Stephan Russenschuck

Field Computation for Accelerator Magnets

Analytical and Numerical Methods for Electromagnetic Design and Optimization

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