LONGITUDINAL SCHOTTKY SPECTRUM OF THE PEAK BUNCH AMPLITUDE SIGNAL

E. Shaposhnikova, T. Bohl and T. Linnecar, CERN, Geneva, Switzerland

Abstract

Diagnostic techniques based on the Schottky spectrum of the peak detected signal have been used at CERN for a long time to study the behaviour of bunched beams. In this paper it is shown how the measured spectrum is related to the particle distribution in synchrotron frequency. The experimental set-up used and its limitations are also presented together with examples of beam measurements in the SPS and LHC.

INTRODUCTION

The so called "peak detected Schottky" (PD Schottky) signal is a beam diagnostics tool developed and used extensively in the SPS [1], [2] since the late seventies, especially during $p\bar{p}$ operation. This technique has already been used in the LHC during the first coasts at 450 GeV.

The theory of Schottky signals for unbunched and bunched beams both in the longitudinal and transverse plane is well developed (e.g. [3]-[5]). In the case of an unbunched beam the longitudinal Schottky spectra gives the particle distribution in revolution frequencies and therefore in particle momentum. For the bunched beam, information about the momentum spread (dispersion) can also be extracted in most cases [6].

The PD Schottky is a special case of the bunched beam longitudinal Schottky signal, different from the usual technique since it uses only one selected piece of information from the beam current - its peak amplitude. This method is in fact closer to the unbunched beam Schottky spectra in that it also provides almost direct information about the particle distribution in oscillation frequency, which for an unbunched beam is the revolution frequency and for a bunched - the synchrotron frequency [7]. The deviation of the PD Schottky spectrum from the synchrotron frequency distribution is mainly defined by the experimental set-up.

PEAK DETECTED SIGNAL

In the SPS and LHC a simple circuit, Fig. 1, consisting of fast switching diode and capacitor detects the peak of the bunch current signal from the wide-band pick-up. The spectrum is obtained using the dynamic spectrum analyser. The parameters relevant to the Schottky measurements in the SPS and LHC are presented in Table 1.

Assuming that the fast diode is open during the bunch passage, current I_b, when $V_b = I_b R_1 \geq V$, the voltage V measured at resistance R_2 during this time interval $(-T_1, T_2)$ can be found from the following equation (valid

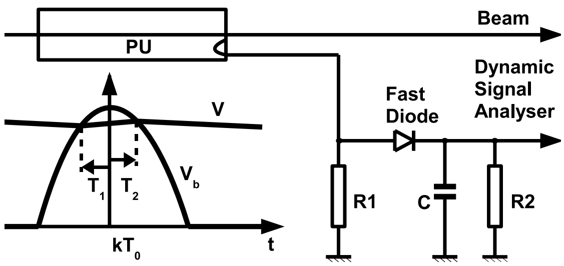

Figure 1: The simplified scheme of the bunch peak detection used for longitudinal Schottky signal in the SPS.

Table 1: SPS and LHC parameters relevant to the PD Schottky measurements and used in numerical examples.

Parameter			SPS	LHC
revol. period	T_0	μs	23.0	88.9
RF harmonic	h		4620	35640
resistance	R_1	Ω	50	50
resistance	R_2	MΩ	1.0	1.0
capacitance	C	pF	240	920
PD decay time	$1/\mu$	μs	240	920
PD growth time	$1/\alpha$	ns	12	12
acquisition time	T_a	s	1.6	3.2

for $R_2 \gg R_1$)

$$\frac{dV}{dt} = \alpha(V_b - V), \quad (1)$$

where $\alpha C = 1/R_1 + 1/R_2$. The solution of eq. (1) is

$$V(t) = \alpha \int_{-T_1}^{t} V_b(t')\, \mathrm{e}^{-\alpha(t-t')}\, dt' + V(-T_1)\mathrm{e}^{-\alpha(t+T_1)}. \quad (2)$$

The diode is off for the rest of the revolution period and

$$\frac{dV}{dt} = -\mu V, \qquad V(t) = V(T_2)\, \mathrm{e}^{-\mu(t-T_2)}.$$

where $\mu = 1/(R_2 C)$. The voltage is sampled during this period (typically 2048 points). The signal has some additional noise since sampling is not at a multiple of the revolution period $T_0 = 2\pi/\omega_0$ [7].

After a transient period, in the quasi-stationary situation variations of T_1 and T_2 from turn to turn are small and defined only by statistical fluctuations (Schottky noise). Then $V(-T_1) \simeq V(T_2)\, \mathrm{e}^{-\mu T_0}$. Taking into account that $V(-T_1) = V_b(-T_1)$ and $V(T_2) = V_b(T_2)$ together with (2) allows the stationary values of T_1 and T_2 to be found

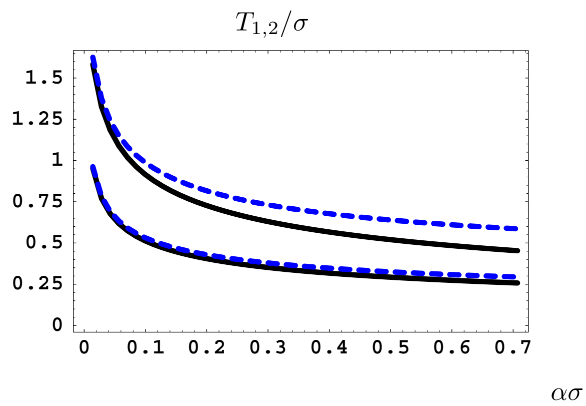

Figure 2: T_1/σ (dashed line) and T_2/σ (solid line) for $\mu T_0 = 0.07$ (upper curves, SPS values) and $\mu T_0 = 0.01$ (lower curves) found for a Gaussian line density.

as functions of beam (bunch length for a given particle distribution) and experimental set-up (α and μ) parameters. They are shown in Fig. 2 for a Gaussian line density with rms bunch length σ. One can see that in this model for the SPS set-up $T_1 \simeq T_2 \simeq \sigma$.

The signal detected at the moment t, after the k-th bunch passage, is $V_k e^{-\mu(t-t_k)}$, where $t_k = kT_0$ and

$$V_k = \Delta V_k + V_{k-1} e^{-\delta} = \sum_{q=0}^{k} \Delta V_{k-q}\ \mathrm{e}^{-q\delta}, \qquad (3)$$

Here $\delta = \alpha(T_2 + T_1) + \mu T_0$. In the SPS set-up $\alpha T_2 = 0.083$, $\mu T_0 = 0.077$ and $\delta \simeq 0.25$ for $T_2 = T_1 = 1$ ns and the increase in voltage at each revolution turn

$$\Delta V_k = R_1 \alpha \int_{t_k - T_1}^{t_k + T_2} I_b(t_k - t')\ \mathrm{e}^{-\alpha(t_k + T_2 - t')} dt'. \quad (4)$$

is proportional to the average bunch peak amplitude.

PD SCHOTTKY SPECTRUM

A particle will be detected at the azimuthal position ϕ (RF phase) twice per synchrotron period $2\pi/\Omega_n$ at time t_1 and t_2, when its phase $\psi_n = \Omega_n t + \psi_{n0}$ is equal to $\Omega_n t_\phi + 2\pi m$ or $\pi - \Omega_n t_\phi + 2\pi m$, $m = \pm 0, 1, ..., \infty$, and

$$t_\phi = t_\phi(\mathcal{E}_n, \phi) = \int_0^\phi \frac{d\phi'}{\sqrt{2[\mathcal{E}_n - W(\phi')]}}.$$

We consider below a single RF system with potential well $W(\phi) = \Omega_{s0}^2(1 - \cos\phi)$, where $\Omega_{s0} = 2\pi f_{s0} = 2\pi/T_{s0}$ is a linear synchrotron frequency and for a particle with phase oscillation amplitude ϕ_a the synchrotron energy $\mathcal{E} = W(\phi_a)$. The particle contribution to a bunch current at ϕ is

$$I_n(t,\phi) = \frac{e}{2} \sum_m [\delta(t - t_1) + \delta(t - t_2)] =$$

$$= \frac{e\Omega_n}{4\pi} \sum_m [\mathrm{e}^{im\Omega_n t_\phi} + \mathrm{e}^{im(\pi - \Omega_n t_\phi)}]\ \mathrm{e}^{-im(\Omega_n t + \psi_{n0})}. \quad (5)$$

Collecting contributions at ϕ from all particles the increase in voltage (4) can be written in the form

$$\Delta V_k = \frac{e}{2\pi} B \sum_n \sum_{m=-\infty}^{\infty} \Omega_n\, A_m(\mathcal{E}_n)\, \mathrm{e}^{-im(\Omega_n t_k + \psi_{n0})}, \qquad (6)$$

where $B = 2R_1 \alpha T \mathrm{e}^{-\alpha T}$ and $A_m = A_m(\mathcal{E}_n)$ is

$$A_m = \frac{1}{2\Phi} \int_0^{\Phi_{max}} \cosh\frac{\alpha\phi}{h\omega_0} [\mathrm{e}^{im\Omega_n t_\phi} + \mathrm{e}^{im(\pi - \Omega_n t_\phi)}] d\phi.$$

Here $\Phi = h\omega_0 T_2$, $\Phi_{max} = \Phi$ for $\mathcal{E}_n > W(\Phi)$ and particles with $\mathcal{E}_n < W(\Phi)$ contribute to measurements only up to $\Phi_{max} = \Phi_n(\mathcal{E}_n)$, where $\mathcal{E}_n = W(\Phi_n)$.

Finally from (3) for the PD signal we obtain

$$V_k = \frac{eB}{2\pi} \sum_n \sum_m \Omega_n\, A_m(\mathcal{E}_n)\, Q_m(\Omega_n)\, \mathrm{e}^{-im(\Omega_n t_k + \psi_{n0})}$$

The sum Q_m, which for $k\delta \gg 1$ has a form

$$Q_m(\Omega_n) = \sum_{q=0}^{k} \mathrm{e}^{im\Omega_n T_0 q - q\delta} \simeq \frac{1}{1 - \mathrm{e}^{im\Omega_n T_0 - \delta}} r,$$

is shown in Fig. 3 for $m = 2$ and different δ.

Following a procedure similar to that for unbunched beam Schottky (averaging over initial phase ψ_{n0}) [4] and replacing the sum over all particles by the integral over the distribution function $F(\Omega) = dN/d\Omega$ (normalised to unity) the power spectral density of the PD signal is [7]

$$P(\omega) = \frac{P_0}{\Omega_{s0}^2} \sum_{m=1}^{\infty} \int \Omega^2 F(\Omega)\, |A_m(\Omega)|^2 |Q_m(\Omega)|^2\, S^2 d\Omega,$$

where $P_0 = e^2 N f_{s0} B^2$ and for an acquisition time T_a

$$S^2 = |S(\omega - m\Omega)|^2 = \frac{2T_a}{T_{s0}} \frac{\sin^2 [(\omega - m\Omega)T_a/2]}{[(\omega - m\Omega)T_a/2]^2}.$$

Examples of the first four lines in the PD Schottky spectrum for $\sigma_\phi = h\omega\sigma = \pi/4$ and $\Phi = \pi/8$ are shown

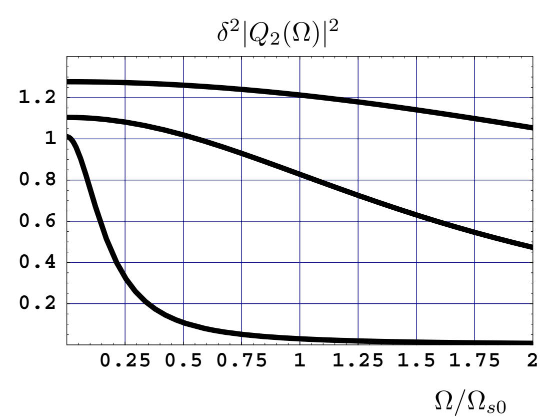

Figure 3: Function $\delta^2|Q_2(\Omega)|^2$ for $\delta = 0.25$ (top), $\delta = 0.1$ (middle), $\delta = 0.01$ (bottom) with $T_0\Omega_{s0} = 0.03$.

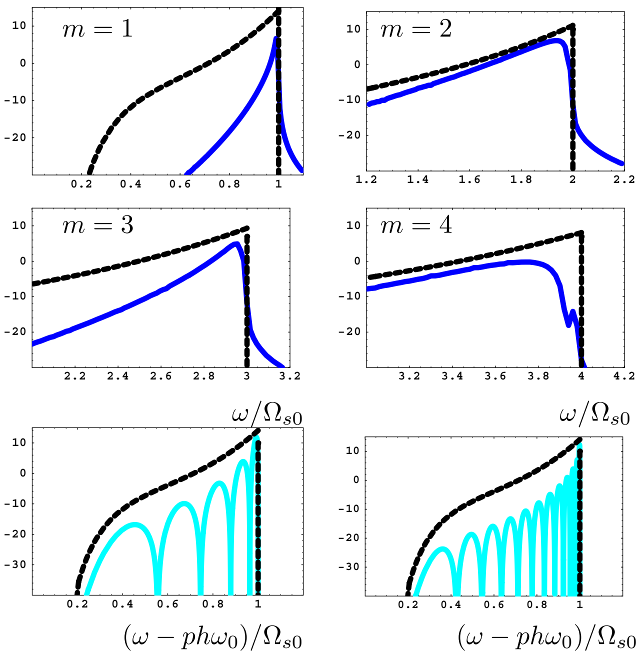

Figure 4: PD Schottky bands P/P_0 for $m = 1, 2, 3, 4$ (blue) for $\Phi = \pi/8$, $T_a = 320T_s$. Bottom: dipole sideband (a.u.) in the traditional Schottky spectrum (cyan) for $T_a = \infty$, $p = 5$ (left, SPS 1 GHz system) and $p = 12$ (right, LHC 4.8 GHz system). In all figures logarithmic scale (dB), $2\Omega_{s0}/mF(\omega/m)$ as dashed line, $\sigma_\phi = \pi/4$.

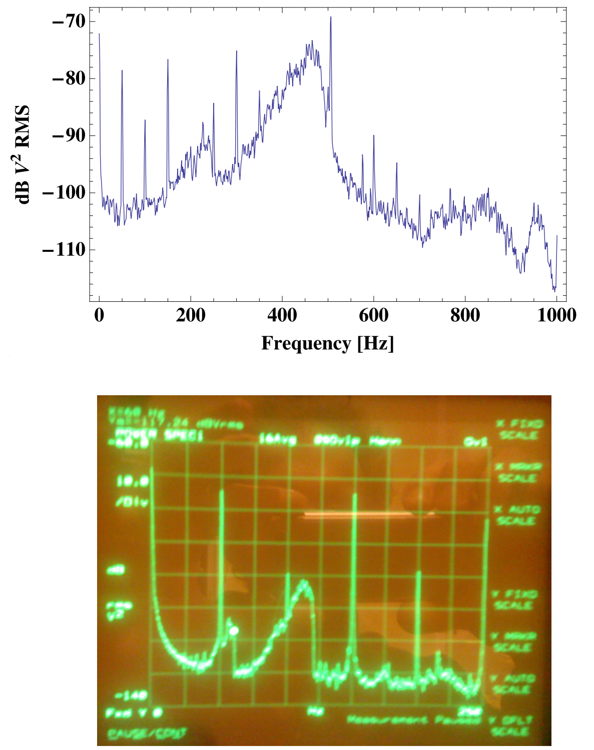

Figure 5: Measured PD Schottky spectrum. Top: **SPS** at 26 GeV, $f_{s0} = 240$ Hz, bunch with $\sigma_\phi = \pi/4$. Bottom: **LHC** at 450 GeV, bunch with $\sigma_\phi = \pi/7$, $f_{s0} = 66$ Hz; scales: 10 dB V^2rms/div - vertical, 25 Hz/div - horizontal.

in Fig. 4. These spectra can be compared with the corresponding distribution function in synchrotron frequency as well as with Schottky spectra of the peak amplitude signal (”ideal” case, only taking the finite acquisition time T_a into account). As Q_m is a fairly flat function of Ω for not too small δ, the measured PD Schottky spectrum deviates from $F(\omega/m)$, mainly due to $A(\Omega)$. The distortion is smaller for smaller Φ and in the limit of $t_\phi = 0$, $A_m = (1+(-1)^m)/2$, so that only even multipoles (reducing as $1/m$ in amplitude) are present in the spectrum. The quadrupole line gives the best reproduction of particle distribution.

For comparison, the dipole sideband at revolution harmonic ph in the traditional Schottky spectra (e.g. [4]), found by replacing the function $(Q_mA_m)^2$ in $P(\omega)$ by the Bessel function $J_m^2(p\phi_a)$, with $p = 5$ in the SPS and $p = 12$ in LHC [8], is also shown in Fig. 4 (bottom).

The calculated spectrum from Fig. 4 finally can be compared with the measured PD Schottky spectrum, Fig. 5. The shapes of dipole and quadrupole lines are very close, the octupole line even showing the double hump. The amplitude of the dipole line is much smaller than a quadrupole line as could be expected for a small integration distance Φ. This distance is further reduced if the finite reaction time of the fast diode is taken into account.

Summary. The quadrupole line of the PD Schottky spectrum represents the particle distribution in synchrotron frequency modified by nonlinearity of the synchrotron frequency (factor Ω^2) and experimental set-up (function A_2Q_2). The deviation introduced by the latter is mainly defined by the distance (phase Φ) over which the bunch peak amplitude averaging is performed. The connection between Φ and bunch parameters obtained allows the existing Schottky measurements to be understood and possible improvements to be foreseen. In particular a reduction in Φ can be achieved by decreasing the signal decay during the revolution turn (μ) and increasing the signal growth during the bunch passage (α).

Acknowledgments We are grateful to U. Wehrle for his help in system development and studies.

REFERENCES

[1] D. Boussard *et al.*, SPS Improvement Reports No. 154, 162, 167, 1979.

[2] T. Linnecar *et al.*, AIP Conf. Proc. - June 8, 2005, Volume 773, p. 345.

[3] J. Borer *et al.*, Proc. 9th Int. Conf. on High-Energy Acc., SLAC, 1974.

[4] S. Chattopadhyay, CERN-84-11, 1984.

[5] D. Boussard, Proc. CAS, CERN-87-03, p. 416, 1987.

[6] V. Balbekov and S. Nagaitsev, Proc. EPAC04, Lucerne, Switzerland, p.791 (2004).

[7] E. Shaposhnikova, CERN-BE-2009-010, 2009.

[8] F. Caspers *et al.*, LHC Project Report 1031, 2007.