COORDINATION OF INVENTORY DISTRIBUTION & PRICE MARKDOWNS FOR CLEARANCE SALES AT ZARA

By

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B.S. Industrial Engineering
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Submitted to the MIT Sloan School of Management and
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ABSTRACT

There is an essential need in the retail industry, of integrating inventory planning and pricing strategies. In the fast-fashion world of retail, inventory is treated as a perishable item leading to short selling periods. It is a common practice for retailers to liquidate unsold merchandise via clearance markdown policies. Joint marketing and production decisions are important and challenging in retailing. Clearance sales depend on the pricing, seasonal effects, and the assortment of goods available to the customer. Errors in inventory distribution and clearance pricing result in loss of potential revenue or excess inventory to be salvaged.

In the case of Spanish-based retailer Zara, thirteen percent of annual revenues are attributed to clearance sales. To maximize these revenues a supply chain tool is designed to facilitate the inventory distribution decisions for the clearance season while considering price markdowns. A two part linear optimization model considers the demand forecast, pricing decisions, and logistic costs in determining the allocation of excess inventory.

The business case is very similar to other retailers where revenues need to be maximized. However, Zara's business model and vertically integrated supply chain makes this case very unique. In a forecast error comparison test, the proposed solution improved the forecast error from 8 to 4 percent in respect to the current forecast process.

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I would like to extend my sincere appreciation to my advisor Professor Felipe Caro who mentored and encouraged me throughout the project. It was an honor working side by side with him and I will forever by grateful. For Professor Jérémie Gallien who inspired me take a supply chain adventure in fast fashion. To Professor Chris Caplice for reminding me that business solutions are often practical and simple.

For my family and friends who have helped guide me throughout my life and whose love provides fuel for my soul.

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1 Introduction

There is an essential need in the retail industry, of integrating inventory planning and pricing strategies. In the fast-fashion world of retail, inventory is treated as a perishable item leading to short selling periods in which "inventory and pricing strategies are central to success."[1] It is a common practice for retailers to liquidate unsold goods via clearance pricing[2]. From 1925-1965 average markdowns were stable at 6% but by the mid-1990's they had grown to over 26%[3].

The forefront of the problem lies with the demand uncertainty followed by discount decisions made dynamically. Mantrala and Rao (2001), state that in the 1990's there had been an explosion of product variety on the supply side and consumer tastes had diversified on the demand side increasing demand volatility. With excess inventory on hand, retailers focus on maximizing revenues by improving clearance markdown policies, which has caused an increase in dynamic pricing strategies in recent years to facilitate these needs[4].

Integrating marketing and inventory control was first proposed by Whitin in 1955 and has since received broad attention in the marketing, economics and inventory management literature[5]. In 1991, for example, Eliasberg and Steinberg designed an integrated joint marketing production decision model. There has also been analysis of pricing strategies in respect to clearance markdowns found in (Gallego & van Ryzin, 1994) as well as (Feng & Gallego, 1995) among others.

The following will discuss a case study on Zara, a Spanish based fast-fashion retailer. It will address a business case, a proposed solution, which will include a demand forecast and a two-part optimization model, and an analysis of the findings with further recommendations. The business case is very similar to other retailers where revenues need to be maximized. However, Zara's business model and vertically integrated supply chain makes this case very unique. The proposed solution suggests a more sophisticated methodology in coordinating inventory distribution and discount decisions than the current business policy. In a comparison test for forecast accuracy the proposed solution improved the current business process by four percentage points.

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1 (Bitran, Caldentey, & Mondschein, 1998)

2 (Wang & Webster, 2009)

3 (Mantrala & Rao, 2001)

4 (Keskinocak & Elmaghraby, 2003)

5 (Smith & Achabal, Clearance Pricing and Inventory Policies for Retail Chains, 1998)

1.1 Overview of Problem Statement

End-of-season clearance sales is a strategy retailers use to liquidate excess inventory. Each year Inditex S.A. sells less than twenty percent of regular season inventory during clearance sales shown in Figure 1. Within the first seven days, thirty percent of this inventory is sold. Inditex S.A. is one of the largest fashion distributers in the world with revenues of €10.4 billion[6]. Zara is Inditex's largest of eight commercial brands and accounts for 65.5% of the total contribution. In 2008, thirteen percent of Zara's revenues were generated during the clearance season. The objective of this project is to maximize clearance profits. In order to facilitate this goal, merchandise should be distributed to stores with the highest potential of sale and employ the proper discount price.

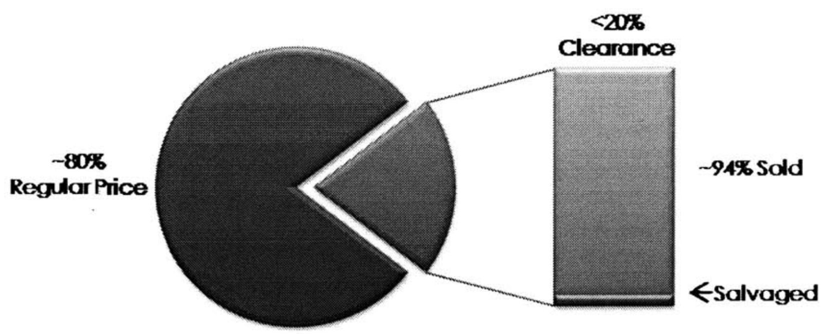

Figure 1: Inventory Distribution

One of the biggest challenges Zara faces in preparation for the clearance season is determining how to distribute 11,000 different fashion designs to over 1,200 stores worldwide. To accommodate for limited store capacity, shipments of clearance inventory must begin one month before the start of the sale. Once the sale begins, the ability to react and transfer merchandise is limited.

In 2007, Professor Jérémie Gallien, Professor Felipe Caro and Rodolfo Carboni developed a methodology for Zara to determine the price discounts that should be applied to each fashion design during the clearance season[7]. This model assumed that the physical distribution of the inventory was already at each store. The purpose of this project is to develop a methodology that will optimize store merchandising during the clearance season to later facilitate pricing decisions in which the combination of the two would lead to maximizing revenues. In other words, each store would have

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6 (Grupo Inditex 2008 Annual Report)

7 (Carboni, 2008)

the right merchandise, at the right time, with the right pricing and gain higher profits during the clearance season.

1.2 Project Methodology

The approach of the project was composed of two parts: determining a demand forecast for end-of-season and clearance season sales and developing an optimization model prototype that coordinates pricing decisions with store merchandising during the clearance season.

The project began by trying to improve the current clearance pricing demand forecast model developed by Caro and Gallien. This forecast considers the size of the purchase, age, demand rate of the previous period, broken assortment, and price discount as determining factors. The model was adjusted to develop an end-of-season demand forecast. The main adjustment was removing demand elasticity as the price remains steady during the regular season. Next various tests were conducted that determined that the elasticity of demand or the price discount variable was the most sensitive in estimating the demand during the clearance season. Therefore various factors were tested to try to improve the elasticity of demand such as unemployment rate, store to city population ratio, and price demand elasticity of regular season discounts. Though all of these variables and experiments seemed promising, it was concluded that the current model's prediction is the most favorable for both end-of-season and clearance season sales.

Next, two optimization models were developed to determine end-of-season replenishments as well as clearance inventory distribution. The first optimization model used the input from the demand forecasting model to decide how much merchandise to ship to each of the 72 counties to maximize global revenues. Shipment costs from the central distribution centers to each country were taken into account as well as the cost of transshipment between each distribution center. The second optimization model used the output from the first model to disaggregate the country shipments to the store and article level while minimizing costs. In this model, shipment costs to the stores from the distribution centers, country warehouses, and other stores were also considered. The advantages of the model are that it uses a demand forecast, considers discount prices and costs, and determines shipments to a store at a fashion design level. Figure 2 illustrates an overview of the project approach.

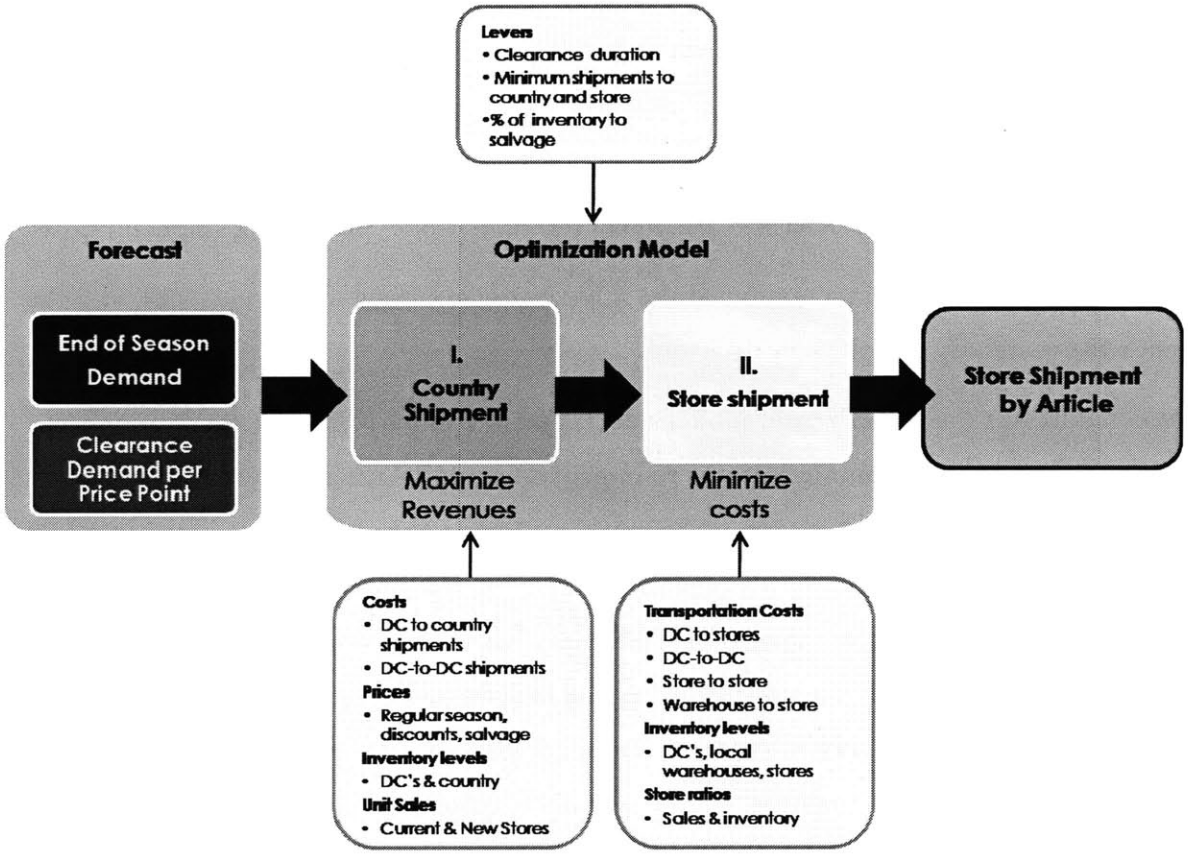

Figure 2: Project Overview

Lastly, a comparison of the current merchandising process with the proposed model will be discussed. An extensive analysis of the model will be given along with the motivation of the decisions of the proposed model. Recommendations on how to improve the model and how to utilize its methodology in future research will also be discussed.

1.3 Chapter Outline

This thesis is divided into seven chapters as discussed below.

Chapter 2 provides company background and introduces the clearance season and its key components.

Chapter 3 explains the current forecasting and inventory distribution process for the clearance season as well as the description of the proposed model.

Chapter 4 describes the pricing demand forecasting model and outlines the various hypothesis tests conducted to improve and modify the forecast to utilize in inventory planning and pricing decisions.

Chapter 5 outlines the two linear optimization models and its formulations that work in conjunction with a demand forecast to maximize revenues while minimizing shipment costs.

Chapter 6 discusses the implementation of the integer programming model in AMPL and the present's the inventory distribution decisions.

Chapter 7 addresses the optimization model's decisions in comparison with the current processes and provides recommendations to improve the model use it in future research.

2 Company Background

In 1975, Amancio Ortega Gaona opened the first Zara store in La Coruna, Spain. Zara provided high quality designer clothing at reasonable prices and quickly gained customer popularity. By 1989, there were 82 stores in Spain with international presence in Portugal, Paris, and New York City. Inditex Group, Zara's parent company, became a publically traded company in 2001 and was the third largest clothing retailer[8]. Today, Inditex is the world's largest fast fashion retailer by sales, overtaking GAP Inc. in 2008. Inditex has eight commercial formats including Zara, Pull & Bear, Massimo Dutti, Bershka, Stradivarius, Oysho, Zara Home and its newest brand Uterqüe. In 2008, Inditex had 89,122 employees in 72 countries with 4,264 stores and over 140 nationalities[9].

Zara is the flagship chain for Inditex with €6.8 billion in revenues. It is known for its innovative trends in fashion and rapid response to market demand. The key to Zara's success and rapid growth has been due to its virtually integrated supply chain that includes textile sourcing, design, production, distribution and retail. Zara designers look to luxury fashion designers for inspiration for new trends and work together with the fashion savvy regional sales teams to respond to customer demands. The supply chain flexibility, from procurement to distribution, enables new fashion trends to reach stores on a weekly basis. For example, the logistic team is able to take orders from a store and deliver the merchandise within 24 hours for European stores and within 48 hours to American or Asian stores.

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[8] (Fraiman, Medini, Arrington, & Paris, 2002)

[9] (Grupo Inditex 2008 Annual Report)

In the past decade, Zara has grown from 507 stores in 39 countries[10] to over 1300 stores in 73 countries tripling its revenues. To thrive with the large expansion, Zara expanded its network of vendors to other European and Asian suppliers. With the rapid growth Zara was forced to rescale their supply chain and improve their current process to maintain their competitive advantage. Figure 3 is a picture of one of the Zara stores located in Greece.

Figure 3: Zara Store in Greece

To facilitate growth strategies, in 2006, Zara began a partnership with the Leaders for Global Operations (LGO) Program at MIT to develop and incorporate novel forecasting, inventory, pricing and distribution models to their supply chain. Zara stores have three departments Women´s, Men´s and Kids´. Zara Woman represents sixty percent of Zara's revenues and hence has been selected to pilot new supply chain tools, including those presented here. Past projects with MIT included regular season replenishment and clearance pricing. The project described here combines merchandising, replenishment and pricing in an optimization that will maximize clearance season revenues.

2.1 Sales Campaign and Regular Season

Zara divides its design collections into two campaigns each year the winter campaign and the summer campaign. Each campaign lasts approximately six months and at the end of each campaign a clearance season is followed that serves as a transition from one campaign to the next. During the first two weeks of the clearance season the stores position themselves to focus primarily on clearance sales in order to liquidate the old trends. As the clearance season progresses, less and less of the

[10] (Grupo Inditex 2001 Annual Report)

store´s real estate is dedicated to the clearance items. The sale signs in the display windows come down and the new campaign designs are displayed. The clearance season ranges from two to eight weeks and varies in start dates by country and store.

2.2 Organization Team

There are three groups that collaborate to determine the distribution of inventory during the clearance season. The first group is the Performance Management Team (PMT) who estimates the expected sales of each store in both revenues and unit sales. They are responsible for maintaining and updating the store shipment schedule each week.

The second team involved in the inventory distribution process is the distribution group. They forward the schedule of shipments to the distribution centers (DC) to create work orders. Because the schedule specifies number of units for each store it is up to the DC to pick and choose the stock keeping units (SKUs) to distribute to each store. A SKU is a particular size and color of an article.

Lastly, commercial managers are responsible for store sales and are the market experts in the countries and regions they manage. They are in contact with store managers on a daily basis and have familiarity with store inventory needs. Once the schedule of shipments is released, they work with the PMT and the distribution group to make any adjustments to the schedule. Commercial managers can also block shipments of certain trends to a given region. A common example is blocking the distribution of heavy coats to the Middle East during the summer months. Due to hot weather conditions, customers are less likely to buy coats even at a discounted price.

2.3 Product Lines

The Women´s Department is separated into six product lines that have a team of designers and buyers who are responsible for the quality and success of the designs. The product lines vary in price, design, and fit. Some for example have more trendy designs while others have basic trends for casual use. Within each product line there are groups that further categorize each segment such as dresses or pants.

In each group there are references or articles, which are design trends that are categorized by a model and textile quality. For example, a reference such as a denim jacket would be under the outerwear group for product line A.

2.4 Regular Season Merchandising and Distribution

Merchandising decisions are essentially investment decisions made by retail managers[11]. From a marketing prospective, there are many advantages to offering a wide assortment of products. Van Ryzin and Mahajan suggest that having such an assortment will increase the likelihood a customer will purchase something from that assortment (1999). Merchandising is centralized at Zara from inventory distribution to store merchandising.

During the regular season, Zara's inventory distribution philosophy uses a forecast to predict the following week's sales based on historic sales. A demand forecast and optimization model created by Gallien and Caro is used to maximize revenue for a reference and determines how to allocate the inventory to each store[12]. Zara's information technology system also enables store managers to place any additional merchandise orders from a personal digital assistant (PDA). m

Figure 4 : Zara Merchandizing

Store merchandising is fundamental to Zara's unique customer experience. At headquarters, Zara has mock stores where visual merchandisers design store layouts including window and in-store displays as well as a design assortment to facilitate a standard Zara appearance worldwide. Store dynamics includes a very detailed process to ensure a well assorted collection of fashion designs shown in Figure 4. This includes a strict rule on the number of sizes that can be displayed at a time for a given article and a quick replenishment process once an item is sold.

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[11] (Sweeney, 1973)

[12] (Correa, 2007)

2.5 Clearance Season

Zara approached Professors Gallien and Caro to assist them once again in an optimization model but this time for clearance season sales. The approach taken by the professors was to develop a two-step process. The first was to establish a pricing policy to ensure articles during the clearance season were priced in a manner that maximized revenues. Zara's current pricing policy is managed by the Pricing Team and country managers. Each week during the clearance season the sales team reviews its sales at the current price and based on experience and instinct make pricing decisions. In 2007, LGO Rodolfo Carboni in conjunction with Gallien and Caro worked on an optimization model that would facilitate the pricing decisions[13].

The second step, which will be discussed here, is ensuring stores receive the proper merchandise for the clearance season. The combination of having stores with the right merchandise and priced at the right levels would facilitate Zara's goal of maximizing clearance revenues. Similar methodologies are used in both projects purposely in order to facilitate integration.

3 Clearance Distribution Problem

One month before the start of the clearance sale the Performance Management Team estimates store sales for the remainder of the regular season merchandise. This includes sales from both the last month of regular season and those accumulated from the clearance season. With this estimate a store shipment schedule is created and is used by the business as the main distribution plan. This chapter discusses the clearance season in more detail focusing on the current inventory distribution decision process. The process includes the existing clearance forecasting method as well as the allocation of goods to each store.

3.1 Legacy Clearance Forecasting

The current forecasting method used by the PMT is based on data from similar campaigns and seasons. Therefore, when estimating sales for the current winter clearance season the previous year's winter clearance sales data is used. The forecast is calculated by taking into account the sales from the previous year and the current sales trend for that campaign and season. These estimations are in both revenues and unit sales.

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[13] (Carboni, 2008)

The forecasting method ensures all merchandise in the DC's is distributed to the stores. Therefore, stores are expected to make an effort to sell the excess inventory. A load factor that proportionately allocates the inventory is used to calculate the expected effort of the store. The load factor considers the inventory levels in the network including country warehouse, distribution centers, and stores. If there is sufficient inventory in the store to cover the estimated sales, the store will be blocked from additional shipments. Once the store estimations are complete they are aggregated at a country level to identify if a country needs to be blocked.

The forecast drives the store shipment schedule by determining total store sales. The shipment schedule uses this information to breaks down the total shipment by weekly shipments. Weekly shipments are derived by a combination of store inventory capacity and maximum shipment levels.

3.2 Legacy Clearance Distribution Process

The PMT manages a schedule of shipments that is sent to the distribution center to decide what merchandise each store will receive. A three step process is used by the distribution center to determine the shipments. These steps are first selecting a reference, determining which stores to ship the reference, and calculating the shipment size.

Figure 5: Distribution Center

3.2.1 Selecting a Reference

The first step is selecting a reference to distribute. Zara's picking system is based on inputting a reference and then assigning the quantity to ship to all stores. DC personnel pick at a reference level and then the item is physically sorted by store. This is different than picking a set of references for a given store.

To choose which reference r , the distribution team considers merchandise with the highest success ratio, lowest stock to sales ratio, and most inventories. Each reference has a success ratio SS_r, which is the ratio of total sales of a reference divided by the total opportunity of sales for the current season. This estimates the success of sales over the life of the item. The range of the success ratio is from 0 to 1 and used as a percentage of inventories that has been sold at a given store. A success ratio of 0.8 signifies that 80 percent of the total inventory available to the store has been sold. Equation 3-1 describes the success ratio in which $Inventory_r$ is the current stock on hand at a given store and $Sales_r$ is the total sales of that reference.

$$SS_r = \frac{Sales_r}{Invenotry_r + Sales_r}$$

Equation 3-1: Success Ratio

Next, references are sorted by those with a low stock to sales ratio. The stock to sale ratio or rotation Rot_r is "a direct index of the stock which should be on hand at each store in a given time.[14]" It divides the stock at the end of a period with the sales during that period. Rotation is the number of days the current inventory will remain on hand at a store if the sales rate remains the same for that period. This differs from the success ratio as it is only captured at a given period and not the entire season. At Zara, the stock to sales ratio is estimated on a daily or weekly basis. Equation 3-2 is the rotation calculation for a given period.

$$Rot_r = \frac{Inventory_r}{Sales_r}$$

Equation 3-2: Rotation

By considering the success ratio SS_r and the stock to sale ratio Rot_r the DC is ultimately choosing to ship those items that are selling very well first. The sooner these items are in the store the more likely they will be sold during the first week of clearance and will not have to be discounted numerous times.

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14 (Schmalz, 1928)

Lastly, references with the most inventory on hand take priority shipment. Items with the most stock on hand are those that did not have success during the regular season. If there is an excess amount of these items in the distribution centers they are likely to be used to fill store orders due to the volume of the merchandise.

3.2.2 Choosing a Store

To choose which stores should receive a shipment of a chosen reference, the DC team considers first those stores that have the highest total unit sales of the reference and the highest sales of that reference the previous week. The information system, based on point-of-sale, has this information easily accessible for the user to monitor. Stores that have been blocked by the commercial manager will have an alert in the system and will not allow the user to assign a pick list to that store. A final method of choosing a store is based on the DC's knowledge of that market. For instance, if the reference is a pair of Bermuda shorts and it is known that historically Japan sells many Bermudas, Japanese stores will have a priority of these shipments with respect to another country.

3.2.3 Store Shipment Size

The size of the shipment to each store is based on the current inventory level of that store and its sales the previous week. Conventionally store shipments are larger for the clearance season than for the regular season since the merchandise is to replenish stock for the entire clearance season and not just one week of sales. If there is an item in which there are less than a few thousand articles available, the DC will ship to only those few stores with the highest historical sales. Once the top references have been evaluated and assigned to a store the team sums the total shipments quantity per store. A report is printed out with each stores shipment size and compared to the schedule created by the PMT team to ensure the proper shipment sizes are implemented.

3.3 Proposed Solution

The proposed solution to this process is to create a demand forecast that will be use to determine sales for both end-of-season and clearance season. It will be an automated process in which point-of-sale data would be used for inventory and past sale records, making the forecast more viable. An optimization model based from linear programming will be implemented to make distribution decisions for each reference by maximizing profits for the global network and then minimizing costs for distribution within each country.

4 Forecasting Model

The fashion industry relies heavily on sales forecasting for operational decisions. Demand is very volatile and "any gaps between supply and demand leave stores holding too much of what customers don't want and too little of what they do"[15]. In Chapter 3 the traditional forecasting tools used by Zara and how historical data is used to reflect current trends are discussed.

In this chapter the current forecasting model created by Caro and Gallien in 2007 is addressed. This model that will be referred to as the pricing forecast, considers five variables to estimate demand including the elasticity of demand in each country. It encompasses data from both the current season as well as historical sales from the previous two years. Next, the model is analyzed by running various simulations and hypothesis testing to determine key decision variables to improve forecasting accuracy. Finally, a forecasting model for end-of-season demand is formulated.

4.1 Clearance Pricing Forecasting Model

In 2007, Caro and Gallien developed a forecast to estimate demand of a given reference at different price points during the clearance season. The pricing forecast estimates expected unit sales for a given reference at a group and country level. In Carboni's case study, he found that the sales data was most accurate when observed at an aggregated reference level. The pricing forecast is described in Equation 4-1.

$$\lambda_r^w = e^{(\beta_{0r})} * e^{(\beta_1 LN(Purchase_r))} * e^{\left(\beta_2 Age_r^w\right)} * e^{\left(\beta_3 LN\left(\lambda_r^{w-1}\right)\right)} * e^{\left(\beta_4 LN\left(MIN\left(1, \frac{InvPos_r^w}{f * C_r * S_r}\right)\right)\right)} * e^{\left(\beta_5 LN\left(\frac{P_r^w}{P_r^T}\right)\right)}$$

Equation 4-1: Pricing Forecast[16]

There are five explanatory variables, purchase, age, demand level, broken assortment, and price discount. Purchase represents the global purchase in units of the given reference. Age refers to the number of days the reference has been in the store starting from the day it shipped out of the distribution center. The demand level is an estimation of demand based on sales from the week before. Broken assortment refers to the variety colors and sizes available for a given reference. Finally, the price discount is the price elasticity of demand also referred as β_5.

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[15] (Friend & Walker, 2001)

[16] (Carboni, 2008)

Regular season data is used to calculate β_0 (the intercept) through β_3 (demand level) in a minimum squared error linear regression. In order to normalize the regression, the model removes the effects of seasonality and product assortment by using intra-week (*delta*) and inter-week seasonality. Intra-week refers to seasonality observed from one day of the week to the next. Inter-week seasonality measures sales patterns from one week in the regular season to another.

The broken assortment (β_4) and price discount (β_5) is calculated using historical regular season and clearance season data. In order to estimate the price elasticity of demand, past data is important when capturing the price sensitivity of each reference by country. the broken assortment also changes during the clearance season as the majority of the reference collection was sold during the regular season. β_4 and β_5 are calculated through a series of residuals and then are fitted in the linear regression with the other variables.

Once the demand forecast is calculated it is must be converted into unit sales. The demand is than multiplied by an *alpha* parameter that disaggregates the demand to a color, size, and store level. Currently the demand is at a reference by country level. Next, it is multiplied by the *delta* intra-week seasonality that gives an estimated demand by the number of days of the clearance period. During the clearance season, each period of sales may vary in number of days as it is dependent on when the business decides to make a pricing decision. Finally, the Gamma Distribution is used to convert this calculation into unit sales.

4.2 Regular Season Forecasting

The pricing forecast created by Caro and Gallien estimates clearance sales using both historical and current season data. Conceptually, the fitted model should be able to predict regular season demand by using β_0 through β_4. In this section this hypothesis is tested by predicting one week in the regular season and then using the model to estimate sales for the remainder of the regular season. As mentioned in Section 3.1, the current forecast is calculated four weeks before the clearance season. In order to allow decision flexibility the forecast analysis in this chapter is based on an eight week horizon.

Belgium's winter 2008 regular season data was used to test the model. Belgium was chosen as it is a medium size country with 25 stores and has been the pilot country chosen by Zara. The broken assortment parameter will be calculated with the regular season data through the linear regression.

Historical data will not be used as the price discount parameter will not be incorporated since there is no change in prices during the regular season.

4.2.1 Forecasting for Weekly Regular Season Sales

The first test conducted was to understand if the pricing model, with modifications, would predict weekly sales during the regular season. Meaning, can the pricing model determine sales for week *t+1* with current regular season data through week *t*. Modifications to the pricing demand model were calculating β_4 with regular season data and removing β_5. Another alteration needed for the model was to use seven days for the *delta* intra-week seasonality used to transform the demand to sales. Seven days represents the duration of a regular season sales week.

The sales forecast in comparison with the true sales resulted in a forecasting error of -1 percent. With such a low error it was concluded that the modified forecast gave an acceptable estimation of weekly sales. However, it was unclear if the model could estimate the eight weeks leading up to the clearance sales. This is imperative to have when making the inventory distribution choices.

4.2.2 Forecasting for the Entire End-of-Season Sales

To determine if the same approach could be used to estimate the last eight weeks of the regular season two methodologies were tested to calculate the intra-week seasonality. The first approach was a straight line estimation using 56 days (7 days x 8 weeks) as it represents the remaining days left before the clearance period. The second approach was a calculation based on the number of days a SKU was on display at each store. In this latter method, it took the average days a given color and size of a reference was on display per store. The number of days was calculated based on the day of the first shipment to the store.

Though the average number of days on display of a SKU appeared to be logical, when testing both methods this approach gave a large underestimation compared to the straight line approach. There were many doubts about the straight line approach as sales tend to drop as the regular season comes to an end. The results however proved to give a feasible prediction for two months in the future. The new regular season forecast is shown in Equation 4-2. This equation is the same as Equation 4-1: Pricing Forecast but does not contain the price discount (β_5) variable.

$$\lambda_r^w = e^{(\beta_{0r})} * e^{(\beta_1 LN(Purchase_r))} * e^{(\beta_2 Age_r^w)} * e^{(\beta_3 LN(\lambda_r^{w-1}))} * e^{\left(\beta_4 LN\left(MIN\left(1, \frac{InvPos_r^w}{f*C_r*S_r}\right)\right)\right)}$$

Equation 4-2: Regular Season Forecast

4.2.3 Regular Season Comparison Test

After testing the regular season forecast, it was compared to the legacy forecasting method discussed in Section 3.1. In the legacy method, the forecasts are estimated at a store level. The regular season forecast, on the other hand, is based on sales for a reference therefore this estimation was disaggregated to a store level in order to compare both techniques. The forecast error is calculated by taking the difference between the forecast and the actual sales and dividing by the actual sales as shown in Equation 4-3.

$$Forecast\_Error = \frac{Forecast - True\_Sales}{True\_Sales}$$

Equation 4-3: Forecast Error

Figure 6 shows the forecast error of both methods for each Belgian store. A negative error represents an underestimation of the forecast where a positive error represents an overestimation. From a store level there is more variation in the forecast error using the regular season forecast. However, at the aggregated country level, the regular season forecast gives a closer estimation of sales than the legacy method. The ideal forecast the organization would like to obtain would be one that decreases both store and country variation.

Store	Error Old	Error New
337	-10%	6%
339	-5%	-1%
340	-8%	-10%
346	-7%	12%
348	-1%	5%
376	-11%	9%
377	-20%	2%
378	-4%	8%
388	-8%	-8%
392	-5%	-3%
3014	2%	6%
3139	-12%	-18%
3141	-15%	-19%
3168	-9%	9%
3193	-8%	-11%
3295	-12%	-10%
3332	-1%	34%
3542	2%	-5%
3578	-11%	73%
3589	-6%	10%
3597	-1%	0%
3661	-10%	17%
3743	-28%	-5%
3810	-11%	-9%
3824	-1%	-6%
Total	-8%	-1%

Belgium-W08	Old	New
Total	-8%	-1%

Figure 6: Regular Season Forecast Comparison Test

With the regular season trials, the team was comfortable with the formulation of the regular season forecast and the results of the comparison test. The next step was to focus on estimating sales for the clearance season followed by experimenting with adding additional variables to improve the demand forecast.

4.3 Clearance Season Forecasting

The pricing forecast is based on all data leading up to the clearance season. In other words, it captures the entire regular season information to estimate clearance sales. Smith and Achabal's (1998) research demonstrates that the first clearance markdown tends to be the dominant decision economically. With this in mind, Caro and Gallien designed the pricing forecast model to estimate the first period of the clearance season and through a pricing optimization model to calculate the total clearance sales[17].

The following tests focus on improving the first period of the clearance season by using the pricing forecast. The first experiment was to test the forecast error using the data that would be available eight weeks prior to the start of the clearance season. The second experiment was testing other variables that may affect the elasticity of demand for the pricing discounts.

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[17] (Carboni, 2008)

4.3.1 Forecasting Tests for the First Clearance Period

Estimating clearance sales two months before the sales begin is challenging. One aspect to consider is that the inventory position in the current period will be different once the clearance season begins. The current inventory position changes as there will be both sales and merchandise replenishment in the eight weeks leading up to the clearance season, both of which are unknown. Inventory position affects the calculation of the broken assortment variable in the linear regression. Moreover, it impacts the conversion of the demand to sales through the Gamma Distribution. The Gamma Distribution has two parameters *k* and *theta*. *Theta* is the scale parameter and in the Caro and Gallien pricing model is the demand multiplied by the intra-week *delta* factor and an *alpha* that disaggregates the reference to a SKU and store level. Current stock level or inventory position is the parameter *k* used to shape the Gamma Distribution. Changes to the stock level will impact the sale estimation. Various iterations of the inventory position were tested to analyze how feasible it would be to estimate the first week of clearance sales.

In this analysis winter 2008 data for all stores in Belgium were used. The clearance period duration for the winter 2008 was five days, therefore the intra-week seasonality resulted in a 4.8 value[18]. This of course is an after-the-fact data point but was used to understand the viability of the model.

The various inventory positions are described below:

- **Inventory Position I:** The inventory position eight weeks prior to the start of the clearance season, this would reflect the stock levels the business can identify.
- **Inventory Position II:** The stock level eight weeks prior to the clearance period plus all shipments within the eight weeks. In practice an estimation of all the shipments would not be feasible but was tested to determine the significance of these shipments.
- **Inventory Position III:** The inventory position at the start of the clearance season. The inventory level at the beginning of the clearance season would be unknown in practice as both shipments and generated sales would have to be considered.

The experiments showed that there was little difference with the forecast errors when using Inventory Position I versus Inventory Position III. The second inventory position did not offer any convincing insights and was quickly discarded. It was concluded that using stock levels two months

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[18] Each day of the week has its own sales weight (seasonality), the five days in this case had a 4.8 value.

prior to the clearance season provides a demand forecast similar to one when knowing the true stock level at the start of the clearance season. Consequently the pricing forecast could be used to determine the first period of the clearance season given a shortage on regular season data.

4.3.2 Estimating the Elasticity of Demand

Price elasticity of demand is "a measure of the sensitivity of quantity demanded to changes in price"[19]. The current model estimates the elasticity as a function of historical sales in the previous two years. However, it fails to test if there are external factors that can influence the ratio between quantity demanded and change in price. For the following experiment three years worth of data (2006-2008) for the summer clearance season was used to test the variables for 10 countries using a linear regression approach. The next section describes the factors considered as viable variables to test.

4.3.2.1 Test Variables

1) **Country Growth Domestic Product rate year over year[20]**: GDP was used to understand if there was a trend with a countries growth that it may affect the elasticity of demand.
2) **Country unemployment rate year over year[21]**: Unemployment rate was considered due to the social impacts and changes in spending trends in consumers.
3) **Store to city population ratio[22]:** The ratio of population of each city to the number of stores in each country was considered to understand the dynamics of any cannibalization between stores.
4) **Country economic position:** Economic position of a country was a measure of how many hours a store employee would have to work to be able to purchase an article from the store.
5) **Regular Season Markdowns:** The price elasticity of demand of regular season mark downs.

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[19] (Wikipedia Elasticity of Demand)

[20] (European Commission Eurostat)

[21] (European Commission Eurostat)

[22] (City Population,)

4.3.2.2 Elasticity of Demand Regression Analysis

The pricing forecast focuses on estimating sales for each of the 21 groups at a time. As mentioned in Section 2.3, groups are categorized into product lines. Through analyzing historical sales, there was evidence in trends amongst product lines. Therefore, the elasticity of demand was estimated at the product line level. Using the true β_5 for the summer clearance season, the elasticity of demand for the product line was calculated by taking the average β_5 of the groups within the product line.

To test the correlation of the variables with the elasticity of demand, three years worth of data was used for 10 countries with a total sample size of 30 per product line. The linear regression tested each of the variables independently with only unemployment rate and regular season markdowns proving to be statistically significant. The relationship with unemployment rate and the elasticity of demand signified that consumers were more sensitive to price mark downs as the unemployment rate inceased. However, using unemployment rates to determine the elasticity of demand for the clearance sales has its short comings as these figures may not be readily accessible for all countries. The elasticity of demand from the regular season markdowns, on the other hand, is a more practical solution. The next step was to test the theory by estimating the elasticity of demand using current season data.

4.3.2.3 Estimating β_5 with Regular Season Markdowns

The plot of the linear regression using three years worth of data can be found in Figure 7. The dependent variable β_5 product line was estimated by taking the weighted average elasticity of each group within the product line. Markdown elasticity was measured using the definition of price elasticity found in Equation 4-4 were D is unit sales and P is price. Since the unit sales are known for the price markdown during the regular season, demand was not necessary.

$$Elasticity = \frac{\Delta D / D}{\Delta P / P}$$

Equation 4-4: Markdown Elasticity[23]

The change in sales was determined by the sales of the last seven days at the regular season price denoted by variable e and the sales for the first seven days at the markdown price denoted by variable

[23] (Wikipedia Elasticity of Demand)

b. The price ratio was calculated by using a weighted average that was dependent on the inventory volume for each reference r. Equation 4-5 and Equation 4-6 illustrate the sales and price ratio respectively.

$$\Delta D / D = \frac{(D_b - D_e)}{D_e}$$

Equation 4-5: Change in Sales Ratio

$$\Delta P / P = \frac{\sum_r Stock_r * \left(\frac{(P_b - P_e)}{P_e} \right)_r}{\sum Stock_r}$$

Equation 4-6: Change in Price Ratio

To estimate the elasticity of demand for summer 2009, a simple linear regression was used given by the regression analysis of the historical data. The relationship between the two variables can be found in Figure 7 where y is the β_5 product line for 2009 and x is the markdown elasticity for 2009.

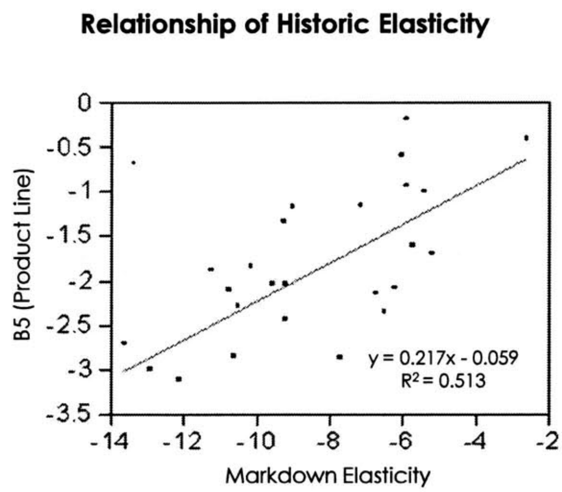


Figure 7: Demand Elasticity Plot

This linear relationship will give the elasticity of demand at the product line level, therefore needs to be disaggregated at a group level for the pricing forecast. To estimate the β_5 group ($B5_G$) we took the weighted average from the previous year as denoted in Equation 4-7. Where $B5_{PL}$ represents β_5 product line and t is the current year.

$$\hat{B}5_G^{t-1} = \frac{B5_G^{t-1}}{B5_{PL}^{t-1}}$$

Equation 4-7: Beta-5 Weighted Average

Equation 4-8 portrays the final calculation of $B5_G$ by multiplying the group weighted average by $B5_{PL}$.

$$B5_G^t = B5_{PL}^t * \hat{B}5_G^{t-1}$$

Equation 4-8: Beta-5 Group Estimation

With the estimated β_5 for summer 2009, the pricing forecast was tested with these new values versus the values given though the residual analysis discussed in Section 4.1.

Next, 10 countries with regular season data up to two months prior to the clearance season was used to test the pricing forecast. The forecast errors using both methods of calculating the elasticity of demand were compared and are shown in Table 1.

Country ID	Pricing Forecast	Pricing Forecast + Markdowns
2	-4%	-11%
3	-8%	-23%
7	-3%	-10%
8	-4%	11%
18	-2%	-12%
28	-2%	1%
30	-26%	-41%
38	-3%	23%
39	-21%	-22%
101	25%	-22%

Table 1: Demand Forecast Beta-5 Comparison

The results showed that estimating the β_5 using regular season markdowns improved the pricing forecast by an absolute error in country 28 and 101. However, the pricing forecast overall had a smaller error percentage for 80 percent of the countries tested. Another insight from this test was that the model was successful in terms of forecast error for multiple countries not just one as tested in Section 4.3.1.

4.4 New Variables for Demand

The last hypothesis in improving the pricing forecast was to determine if adding an additional variable to the current five variable pricing forecast would decrease the overall forecasting error. In Section 3.2.1 the stock to sales ratio called rotation was discussed as a factor that Zara uses to measure how well a given article is performing. A rotation of 5 signifies that the current inventory for a reference would last 5 days if the store continues to sell at that rate. The data used in this experiment was winter 2008 figures for three medium size countries. To test if the Rotation variable was significant in determining demand, it was added to the linear regression as β_6 and calculated using regular season data. Given that the rotation is the stock to sales ratio, there are various techniques that can measure this variable. Therefore, three distinct rotation measurements were tested by adding them to the regular season forecast and the pricing forecast. Below are the rotation calculations and results of these experiments.

1. Rotation of the first day of sale
2. Rotation of the first week of sale
3. Rotation of the total regular season

Collecting the data for the first two rotations was a tedious task as each article had its individual first day of sale. The third rotation measurement took less effort to collect and calculate as it was the ratio of the sum of total inventory and total sales for the regular season. Table 2 is a comparison of each rotation measurement for a sample reference.

	Rotation
First Day of Sale	4.35
First Week of Sale	3.81
Total Season Sale	4.12

Table 2: Rotation Comparison

The rotation tests were tested on both the regular season forecast and the pricing forecast. In each case, the results were not compelling. In the case of the pricing forecast, all three rotation calculations were not statistically significant in the linear regression. The third rotation calculation, however, was statistically significant in the regular season linear regression. The sales estimation for the regular season with the rotation variable was an improvement from the legacy forecasting method but was not the case in comparison to the regular season forecast as shown in Equation 4-2. It was therefore concluded that rotation is not a significant variable and will not be added to the two forecasting methods.

4.5 Forecasting Process Summary

Caro and Gallien's pricing forecast proved to be sufficient in determining both regular season sales and clearance season sales with limited data. The forecast is aggregated at each reference and predictions are arranged at a group and country level. A linear regression analysis is used to determine demand and finally converted to sales though the Gamma Distribution.

The regular season forecast is a modification of the pricing forecast in that it did not contain the price discount variable. A straight line estimation approach was used to estimate the sales for the ending weeks of the regular season. The demand forecast is then converted to a sales forecast for the optimization models discussed in Chapter 1.

H1: Pricing forecast can predict regular season sales.		
	Experiement	Results
1	• Intra-week delta = Straight line method	✓
	• Intra-week delta = Days on display	✗
H2: External factors affect price elasticy and can improve pricing forecast.		
	Experiement	Results
1	External factors can predict beta-5	
	• Regular season markdowns	✓
	• Unemployment rate	✗
	• Country GDP	✗
	• Store to city population	✗
	• Country economic position	✗
2	Regular season markdowns improve forecast	✗
H3: Adding Rotation as a variable will improve forecast.		
	Experiement	Results
1	• Rotation = First day of sale	✗
	• Rotation = First week of sale	✗
	• Rotation = Total season sale	✗

Figure 8: Forecast Experiment Summary

In estimating demand for the clearance season, the pricing forecast proved to be sufficient. Various tests were conducted to improve the model as shown in Figure 8 but all failed in providing compelling evidence to change the current model. The demand for the first week of the clearance season at various discounted prices is determined by using the pricing forecast and an optimization model discussed in Carboni's case study. To estimate inventory distribution for the clearance season, a demand estimate is used for the clearance season rather than a sales estimate as used in the regular season forecast. This is due to the optimization models that will be discussed in the following chapter that will determine the shipment based on the available inventory and demand.

5 Model Formulation: Clearance Material Distribution Optimization

The optimization model was broken down into two models in order to be able to manage the size of the data of 11,000 SKUs and 73 countries. The objective of the first model is to maximize global profits for each group and pricing cluster by determining the allocation of inventory to each country. A pricing cluster is a set of references that have the same regular season price. It uses the demand forecast at each of the various price points from the regular season price to the distinct price discounts during the clearance season. The output of this model will be the number of units to ship to a county in a given group and pricing cluster.

In the second model it will take each group and cluster and allocate these at a reference level to each store while minimizing shipping costs. With this model users can determine what reference to ship to each store from the distribution centers and local warehouses as well as which references to transfer from one store to another. These two models are designed to run a few weeks before the clearance season begins to assign the total number of units by reference to ship to each store for both end-of-season and clearance season sales. The two models were formulated by Professor Felipe Caro[24].

In order to distinguish the variables between regular season and clearance season, in the following section variables and parameters with a tilde $\sim$ will denote regular season. For example $\tilde{F}_{mgn}$ and F_{mgn} denote the sales forecast for a given country m, group g, and cluster n during the regular season and the clearance season respectfully.

5.1 Model I: Maximizing Revenues Formulation

The purpose of this optimization model is to maximize global profits by making inventory distribution decisions based on a dynamic pricing system. The model considers each product

[24] (Caro F. , In preparation for: Coordination of Inventory Distribution and Price Markdowns for Clearance Sales, 2009, November 15)

group and pricing cluster in determining the allocation of the inventory to each country. Inventory and demand estimates are taken from the POS system each period for data accuracy. For clearance markdowns it is assumed that pricing decisions are permanent, therefore cannot increase overtime. The general notations of the model will be introduced first followed by the definition of the parameters.

5.1.1 Indices and Index Sets

- $m \in M$: Each county within the distribution network. The index m will also be used to denote local warehouses for those countries in which it exists.
- $a \in A$: Main distribution centers located in Spain. Thus, $a(m)$ denotes the distribution center that supplies country m and $a(j)$ the distribution center that supplies store j . Let $P(a) \subseteq M$ and $T(a) \subseteq J$ denote the set of countries and stores respectfully that are supplied by the distribution centers.
- $g \in G$: Clearance sales groups that represent each product line and are numbered from 1 to 21.
- $n \in N(g)$: Price clusters in a given group g. each group g has distinct price clusters that represent several references r.
- $k \in K = \{k : k \geq 1\}$: Clearance prices are indexed by (k). Let p_k denote the set of clearance prices that increases by convention, i.e. $p_1 \leq p_2 \leq \ldots \leq p_k$, where p_1 is the final liquidation price.
- $w \in W = \{w : 0 \leq w < W\}$: Periods within the clearance season, where $w = 0$ represents the first period of the clearance season and $w = W$ the last period of the season.

5.1.2 Parameters

Each country has different price clusters based on their currency as well as their local market demand. For example, an article in the distribution center may be categorized in price cluster 9.99 Euros. That same article in a different country might have a different price point in the local country currency. In order to allocate the inventory the clusters were standardized in the pricing categories of the distribution centers located in Spain. Therefore, the clusters were grouped in the same grouping as the distribution centers and prices were converted into Euros.

- $\widetilde{p}_{mgn}$: Regular season price in Euros, for cluster n in country m and group g.
- $psalvage_g$: Liquidation price for each group g at the end of the clearance season.

When allocating inventory from the distribution centers, the model includes merchandise for the current season that is in transit or on order from the vendor. It is important to count this inventory as it will only be sold during the current season and will be distributed for the current period as well as the clearance season. Any changes to vendor fulfillment is updated on a weekly basis and reflected in the inventory count. Therefore the inventory on hand in the current period is denoted as follows:

- $\widetilde{I}_{agn}$: Inventory on hand in units, at the beginning of the current period at distribution center a and in group g and cluster n. The inventory also includes incoming merchandise from vendors for the current regular season.
- $\widetilde{I}_{mgn}$: Inventory on hand in units, at the beginning of the current period at country m and in group g and cluster n. This is the total inventory in each country which represents the total inventory on hand in both the stores and the local warehouse (if it exists) for a given country.

The forecast for each country, specified by the regular season forecast and pricing forecast discussed in Chapter 4 are denoted below.

- F_{mgn} : Estimated demand in units for the clearance season at country m and in group g and cluster n.
- $\widetilde{F}_{mgn}$: Estimated sales in units for the end-of-season sales during the regular season at country m and in group g and cluster n. This represents estimated sales as discussed in Section 4.6.

To ensure the inventory is distributed across various countries a maximum shipment constraint is established. The maximum shipment constraint is based on the success ratio SS_{mgn} for each group g and cluster n for country m. There are four steps to calculate this parameter.

- B_{mgn} : Maximum shipment of a given group g and cluster n for country.

Step 1 – Calculating a countries success ratio

In order to calculate the country success ratio, the success ratios of all new store openings, closing, or expansions must first be estimated. Each year there are new store openings and to account for expected sales from these stores a sales level of an equivalent store that has been opened for at least one year must be assigned to the new store.

Step 2 – Eliminating counties with low success ratios

For a country m that has a success ratio SS_{mgn} that is 1.5 standard deviations below the mean μ_{mgn} of all countries in the set of M_{gn}, let $B_{mgn} = 0$ and remove that country from M_{gn}.

Step 3 –Calculating the maximum shipment B_{mgn}

For all countries $m \in M_{gn}$, let the maximum shipment be calculated by the following equation:

$$B_{mgn} = \sum_{a \in A} \widetilde{I}_{agn} \bullet d_{mgn}, \text{ where } d_{mgn} = \left(\frac{Sales_{mgn}}{\sum_{m' \in M_{gn}} Sales_{m'gn}} \right)$$

Equation 5-1: Maximum Shipment Estimation

Step 4 – Recalculating B_{mgn}

Let X_{mgn} be the minimum shipment quality, which can be country, group, or cluster specific if necessary. If $B_{mgn} < X_{mgn}$ for country m, set $B_{mgn} = 0$ and remove that country from M_{gn}. For those countries that remain in M_{gn} recalculate B_{mgn}.

5.1.3 Decision Variables

- q_{mgn}: Inventory flow in units from the distribution centers to country m for group g and cluster n.
- I^{w}_{mgn}: Inventory on hand in units for period w during the clearance season at country m in group g and cluster n.

- $\tilde{\lambda}_{mgn}$: Expected sales in units during the regular season for country m in group g and cluster n.
- λ^{w}_{mgnk}: Expected sales in units for period w during the clearance season for country m in group g and cluster n at clearance price k.
- $q_{aa'gn}$: Inventory flow in units from distribution center a to a' for group g and cluster n.
- $x^{w}_{mgnk} \in \{0,1\}$: Binary decision variable indicating whether the clearance price for cluster n in group g and county m should be less than or equal p_k to during the clearance period w.
- $y^{w}_{mgnk} \in \{0,1\}$: Binary decision variable associated with x^{w}_{mgnk}, that assumes the value of 1 if the price p_k is assigned to cluster n in period w.
- $Lsaldero_{mg}$: Total inventory that will be salvaged in country m and group g.

5.1.4 Objective Function

The objective function in Model I is to maximize sales by taking the sum of total sales and subtracting the associated costs. There are three elements in which profits can be generated and two shipment costs that are being considered.

$$\textbf{Max} \quad EoS\_SALES + C\_SALES + S\_SALES - CS\_COSTS - DCS\_COSTS$$

Equation 5-2: Maximizing Revenues

- Variable EoS_SALES: Represents the total revenue generated from the end-of-season or regular season sales. This corresponds to the sales forecast made by the forecast model at the regular season price.

$$EoS\_SALES = \sum_{\substack{(m,g)\in M\times G,\\ n\in N(g)}} (\tilde{p}_{mgn} \times \tilde{\lambda}_{mgn})$$

Equation 5-3: End-of-Season Sales

- Variable C_SALES: This variable is the revenue generated during the clearance season.

$$C\_SALES = \sum_{\substack{(m,g)\in M\times G,\\ n\in N(g)\\ 0\le w\le W,k\ge 1}} p_k \times \lambda^{w}_{mgnk}$$

Equation 5-4: Clearance Season Sales

- Variable S_SALES: Represents the revenues generated by selling the excess inventory in each country and distribution centers at the liquidation price.

$$S\_SALES = \sum_{\substack{(m,g)\in M\times G,\\ n\in N(g)}} (I^{W}_{mgn} + I_{agn}) \times psalvage_g$$

Equation 5-5: Liquidation Sales

- Variable CS_COSTS: This variable is the cost of shipping a unit from the distribution center a to country m.

$$CS\_COSTS = C_M \bullet \sum_{\substack{(m,g)\in M,\\ g\in G,n\in N(g)}} (\tilde{p}_{agn} \times q_{mgn})$$

Equation 5-6: Country Shipment Cost

- Variable DCS_COSTS: Represents the cost of shipping a unit from the distribution center a to distribution center a'.

$$DCS\_COSTS = C_A \bullet \sum_{\substack{a,a'\in A,\\ g\in G,n\in N(g)}} (\tilde{p}_{agn} \times q_{aa'gn})$$

Equation 5-7: Intra-DC Shipment Cost

5.1.5 Model I Constraints

The following constraints are inventory flow constraints for the distribution centers and the countries. The first constraint ensures that the shipments made from the distribution centers do not exceed the inventory available. In Constraint 5-2, it ensures a balance of inventory flow for each country. The third constraint models the initial inventory levels of each clearance period by discounting the sales from the previous period.

$$\tilde{I}_{agn} + \sum_{a'\in A} q_{a'agn} \ge \sum_{m\in P(a)} q_{mgn} + \sum_{a'\in A} q_{aa'gn} \forall (a,g,n) \in AGN$$

Constraint 5-1: DC Inventory Balance

$$I^0_{mgn} = \tilde{I}_{mgn} + q_{mgn} - \tilde{\lambda}_{mgn} \forall (m,g,n) \in MGN$$

Constraint 5-2: Country Inventory Balance

$$I^{w+1}_{mgn} = I^w_{mgn} - \sum_{k \geq 1} \lambda^w_{mgnk} \forall (m,g,n,w) \in MGNW$$

Constraint 5-3: Clearance Inventory Flow

The expected sales estimation for the Optimization Model I is limited by sales forecast shown in Constraint 5-4 and Constraint 5-5.

$$\tilde{\lambda}_{mgn} \leq \tilde{F}_{mgn} \forall (m,g,n) \in MGN$$

Constraint 5-4: Expected Regular Season Sales

$$\lambda^w_{mgnk} \leq F^0_{mgn}(p_k) y^w_{mgnk} * tiempo^{w-1} \forall (m,g,n,k,w) \in MGNKW$$

Constraint 5-5: Expected Clearance Season Sales

As calculated in Section 5.1.2, the maximum shipment constraint is used to ensure merchandise is allocated to multiple countries. This calculation serves as an essential factor for blocking countries from receiving a shipment for a given group or cluster.

$$\sum_{g \in G, n \in N(g)} q_{mgn} \leq B_{mgn} \forall (m,g,n) \in MGN$$

Constraint 5-6: Maximum Shipment

A minimum shipment constraint is necessary to ensure single digit shipments are not sent to each country. For any item in the distribution centers that has a stock level below this minimum shipment, these items would not ship to the countries and would ultimately be sold at the liquidated price. The minimum shipment requirement is determined by the distribution center policy.

$$\sum_{g \in G, n \in N(g)} q_{mgn} \geq Q_m \forall (m,g,n) \in MGN$$

Constraint 5-7: Minimum Shipment

Constraint 5-8 ensures that the initial ordering of each price cluster is maintained throughout the clearance period. For example, if reference r was priced higher during the regular season than reference r', it will never be price lower than reference r' during the clearance season.

$$x^{w}_{mgnk} \leq x^{w}_{mgn+1k} \forall (m,g,n,k,w) \in MGNKW$$

Constraint 5-8: Price Cluster Order

In order to follow Zara's clearance pricing policy, Constraint 5-9 ensures that the price of an article can never increase as the clearance period progresses. Meaning that the clearance price for period $w+1$ can only be equal or below the clearance price in week w.

$$x^{w}_{mgnk} \leq x^{w+1}_{mgnk} \forall (m,g,n,k,w) \in MGNKW$$

Constraint 5-9: Pricing Restriction

Constraint 5-10 and Constraint 5-11ensure the all clusters will be discounted during the first week of the clearance period.

$$x^{1}_{mgnk} = 1 \forall \left(m,g,n,\max(k) \ni p_k < \widetilde{p}_{mgn} * (1 - Min\_Discount)\right) \in MGNK$$

Constraint 5-10: Cluster Discounts

$$y^{1}_{mgnk} = 1 \ \forall (m,g,n,k) \in MGNK$$

Constraint 5-11: First Week Discounts

The total liquidation revenues that can be generated are restricted by the amount of excess inventory in the distribution centers and the countries given in Constraint 5-12. The liquidated inventory for each country is restricted by the amount of inventory left unsold at the end of the clearance season as denoted in Constraint 5-13.

$$\sum_{(a,g,n) \in AGN(g)} (\widetilde{I}_{agn} - \sum_{m \in M} q_{mgn}) + \sum_{(m,g,n) \in MGN(g)} I^{W}_{mgn} \leq \left(\sum_{(a,g,n) \in AGN(g)} \widetilde{I}_{agn} + \sum_{(m,g,n) \in MGN(g)} I^{W}_{mgn} \right) \bullet S_T \ \forall (a,m,g,n) \in AMGN$$

Constraint 5-12: Liquidation Revenues

$$Lsaldero_{mg} \leq \sum_{n \in N(g)} I^{w}_{mgn} \forall (m,g,n,w) \in MGNW$$

Constraint 5-13: Country Liquidation

The three following constraints are structural constraints to facilitates the binary variables x and y.

$$x^{w}_{mgnk-1} \leq x^{w}_{mgnk} \forall (m,g,n,k,w) \in MGNKW$$

Constraint 5-14: Structural Constraint I

$$y^{w}_{mgnk} = x^{w}_{mgnk} - x^{w}_{mgnk-1} \forall(m,g,n,k,w) \in MGNKW$$

Constraint 5-15: Structural Constraint II

$$y^{w}_{mgnk}, x^{w}_{mgnk} \in \{0,1\} \forall(m,g,m,k,w) \in MGNKW$$

Constraint 5-16: Binary Variables

Constraint 5-17 and Constraint 5-18 enforces all shipments, expected sales and stock levels to have positive values.

$$\lambda^{w}_{mgnk}, I^{w}_{mgnk} \geq 0 \forall(m,g,n,k,w) \in MGNKW$$

Constraint 5-17: Non-Negative Clearance Values

$$q_{aa'gn}, q_{mgn}, \widetilde{\lambda}_{mgn} \geq 0 \forall(a,a',m,g,n) \in AAMGN$$

Constraint 5-18: Non-Negative Shipments and Demand

5.1.6 Levers

The following are levers that can be adjusted based on business policy and standards.

- C_M: Shipment costs by unit from distribution centers to country m.
- C_A: Shipment costs by unit from one distribution center to another.
- D: Number of days in a clearance period.
- W_C: Number of periods in the clearance season in which W_1 represents the first period in the clearance season and W_C the last period.
- S_A: Percent of stock in the distribution centers that can be salvaged.
- S_T: Percent of total global stock that can be salvaged.
- Q_m: Minimum shipment quantity in units per country m.
- $Min\_Discount$: Minimum discount for the first week of sale. This forces the model to make a minimum discount for which price clusters must be marked down in respect to the regular season.

- *tiempo*: Time factor that decreases the amount country clearance sales each week. A time factor of 0.80 represents that the maximum amount of sales the current clearance period can be is 80% of the estimated sales forecast from the previous week.

5.2 Model II: Minimizing Costs Formulation

The purpose of Model II is to distribute the merchandise to each store by minimizing shipment costs. With the output of Model I that dictates the total quantity to ship to each country at a group and cluster level, this model disaggregates the shipments to an article and store level. This optimization model focuses on one group and cluster at a time due to the excess amount of data. The number of references in a group and cluster can range from 1 to 100 and they are distributed to over 1200 stores in 73 counties.

5.2.1 Indices and Index Sets

The following are additional references that are introduced in Model II in addition to those from the first model.

- $r \in R(n)$: References in a given price cluster n. A reference is a given fashion design that is categorized by their original regular season price and grouped by a price cluster n.
- $j \in J$: Stores within the distribution network. Let $m(j)$ denote the country of store j and $T(m) \subseteq J$ denote the set of stores in country m.
- $\ell \in L$: Logistic platforms for each store. Stores are able ship and receive clearance merchandise from stores within their county or within a region of the country. Therefore, $\ell(j)$ denotes the logistic platform for a given store and $T(\ell) \subseteq J$ denotes the set of stores associated with platform ℓ.

5.2.2 Parameters

The following initial inventory parameters similar to those in Model I, however these are at a reference level not at a group and cluster level.

- $\widetilde{I}_{ra}$: Inventory in units at the beginning of the current period for distribution center a and reference r.
- $\widetilde{I}_{rj}$: Inventory in units at the beginning of the current period for store j and reference r.

- $\tilde{I}_{rm}$: Inventory in units at the beginning of the current period for country m and reference r.

The first optimization model gives an output of the total shipment to a country by group and cluster in the decision variable q_{mgn}. In this model it is important to first determine how to distribute this quantity to each store. The total inventory in the country plus the incoming shipment from the distribution center is allocated to each store based on the percentage of sales of each store in its respected country. The store's current inventory is also taken into consideration as shown in Equation 5-8.

- λ_j: Total shipment quantity in units allocated to each store j.

$$\lambda_j = \left(\tilde{I}_{mgn} + q_{mgn}\right) \cdot d_{jgn} - \sum_{r \in R} \tilde{I}_{rj} \text{ where, } d_{jgn} = \left(\frac{Sales_{jgn}}{\sum_{j' \in T_{M(n)}} Sales_{j'gn}} \right)$$

Equation 5-8: Store Shipment Quantity

Similar to the maximum shipment calculation of B_{mgn} in Model I, there is a similar variable for the maximum shipment per store and reference. This parameter ensures references are well allocated to each store and allows certain references to be blocked at a store level. There are six steps in the calculation of this parameter as discussed below:

- B_{rj}: Maximum shipment quantity in units for store j and reference r.

Step 1: Set maximum shipment to zero if country shipment is zero

Let J_r be the set of all stores for reference r. For any country m that $q_{mgn} = 0$, set $B_{rj} = 0$ for all $j \in T(m)$ and remove those stores from J_r.

Step 2: Remove stores from J_r that have a low success ratio

For a store j that has a success ratio SS_{rj} that is 1.5 standard deviations below the mean μ_{rj} of all stores j in the set J_r, let $B_{rj} = 0$ and remove that store from J_r.

Step 3: Verify if the shipment meets the minimum shipment requirement

Let H represent the maximum number of stores that can receive a reference r. Let $|J_r|$ represent the number of stores in set J_r and X_r the minimum shipment requirement that may be reference specific. Calculate H by using the following Equation 5. 8. If $H < |J_r|$, then remove $|J_r| - H$ stores from J_r that have the lowest success ratio SS_{rj} and set $B_{rj} = 0$.

$$H = \left(\frac{\sum_{a \in A} \tilde{I}_{ar}}{X_r} \right)$$

Equation 5-9: Maximum Number of Stores

Step 4: Calculating B_{rj}

Let $[\lambda_j]^+ = \max\{\lambda_j, 0\}$ denote the positive part of λ_j. For every store $j \in J_r$ calculate B_{rj} using Equation 5.9.

$$B_{rj} = \frac{[\lambda_j]^+}{\sum_{j' \in J_r} [\lambda_{j'}]^+} \bullet \sum_{a \in A} \tilde{I}_{ar}$$

Equation 5-10: Store Maximum Shipment

Step 5: Ensuring B_{rj} *meets the minimum shipment requirement*

For a store j that does not meet the minimum shipment requirement $B_{rj} < X_r$, set $B_{rj} = 0$ and remove the store from J_r.

Step 6: Recalculate B_{rj} *and verify feasibility*

Recalculate B_{rj} for those stores $j \in J_r$ using Equation 5.10. To avoid infeasibility, verify the following condition is met for each store.

$$\sum_{r \in R_{(n)}} B_{rj} \geq q_{mgn} \bullet d_{jgn}$$

Equation 5-11: Feasibility Calculation

5.2.3 Decision Variables

All the decision variables in this model are shipment quantities from one point to another. The variables below are those shipments from the distribution center and country warehouses.

- q_{raj}: Shipment quantity in units for reference r from distribution center a to store j.
- q_{rmj}: Shipment quantity in units for reference r from country warehouse m to store j.
- $q_{raa'}$: Transshipment quantity in units for reference r between distribution center a and a'.

During the clearance season, stores may receive merchandise from other stores within its country. Most countries do not have a country warehouse that can facilitate store to store transfers, therefore there are logistic platforms in each country were shipments are sent and cross docked to another site. Each store has only one logistic platform and shipments can be transferred between one logistic platform and another. The following decision variables facilitate store to store transfers using these logistic platforms.

- $q_{r\ell j}$: Shipment quantity in units for reference r from logistic platform ℓ to store j.
- $q_{rj\ell}$: Shipment quantity in units for reference r from store j to logistic platform ℓ.
- $q_{r\ell\ell'}$: Transshipment quantity in units for reference r between logistic platform ℓ and ℓ'.

5.2.4 Objective Function

The objection function is to minimize shipment costs when distributing merchandise to each store. There are five basic elements in the optimization Model II, each represents costs associated with moving merchandise between distribution centers, stores and logistic platforms. All costs are based on unit costs per reference.

$$\min \sum_{r \in R} \left(c_A \cdot \sum_{a,a' \in A} q_{raa'} + c_S \cdot \sum_{j \in J} (q_{rl(j)j} + q_{rjl(j)}) + c_L \cdot \sum_{l,l' \in L} q_{rll'} + c_{ML} \cdot \sum_{j \in J} q_{rm(j)j} + c_M \sum_{j \in J} q_{ra(j)j} \right)$$

Equation 5-12: Minimizing Costs

- Variable $DC2DC$: The cost of shipping a reference between one distribution center to another. This cost includes handling costs as in many instances price tags must be removed and replaced if the merchandise is shipping to a country with a different price. It is the same cost as the cost in Model I.

$$DC2DC = \sum_{r \in R} \left(c_A \bullet \sum_{a,a' \in A} q_{raa'} \right)$$

Equation 5-13: DC-to-DC Shipment Cost

- Variable $Store\_LP$: The shipping cost variable of distributing merchandise from one store to another through a logistic platform. The shipping cost includes material handling costs of preparing the shipment in one store and receiving the shipment in another store. Currently, commercial managers are responsible in making the decision of what fashion items to transfer store to store. This variable helps facilitate that decision based on transportation costs.

$$Store\_LP = \sum_{r \in R} \left(c_S \bullet \sum_{j \in J} (q_{rl(j)j} + q_{rjl(j)}) \right)$$

Equation 5-14: Store-to-Store Shipping Cost

- Variable $LP2LP$: The cost of shipping one item from a logistic platform to another. One example of is a store in New York has excess merchandise that can be sold in a store in Miami will ship the articles to its northeast logistic platform. That platform would then ship the material to the southern logistic platform and enable the merchandise to be distributed to stores that support the southern region of the United States.

$$LP2LP = \sum_{r \in R} \left(c_L \bullet \sum_{l,l' \in L} q_{rll'} \right)$$

Equation 5-15: LP-to-LP Shipping Cost

- Variable DC_Store: The cost to ship from the distribution centers to the stores in which the cost C_M from Model I is similar. Shipment cost includes both transportation and material handling costs.

$$DC\_Store = \sum_{r \in R} \left(c_M \bullet \sum_{j \in J} q_{ra(j)j} \right)$$

Equation 5-16: DC-to-Store Shipping Cost

- Variable $Warehouse\_Store$: Represents the cost of shipping merchandise from the local country warehouse to a store. There are only a few countries that have a large number of stores that need a warehouse to support sales.

$$Warehouse\_Store = \sum_{r \in R} \left(c_{ML} \bullet \sum_{j \in J} q_{rm(j)j} \right)$$

Equation 5-17: Warehouse-to-Store Shipping Cost

5.2.5 Constraints

There are five main constraints in Model II that are primarily based on inventory flow and shipment constraints. Constraint 5-19 ensures shipments from the distribution centers do not exceed the inventory available.

$$\widetilde{I}_{ra} + \sum_{a' \in A} q_{ra'a} \geq \sum_{j \in T(a)} q_{raj} + \sum_{a' \in A} q_{raa'} \forall r \in R(n), a \in A$$

Constraint 5-19: DC Inventory Balance

Constraint 5-20 ensures that all merchandise flows through a logistic platform and is not stored at the logistic platform.

$$\sum_{l' \in L} q_{rl'l} + \sum_{j \in T(l)} q_{rjl} = \sum_{l' \in L} q_{rll'} + \sum_{j \in T(l)} q_{rlj} \forall r \in R(n), l \in L$$

Constraint 5-20: LP Inventory Flow

Constraint 5-21 is an inventory balance equation that ensures shipments from country warehouses do not exceed the inventory on hand.

$$\widetilde{I}_{rm} \geq \sum_{j \in T(m)} q_{rmj} \forall r \in R(n), m \in M$$

Constraint 5-21: Warehouse Inventory Balance

Constraint 5-22 ensures that the inflow of inventory to each store is at least the total shipment quantity allocated to that store.

$$\sum_{r\in R(n)}(q_{ra(j)j}+q_{rm(j)j}+q_{rl(j)j})\geq \lambda_j+\sum_{r\in R(n)}q_{rjl(j)}\forall j\in J$$

Constraint 5-22: Store Inventory Flow

Constraint 5-23 ensures shipments of a reference are distributed to multiple stores by the maximum shipment requirement.

$$(q_{ra(j)j}+q_{rm(j)j}+q_{rl(j)j})\leq B_{rj}\forall r\in R(n),j\in J$$

Constraint 5-23: Maximum Shipment

Constraint 5-24 requires all shipments to be positive.

$$q_{raa'},q_{raj},q_{rmj},q_{rlj},q_{rjl},q_{rll'}\geq 0\forall r\in R(n),j\in J,a\in A,l,l'\in L$$

Constraint 5-24: Non- Negative Shipments

5.2.6 Levers

The only levers in this model are the costs associated with shipping and handling.

- C_S: Transportation cost between logistic platforms to stores.
- C_L: Transportation cost between logistic platforms to logistic platforms.
- C_{ML}: Transportation cost from country warehouse to store.

6 Implementation

The implementation and results of the optimization models will be discussed in this chapter. The integer programming language AMPL was used to execute the optimization model. The results of the model are exported to a database where SQL queries are utilized to manage the data. In this chapter an example of the output information provided by the optimization model will be provided. Finally, an analysis of the model results will be discussed.

6.1 AMPL Files

The optimization models were formulated using a mixed integer program model coded in the programming language AMPL and solved using IBM ILOG CPLEX linear programming Solver. Each optimization model contains three files in which MIT4_PAIS represents Model I and

MIT4_TIENDA represents Model II. The AMPL code for all six files can be found in the Appendix Section 8.1 and 0.

- MIT4_PAIS.mod and MIT4_TIENDA.mod: These files contain the optimization model formulation including variables, parameters, objection functions, constraint declarations and definitions.
- MIT4_PAIS.dat and MIT4_TIENDA.dat: The purpose of this module is to load external input parameters into the model through SQL queries. Input data includes the forecasts, inventory position, maximum shipment requirement and related variables.
- MIT4_PAIS.run and MIT4_TIENDA.run: The run file invokes the model and data components through the Solver and exports the results to an internal database. This file serves as the working file once the model is complete. Any adjustments to the levers such as costs, clearance group, length of the clearance period and others mentioned in Section 1 are configured in the run file.

6.2 Model I Output

Model I is managed at a group level for all countries in the network and the objective function is to maximize global revenues. The output of the model provides estimated country sales and shipment recommendations at a group cluster level from DC to DC and DC to country. In this first model, the most important decision variable is q_{mgn}, as it determines the total DC shipments to a country and is the main input for Model II.

Table 3 represents the inventory flow for group 8 and cluster 4590. In this example, the model lists each country and its inventory position from the current date (Inv_RS) to the end of clearance season (Final_Inv). Sale_RS and Sales_CS are the predicted sales for the regular and clearance season respectfully. Inv_CS is the estimated inventory at the start of the clearance season. For Country 1, the model suggests to ship 3538 units (q_{mgn}) to cover all sales leaving no excess inventory in the country. If the user wants the inventory in the distribution center to be depleted, some countries may receive excess merchandise as the case of Country 4.

Country	Group	Cluster	Inv RS	Sales RS	q_{mgn}	Inv CS	Sales CS	Final Inv
1	8	4590	17331	3539	3528	17320	17320	0
2	8	4590	1503	154	479	1828	1828	0
3	8	4590	5293	254	992	6031	6031	0
4	8	4590	7883	741	851	7993	7911	82

Table 3: Model I Country Detailed Shipments

6.3 Model II Output

Model II uses the shipment recommendation for each country from Model I and determines the shipment size by store and article. The objective function is to minimize shipment costs between stores, warehouses and distribution centers. Table 4 is an example of the distribution decision for article 318990. In the example below, Store 64 will not receive any inbound shipments and has excess inventory that will be shipped to another store. Store 3230 on the other hand, has expected sales of 37 units and will be replenished by multiple locations.

Country	Store	Group	Cluster	Article	Stock	DC Shipments	CW Shipments	Store Shipments	Store Outbound Shipment
11	64	11	2990	318990	30	0	0	0	10
6	3230	11	2990	318990	2	20	10	5	0

Table 4: Model II Store Detailed Shipment

Distribution center reports are also available at a detailed store and article level. The reports primarily demonstrate the inventory flow between distribution centers and store shipments. Table 5 is an example of the aggregated inventory position for two distribution centers.

DC	Stock	Store Shipments	DC Shipments	Final Stock
1	954913	682939	202221	122076
2	518052	1136758	656	18555

Table 5: Model II DC Total Shipments

6.4 Optimization Model Analysis

The optimization model was tested to predict the end of the regular season and clearance season sales for the winter 2010 campaign. This prediction occurred in December 2009, a few weeks prior to the start of the clearance season. In the following sections an analysis of the forecast errors of the legacy process and the optimization model in respect to actual sales at the closing of the campaign is discussed.

6.4.1 Legacy Process and Proposed Model Fit

To ensure the model aligned with the decision of the current legacy process, both country forecasts were plotted in a bivariate fit. As illustrated in Figure 9, the MIT solution in total unit sales has a linear fit with the legacy process. The correlation of the models signified that the proposed solution supports the current practice and there is not a drastic difference.

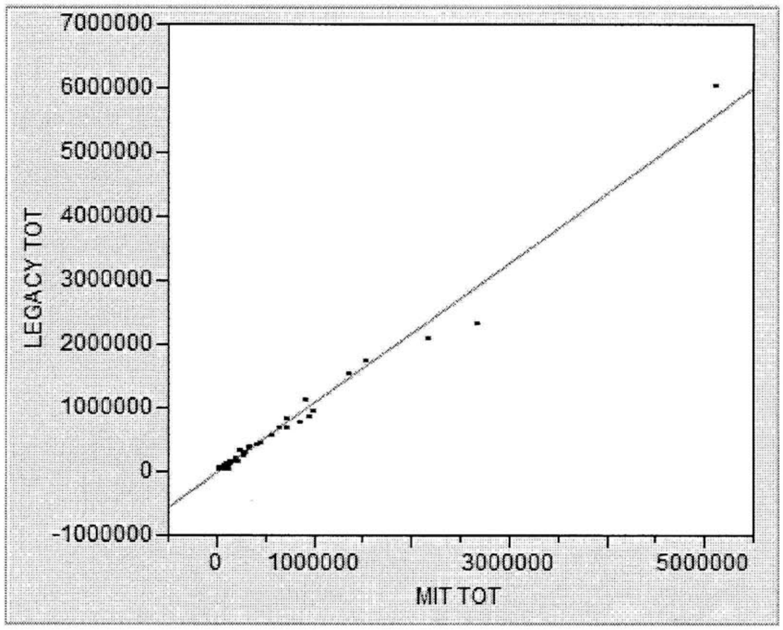


Figure 9: Unit Sales by Country Linear Fit

6.4.2 Winter 2010 Forecast Error Overview

The forecast error for total global unit sales was 4 percent for the optimization model and 8 percent for the legacy model. The distribution of the country errors for the optimization model and the legacy model can be found in Figure 10 and Figure 11respectfully.

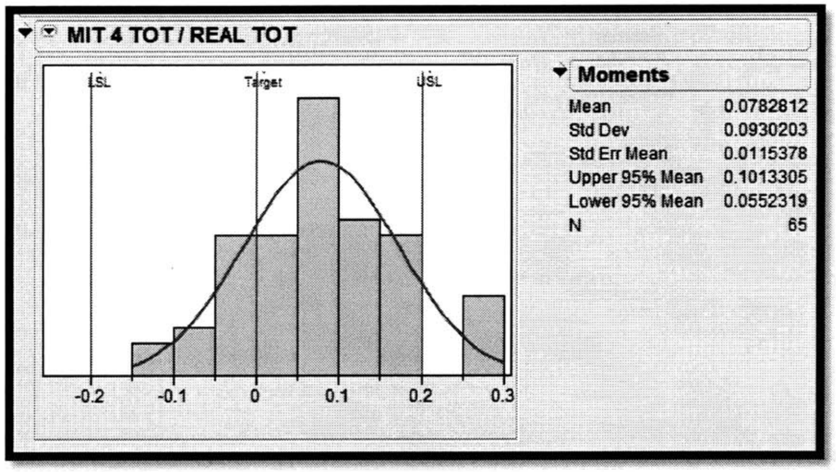


Figure 10: Optimization Model Descriptive Statistics

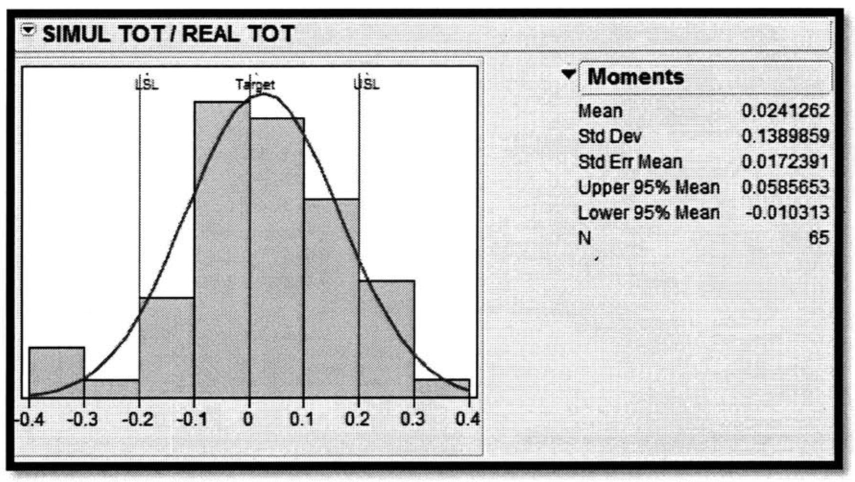


Figure 11: Legacy Process Descriptive Statistics

Both processes are normally distributed, with the average country error for the test model was 8 percent in comparison to a 2 percent average for the legacy process. The optimization model on the other hand, had a much smaller standard deviation than the legacy model at 0.09 compared to 0.14. Through a capability test analysis, with a lower specification limit (LSL) of -0.2 and an upper specification limit (USL) of 0.2, 92 percent of the countries fell within this error range with the optimization model predictions. For the legacy process, only 82 percent of the forecast errors are within this span as seen in Figure 12 and Figure 13.

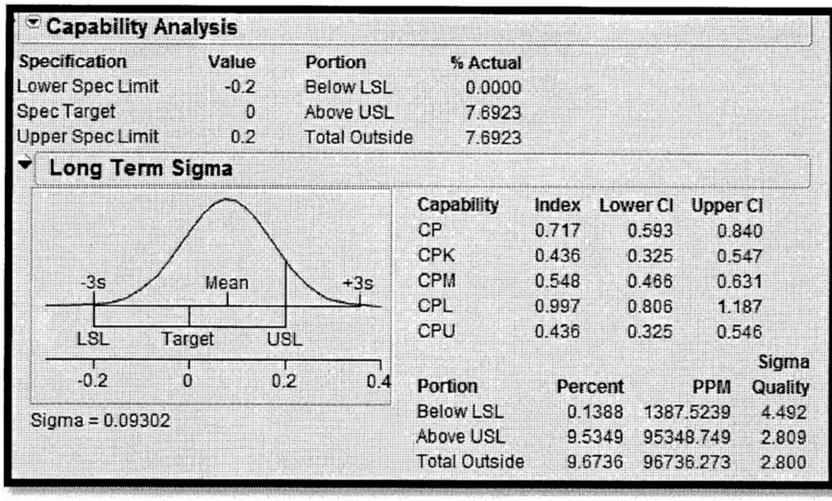


Figure 12: Optimization Model Capability Analysis

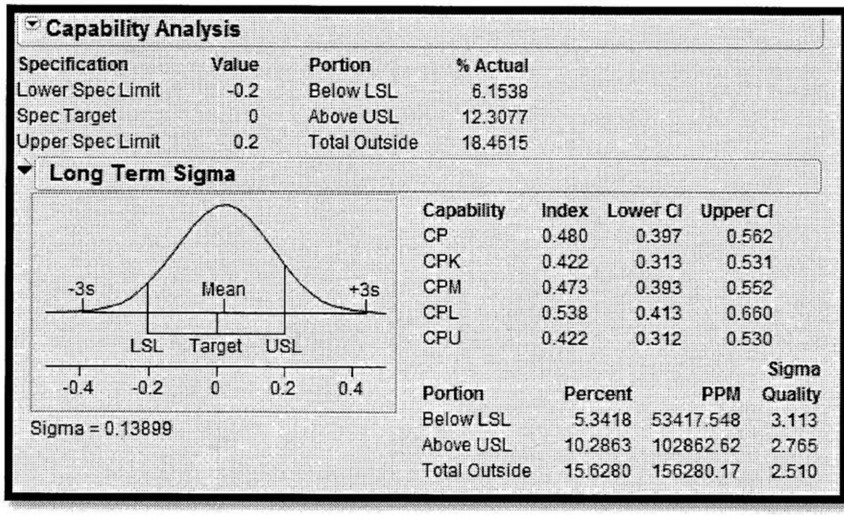


Figure 13: Legacy Process Capability Analysis

There were a total of 70 countries tested, through statistical analysis five countries were identified as outliers. These outliers represented countries in which the forecast was unable to give an adequate estimation. In some cases it was because the country was a new market entry for Zara and did not have any historical sales data. While other cases had external factors that affected country sales creating a large gap between the forecast.

The regular season forecasts errors for the optimization model (MIT) did not verify the improvement to the legacy process as it did in the test discussed in Section 4.2.3. As shown in Table 6, the legacy process had a 3 percent error for global sales during the regular season while the optimization model was 76 percent. If one looks at the average error per country the error value is extremely high over 90% in both models with very high standard deviations. Therefore, the current methodology for estimating the end-of-season sales needs to be adjusted to provide a more accurate estimation in both models.

	REGULAR SEASON			CLEARANCE SEASON		
	Global	Average	Std Dev	Global	Average	Std Dev
LEGACY	3%	94%	2.4	8%	2%	0.14
MIT	76%	118%	1.09	-2%	3%	0.09

Table 6: Legacy vs. MIT Forecast Error

Clearance season estimations were improved using the MIT optimization model. The total global sales error was -2 percent for the optimization model and 8 percent for the legacy process. The average error per store was similar at 2 and 3 percent. The main difference is that the variability of the error was reduced using the optimization model making the spread much smaller. To view the descriptive statistics on both the regular and clearance season data refer to Appendix 8.1.

6.4.3 Store Forecast Error

As discussed in Section 3, the legacy process is based on a top down approach in which the store forecast is estimated followed by the detailed article shipments. The pricing forecast however is a bottom up approach that focuses on estimating sales for each group and article then sorted at a store level. Viewing the descriptive statistics of the store forecast errors, the average error for the legacy process was about 13 percent with a standard deviation of 0.22 as shown in

Figure 14. The mean error for the optimization model was 5 percent with a standard deviation of 0.38 illustrated in Figure 15.

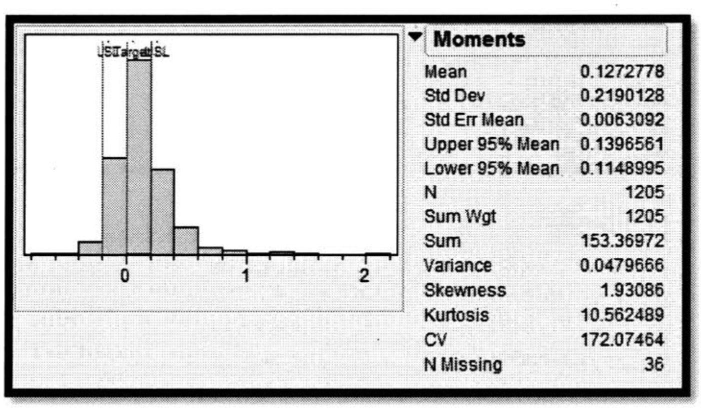
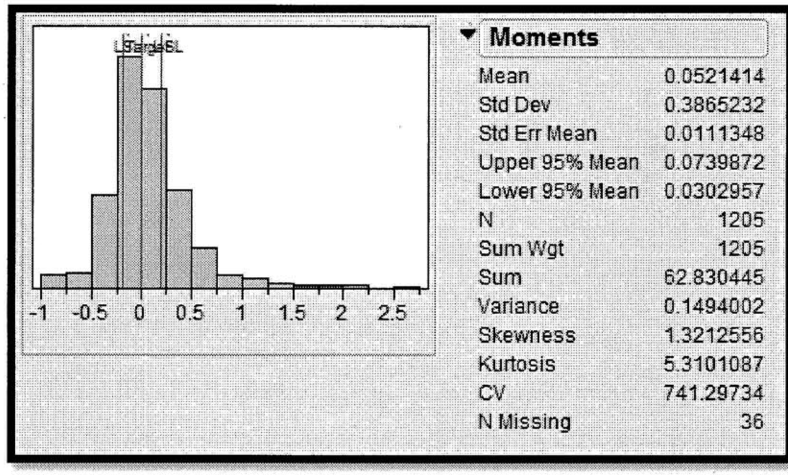


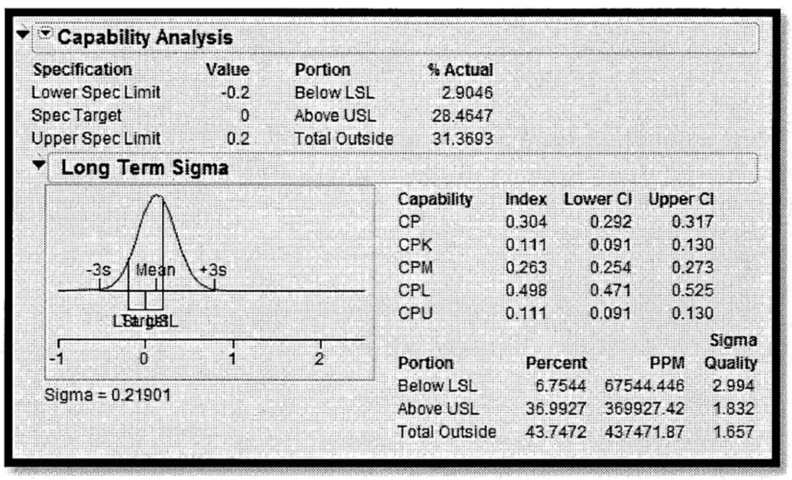
Figure 14: Legacy Store Descriptive Statistics



Moments	
Mean	0.0521414
Std Dev	0.3865232
Std Err Mean	0.0111348
Upper 95% Mean	0.0739872
Lower 95% Mean	0.0302957
N	1205
Sum Wgt	1205
Sum	62.830445
Variance	0.1494002
Skewness	1.3212556
Kurtosis	5.3101087
CV	741.29734
N Missing	36

Figure 15: Optimization Model Store Descriptive Statistics

The capability analysis of the models in Figure 16 and Figure 17 show that the legacy process mean is further away from the target value of zero error in comparison to the optimization model. However, 30 percent of the stores in the legacy process have errors outside of the specification limits while 50 percent of stores fall outside in the optimization model. The large variance in the optimization model is the driving force behind this difference.



Specification	Value	Portion	% Actual
Lower Spec Limit	-0.2	Below LSL	2.9046
Spec Target	0	Above USL	28.4647
Upper Spec Limit	0.2	Total Outside	31.3693

Capability	Index	Lower CI	Upper CI
CP	0.304	0.292	0.317
CPK	0.111	0.091	0.130
CPM	0.263	0.254	0.273
CPL	0.498	0.471	0.525
CPU	0.111	0.091	0.130

Portion	Percent	PPM	Sigma Quality
Below LSL	6.7544	67544.446	2.994
Above USL	36.9927	369927.42	1.832
Total Outside	43.7472	437471.87	1.657

Figure 16: Legacy Store Capability Analysis

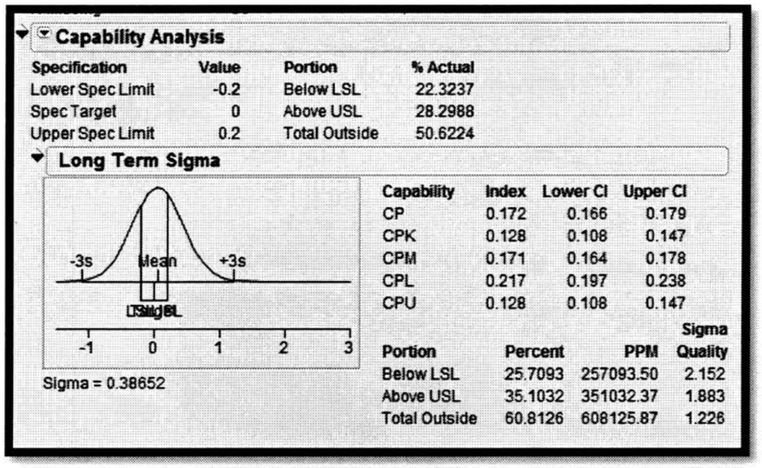


Figure 17: Optimization Model Store Capability Analysis

6.4.4 Product Line Forecast Error

One of the advantages of the optimization model to the legacy forecasting method is that predictions can be evaluated at a product line, group or article level. The current process only estimates total unit sales at a store level. As mentioned in Section 2.3 there are six product lines with a total of 21 groups. Table 7 exhibits the forecast errors at a global product line level and the average error per group within the product line.

	REGULAR SEASON			CLEARANCE SEASON			TOTAL SALES		
Product Line	Total Global	Average	Std Dev	Total Global	Average	Std Dev	Total Global	Average	Std Dev
A	-8%	81%	1.7	37%	27%	0.19	29%	27%	0.15
B	86%	161%	4	-7%	0%	0.18	0%	6%	0.15
C	42%	64%	0.77	-1%	3%	0.12	4%	5%	0.11
D	48%	130%	2.5	-8%	-6%	0.23	-3%	-2%	0.22
E	192%	237%	2.13	-3%	7%	0.14	9%	14%	0.14
F	77%	213%	5.2	0%	7%	0.19	6%	13%	0.18

Table 7: Product Line Forecast Error Summary

The table demonstrates once again that regular season forecasts had large over estimations of sales and must be adjusted to reduce the gap. The clearance season estimations as well as the total unit sales estimations are in acceptable ranges, mainly single digit differences. Product line A has a significantly higher disparity with the sales prediction. This is primarily due to the product line's characteristics as it carries a greater assortment of articles and there is very low volume of each article in comparison with the other lines. In viewing the total unit sales by product line in Figure 18 it is apparent that these errors for produce line A are not significant as it only represents 4 percent of the total unit sales.

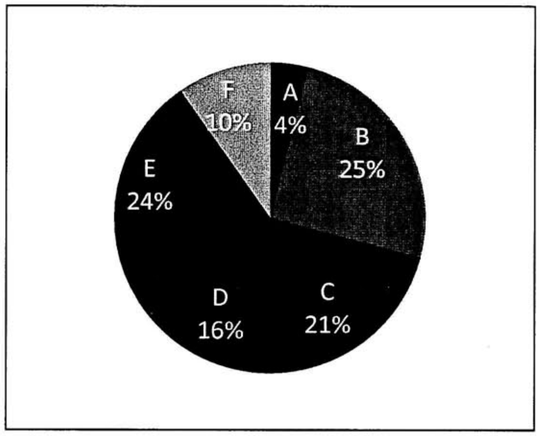


Figure 18: Product Line Unit Sales

Viewing the forecast errors at a group level, it is evident that some groups such as 2, 13, and 14 have a wider error span. These groups belong to product line D and F, the two that have the largest standard deviation. Adjustments to the forecasting model will be necessary for these individual groups to be able to reduce the variability for each product line.

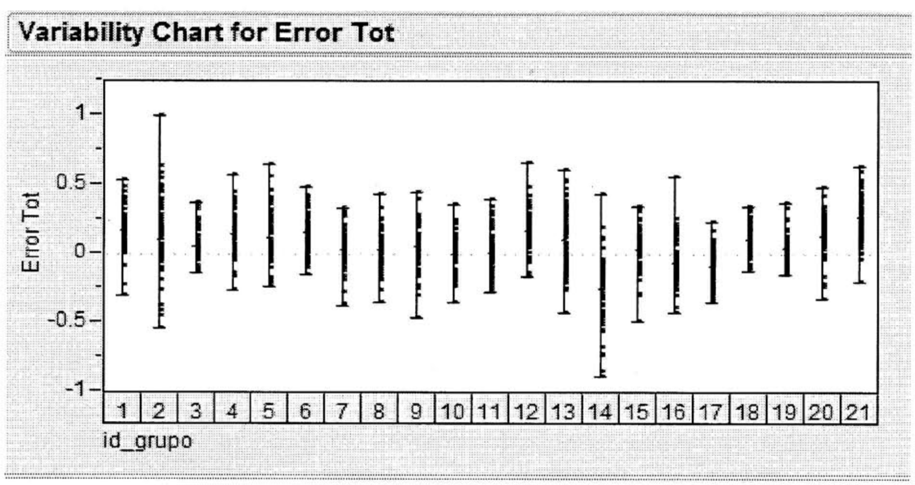


Figure 19: Group Error Chart

7 Conclusions

Clearance inventory management and pricing are important and challenging problems in retailing. Final sales depend on the pricing, seasonal effects, and the assortment of goods available to the customer. Errors in inventory distribution and clearance pricing result in loss of potential revenue or excess inventory to be liquidated. Supply chain models often concentrate on logistic costs and inventory management. Sequentially an organization may use a marketing model that focus on pricing strategies and impact on sales volume and revenues. In order to effectively and efficiently maximize clearance revenues an integrated model must be developed to make joint marketing and production decisions.

The case study on Zara was an 11 month analysis on improving the inventory distribution decisions for the clearance season while considering price markdowns. An optimization model was created that maximized revenues by considering potential pricing decisions with the current inventory position. The model used a forecast that predicted sales at a product line and group level for each country and then desegregated the sales to a store and article level. Both the forecast and the optimization models are designed to facilitate distribution decisions up to two months prior to the start of the clearance season. It can be updated on a daily or weekly basis to capture accurate point of sale data.

7.1 Recommendations for Zara

At the end of the winter 2010 clearance season a forecast error analysis was conducted to compare the legacy process and the proposed solution. It was found that the optimization model had a global sales error of 4 percent an improvement to the 8 percent of the legacy process. The data showed that only five countries had forecast errors above 20 percent in the proposed solution. Whereas the legacy process had 11 countries outside the upper and lower specification limits.

In the capability analysis at a store level showed that the optimization model had 50 percent of stores outside of the LSL and USL. When viewing the capability analysis of each product line, found in Section 8.1, the percentages of outliers ranged from 7 to 34. This of course is not including product line A. The significance of this analysis is that the model is more capable of predicting sales at a product line and group level than at the store level. The conclusion aligns with the models bottom up approach. The question is what is more important to be precise on store sales or group sales.

To improve the optimization model, the recommendation to Zara is to improve the regular season sales forecast and decrease the variance in the store and group errors. Regular season forecasts had a tendency to overestimate sales. When developing the regular season forecast discussed in Section 4.2 a straight line *delta* approach was used and proved to be sufficient. However, the initial test was conducted to estimate sales two months in advance. The forecast was tested in December, a couple of weeks to a few days before clearance season started for the various countries. Therefore, the *delta* needs to consider the drop in sales that occurs at the tail end of the regular season as the clearance season approaches to capture a true sales estimate. There was a large variance in the forecast errors at a store level for the optimization model. This variance can be reduced by changing the calculation of parameter B_{rj} in Model II that desegregates the reference shipments to a store level. Another option is to develop a forecast that is based on store sales not group sales and use that forecast as an input to the optimization models. Lastly, there is room to improve the product line errors by understanding group sales trends and modifying the forecast model to adjust for these changes.

It is important to note that the model has a disadvantage when comparing to the actual sales. This is because the pricing and inventory distribution decisions were based from the legacy process and the model did not play a role in these decisions. Therefore these large errors at a store or group level are arbitrary and do not confirm the models validity.

Overall, the model proves to be an improvement over the legacy process in estimating country unit sales. An advantage of the model is that the forecast is derived from point of sale data and considers price elasticity of demand. The optimization model makes inventory distribution decisions based on dynamic pricing and logistic costs. All of which is currently not available in the legacy process.

Another advantage of the model is its automation that will allows the organization to create and test various scenarios such as modifications to the inventory levels, duration of the clearance periods, or a new pricing policy. Using historical data the model can be used to predict clearance sales one to ten weeks prior to the start of the sale and the organization can determine the optimal week the distribution decisions should begin based on the forecast errors.

A pilot test needs to be conducted to test the legitimacy of the model. Measuring success of the optimization model can be determined by the duration of the clearance season, if it lasts longer or shorter than average in a given country. Another method of measuring success is the number of price markdowns that occurred during the clearance season and the average total discount per country. The true measure of success however, is if the optimization model helped increase revenues. There is a pilot test scheduled for the summer 2010 campaign to further analyze the benefits of the linear optimization model.

7.2 Future Work

Clearance pricing and distribution model are important for the retail industry especially those that sell perishable items that quickly become obsolete such as fashion goods and electronics. In these industries it is important to create supply chain tools that facilitate current business practices. Does the company make decisions based on store sales or product line sales? What are the inventory and ordering policies?

Further related theoretical work can be expanded from this research by exploring solutions to forecast discount sales during the regular season such as a weekend holiday promotion. The same methodology can also be incorporated for retailers who want to markup merchandise that are hot sellers during a campaign. Finally the work can be expanded to traditional retailers outside of fast fashion to department stores and other mass retailers.

Supply chain tools such as a linear optimization model can save organizations millions of dollars if a company is willing to change the decision making methodology within the company culture. The most challenging part of a solution is implementation and having a leadership team that is receptive to change makes a difference in the final outcome.

8 Appendix

8.1 Descriptive Statistics for Forecast Errors

8.1.1 Regular Season Statistics

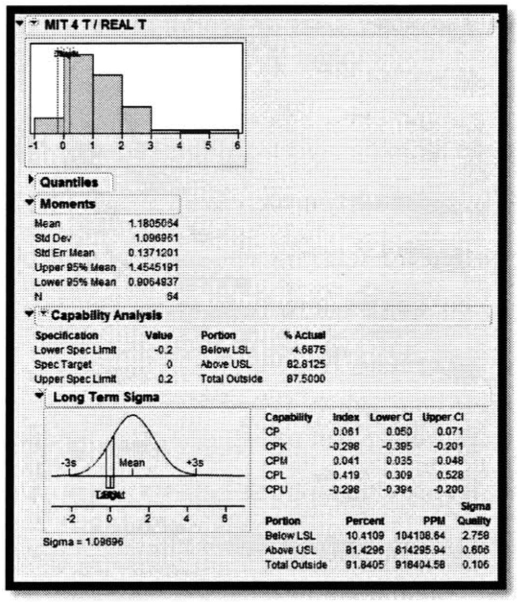


Figure 20: Optimization Model Regular Season Statistics

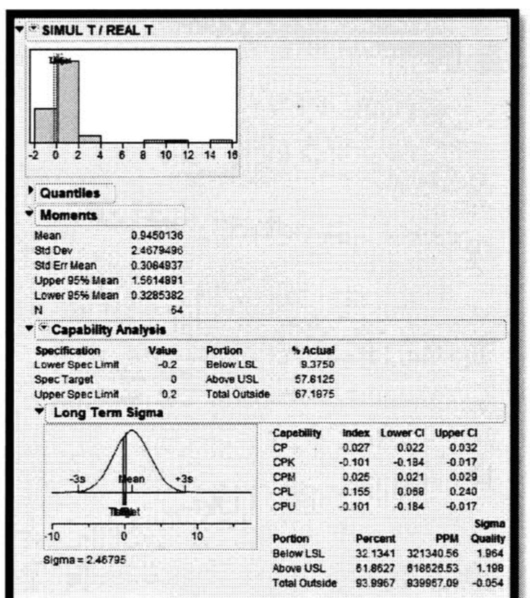


Figure 21: Legacy Regular Season Statistics

8.1.2 *Clearance Season Statistics*

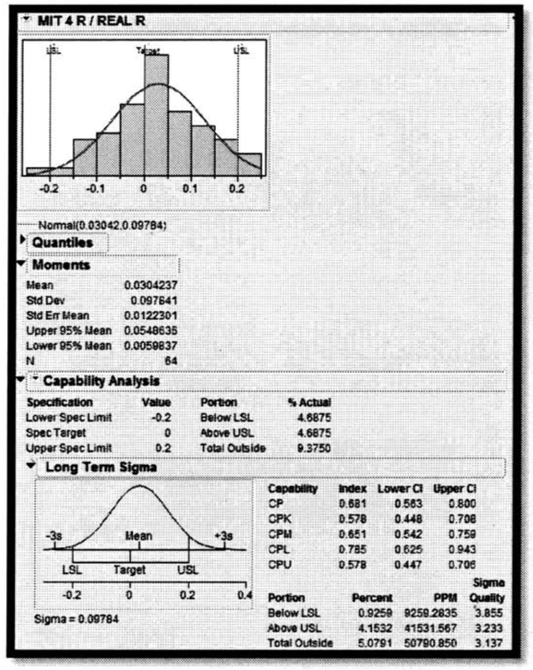


Figure 22: Optimization Model Clearance Season Statistics

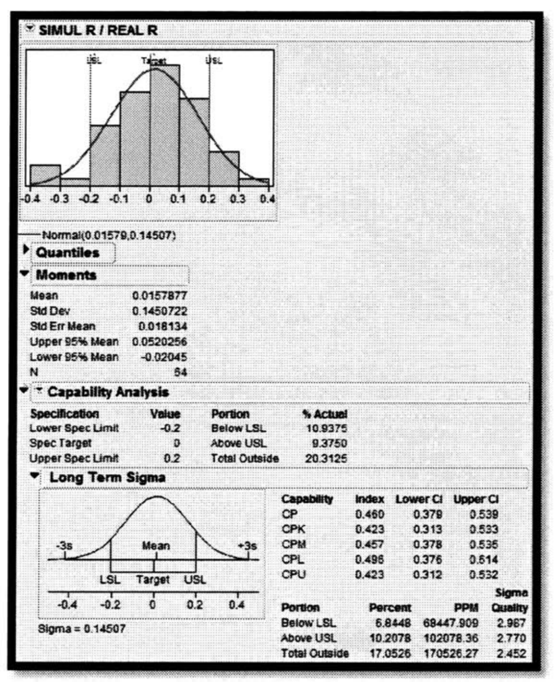


Figure 23: Legacy Clearance Season Statistics

8.1.3 Product Line Statistics

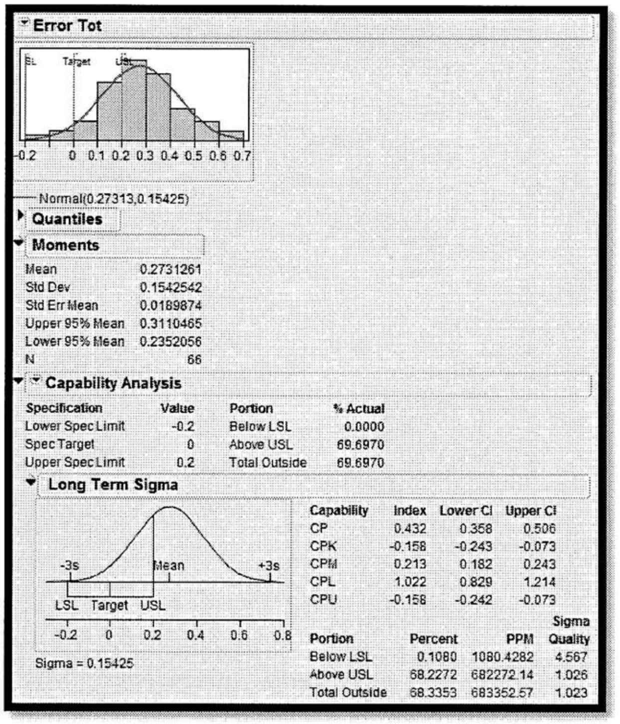


Figure 24: Product Line A Statistics

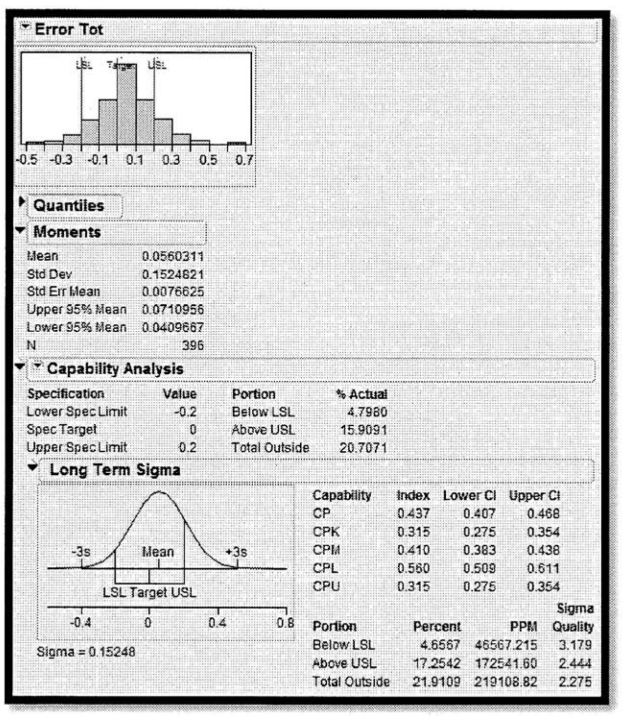


Figure 25: Product Line B Statistics

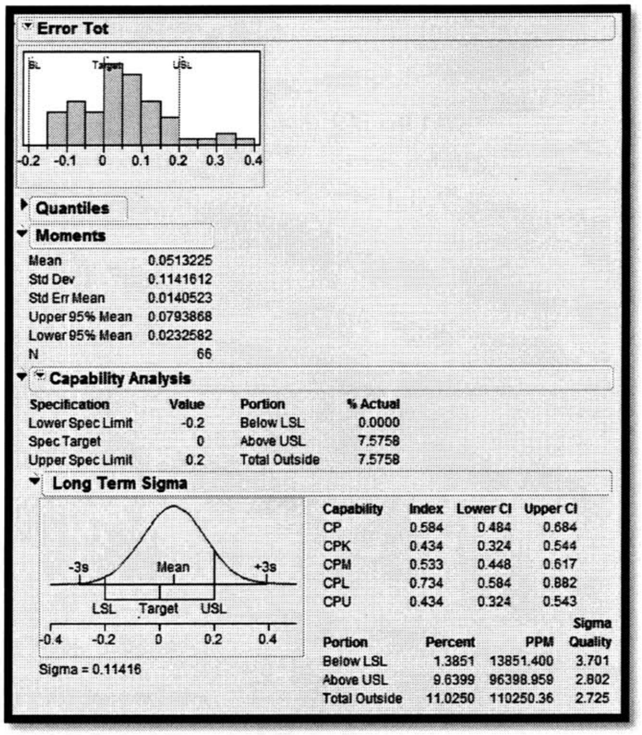


Figure 26: Product Line C Statistics

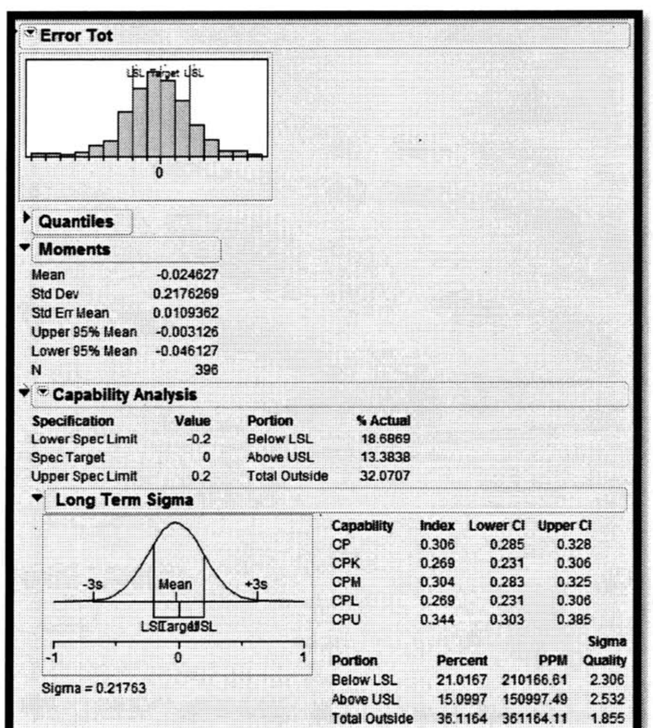


Figure 27: Product Line D Statistics

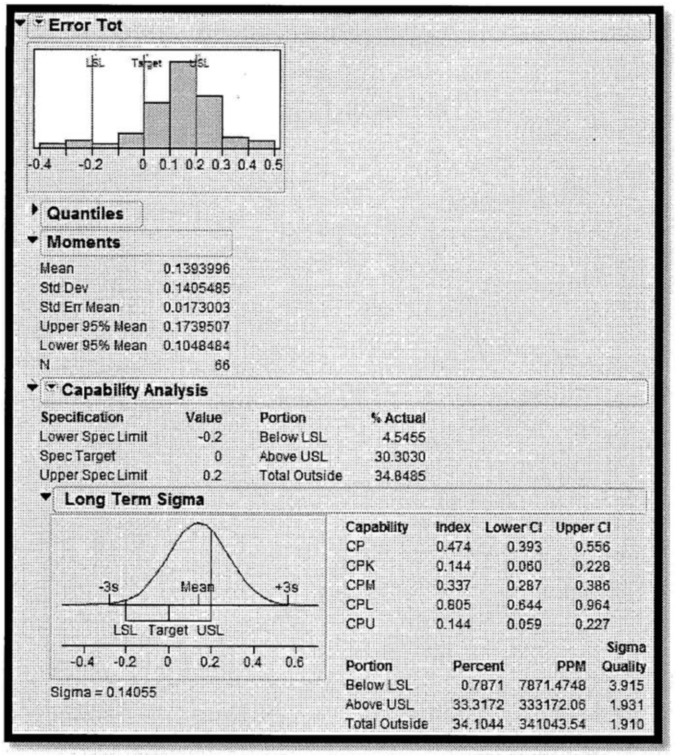


Figure 28: Product Line E Statistics

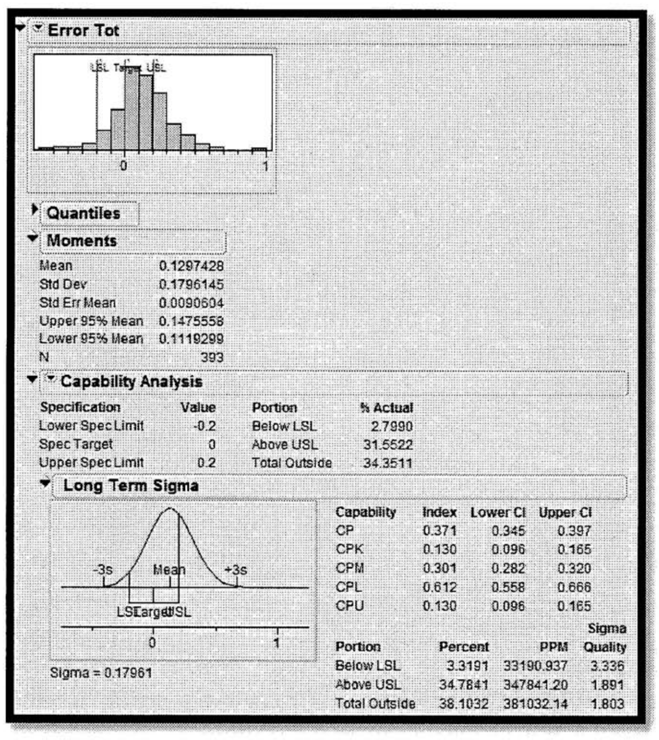


Figure 29: Product Line F Statistics

8.2 Model I AMPL Files

8.2.1 Model I Data File

MIT4_PAIS.dat

```
#**********************************************************************************************
# MIT4 CARGA DE DATOS MODELO II - COUNTRY MODEL WITH MIT2                                      *
# FECHA: 11/11/2009                                                                            *
# VERSION: 2                                                                                   *
#                                                                            ORIETTA VERDUGO *
#**********************************************************************************************

# UPDATES
# 10/11/09 -  UPDATED TABLES


###     DECLARATION OF TABLES
###-----------------------------------------------


table PaisesDC {dc in DATA_DC} IN "ODBC" "MIT4" ("SQL=select distinct b.id_pais from
maestros.centro_distribucion_tienda_seccion a inner join maestros.tienda b on a.id_tienda = b.id_tienda and a.id_tipo_seccion =
" & $SECCION & " and a.id_campana = " & $ANO_CAMPANA & " and a.id_centro_distribucion = " & dc & " inner join
mit4.stock_pais_cat s on b.id_pais = s.id_pais"  & " where b.id_pais in (" & $PAISES & ")"): PAIS_DC[dc] <- [id_pais];

table Paises IN "ODBC" "MIT4" ("SQL=select distinct id_pais from mit4.stock_pais_cat where id_tipo_seccion = " & $SECCION & "
and id_campana = " & $ANO_CAMPANA & " and id_pais in (" & $PAISES & ")"): PAIS <- [id_pais];

read table PaisesDC;
read table Paises;

table Clusters {m in PAIS, g in GROUP} IN "ODBC" "MIT4" ("SQL=select distinct categoria from mit4.stock_pais_cat where id_grupo
= " & g  & " and id_centro_compra = " & $CENTROCOMPRA & " and id_pais = " & m): CLUSTER[g,m] <- [categoria];

table ClustersAlmacen {dc in DATA_DC,g in GROUP} IN "ODBC" "MIT4" ("SQL=select distinct categoria from (select distinct
categoria from tmp.stock_almacen_cat where id_grupo = " & g  & " and id_centro_compra = " & $CENTROCOMPRA & " and
id_centro_distribucion in (" & $DCS & ") union select distinct categoria from stock_pais_cat where id_grupo =  " & g  & " and
id_centro_compra = " & $CENTROCOMPRA & " ) as a"): CLUSTER_ALMACEN[g, dc] <- [categoria];

table Inventario IN "ODBC" "MIT4" ("SQL=select id_pais, id_grupo, categoria, stock from mit4.stock_pais_cat where " & $where ):
[id_pais, id_grupo, categoria], Imgn ~ stock;

#Precios of Demand
table Precios  {m in PAIS, g in GROUP} IN "ODBC" "MIT4" ("SQL=select distinct precio from mit4.prediccion_saldo_cat_precio
where id_pais = " & m &" and  id_grupo = " & g ): PRECIOS [m,g] <- [precio];

#Precios of Sales
#table Precios  {m in PAIS, g in GROUP} IN "ODBC" "MIT4" ("SQL=select distinct precio from
mit4.prediccion_saldo_cat_precio_ventas where id_pais = " & m &" and  id_grupo = " & g ): PRECIOS [m,g] <- [precio];

table PrecioSaldero {g in GROUP} IN "ODBC" "MIT4" ("SQL=SELECT id_grupo, precio FROM mit2.opt_precios_comerciales WHERE
id_grupo = " & g & " AND ESPRECIOSALDERO = 1 and id_pais = 2"): [id_grupo], psaldero[g] ~ precio;

read table Clusters;
read table ClustersAlmacen;
read table Inventario;
read table Precios;
read table PrecioSaldero;


# Ratio
for {m in PAIS, g in GROUP, n in CLUSTER[g,m]} {
        let Bmgn[m,g,n] := 1;
}

table Ratio IN "ODBC" "MIT4" ("SQL=SELECT id_pais, id_grupo, categoria, ratio FROM mit4.ratio WHERE " & $where): [id_pais,
id_grupo, categoria], Bmgn ~ ratio;

read table Ratio;



#***** MIT 4 ESTIMACION (Venta en temporada y Demanda con cortes medios y kappa en saldo)
table PF_Temporada  IN "ODBC" "MIT4" ("SQL=select id_pais, id_grupo, categoria, precio, prediccion as prediccion_temporada from
mit4.prediccion_temporada_cat_venta where " & $where ): [id_pais, id_grupo, categoria], Pmgn ~ precio, Fmgn ~
prediccion_temporada;

#Initially assignes a zero demand for all pricing possibilities then table F_Saldo will give demand to the prices that are
available leavign all others at zero
for {m in PAIS, g in GROUP, n in CLUSTER[g,m], k in PRECIOS[m,g]} {
        let F_mgn[m,g,n,k] := 0;
}

#F of Demand
table F_Saldo  IN "ODBC" "MIT4" ("SQL=select id_pais, id_grupo, categoria, precio, f as prediccion_saldo from
mit4.prediccion_saldo_cat_precio where " & $where ): [id_pais, id_grupo, categoria,precio], F_mgn ~ prediccion_saldo;

#F of Sales
#table F_Saldo  IN "ODBC" "MIT4" ("SQL=select id_pais, id_grupo, categoria, precio, f as prediccion_saldo from
mit4.prediccion_saldo_cat_precio_ventas where " & $where ): [id_pais, id_grupo, categoria,precio], F_mgn ~ prediccion_saldo;

# ****Table with inventory

#table InventarioAlmacenes IN "ODBC" "MIT4" ("SQL=select id_centro_distribucion, id_grupo, categoria, stock from
tmp.stock_almacen_cat where " & " id_centro_compra = " & $CENTROCOMPRA & " and id_tipo_seccion = " & $SECCION &" and id_campana
= " & $ANO_CAMPANA & " and id_centro_distribucion in (" & $DCS & ")"): [id_centro_distribucion, id_grupo, categoria], Iagn ~
stock;

# ****Stock with real shipments (imaginary wareohouse) use for testing only***

table InventarioAlmacenes IN "ODBC" "MIT4" ("SQL=select id_centro_distribucion, id_grupo, categoria, stock from
mit4.stock_almacen_cat_envios where " & " id_centro_compra = " & $CENTROCOMPRA & " and id_tipo_seccion = " & $SECCION &" and
id_campana = " & $ANO_CAMPANA & " and id_centro_distribucion in (" & $DCS & ")"): [id_centro_distribucion, id_grupo,
```

le

```
categoria], Iagn ~ stock;

table PrecioAlmacenes {dc in DATA_DC} IN "ODBC" "MIT4" ("SQL=select distinct " & dc &" as id_centro_distribucion, id_grupo,
categoria, precio as precio_almacen from tmp.stock_almacen_cat where " & " id_centro_compra = " & $CENTROCOMPRA & " and
id_tipo_seccion = " & $SECCION &" and id_campana = " & $ANO_CAMPANA & " and id_centro_distribucion in (" & $DCS & ")"):
[id_centro_distribucion, id_grupo, categoria], Pagn ~ precio_almacen;


###     READING TABLES
###-------------------------------------------------
read table PF_Temporada;
read table F_Saldo;

#Gives an inventory value of zero for all clusters in stores that are not in DCs
for{dc in DATA_DC, g in GROUP, n in CLUSTER_ALMACEN[g, dc]} {
        let Iagn[dc,g,n] := 0;
        let Pagn[dc,g,n] := 0;
}

read table InventarioAlmacenes;
read table PrecioAlmacenes;



###     TEST LEVERS
###-------------------------------------------------

#This is to assign a percentage of DC inventory to the 10 countries
#let {dc in DATA_DC, g in GROUP, n in CLUSTER_ALMACEN[g, dc]} Iagn [dc,g,n] := Iagn [dc,g,n]/24.31;


###     OTHER COMMANDS
###-------------------------------------------------


# If inventory in country is less than zero set it to zero
for {m in PAIS, g in GROUP, n in CLUSTER[g,m]} {
        if Imgn[m,g,n] < 0 then let Imgn[m,g,n] := 0;
}


#If inventory in DC is less than zero set it to zero
for {dc in DATA_DC, g in GROUP, n in CLUSTER_ALMACEN[g, dc]} {
        if Iagn [dc,g,n] <0 then let Iagn[dc,g,n] := 0;
}


#If DC inventory is less than or equal to zero set shipments from that DC to zero and set shipments within the same DC to zero
for {dc in DATA_DC, g in GROUP, n in CLUSTER_ALMACEN[g, dc]} {
        if Iagn [dc,g,n] <=0 then {
                for {m in PAIS} {
                        fix Q_mgn[dc,m,g,n] := 0;
                }
                for {dc2 in DATA_DC} {
                        fix Q_agn[dc,dc2,g,n] := 0;
                }
        }
        for {dc2 in DATA_DC} {
                if dc = dc2 then {
                        fix Q_agn[dc,dc2,g,n] := 0;
                }
        }
}

# Restricts DC's to only ship directly to county within its domain
for {dc in DATA_DC, m in PAIS} {
        if m not in PAIS_DC[dc] then {
                for  {g in GROUP, n in CLUSTER_ALMACEN[g,dc]} {
                        fix Q_mgn[dc, m, g, n] := 0;
                }
        }
}
```

8.2.2 Model I Run File

```
                                        MIT4_PAIS.run
#*******************************************************************************************
# MIT4 PLANTILLA DE EJECUCION MODELO II- COUNTRY MODEL WITH MIT2                           *
# FECHA: 11/11/2009                                                                         *
# VERSION: 2                                                                                *
#                                                                         ORIETTA VERDUGO *
#*******************************************************************************************

#Version changes and updates
#11/11/09: Updated reports to relfect true revenues and inventory
#10/11/09: Included AMPL simulation reports

reset;

model MIT4_PAIS.mod;


###     LEVERS
###-----------------------------------------------------
let D := 4;                             # Duration of each period (multiplied by 7 days a week)


include MIT4_PAIS_I09.dat;


###     LEVERS
###-----------------------------------------------------

let CA := .01;                          # Cost of shipments between DCs
let CS := .02;                          # Cost of shipments to countries

let SA := .01;                          # Percent of DC inventory to salvage
let ST := .10;                          # Percent of total inventory to salvage

let Qm := 10;                           # Minimum shipment to each counry

let mindiscount := 0;                   # Minimum discount
let WC := 4;                            # Number of periods during clearance sales

let tighten_slack := 0.9999;            # Discount factor that gives an upper bounds to L_mgn
let tiempo := 0.82;                     # *******Need to make a table??


###     AMPL SIMULATION OPTIONS
###-----------------------------------------------------
option show_stats 1;
option cplex_options ("bestnode ");
option cplex_options ("bestnode mipgap= 0.0020 timelimit= 5400 mipdisplay= 2");

solve;

printf "Ampl Time: %0.2f\tSolver Time: %0.2f\n\n",_ampl_time, _solve_time;


###     RESULTS
###-----------------------------------------------------;

print;
printf "========Cash Flow========\n\n";
printf "Objective Function: %.2f \n\n", REVENUES + DISC_FUT_PROFIT + DISC_PROFIT_SALDERO - DC2DC_COSTS - SHIPMENT;
printf "Total Revenue: %.2f \n\n",REVENUES + DISC_FUT_PROFIT + DISC_PROFIT_SALDERO;
printf "Total DC2DC Costs: %.2f \n\n", DC2DC_COSTS;
printf "Total Shipment Costs: %.2f \n\n", SHIPMENT;

printf "=====Inventory Report (Units)=====\n\n";

printf "Total Initial Inventory: %.0f \n\n", sum {dc in DATA_DC, g in GROUP, n in CLUSTER[g]} Iagn [dc,g,n]
        + sum {m in PAIS, g in GROUP, n in CLUSTER[g]} (Imgn [m,g,n]);

printf "Total DC Transfers: %.0f \n\n", sum {dc in DATA_DC, dc2 in DATA_DC, g in GROUP, n in CLUSTER[g]} Q_agn [dc,
dc2,g,n];

printf "Total Salvaged: %.0f \n\n", sum {m in PAIS, g in GROUP, n in CLUSTER[g]}
        (I_mgn [WC+1,m,g,n])+(sum {dc in DATA_DC, g in GROUP, n in CLUSTER[g]} Iagn [dc,g,n])
        - (sum {dc in DATA_DC,m in PAIS, g in GROUP, n in CLUSTER[g]} Q_mgn [dc,m,g,n]);

printf "Total % Salvaged: %.2f \n\n", (sum {m in PAIS, g in GROUP, n in CLUSTER[g]}
        I_mgn [WC+1,m,g,n]+ sum {dc in DATA_DC, g in GROUP, n in CLUSTER[g]} Iagn [dc,g,n]
        - sum {dc in DATA_DC,m in PAIS, g in GROUP, n in CLUSTER[g]} Q_mgn [dc,m,g,n])
        /(sum {dc in DATA_DC, g in GROUP, n in CLUSTER[g]} Iagn [dc,g,n]
        + sum {m in PAIS, g in GROUP, n in CLUSTER[g]} Imgn [m,g,n]);

printf "DC % Salvaged: %.2f \n\n", (sum {dc in DATA_DC, g in GROUP, n in CLUSTER[g]} Iagn [dc,g,n]
        - sum {dc in DATA_DC, m in PAIS, g in GROUP, n in CLUSTER[g]} Q_mgn [dc,m,g,n])
        /(sum {dc in DATA_DC,g in GROUP, n in CLUSTER[g]} Iagn [dc,g,n]);


###     EXCEL REPORTS
###-----------------------------------------------------

# Table of shipments to and from each DC
table Envios_DC2DC OUT "ODBC" ('c:\mit4\MIT4_Excel\Pais_Campana_' & $CAMPANA & '_' & $ANO & '_G' & $GRUPO & '.xls'):
{dc in DATA_DC, dc2 in DATA_DC, g in GROUP, n in CLUSTER[g]} -> [DC_Salida, DC_Entrada, Grupo, Cluster],
Q_agn [dc, dc2,g,n] ~ Tot_Unidades, CA *  Pagn [dc, g, n] * Q_agn [dc, dc2, g, n] ~ Tot_Coste;

                                        Page 1
```

```
# Table of total units shipped to each country
table Envios_Pais OUT "ODBC" ('c:\mit4\MIT4_Excel\Pais_Campana_' & $CAMPANA & '_' & $ANO & '_G' & $GRUPO & '.xls'):
{dc in DATA_DC, g in GROUP, n in CLUSTER[g]}-> [DC, Grupo, Cluster],  Iagn [dc,g,n] ~ DC_INV_Initial,
sum {m in PAIS} Q_mgn [dc,m,g,n] ~ Envios,
(Iagn [dc,g,n] - sum {m in PAIS} (Q_mgn [dc,m,g,n])+ sum{dc2 in DATA_DC} (Q_agn [dc2, dc, g, n])- sum{dc2 in DATA_DC}
(Q_agn [dc, dc2, g, n])) ~ DCINV_Final;

# Table of inventory on hand at the beginning of clearance sales and expected sales for both regular and clearance season
table Pais_Ventas_Inv OUT "ODBC" ('c:\mit4\MIT4_Excel\Pais_Campana_' & $CAMPANA & '_' & $ANO & '_G' & $GRUPO & '.xls'):
{m in PAIS, g in GROUP, n in CLUSTER[g]} -> [Pais, Grupo, Cluster],
Imgn [m,g,n]~Inv_Temporada,
Lmgn [m,g,n] ~ Ventas_Temp,
sum {dc in DATA_DC} Q_mgn [dc,m,g,n] ~ Envios,
I_mgn [1,m,g,n] ~ Inv_Saldo,
sum {w in 1..Wc,k in PRECIOS[m,g]} L_mgn [w,m,g,n,k] ~ Ventas_Saldo,
I_mgn [Wc+1,m,g,n] ~Inv_Final;

# Table total revenues by country
table Ingreso_Pais OUT "ODBC" ('c:\mit4\MIT4_Excel\Pais_Campana_' & $CAMPANA & '_' & $ANO & '_G' & $GRUPO & '.xls'):
{m in PAIS} -> [Pais],

sum {g in GROUP, n in CLUSTER[g]}  Pmgn [m, g, n]* Lmgn [m,g,n] ~ Ingreso_Temporada,

sum{w in 1..Wc,g in GROUP, n in CLUSTER[g],k in PRECIOS [m,g]} L_mgn[w,m,g,n,k] * k ~ Ingreso_Saldo,

sum {g in GROUP, n in CLUSTER[g]} (Pmgn [m, g, n]* Lmgn [m,g,n]
+ sum{w in 1..Wc, k in PRECIOS [m,g]} L_mgn[w,m,g,n,k] * k)~ Ingreso_Tot,

sum {g in GROUP, n in CLUSTER[g]} Pmgn [m,g,n]*I_mgn [Wc+1,m,g,n] ~ Inventoria_Tot;

# Table total inventory by country
table Resume_Inv_Pais OUT "ODBC" ('c:\mit4\MIT4_Excel\Pais_Campana_' & $CAMPANA & '_' & $ANO & '_G' & $GRUPO & '.xls'):
{m in PAIS} -> [Pais],
sum {g in GROUP, n in CLUSTER[g]} Imgn [m,g,n]~Inv_Hoy,
sum {dc in DATA_DC, g in GROUP, n in CLUSTER[g]} Q_mgn [dc,m,g,n] ~ Envios_Unidades,
sum {g in GROUP, n in CLUSTER[g]}(Lmgn [m,g,n]) ~ Venta_Temporada,
sum {g in GROUP, n in CLUSTER[g]} (I_mgn [1,m,g,n]) ~ Inv_Rebajas,
sum {w in 1..Wc,g in GROUP, n in CLUSTER[g], k in PRECIOS[m,g]} L_mgn [w,m,g,n,k] ~ Venta_Saldo,
sum {g in GROUP, n in CLUSTER[g]} I_mgn [Wc+1,m,g,n] ~ Inv_Saldero;

# Table total inventory by DC
table Resume_Inv_DC OUT "ODBC" ('c:\mit4\MIT4_Excel\Pais_Campana_' & $CAMPANA & '_' & $ANO & '_G' & $GRUPO & '.xls'):
{dc in DATA_DC} -> [DC], sum {g in GROUP, n in CLUSTER[g]} Iagn [dc,g,n] ~ DC_Initial_Inv,
sum {dc2 in DATA_DC, g in GROUP, n in CLUSTER[g]} Q_agn [dc, dc2,g,n] ~ Envios_DC2DC,
sum {m in PAIS, g in GROUP, n in CLUSTER[g] } Q_mgn [dc,m,g,n] ~ Envios_Pais,
((sum {g in GROUP, n in CLUSTER[g]} Iagn [dc,g,n])
        - (sum {m in PAIS, g in GROUP, n in CLUSTER[g]} Q_mgn [dc,m,g,n])
        + (sum{dc2 in DATA_DC, g in GROUP, n in CLUSTER[g]} (Q_agn [dc2, dc, g, n]))
        - (sum{dc2 in DATA_DC,g in GROUP, n in CLUSTER[g]} (Q_agn [dc, dc2, g, n]))) ~ DCInv_Final;


###     EXECUMENT OF REPORTS
###----------------------------------------------------------
write table Envios_DC2DC;
write table Envios_Pais;
write table Pais_Ventas_Inv;
write table Ingreso_Pais;
write table Resume_Inv_Pais;
write table Resume_Inv_DC;


#option NOMBRE_FICHERO ('results_campana_' & $CAMPANA & '_' & $ANO & '_G' & $GRUPO & '.sql');

### OUTPUT TO DATABASE
###---------------------------------------------------------------------------------
#printf {g in GROUP} "delete from d_mit4.salida_pais_grupo_cluster_mini_mit2 where id_grupo = %s;\n",g >
($NOMBRE_FICHERO);
#printf {m in PAIS, g in GROUP, n in CLUSTER[g]}: "insert into d_mit4.salida_pais_grupo_cluster_mini_mit2 (id_pais,
id_grupo, categoria, stock, ventas_temporada, envios, ventas_saldo) values (%s, %s, %s, %s, %s, %s, %s);\n", m, g, n, Imgn
[m,g,n], Lmgn [m,g,n], sum {dc in DATA_DC} Q_mgn [dc,m,g,n], sum {w in 1..Wc,k in PRECIOS[m,g]} L_mgn [w,m,g,n,k] >
($NOMBRE_FICHERO);
#printf {g in GROUP} "delete from d_mit4.salida_almacen_grupo_cluster where id_grupo = %s;\n",g > ($NOMBRE_FICHERO);
#printf {dc in DATA_DC, g in GROUP, n in CLUSTER[g]}: "insert into d_mit4.salida_almacen_grupo_cluster
(id_centro_distribucion, id_grupo, categoria, stock, envios_pais, envios_almacenes, stock_final) values (%s, %s, %s, %s,
%s, %s, %s);\n", dc, g, n, Iagn [dc,g,n], sum {m in PAIS} Q_mgn [dc,m,g,n], sum {dc2 in DATA_DC} Q_agn [dc, dc2, g, n],
(Iagn [dc,g,n] - sum {m in PAIS} (Q_mgn [dc,m,g,n]) + sum{dc2 in DATA_DC} (Q_agn [dc2, dc, g, n]) - sum{dc2 in DATA_DC}
(Q_agn [dc, dc2, g, n])) > ($NOMBRE_FICHERO);
#close ($NOMBRE_FICHERO);

#shell("mysql -h axincloud1 -u ccsa -pdMhqPeGX < " & ($NOMBRE_FICHERO));
```

8.2.3 Model I Mod File

```
                                                           MIT4_PAIS.mod
#******************************************************************************
# MIT4 OPTIMIZACION MODELO II - COUNTRY MODEL WITH MIT2                       *
# FECHA: 11/11/2009                                                           *
# VERSION: 2                                                                  *
#                                                            ORIETTA VERDUGO *
#******************************************************************************

#Version changes and updates
#11/11/09 Included salvage value of excess inventory in warehouse
#10/11/09 took out Rmgn and updated the objection funtion with the revenues from clearance sales


###     OPTIONS FOR DATA MINING
###-------------------------------------------------

option CENTROCOMPRA 1;
option SECCION 1;
option ANO 2009;
option CAMPANA 2;
option ANO_CAMPANA 5;
option SECCION 1;
option DCS '10001,10018,10026';
option where (" id_centro_compra = " & $CENTROCOMPRA & " and id_tipo_seccion = " & $SECCION &" and id_campana = " &
$ANO_CAMPANA);


###     DECLARATION OF SETS AND PARAMETERS MIT4
###-------------------------------------------------
#option GRUPO WOMAN;

set GROUP;                          # Definition of groups
set DATA_DC:= {10001,10018,10026};              # DC codes
set PAIS_DC {DATA_DC};                     # Set of countries that each DC supports
set PAIS  ;                                # Set of all countries
set CLUSTER{GROUP} ordered;      # Clusters in each group within a country

param CA >=0;                              # DC costs
param CS >=0;                              # Country shipment cost
param ST >=0, <=1;                         # Salvage percentage
param SA >=0, <=1;                         # Salvage percentage
param Qm >=0;                              # Min shipment

# Inventory in the beginning of the current period of the regular season
param Imgn {m in PAIS, g in GROUP, n in CLUSTER[g]};

# Price of each cluster per group by country during the regular season
param Pmgn {m in PAIS, g in GROUP, n in CLUSTER[g]} >= 0;

# Forecast for end of regular season
param Fmgn {m in PAIS, g in GROUP, n in CLUSTER[g]};

# Inventory at DC at the beginning of the current period in the regular season
param Iagn {dc in DATA_DC, g in GROUP, n in CLUSTER[g]};

# Price of cluser at DC
param Pagn {dc in DATA_DC, g in GROUP, n in CLUSTER[g]};

# Ratio
param Bmgn {m in PAIS, g in GROUP, n in CLUSTER[g]};


###     DECLARATION OF SETS AND PARAMETERS MIT2
###-------------------------------------------------

# Pricing options
set PRECIOS {m in PAIS, g in GROUP} ordered default {};

param psaldero {g in GROUP};               # Salvage Price by group
param WC >= 1;                             # Number of periods during clearance
param D >= 0;                              #Number of days in each period

# Need Tiempo in by country
# param tiempo {m in PAIS} > 0, <= 1;      # Factor that decreases weekly sales
param tiempo > 0;

# Minimum discount for first week of sale
param mindiscount  >= 0, <= 1 default 0;

# Discount Factor to bound weekly sales
param tighten_slack >= 0, <= 1 default 0.9999;

# Forecast for clearance season
param F_mgn {m in PAIS, g in GROUP, n in CLUSTER[g], k in PRECIOS [m,g]} >= 0 ;


###            DECISION VARIABLES MIT4
###------------------------------------------

# Inventory flow from DC to country m for cluster n of group g
var Q_mgn {dc in DATA_DC, m in PAIS, g in GROUP, n in CLUSTER[g]} >=0;

# Expected sales for regular season
var Lmgn  {m in PAIS, g in GROUP, n in CLUSTER[g]} >=0;
```

```
# Flow from DC to DC for cluster n in group g
var Q_agn {dc in DATA_DC, dc2 in DATA_DC, g in GROUP, n in CLUSTER[g]} >=0;

# Total revenues of regular and clearance season
var REVENUES =
        sum {m in PAIS, g in GROUP, n in CLUSTER[g]}
        (Pmgn [m, g, n]* Lmgn [m,g,n]);

# Total DC transfer costs
var DC2DC_COSTS =
        CA * sum {dc in DATA_DC, dc2 in DATA_DC, g in GROUP, n in CLUSTER[g]}
                Pagn [dc, g, n] * Q_agn [dc, dc2, g, n] ;

# Total country shipment costs
var SHIPMENT =
        CS * sum {dc in DATA_DC, m in PAIS, g in GROUP, n in CLUSTER[g]}
                Pagn [dc, g, n] * Q_mgn [dc, m, g, n];


###             DECISION VARIABLES MIT2
###--------------------------------------------

# Selects prices that are >= to the decision price
var X {1..Wc,  m in PAIS, g in GROUP, n in CLUSTER[g], k in PRECIOS [m,g] } binary;


# Decides the price for clearance sale
var Y {w in 1..Wc, m in PAIS, g in GROUP, n in CLUSTER[g], k in PRECIOS [m,g]} =
        if k = first(PRECIOS [m,g]) then
                (X[w,m,g,n,k])
        else
                (X[w,m,g,n,k] - X[w,m,g,n,prev(k)]);

# Expected sales during clearance sales in country m for cluster n of group g
var L_mgn {1..Wc, m in PAIS, g in GROUP, n in CLUSTER[g],k in PRECIOS [m,g]} >=0;

# County inventory that will be salvaged
var Lsaldero {m in PAIS, g in GROUP} >= 0;

# Inventory of each country at the beginning of clearance season
var I_mgn {1..(Wc+1), m in PAIS, g in GROUP, n in CLUSTER[g]} >=0;

# Total clearance profits
var CLEARANCE_PROFIT =
        sum{w in 1..Wc,  m in PAIS, g in GROUP, n in CLUSTER[g],k in PRECIOS [m,g]}
        L_mgn[w,m,g,n,k] * k;


# Clearance profits for modeling purposes
var DISC_FUT_PROFIT =
        sum {w in 1..Wc,  m in PAIS, g in GROUP, n in CLUSTER[g],k in PRECIOS [m,g]}
        L_mgn[w,m,g,n,k] * k * tighten_slack^(w-1);

# Total profit from salvage inventory
var PROFIT_SALDERO =
          sum {m in PAIS, g in GROUP} (Lsaldero [m,g] * psaldero [g])
          + (sum {dc in DATA_DC, g in GROUP, n in CLUSTER[g]} (Iagn [dc,g,n]
         - sum {m in PAIS} Q_mgn [dc,m,g,n])* psaldero [g]);

# ProfiTs for salvage inventory for modeling purposes
var DISC_PROFIT_SALDERO =
         (sum {m in PAIS, g in GROUP} (Lsaldero [m,g] * psaldero [g])) * tighten_slack^(Wc)
         + (sum {dc in DATA_DC, g in GROUP, n in CLUSTER[g]} (Iagn [dc,g,n]
         - sum {m in PAIS} Q_mgn [dc,m,g,n])* psaldero [g]);


###             OBJECTIVE FUNCTION
###--------------------------------------------

maximize PAIS_REVENUES:

        REVENUES + DISC_FUT_PROFIT + DISC_PROFIT_SALDERO - DC2DC_COSTS - SHIPMENT;


###             CONSTRAINTS MIT4
###--------------------------------------------

#* Ensures shipments between DCs do not exceed the inventory available
subject to dc_shipments {dc in DATA_DC, g in GROUP, n in CLUSTER[g]}:
        Iagn [dc,g,n] + sum {dc2 in DATA_DC} Q_agn [dc2,dc,g,n] >=
        (sum {m in PAIS} Q_mgn [dc,m,g,n]) + (sum {dc2 in DATA_DC} Q_agn [dc,dc2, g, n]);

# *Inventory balance equation
subject to inv_balance {m in PAIS, g in GROUP, n in CLUSTER[g]}:
        I_mgn [1,m,g,n] =
        Imgn [m,g,n] + (sum {dc in DATA_DC} Q_mgn [dc,m,g,n]) - Lmgn [m,g,n];

# *Ensures the expected sales for the regular season does not exceed expected demand
subject to reg_season_sales {m in PAIS, g in GROUP, n in CLUSTER[g]} :
        Lmgn [m,g,n] <= Fmgn [m,g,n];

# *Imposes a maximum excess inventory quantity to salvage
```

```
subject to excess_inv_total:
        sum {m in PAIS, g in GROUP, n in CLUSTER[g]} I_mgn[WC+1,m,g,n] +
        sum {dc in DATA_DC, g in GROUP, n in CLUSTER[g]} (Iagn [dc,g,n] - sum {m in PAIS} Q_mgn [dc,m,g,n])
        <= ((sum {dc in DATA_DC, g in GROUP, n in CLUSTER[g]} Iagn [dc,g,n]) +
           (sum {m in PAIS, g in GROUP, n in CLUSTER[g]} Imgn [m,g,n])) * ST;

# *Distributes inventory proportionately to each country based on a sales ratio
subject to max_capacity  {m in PAIS, g in GROUP, n in CLUSTER[g]}:
        sum {dc in DATA_DC}Q_mgn [dc,m,g,n] <= sum{dc in DATA_DC} Iagn [dc,g,n] * Bmgn[m,g,n];


# *Imposes a maximum excess inventory quantity to salvage in DC
subject to excess_inv:
        sum {dc in DATA_DC, g in GROUP, n in CLUSTER[g]} (Iagn [dc,g,n] - sum {m in PAIS} Q_mgn [dc,m,g,n])
        <= sum {dc in DATA_DC, g in GROUP, n in CLUSTER[g]} Iagn [dc,g,n] * SA;

# *Imposes a minimum shipment quantity for each country
subject to min_shipment  {m in PAIS, g in GROUP, n in CLUSTER [g]: Bmgn[m,g,n] > 0}:
        sum {dc in DATA_DC} Q_mgn [dc, m, g, n] >= Qm;


###     CONSTRAINTS MIT2
###-------------------------------------------


# *Ensures expected sales for clearance season does not exceed expected demand
subject to price_demand
        {w in 1..WC, m in PAIS, g in GROUP, n in CLUSTER[g], k in PRECIOS [m,g]}:
                L_mgn[w,m,g,n,k] <= F_mgn [m,g,n,k] *Y[w,m,g,n,k]*tiempo^(w-1);

# *Ensures that salvage inventory is less than the excess inventory
subject to remain_inv {m in PAIS, g in GROUP}:
        Lsaldero [m,g] <= sum{n in CLUSTER[g]} I_mgn[WC+1,m,g,n];


#* Computes the estimated inventory for the subsequent period
subject to estimated_inventory {w in 1..WC, m in PAIS, g in GROUP, n in CLUSTER[g]}:
       I_mgn[w+1,m,g,n] = I_mgn[w,m,g,n] - (sum{k in PRECIOS [m,g]} L_mgn[w,m,g,n,k]);


# *Selects prices greater or equal to the selected price
subject to price_k_or_higher
        {w in 1..WC, m in PAIS, g in GROUP, n in CLUSTER[g], k in PRECIOS [m,g]}:
                X[w,m,g,n,k] <=
                        if k = last(PRECIOS [m,g]) then
                                1
                        else
                                X[w,m,g,n,next(k)];

#* Ensures price clusters are in decreasing order
subject to ordered_clusters
        {w in 1..WC, m in PAIS, g in GROUP, n in CLUSTER[g], k in PRECIOS [m,g]}:
                X[w,m,g,n,k] <=
                        if n = last(CLUSTER[g]) then
                                1
                        else
                                X[w,m,g,next(n),k];

#* Prices are equal to or less than the price of the last period
subject to decreasing_prices
        {w in 1..WC,m in PAIS, g in GROUP, n in CLUSTER[g], k in PRECIOS [m,g] : ord(w) < WC}:
                X[w,m,g,n,k] <= X[w+1,m,g,n,k];

# Ensures there is a discount the first week of sale
subject to choose {m in PAIS, g in GROUP, n in CLUSTER[g] : card(PRECIOS[m,g]) > 0}:
        sum{k in PRECIOS [m,g]} Y[1,m,g,n,k] = 1;

# Ensures all clusters are discounted during the sale
subject to initial_discount {m in PAIS, g in GROUP, n in CLUSTER[g] : card(PRECIOS[m,g]) > 0}:
        X[1,m,g,n,max {k in PRECIOS [m,g]: k <= Pmgn[m,g,n]*(1-mindiscount) || k = first(PRECIOS[m,g])} k] = 1;
```

8.3 Model II AMPL Files

8.3.1Model II Data File

MIT4_TIENDA.dat

```
#*****************************************************************************************
# MIT4 CARGA DE DATOS MODELO III -  DESAGREGACION A TIENDA                               *
# FECHA: 12/11/2009                                                                      *
# VERSION: 1                                                                             *
#                                                                        ORIETTA VERDUGO *
#*****************************************************************************************

# UPDATES

###     DECLARATION OF TABLES
###-----------------------------------------------


table Tienda IN "ODBC" "MIT4" ("SQL=select distinct b.id_tienda from maestros.centro_distribucion_tienda_seccion a inner
join maestros.tienda b on a.id_tienda = b.id_tienda and a.id_tipo_seccion = " & $SECCION & " and a.id_campana = " &
$ANO_CAMPANA & " inner join mit4.stock_pais_cat s on b.id_pais = s.id_pais"  & " where b.id_pais in (" & $PAISES & ")"):
TIENDA <- [id_tienda];

table TiendaDC {dc in DATA_DC} IN "ODBC" "MIT4" ("SQL=select distinct b.id_tienda from
maestros.centro_distribucion_tienda_seccion a inner join maestros.tienda b on a.id_tienda = b.id_tienda and
a.id_tipo_seccion = " & $SECCION & " and a.id_campana = " & $ANO_CAMPANA & " and a.id_centro_distribucion = " & dc & "
inner join mit4.stock_pais_cat s on b.id_pais = s.id_pais"  & " where b.id_pais in (" & $PAISES & ")"): T_DC[dc] <-
[id_tienda];

table DCTienda IN "ODBC" "MIT4" ("SQL=select distinct a.id_tienda, a.id_centro_distribucion from
maestros.centro_distribucion_tienda_seccion a inner join maestros.tienda b on a.id_tienda = b.id_tienda and
a.id_tipo_seccion = " & $SECCION & " and a.id_campana = " & $ANO_CAMPANA & " inner join mit4.stock_pais_cat s on b.id_pais
= s.id_pais"  & " where b.id_pais in (" & $PAISES & ")"): [id_tienda], DC_T ~ id_centro_distribucion;

table Paises IN "ODBC" "MIT4" ("SQL=select distinct id_pais from mit4.stock_pais_cat where id_tipo_seccion = " & $SECCION
& " and id_campana = " & $ANO_CAMPANA & " and id_pais in (" & $PAISES & ")"): PAIS <- [id_pais];

read table Tienda;
read table TiendaDC;
read table DCTienda;
read table Paises;


table TiendaPais {m in PAIS} IN "ODBC" "MIT4" ("SQL=select distinct b.id_tienda from
maestros.centro_distribucion_tienda_seccion a inner join maestros.tienda b on a.id_tienda = b.id_tienda and
a.id_tipo_seccion = " & $SECCION & " and a.id_campana = " & $ANO_CAMPANA & " inner join mit4.stock_pais_cat s on b.id_pais
= s.id_pais"  & " where b.id_pais = " & m): T_M[m] <- [id_tienda];

table PaisTienda IN "ODBC" "MIT4" ("SQL=select distinct b.id_tienda, b.id_pais from
maestros.centro_distribucion_tienda_seccion a inner join maestros.tienda b on a.id_tienda = b.id_tienda and
a.id_tipo_seccion = " & $SECCION & " and a.id_campana = " & $ANO_CAMPANA & " inner join mit4.stock_pais_cat s on b.id_pais
= s.id_pais"  & " where b.id_pais in (" & $PAISES & ")"): [id_tienda], PAIS_T ~ id_pais;

read table TiendaPais;
read table PaisTienda;


table LogisticPlatform IN "ODBC" "MIT4" ("SQL=select distinct pl.id_plataforma_logistica from
maestros.centro_distribucion_tienda_seccion a inner join mit4.plataforma_logistica_tienda pl on a.id_tienda = pl.id_tienda
and a.id_tipo_seccion = " & $SECCION & " and a.id_campana = " & $ANO_CAMPANA & " inner join maestros.tienda t on
t.id_tienda = pl.id_tienda and t.id_pais in (" & $PAISES & ") inner join mit4.stock_pais_cat s on t.id_pais = s.id_pais"):
LP <- [id_plataforma_logistica];

table TiendaWithLP IN "ODBC" "MIT4" ("SQL=select distinct pl.id_tienda from maestros.centro_distribucion_tienda_seccion a
inner join mit4.plataforma_logistica_tienda pl on a.id_tienda = pl.id_tienda and a.id_tipo_seccion = " & $SECCION & " and
a.id_campana = " & $ANO_CAMPANA & " inner join maestros.tienda t on t.id_tienda = pl.id_tienda and t.id_pais in (" &
$PAISES & ") inner join mit4.stock_pais_cat s on t.id_pais = s.id_pais"): TIENDA_WITH_LP <- [id_tienda];

table TiendaLogisticPlatform {l in LP} IN "ODBC" "MIT4" ("SQL=select distinct a.id_tienda from
maestros.centro_distribucion_tienda_seccion a inner join mit4.plataforma_logistica_tienda pl on a.id_tienda = pl.id_tienda
and a.id_tipo_seccion = " & $SECCION & " and a.id_campana = " & $ANO_CAMPANA & " where id_plataforma_logistica = " & l):
T_LP[l] <- [id_tienda];


table LogisticPlatformTienda IN "ODBC" "MIT4" ("SQL=select distinct pl.id_tienda, pl.id_plataforma_logistica from
maestros.centro_distribucion_tienda_seccion a inner join mit4.plataforma_logistica_tienda pl on a.id_tienda = pl.id_tienda
and a.id_tipo_seccion = " & $SECCION & " and a.id_campana = " & $ANO_CAMPANA & " inner join maestros.tienda t on
t.id_tienda = pl.id_tienda and t.id_pais in (" & $PAISES & ") inner join mit4.stock_pais_cat s on t.id_pais = s.id_pais"):
[id_tienda], LP_T ~ id_plataforma_logistica;


read table LogisticPlatform;
read table TiendawithLP;
read table TiendaLogisticPlatform;
read table LogisticPlatformTienda;


table References IN "ODBC" "MIT4" ("SQL=SELECT distinct id_articulo FROM ( SELECT distinct  id_centro_compra, id_campana,
id_tipo_seccion, id_grupo, categoria, id_articulo FROM  mit4.stock_almacen_articulo UNION SELECT distinct
id_centro_compra, id_campana, id_tipo_seccion, id_grupo, categoria, id_articulo FROM  mit4.stock_tienda_articulo UNION
SELECT distinct id_centro_compra, id_campana, id_tipo_seccion, id_grupo, categoria, id_articulo FROM
mit4.stock_almacen_pais) AS A WHERE id_centro_compra = " & $CENTROCOMPRA & " and id_campana = " & $ANO_CAMPANA & " and
id_tipo_seccion = " & $SECCION & " and id_grupo = " & $GRUPO & " and categoria = " & $CLUSTER): REF <- [id_articulo];


#table ReferencesDC {dc in DATA_DC} IN "ODBC" "MIT4" ("SQL=SELECT distinct  id_articulo FROM  mit4.stock_almacen_articulo
WHERE id_centro_compra = " & $CENTROCOMPRA & " and id_centro_distribucion = " & dc & " and id_campana = " & $ANO_CAMPANA &
" and id_tipo_seccion = " & $SECCION & " and id_grupo = " & $GRUPO & " and categoria = " & $CLUSTER): REF_ALMACEN[dc] <-
[id_articulo];
```


F

```
read table References;
#read table ReferencesDC;


for {r in REF, m in PAIS} {
        let Irm[r,m] := 0;
}

table InventarioPais IN "ODBC" "MIT4" ("SQL=select id_pais, id_articulo, stock from mit4.stock_almacen_pais where
id_centro_compra = " & $CENTROCOMPRA & " and id_tipo_seccion = " & $SECCION & " and id_campana = " & $ANO_CAMPANA & " and
id_pais in (" & $PAISES & ") and id_grupo = " & $GRUPO & " and categoria = " & $CLUSTER): [id_articulo, id_pais], Irm ~
stock;


for {r in REF, j in TIENDA} {
        let Irj[r,j] := 0;
}

table InventarioTienda IN "ODBC" "MIT4" ("SQL=select id_tienda, id_articulo, stock from mit4.stock_tienda_articulo where
id_centro_compra = " & $CENTROCOMPRA & " and id_tipo_seccion = " & $SECCION & " and id_campana = " & $ANO_CAMPANA & " and
id_pais in (" & $PAISES & ") and id_grupo = " & $GRUPO & " and categoria = " & $CLUSTER): [id_articulo, id_tienda], Irj ~
stock;


for{dc in DATA_DC, r in REF} {
        let Ira[dc,r] := 0;
}

table InventarioAlmacenes IN "ODBC" "MIT4" ("SQL=select id_centro_distribucion, id_articulo, stock from
mit4.stock_almacen_articulo where id_centro_distribucion in (" & $DCS & ") and id_centro_compra = " & $CENTROCOMPRA & "
and id_tipo_seccion = " & $SECCION & " and id_campana = " & $ANO_CAMPANA & " and id_grupo = " & $GRUPO & " and categoria =
" & $CLUSTER): [id_centro_distribucion, id_articulo], Ira ~ stock;


read table InventarioPais;
read table InventarioTienda;
read table InventarioAlmacenes;



for{j in TIENDA} {
        let Lj[j] := 0;
#       let Lj_min[j] := 0;
#       let Lj_max[j] := 0;
}

table LTienda IN "ODBC" "MIT4" ("SQL=select id_tienda, L_j from mit4.L_j where id_centro_compra = " & $CENTROCOMPRA & "
and id_tipo_seccion = " & $SECCION & " and id_campana = " & $ANO_CAMPANA & " and id_pais in (" & $PAISES & ") and id_grupo
= " & $GRUPO & " and categoria = " & $CLUSTER): [id_tienda], Lj ~ L_j;
#table LTiendaMin IN "ODBC" "MIT4" ("SQL=select id_tienda, L_j from mit4.L_j_min where id_centro_compra = " &
$CENTROCOMPRA & " and id_tipo_seccion = " & $SECCION & " and id_campana = " & $ANO_CAMPANA & " and id_pais in (" & $PAISES
& ") and id_grupo = " & $GRUPO & " and categoria = " & $CLUSTER): [id_tienda], Lj_min ~ L_j;
#table LTiendaMax IN "ODBC" "MIT4" ("SQL=select id_tienda, L_j from mit4.L_j_max where id_centro_compra = " &
$CENTROCOMPRA & " and id_tipo_seccion = " & $SECCION & " and id_campana = " & $ANO_CAMPANA & " and id_pais in (" & $PAISES
& ") and id_grupo = " & $GRUPO & " and categoria = " & $CLUSTER): [id_tienda], Lj_max ~ L_j;


for{r in REF, j in TIENDA} {
        let Brj[r,j] := 0;
}


table BReferenceTienda IN "ODBC" "MIT4" ("SQL=select id_tienda, id_articulo, B_rj from mit4.B_rj where id_centro_compra =
" & $CENTROCOMPRA & " and id_tipo_seccion = " & $SECCION & " and id_campana = " & $ANO_CAMPANA & " and id_pais in (" &
$PAISES & ") and id_grupo = " & $GRUPO & " and categoria = " & $CLUSTER): [id_articulo, id_tienda], Brj ~ B_rj;

read table LTienda;
#read table LTiendaMin;
#read table LTiendaMax;
read table BReferenceTienda;


for{j in TIENDA} {
        let bj[j] := 0;
}

table BTienda IN "ODBC" "MIT4" ("SQL=select id_tienda, bj from mit4.bj where id_centro_compra = " & $CENTROCOMPRA & " and
id_tipo_seccion = " & $SECCION & " and id_campana = " & $ANO_CAMPANA & " and id_pais in (" & $PAISES & ") and id_grupo = "
& $GRUPO & " and categoria = " & $CLUSTER): [id_tienda], bj ~ bj;

read table BTienda;


for{j in TIENDA} {
        let Vj[j] := 0;
}

table Ventas IN "ODBC" "MIT4" ("SQL=SELECT s.id_pais, id_tienda, s.id_grupo, s.Cluster, (s.Ventas_Temp + s.Ventas_Saldo) *
dj.d as Ventas from mit4.salida_pais_grupo_cluster_mini_mit2 s inner join mit4.d_j dj on s.id_pais = dj.id_pais where
s.id_pais in (" & $PAISES & ") and s.id_grupo = " & $GRUPO & " and s.Cluster = " & $CLUSTER): [id_tienda], Vj ~ Ventas;
```

```
read table Ventas;

###     TEST LEVERS
###-------------------------------------------------

#This is to assign a percentage of DC inventory to the 10 countries
let {dc in DATA_DC, r in REF} Ira[dc,r] := Ira[dc,r] /24.31;


###     OTHER COMMANDS
###-------------------------------------------------


# If inventory in country is less than zero set it to zero
for {r in REF, m in PAIS} {
        if Irm [r,m] < 0 then let Irm [r,m]:= 0;
}

for {r in REF, j in TIENDA} {
        if Irj [r,j] < 0 then let Irj [r,j]:= 0;
}

#If inventory in DC is less than zero set it to zero
for {dc in DATA_DC, r in REF} {
        if Ira[dc,r] <0 then let Ira[dc,r] := 0;
}


#If DC inventory is less than or equal to zero set shipments from that DC to zero and set shipments within the same DC to
zero
for {dc in DATA_DC, r in REF} {
#       if Ira[dc,r] <=0 then {
#               for {j in TIENDA} {
#                       fix Q_raj [dc,r,j] := 0;
#               }
#               for {dc2 in DATA_DC} {
#                       fix Q_raa [dc,dc2,r] := 0;
#               }
#       }
        for {dc2 in DATA_DC} {
                if dc = dc2 then {
                        fix Q_raa [dc,dc2,r] := 0;
                }
        }
}

# Restricts DC's to only ship directly to county within its domain
for {dc in DATA_DC, j in TIENDA} {
        if j not in T_DC[dc] then {
                for  {r in REF} {
                        fix Q_raj[dc,r,j] := 0;
                }
        }
}

# set negitive Lj to 0
#for {j in TIENDA} {
#       let Lj[j] := Lj[j] / 1.05;
#       if Lj_max [j] <0 then let Lj_max [j] := 0;
#       if Lj_min [j] <0 then let Lj_min [j] := 0;
#}
```

8.3.2 Model II Run File

```
                                        MIT4_TIENDA.run
#*****************************************************************************************
# MIT4 PLANTILLA DE EJECUCION MODELO III -  DESAGREGACION A TIENDA                       *
# FECHA: 11/11/2009                                                                      *
# VERSION: 1                                                                             *
#                                                                         ORIETTA VERDUGO *
#*****************************************************************************************

#Version changes and updates


reset;

model MIT4_TIENDA_WITH_LP.mod;
include MIT4_TIENDA_I09.dat;


###     LEVERS
###-----------------------------------------------------

let CA := 50;                       # Cost of shipments between DCs
let CS := 10;                       # Logistic platform to store cost
let CL := 0;                        # Logistic platform to platform cost
let CM := 100;                      # DC shipment cost to each store
let CML := 75;                      # Country warehouse shipment cost to each store
let RS := 1;                        # Max shipment ratio slack

###     AMPL SIMULATION OPTIONS
###-----------------------------------------------------
option show_stats 1;
option cplex_options ("bestnode ");
option cplex_options ("bestnode mipgap= 0.0003 mipdisplay= 2");

solve;

printf "Ampl Time: %0.2f\tSolver Time: %0.2f\n\n",_ampl_time, _solve_time;


###     RESULTS
###-----------------------------------------------------;

print;
printf "Grupo: %s \n",($GRUPO);
printf "Cluster: %s \n\n\n",($CLUSTER);
printf "==========Costs==========\n";
printf "Total Costs: %.2f \n\n",DC2DC_COSTS + STORE_LP_COSTS + LP2LP_COSTS + STORE_SHIPMENTS +
LOCAL_WAREHOUSE_STORE_SHIPMENTS;
printf "Total Shipment Costs: %.2f \n\n", STORE_SHIPMENTS + LOCAL_WAREHOUSE_STORE_SHIPMENTS;
printf "Total DC2DC Costs: %.2f \n\n", DC2DC_COSTS;
printf "Total LP2LP Costs: %.2f \n\n", LP2LP_COSTS;
printf "Total Store2LP Costs: %.2f \n\n",  STORE_LP_COSTS;


###     EXCEL REPORTS
###-----------------------------------------------------

# Table of shipments to and from each DC
table Envios_DC2DC OUT "ODBC" ('c:\mit4\MIT4_Excel\TIENDA_Campana_' & $CAMPANA & '_' & $ANO & '_G' & $GRUPO & '_CL' &
$CLUSTER & '.xls'):
{r in REF, dc in DATA_DC, dc2 in DATA_DC} -> [REF,DC_Salida, DC_Entrada],
Q_raa [dc, dc2, r] ~ Tot_Unidades;


#Table reference inventory by DC
table Inv_DC OUT "ODBC" ('c:\mit4\MIT4_Excel\TIENDA_Campana_' & $CAMPANA & '_' & $ANO & '_G' & $GRUPO & '_CL' & $CLUSTER &
'.xls'):
{dc in DATA_DC, r in REF} -> [DC,REF],
Ira [dc, r] ~ DC_Initial_Inv,
sum {dc2 in DATA_DC} Q_raa [dc, dc2, r] ~ Envios_DC2DC,
sum {j in TIENDA_WITH_LP } Q_raj [dc,r,j] ~ Envios_Tienda,
(Ira [dc, r]
        - sum {j in TIENDA_WITH_LP } Q_raj [dc,r,j]
        + sum {dc2 in DATA_DC} Q_raa [dc2, dc, r]
        - sum {dc2 in DATA_DC} Q_raa [dc, dc2, r]) ~ DCInv_Final;

# Table of total units shipped to each country
table Envios_Tienda OUT "ODBC" ('c:\mit4\MIT4_Excel\TIENDA_Campana_' & $CAMPANA & '_' & $ANO & '_G' & $GRUPO & '_CL' &
$CLUSTER & '.xls'):
{r in REF, j in TIENDA_WITH_LP}-> [REF,TIENDA],
Irj [r,j] ~ INV_Initial,
sum {dc in DATA_DC} Q_raj [dc,r,j] ~ Envios_Almacen_Central,
Q_rmj [r,PAIS_T [j],j] ~ Envios_Almacen_Pais,
Q_rlj[r,LP_T[j],j] ~ Envios_Tienda,
Q_rjl[r,LP_T[j],j] ~ Salidas_Tienda;


# Table total inventory by store
table Resume_Inv_Tienda OUT "ODBC" ('c:\mit4\MIT4_Excel\TIENDA_Campana_' & $CAMPANA & '_' & $ANO & '_G' & $GRUPO & '_CL' &
$CLUSTER & '.xls'):
{j in TIENDA_WITH_LP} -> [Tienda],
PAIS_T [j] ~ Pais,
sum {r in REF} Irj [r,j]~Inv_Hoy,
Vj[j] ~ Venta_Estimada,
sum {r in REF, dc in DATA_DC} Q_raj [dc,r,j] ~ Envios_Almacen_Central,
sum {r in REF} Q_rmj [r,PAIS_T [j],j] ~ Envios_Almacen_Pais,
                                        Page 1
```

```
sum{r in REF} Q_rlj[r,LP_T[j],j] ~ Envios_Tienda,
sum{r in REF} Q_rjl[r,LP_T[j],j] ~ Salidas_Tienda,
-Vj[j] + (sum {r in REF, dc in DATA_DC} Q_raj [dc,r,j] + sum {r in REF} Q_rmj [r,PAIS_T [j],j]
        + sum{r in REF} (Q_rlj[r,LP_T[j],j] - Q_rjl[r,LP_T[j],j]) +sum {r in REF} Irj [r,j]) ~ Position_Inv;


# Table total inventory by country
table Resume_Inv_Pais OUT "ODBC" ('c:\mit4\MIT4_Excel\TIENDA_Campana_' & $CAMPANA & '_' & $ANO & '_G' & $GRUPO & '_CL' &
$CLUSTER & '.xls'):
{m in PAIS} -> [Pais],
sum {r in REF,j in  T_M [m]} Irj [r,j] + sum {r in REF} Irm [r,m]~Inv_Hoy,
sum {j in  T_M [m]} Vj[j] ~ Venta_Estimada,
sum {r in REF, dc in DATA_DC, j in  T_M [m]} Q_raj [dc,r,j]   ~ Envios_Unidades,
-sum {j in  T_M [m]} Vj[j] + (sum {r in REF, dc in DATA_DC, j in  T_M [m]} Q_raj [dc,r,j]
        + sum {r in REF} Irm [r,m]
        + sum {r in REF,j in  T_M [m]} Irj [r,j]) ~ Position_Inv;


# Table total inventory by DC
table Resume_Inv_DC OUT "ODBC" ('c:\mit4\MIT4_Excel\TIENDA_Campana_' & $CAMPANA & '_' & $ANO & '_G' & $GRUPO & '_CL' &
$CLUSTER & '.xls'):
{dc in DATA_DC} -> [DC],
sum {r in REF} Ira [dc, r] ~ DC_Initial_Inv,
sum {r in REF, dc2 in DATA_DC} Q_raa [dc, dc2, r] ~ Envios_DC2DC,
sum {r in REF, j in TIENDA_WITH_LP } Q_raj [dc,r,j] ~ Envios_Tienda,
sum {r in REF, dc2 in DATA_DC} Q_raa [dc2, dc, r] ~ Recibos_DC2CD,
(sum {r in REF} Ira [dc, r]
        - sum {r in REF, j in TIENDA_WITH_LP } Q_raj [dc,r,j]
        + sum {r in REF, dc2 in DATA_DC} Q_raa [dc2, dc, r]
        - sum {r in REF, dc2 in DATA_DC} Q_raa [dc, dc2, r]) ~ DCInv_Final;


# Table total inventory by Country warehouse
for {r in REF, m in PAIS} {
        if Irm [r,m] < 0 then let Irm [r,m]:= 0;
}
table Resume_Inv_Almacen_Pais OUT "ODBC" ('c:\mit4\MIT4_Excel\TIENDA_Campana_' & $CAMPANA & '_' & $ANO & '_G' & $GRUPO &
'_CL' & $CLUSTER & '.xls'):
{m in PAIS} -> [Pais],
sum {r in REF} Irm [r, m] ~ Initial_Inv,
sum {r in REF, j in T_M[m] } Q_rmj [r, m, j] ~ Envios_Tienda,
(sum {r in REF} Irm [r, m]
        - sum {r in REF, j in T_M[m]} Q_rmj [r, m, j]) ~ Inv_Final;


###     EXECUMENT OF REPORTS
###-----------------------------------------------------------------
#write table Envios_DC2DC;
#write table Inv_DC;
#write table Envios_Tienda;
#write table Resume_Inv_Tienda;
#write table Resume_Inv_Pais;
#write table Resume_Inv_DC;
#write table Resume_Inv_Almacen_Pais;



option NOMBRE_FICHERO ('results_tienda_campana_' & $CAMPANA & '_' & $ANO & '_G' & $GRUPO & '_C' & $CLUSTER & '.sql');

### OUTPUT TO DATABASE
###---------------------------------------------------------------------------
printf "delete from d_mit4.salida_tienda_grupo_cluster_articulo where id_grupo = %s and categoria =
%s;\n",($GRUPO),($CLUSTER) > ($NOMBRE_FICHERO);
printf {r in REF, j in TIENDA_WITH_LP: Irj [r,j] or sum{dc in DATA_DC} Q_raj[dc,r,j] > 0 or Q_rmj [r,PAIS_T [j],j] > 0 or
Q_rjl[r,LP_T[j],j] > 0 or Q_rlj[r,LP_T[j],j] > 0}: "insert into d_mit4.salida_tienda_grupo_cluster_articulo
(id_centro_compra, id_campana, id_tipo_seccion, id_pais, id_grupo, categoria, id_tienda, id_articulo, stock,
envios_almacen, envios_almacen_pais, salidas_plataforma, entradas_plataforma) values (%s, %s, %s, %s, %s, %s, %s, %s, %s,
%s, %s, %s, %s);\n", ($CENTROCOMPRA), ($ANO_CAMPANA), ($SECCION), PAIS_T [j], ($GRUPO), ($CLUSTER), j, r, Irj [r,j],
sum{dc in DATA_DC} Q_raj[dc,r,j], Q_rmj [r,PAIS_T [j],j], Q_rjl[r,LP_T[j],j], Q_rlj[r,LP_T[j],j] > ($NOMBRE_FICHERO);


printf "delete from d_mit4.salida_almacen_modelo_tiendas where id_grupo = %s and categoria = %s;\n",($GRUPO),($CLUSTER) >
($NOMBRE_FICHERO);
printf {dc in DATA_DC, r in REF: Ira [dc, r] > 0 or sum {j in TIENDA_WITH_LP } Q_raj [dc,r,j] > 0 or sum {dc2 in DATA_DC}
Q_raa [dc, dc2, r] > 0 or (Ira [dc, r] - sum {j in TIENDA_WITH_LP } Q_raj [dc,r,j] + sum {dc2 in DATA_DC} Q_raa [dc2, dc,
r] - sum {dc2 in DATA_DC} Q_raa [dc, dc2, r]) > 0}: "insert into d_mit4.salida_almacen_modelo_tiendas (id_centro_compra,
id_centro_distribucion, id_campana, id_tipo_seccion, id_grupo, categoria, id_articulo, stock, envios_tienda,
envios_almacenes, stock_final) values (%s, %s, %s, %s, %s, %s, %s, %s, %s, %s, %s);\n", ($CENTROCOMPRA), dc,
($ANO_CAMPANA), ($SECCION), ($GRUPO), ($CLUSTER), r, Ira [dc, r], sum {j in TIENDA_WITH_LP } Q_raj [dc,r,j], sum {dc2 in
DATA_DC} Q_raa [dc, dc2, r], (Ira [dc, r] - sum {j in TIENDA_WITH_LP } Q_raj [dc,r,j] + sum {dc2 in DATA_DC} Q_raa [dc2,
dc, r] - sum {dc2 in DATA_DC} Q_raa [dc, dc2, r])  > ($NOMBRE_FICHERO);


close ($NOMBRE_FICHERO);

shell("mysql -h axincloud1 -u ccsa -pdMhqPeGX < " & ($NOMBRE_FICHERO));

shell("del " & ($NOMBRE_FICHERO));
```

8.3.3 Model II Mod File

```
                                                MIT4_TIENDA.mod
#*****************************************************************************
# MIT4 OPTIMIZACION MODELO III -  DESAGREGACION A TIENDA                     *
# FECHA: 11/11/2009                                                          *
# VERSION: 1                                                                 *
#                                                          ORIETTA VERDUGO *
#*****************************************************************************

#Version changes and updates


###     OPTIONS FOR DATA MINING
###-----------------------------------------------

option CENTROCOMPRA 1;
option SECCION 1;
option ANO 2009;
option CAMPANA 2;
option ANO_CAMPANA 5;
option DCS '10001,10018,10026';
#option PAISES '2,3,7,8,18,28,30,38,39';
option GRUPO '19';
option CLUSTER '590';

option Where (" id_centro_compra = " & $CENTROCOMPRA & " and id_tipo_seccion = " & $SECCION &" and id_campana = " &
$ANO_CAMPANA
#& " and id_pais in (" & $PAISES & ")"
);


###     DECLARATION OF SETS AND PARAMETERS
###-----------------------------------------------

set DATA_DC:= {10001,10018,10026};              # DC codes
set T_DC {DATA_DC};                         # Set of stores that each DC supports
set PAIS;                                   # Set of all countries
set REF;                            # Reference in each group and cluster

set TIENDA;                                 # Set of stores
set TIENDA_WITH_LP default {};                      # Set of stores with logistic platform
set LP default {};
set T_M {PAIS};                     # Set of stores per country
set T_LP {LP};
param LP_T {TIENDA};
param PAIS_T {TIENDA};
param DC_T {TIENDA};

param CA >=0;                               # DC to DC costs
param CM >=0;                               # Country shipment cost to each store
param CS >=0;                               # Logistic platform to store cost
param CL >=0;                               # Logistic platform to platform cost
param RS >=0;                               # Ratio slack for maximum shipment by store or reference

# Inventory in country warehouse in the beginning of the current period of the regular season
param Irm {r in REF, m in PAIS};

# Inventory in store in the beginning of the current period of the regular season
param Irj {r in REF,j in TIENDA};

# Inventory at DC at the beginning of the current period in the regular season
param Ira {dc in DATA_DC, r in REF};

# *********Shipment allocation to each store
#param Lj_max {j in TIENDA};
#param Lj_min {j in TIENDA};
param Lj {j in TIENDA};

# Maximum shipment quantity of each reference to stores
param Brj {r in REF, j in TIENDA};

param bj {j in TIENDA};

param vj {j in TIENDA};

###             DECISION VARIABLES
###----------------------------------------

# Inventory flow from DC to store
var Q_raj {dc in DATA_DC, r in REF, j in TIENDA} >=0;

# Inventory flow from local warehouse to store
var Q_rmj {r in REF, m in PAIS, j in TIENDA} >=0;

# Inventory flow from logistic platform to store
var Q_rlj {r in REF, l in LP, j in T_LP [l]} >=0;

# Inventory flow from store to logistic platform
var Q_rjl {r in REF, l in LP, j in T_LP [l]} >=0;

# Flow from DC to DC for cluster n in group g
var Q_raa { dc in DATA_DC, dc2 in DATA_DC,r in REF} >=0;

# Flow from logistic platform to platform
var Q_rll {r in REF, l in LP, l2 in LP} >=0;
```

```
# Total DC transfer costs
var DC2DC_COSTS =
        CA * sum {dc in DATA_DC, dc2 in DATA_DC, r in REF} Q_raa [dc, dc2, r];


# Total store to logistic platform transfer costs
var STORE_LP_COSTS =
        CS * sum {r in REF,j in TIENDA_WITH_LP} (Q_rlj [r,LP_T [j],j] + Q_rjl [r,LP_T [j],j]);

# Total LP to LP transfer costs
var LP2LP_COSTS =
        CL * sum {r in REF,l in LP, l2 in LP} Q_rll [r,l, l2];

# Total DC transfer costs
var STORE_SHIPMENTS =
        CM * (sum {dc in DATA_DC, r in REF, j in TIENDA} Q_raj [dc, r, j]
             + sum {r in REF, m in PAIS, j in TIENDA} Q_rmj [r,m,j]);


###             OBJECTIVE FUNCTION
###-------------------------------------------

minimize SHIPMENT_COSTS:

        DC2DC_COSTS + STORE_LP_COSTS + LP2LP_COSTS + STORE_SHIPMENTS;


###             CONSTRAINTS MIT4
###--------------------------------------------

# Ensures shipments between DCs do not exceed the inventory available
subject to dc_shipments {dc in DATA_DC, r in REF}:
        Ira [dc, r] + sum {dc2 in DATA_DC} Q_raa [dc2,dc,r] >=
        (sum {j in TIENDA} Q_raj [dc,r,j]) + (sum {dc2 in DATA_DC} Q_raa [dc,dc2,r]);

# Inventory balance equation
subject to inv_balance {r in REF, m in PAIS}:
        Irm [r,m] >= sum {j in TIENDA} Q_rmj [r,m,j];

# Ensures that the inflow to each store is less than the total shipment quantity allocated to that store
subject to min_store_shipments {j in TIENDA}:
        sum {r in REF} (Q_rmj [r,PAIS_T [j],j] + Q_raj [DC_T [j],r,j])
                >= max(Lj [j],0);

# ***********Warehouse shipment requirement
subject to min_excess {dc in DATA_DC}:
        sum{r in REF}(Ira [dc, r]- sum {j in TIENDA } Q_raj [dc,r,j]
        + sum {dc2 in DATA_DC} Q_raa [dc2, dc, r]
        - sum {dc2 in DATA_DC} Q_raa [dc, dc2, r]) <= 0.2 * sum{r in REF} Ira [dc,r];

# ********max ratio per store
subject to max_ratio {r in REF, j in TIENDA}:
        sum{dc in DATA_DC} Q_raj[dc,r,j] + Q_rmj [r,PAIS_T [j],j]
        <= bj[j] * sum{dc in DATA_DC} Ira[dc, r]* RS;
```

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GLOSSARY

Article: Please see *Reference.*

Campaign: Design collections are separated into two campaigns, winter and summer. Campaigns last roughly seven weeks during the regular season and are followed by the clearance season.

Clearance season sales: The number of units sold or total revenues during the clearance season.

Clearance Season: The clearance season is the time period at the end of each campaign in which excess merchandise from the current campaign is sold at a discounted price. This period can last from a few weeks up to two months depending on the country and available merchandise.

Cluster (Pricing Cluster): Groups are segmented into pricing clusters based on the price of an article during the regular season. A cluster therefore can be 990 Euros and will have all the garments within a group that share this price point.

Country Warehouse: A storage facility that serves stores in the local country.

Distribution Center (DC): A storage facility that focuses on warehousing and distribution of merchandise from vendors to stores and country warehouses.

End-of-season sales: The number of units sold or total revenues during the last few weeks of the regular season.

End-of-Season: The last few weeks of the regular season is referred to end-of-season. Merchandise is sold at retail price and clearance season forecasts begin.

Group: A grouping of garments with similar characteristics and within the same product line. An example of a group is dresses.

Point of Sale: Hardware and software used at store registers that manages the selling process. Store sales and returns are accessible at the time of the transaction.

Pricing Forecast: Forecast created by Professor Gallien and Caro for a clearance pricing project. This forecast is used as the baseline forecast to test and improve upon.

Product Line: A set of related garments that share characteristics such as design, style, and market segment.

Reference: Also referred to **article**, is a design trend that is categorized by a model and textile quality. For example, a reference such as a denim jacket would be under the outerwear group for a given product line.

Regular Season: The regular season refers to the selling period in which merchandise is sold at retail price during a campaign.

Rotation: The stock to sales ratio of a reference that represents the amount of stock that should be on hand at each store in a given time.

Stock Keeping Unit: A unique identifier of a reference that includes size and color.

Success Ratio: The estimation of the success of sales over the life of the item. It is the ratio of total sales of a reference divided by the total opportunity of sales for the current season.