
w0-w2 ≥0	58.3
Quantity of q	
q=(w0-w2)/p	194.3
Expected utility function	
π*w1^α+(1-π)*w2^α	7.480

Table II: Comparative Analysis	
Parameters	
π	0.4
p	0.3
W_0	1,000
α	0.3
l	700
Variables	
w1	790.0
w2	790.0
Constraints	
*p*w1+(1-p)*w2-w0+pl=0*	0.0
w0-l-w1 ≤0	-490.0
*wo-w1-p *l ≥0*	0.0
wo-pl -w2 ≤0	0.0
w0-w2 ≥0	210.0
Quantity of q	
q=(w0-w2)/p	700.0
Expected utility function	
π*w1^α+(1-π)*w2^α	7.401

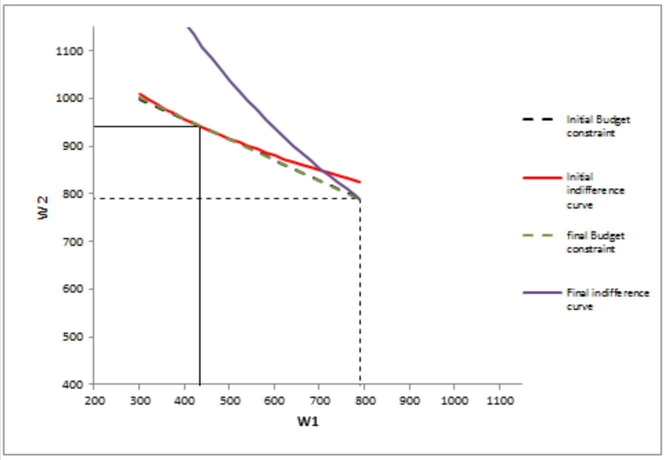

Figure 11: The effect of an increase in the probability of robbery π

Option 2: Utility function $u(w) = ln(w)$

Another type of risk averse agent is the one with logarithm utility function, *u(w)=ln(w)*, which is DARA with $A(w) = w^{-1}$ and CRRA with *R(w)=1*. The result stated at the beginning of Section 5.2 can be applied for this option by letting *a=1*. Thus, fixed all the parameters in the model, we expect a higher amount of insurance *q* for an agent with logarithm utility compared to any agent with utility as in Option 1.

As an example, compare Table I in Figure 10 with Table I in Figure 12 below, where the only difference is the utility function. In the former the utility is defined as in Option 1 where $\alpha = 0.3$ $(1 - \alpha = 0.7)$ and in the latter we have a logarithm utility function. Note that the amount of insurance increases from $q = 194.3$ for utility with $\alpha = 0.3$ to $q = 323.8$ with the logarithm utility.

As previously mentioned, the interactive Excel file can be also used when teaching and solving the comparative static analyses in Section 3. For example, Figure 12 also shows Comparative analysis 1 in Section 4 for this particular utility function. In this case, it is necessary to use the scroll bar below the probability of a robbery π (cell H6), adjusting it from $\pi = 0.2$ to $\pi = 0.4$. Then call Solver and click *Solve* in the *Solver Parameters* window.

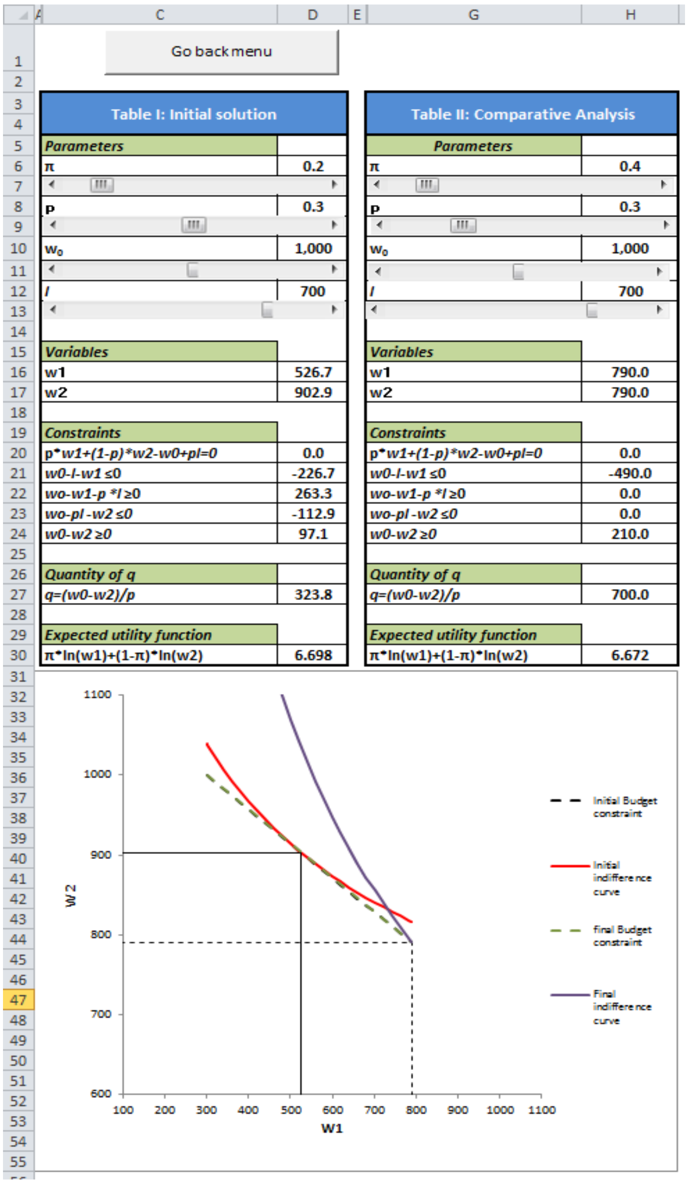

Figure 12: The effect of an increase in the probability of robbery π

Option 3: Utility function $\boldsymbol{u(w) = \frac{w^{1-\sigma}}{1-\sigma}}$ for *σ≠1, σ>0*

In this option, we consider isoelastic utilities with $\sigma>0$ in line with empirical evidence (see Outreville, 2014). This function is DARA with $A(w) = \sigma w^{-1}$ and CRRA with $R(w) = \sigma$. The result stated at the beginning of Section 5.2 can be applied to compare on the one hand, any pair of isoelastic utilities, and on the other hand any isoelastic utility with a utility either in Option 1 or 2. In any case, we expect a positive relationship between the degree of risk aversion and the amount of insurance q, meaning that utilities with higher $R(w)$ will yield higher levels of insurance.

As an example, Figure 13 displays a comparative static analysis adjusting the scroll bar below parameter σ from 0.3 (cell D12) to 1.4 (cell H12), meaning that the agent becomes more risk averse. Next, call Solver and click *Solve* in the *Solver Parameters* window. In this case the amount of insurance increases from $q = 0$ to $q = 420.8$.

Note that any utility function with $\sigma<1$ is equivalent to the utility with $\alpha = 1 - \sigma$ in Option 1. Thus, the decision variables are equal in both cases. Students can check it by setting $\alpha = 0.7$ in Option 1 and comparing the values of w_1, w_2, q with the ones in Table I, Figure 13.

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Table I: Initial solution	
Parameters	
π	0.2
p	0.3
w_0	1,000
σ	0.3
l	700
Variables	
w1	300.0
w2	1000.0
Constraints	
*p*w1+(1-p)*w2-w0+pl=0*	0.0
w0-l-w1 ≤0	0.0
*wo-w1-p *l ≥0*	490.0
wo-pl -w2 ≤0	-210.0
w0-w2 ≥0	0.0
Quantity of q	
q=(w0-w2)/p	0.0
Expected utility function	
π*(w1^(1-σ)/(1-σ))+(1-π)*(w2^(1-σ)/(1-σ)	159.4

Table II: Comparative Analysis	
Parameters	
π	0.2
p	0.3
w_0	1,000
σ	1.4
l	700
Variables	
w1	594.6
w2	873.8
Constraints	
*p*w1+(1-p)*w2-w0+pl=0*	0.0
w0-l-w1 ≤0	-294.6
*wo-w1-p *l ≥0*	195.4
wo-pl -w2 ≤0	-83.8
w0-w2 ≥0	126.2
Quantity of q	
q=(w0-w2)/p	420.8
Expected utility function	
π*(w1^(1-σ)/(1-σ))+(1-π)*(w2^(1-σ)/(1-σ)	-0.2

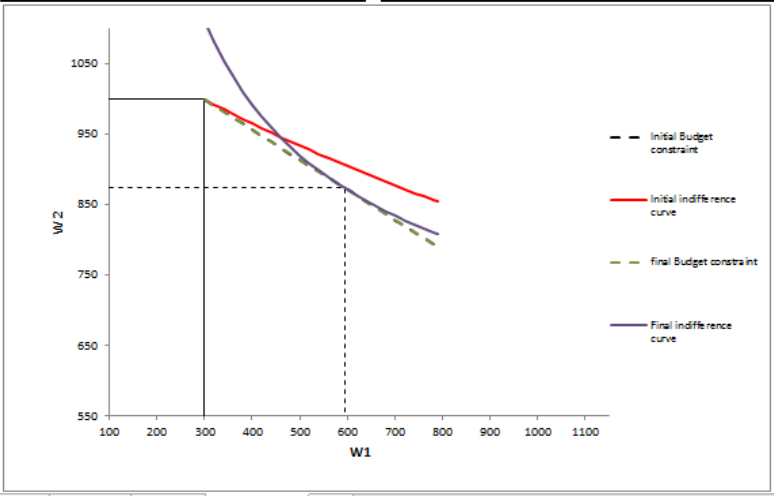


Figure 13: The effect of an increase in the utility parameter σ

6. Concluding Remarks

This paper presents a tutorial exercise for advanced undergraduate students in order to explore the main features of the insurance demand problem using Excel Solver. We first introduced the model and analyzed it mathematically and graphically. Then, we solved the model numerically using Excel Solver and carried out some comparative static analyses by modifying its parameters. These exercises allow students to improve their understanding of what happens to the amount of insurance when the initial wealth and the probability of robbery change. We also analyzed the implications and intuitions for each exercise and get them back to the analytical solutions.

We also presented an interactive Excel file (available as supplementary material) that permits students to use the comparative static and graphical analysis of the insurance model with different well-known utility functions that differ in terms of the attitude toward risk. Comparing the insurance decision in those problems allows students, for example, to see the effect of the attitude toward risk on the amount of insurance.

This paper targets advanced undergraduate students with theoretical background on choice under uncertainty. However, we have included an Appendix that presents an introductory theoretical background on choice under uncertainty for those students not familiar with the topic.

Although we used a traditional classroom instruction approach, alternative instructional strategies, like flipped classroom, problem-based learning, etc., can also be applied to present the model and methodology in this document. However, this was not the objective of this work.

Finally, several exercises for further analyses would be of interest for students. For example, a deeper analysis would consist on incorporating the firm's decision problem and therefore put together the two sides of the market.

REFERENCES

1. Barreto, H. (2009). *Intermediate Microeconomics with Microsoft Excel*. Cambridge University Press. New York, USA.
2. Barreto, H. (2015). "Why Excel?". *The Journal of Economic Education*, 46(3), pp. 300-309.
3. Benninga, S. (2010*). Principles of Finance with Excel*. New York: Oxford University Press.
4. Cahill, M. and Kosicki, G. (2000). "Exploring Economics Models Using Excel". *Southern Economic Journal*, 66(3), pp. 770-792.
5. Gravelle, H. and Rees, R. (2004). *Microeconomics*. 3rd Edition, Prentice Hall.
6. Houston, J. (1997) "Economic Optimisation using Excel's Solver: A Macroeconomic Application and Some Comments Regarding Its Limitations". *Computers in Higher Education Economics Review*, 11(1), pp. 2-5.
7. Jehle, G.A., and Reny, P.J. (2011). *Advanced Microeconomic Theory*. 3rd Edition, Prentice Hall.
8. Kirstein, R. (2000). "Risk Neutrality and Strategic Insurance". *The Geneva Papers on Risk and Insurance*, 25(2): pp. 251-261.
9. MacDonald, Z. (1996) "Economic Optimisation: An Excel Alternative to Estelle et al.'s GAMS Approach". *Computers in Higher Education Economics Review*, 10(3): pp. 2-5.

10. Mas-Colell, A., Whinston, M.D., and Green, J.R. (1995). *Microeconomic Theory*. Oxford University Press.
11. Naevdal, E. (2003). "Solving Continuous-Time Optimal-Control Problems with a Spreadsheet". *The Journal of Economic Education*, 34(2), pp. 99-122.
12. Outreville, J. F. (2014). "Risk Aversion, Risk Behavior, and Demand for Insurance: A Survey". *Journal of Insurance Issues*, 37 (2), pp. 158-186.
13. Pindyck, R.S., D. Rubinfeld (2018). "Microeconomics". Pearson, 9th edition.
14. Silva, J. And Xabadia, A. (2013) "Teaching the two-period consumer choice model with Excel Solver". *Australasian Journal of Economics Education*, 10(2), pp. 24-38.
15. Strulik, H. (2004). "Solving Rational Expectations Models Using Excel". *The Journal of Economic Education*, 35(3), pp. 269-283.
16. Tohamy, Soumaya M., Mixon, J. Wilson, Jr. (2004). "Illustrating Consumer Theory with the CES Utility Function". The Journal of Economic Education, 35 (3), pp. 251-258.

APPENDIX. Choice under Uncertainty: Basic Concepts

The definitions introduced in this section are basic for many problems with uncertainty, in particular, for the analysis of the insurance demand problem we propose in this paper. These relevant concepts can be presented in two hours. The main concepts presented can be found in chapters devoted to choice under uncertainty in microeconomics textbooks, we present them for completeness and to make the paper self-contained.[16]

Choice under uncertainty: The expected utility theory

Many important decisions of agents involve choices with uncertain consequences. For example, to buy insurance to cover the risk for home flooding, to decide the investment in a portfolio containing risky and non-risky assets, etc. In this exercise we concentrate on the insurance model as an application. To handle this problem we need to know the fundamentals of expected utility theory of the market for risk. Expected utility theory extends the model of consumer theory with certainty to choices over risky outcomes. Choices under uncertainty mean that the agent (decision maker) faces a choice among a number of risky alternatives. Each of them may result in one of a number of possible outcomes or payoffs. Which outcome occurs is uncertain at the time of choice. That is people choose among "bundles" that have uncertain payoffs. The uncertainty is associated with different states of the world that can eventually happen, each of which yields a particular outcome or payoff. Preferences are made over "lotteries" instead of consumption bundles, as in deterministic utility theory.

Formally, here is the model. Let W be the set of possible outcomes in a consumer decision problem. The set W could be commodity bundles, as in the classical consumer choice problem with certainty, or amount of wealth as in an insurance decision problem, or monetary payoffs as in the portfolio selection problem. From now on, we focus on wealth, with positive real values, to cope with our insurance problem.

We consider a lottery which specifies a finite set of outcomes $W = (w_1, \dots, w_n)$ and a probability distribution on W, say $P = (p_1, \dots, p_n)$. Formally, we define a *lottery* as $L = (P, W) = (p_1, \dots, p_n; w_1, \dots, w_n)$ with $p_j > 0$ for all $j \in \{1, \dots, n\}$, and $\sum_{j=1}^{n} p_j = 1$, where p_j is the probability of outcome j occurring (equivalently, the probability that state of the world j occurs), and where outcome j provides a monetary payoff w_j. The *expected value of the lottery* L is $E(L) = \sum_{j=1}^{n} p_j\, w_j$.

[16]See, for example, Pindyck and Rubinfeld (2018) for a basic description of the concepts, and Jehle and Reny (2011) and Mas-Colell, Whinston, and Green (1995) for an advanced and more extensive analysis.

We assume that each agent has preferences on wealth that are represented by a utility function: Any outcome $w \in W$ has a utility level $u(w)$, and for the analysis we want to carry out, it is useful to assume that $u(w)$ is at least twice differentiable at all wealth levels, where $u'(w) > 0$ and $u''(w) < 0$.

The utility function over lotteries has the *expected utility property*. Formally, for each lottery $L = (P, W) = (p_1, \dots, p_n; w_1, \dots, w_n)$ its utility is $U(L) = \sum_{j=1}^{n} p_j\, u(w_j)$. That is, the utility assigned to a lottery is the weighted average of the utilities $u(w_j)$ of the n outcomes, weighted by the outcomes' probability. Then, we say that a lottery L_1 is strictly preferred to a lottery L_2 if the utility assigned to L_1 is strictly higher than the utility assigned to L_2.

Risk aversion

Agents' decisions under uncertainty depend on their attitude towards risk. To define risk aversion we compare the utility assigned to a lottery L, $U(L)$ (that is, the expected value of the utilities of outcomes) with the utility assigned to the lottery's expected value, say $U(E(L))$, where the lottery $E(L)$ can be thought of receiving the value $E(L)$ with certainty, with probability 1, thus, $U\big(E(L)\big) = u\left(\sum_{j=1}^{n} p_j\, w_j\right)$.

We say that an agent is *risk averse* if $U(L) < U\big(E(L)\big)$ holds for each lottery L; the agent is *risk loving* if $U(L) > U\big(E(L)\big)$ for each lottery L, and the agent is *risk neutral* in case of equality. In words, for a risk averse agent, the expected utility assigned to the lottery is less than the utility assigned to its expected wealth. The contrary applies to the risk loving agent.

Each one of the three attitudes towards risk is related to the shape of the agent's utility function on wealth u. The utility function of a risk averse agent is strictly concave, that is, $u''(w) < 0$. In this case wealth has diminishing marginal utility, and losses are more costly than gains in terms of utility (see Figure 1 below). A risk loving agent has a strictly convex utility function, that is, $u''(w) > 0$, while a risk neutral agent has a linear utility function ($u''(w) = 0$). In the following Example 1 we show the crucial concepts defined till now and in Figure 1 we depict the case of a risk averse agent.

<u>*EXAMPLE 1*</u>: Consider a lottery defined by the following probability distribution and outcomes: $L = (P, W) = (0.5, 0.5; 2{,}16)$. The expected value of this lottery is 9. Suppose that the agent's utility function on wealth is $u(w) = w^{\frac{1}{2}}$. Then, $U(L) < U\big(E(L)\big)$ as we check below:

$$E(L) = \sum_{j=1}^{n} p_j\, w_j = 0.5 \cdot 2 + 0.5 \cdot 16 = 9,$$

$$U(L) = \sum_{j=1}^{n} p_j\, u(w_j) = 0.5 \cdot u(2) + 0.5 \cdot u(16) = 2.7071, \text{ and}$$

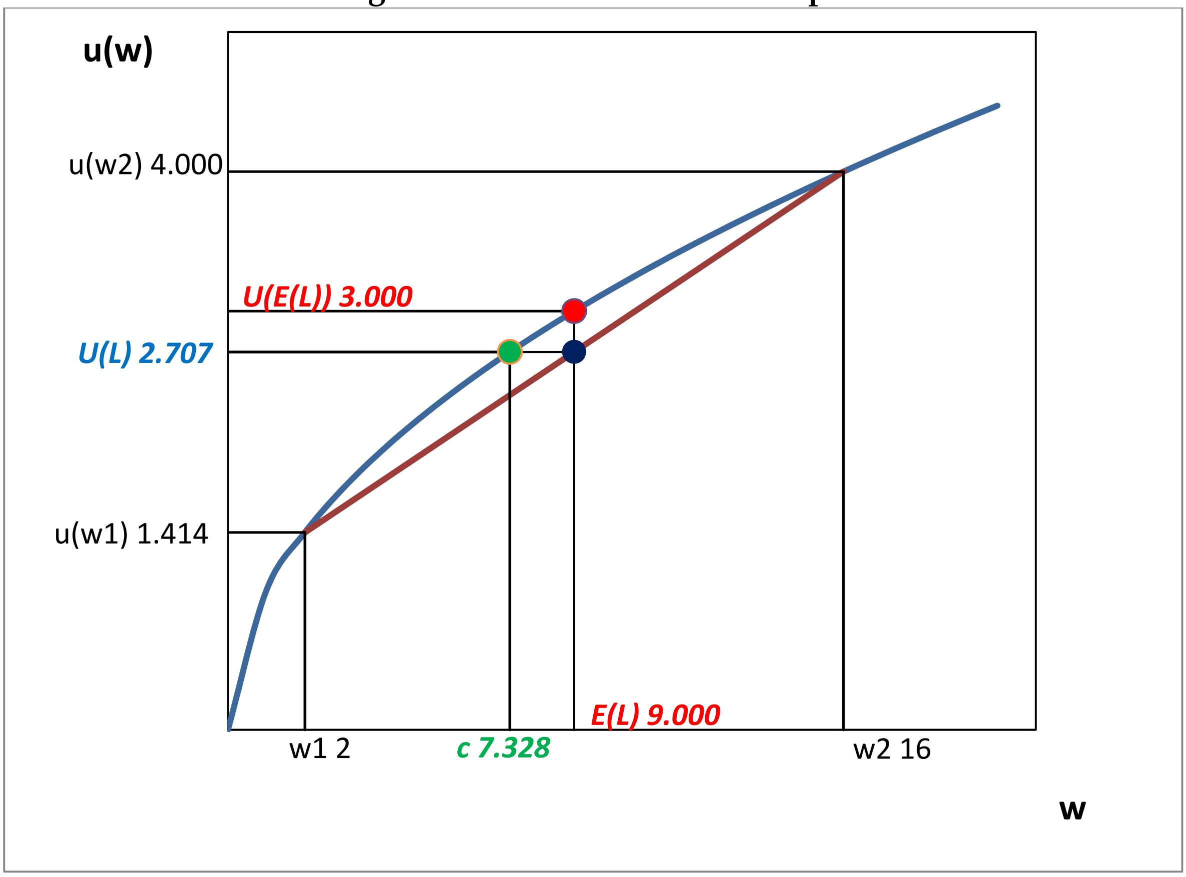
$$U(E(L)) = u\left(\sum_{j=1}^{n} p_j\, w_j\right) = 9^{\frac{1}{2}} = 3.$$

We could check that the same inequality holds for each lottery, and thus, affirm that the agent with $u(w) = w^{\frac{1}{2}}$ is risk averse. Note that to check risk aversion we only need to prove that $u''(w) < 0$ ($u'(w) = \frac{1}{2} w^{\frac{-1}{2}}$ and $u''(w) = -\frac{1}{4} w^{\frac{-3}{2}} < 0$).

Notice also that if the utility is $u(w) = w$, the agent is risk neutral, and for this particular lottery L, the following holds: $U(L) = U(E(L)) = 9$. Moreover, if we consider $u(w) = w^2$, then the agent is risk loving and for the particular lottery L its utility level is $U(L) = 130 > U(E(L)) = 81$.

The interactive graph in our Excel file (Option 4) allows the student to modify the parameters of the lottery as well as the degree of attitude towards risk and, therefore, shows what happens to the utility function under different scenarios, including the ones where individuals are risk loving or risk neutral. See an example in Figure A.

Figure A: Risk aversion in Example 1



Certainty equivalent and risk premium

For the analysis of agents' attitude towards risk, the following concepts are useful. The *certainty equivalent* of a lottery L, denoted as $CE(L)$, is the amount of wealth for which the agent is indifferent between choosing the lottery L and receiving that certainty amount. Formally:

$$U(L) = U\big(CE(L)\big).$$

In words, is the amount of money which, if received for certain would be regarded by the agent as just as good as the lottery L.

Each one of the three attitudes towards risk is related with some relationship between $CE(L)$ and $E(L)$. If $CE(L) = E(L)$ the agent values the lottery as its expected value and it represents risk neutral agents. If $CE(L) < E(L)$ the agent values the lottery less than its expected values and the agent is risk averse. Otherwise, if $CE(L) > E(L)$ the agent values the lottery more than its expected values and the agent is risk loving.

A risk averse agent will sacrifice some positive amount of wealth to avoid the lottery´s inherent risk. The *risk premium* ($r(L)$) is the difference between the lottery's expected value and the certainty equivalent. Therefore, the risk premium determines the amount of wealth the risk averse agent will sacrifice to avoid the risk. Formally,

$$r(L) = E(L) - CE(L),$$
$$U(L) = U(E(L) - r(L)).$$

Again, each one of the three attitudes towards risk is related with the sign of $r(L)$. If $r(L) = 0$, the agent is not willing to give up/sacrifice any amount of wealth to avoid the (risky) lottery, and thus, it represents a risk neutral agent. If $r(L) > 0$, the agent is willing to give up/sacrifice a positive amount of wealth to avoid the (risky) lottery, and thus, it represents a risk averse agent. Otherwise, if $r(L) < 0$, the agent has to be paid to avoid the lottery, this represents a risk loving agent.

Going back to Example 1, notice that the amount of wealth $c = 7.328$ in Figure A is the certainty equivalent of the lottery L. Moreover, the horizontal distance from c to $E(L)$, that is, $E(L) - c = 9 - 7.328 = 1.672$ is the risk premium.