A NUMERICAL INVESTIGATION OF EXTENDED RANGE PREDICTABILITY

by

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ABSTRACT

A numerical investigation of the predictability of idealized hydrodynamic flow is carried out using a model of two-dimensional incompressible flow. The governing equation is the forced, dissipative vorticity equation. The numerical integration is carried out over a square grid with periodic boundaries. The Coriolis parameter is taken as constant over the domain. The initial state of pure zonal flow is perturbed at random grid points and the evolution of the system is observed. Particular attention is paid to the existence of regimes and transitions between various modes of the flow.

The system is determined to be almost-intransitive, with two regimes of motion. The flow is characterized by persistent patterns with rapid transitions in phase angle and less frequent transitions between regimes.

No periodicities are found in the fluid motion and it is determined to be unpredictable at extended range by any method if errors exist in the initial data.

The mechanics of pattern persistence and transitions between modes are explored analytically. Analogies with the large scale atmospheric flow are made.

The role of transitions, regimes, and stability in predictability is discussed.

This work was supervised by Professor Edward N. Lorenz, Department of Meteorology, Massachusetts Institute of Technology, Cambridge, Massachusetts.

I. INTRODUCTION

Advances in the last two decades in our ability to measure the state of the atmosphere and predict its evolution have encouraged consideration of extended range prediction. Of particular interest is the ability to predict at long range those atmospheric features associated with climatic change.

Much of the previous work in determining the practical range of predictability has involved sensitivity studies of numerical models, as described by Charney, et al. (1966). These studies seek to determine the extent to which these models are capable of predicting atmospheric motions at extended range, given that our knowledge of the initial state is uncertain. The limits of predictability are determined in these investigations by the growth rate of small errors introduced in the data.

An investigation by Lorenz (1969a;1973a) looks for analogues in the observed atmospheric states. This study determines the limits of predictability on the basis of error growth and the time required for similar states to evolve to the point that they no longer resemble one another.

Studies by Lorenz (1969b) and Leith (1971) using two-dimensional incompressible flow as a substitute for the actual atmospheric equations estimate predictability limits by examining the nonlinear transfer rates between different scales of motion. These studies combine the estimated spectral distribution of atmospheric energy with the dynamical equations to produce a spectral distribution of errors as a function of time.

With respect to long-range predictability and forecasting climatic change, there remains the fundamental question of whether large variations in the state of the atmosphere are mainly the result of variations in the external forcing. Lorenz (1976b) has hypothesized that long-period fluctuations of the atmosphere-ocean-earth system may be the result of nondeterministic factors, not changes in the external forcing. The implication is that for a given forcing the observed climate need not be unique and that almost-intransitivity may account for the long-period fluctuations.

It is the purpose of this paper to explore extended range predictability with a model of two-dimensional incompressible flow. Models of this flow have found favor in many theoretical examinations of atmospheric motions (cf. Fjortoft, 1953; Arakawa, 1966; Lorenz, 1969b; Batchelor, 1969; Leith, 1971; Lilly, 1972; Knudsen, 1973). This is due in large part to the realization by Fjortoft (1953) that the atmosphere on the largest scales behaves roughly like a two-dimensional incompressible fluid.

It should be noted that no attempt is made here to reproduce observed atmospheric motions. It is our intent to examine the kinematic behavior of the modelled flow, which we believe is analogous to the atmosphere. The model possesses two important characteristics of atmospheric motions in relation to predictability: nonlinearity and randomness. In some respects, the model can be considered a more complex form of the nonlinear functions used by Lorenz (1964;1976b). These functions were used to mathematically illustrate possible climatic regimes and the difficulty in estimating long-term statistical properties of a system which has nonperiodic variations. The equations used here bear a closer resemblance to those describing real fluid motion than those used by Lorenz. Particular attention will be given in this investigation to the existence of regimes, transitions, intransitivity and almost-intransitivity that may occur in this simulated two-dimensional incompressible flow.

II. MODEL

The equation used to describe two-dimensional incompressible flow in this investigation is the forced, dissipative vorticity equation:

$$\frac{\partial \zeta}{\partial t} = -J(\psi, \zeta + f) - k\zeta + F(x,y) \qquad (1)$$

where $\zeta = \nabla^2 \psi$

and ζ = relative vorticity

ψ = streamfunction

$J(,)$ = two-dimensional Jacobian

k = damping constant

$F(x,y)$ = forcing function

The terms contributing to the local change of vorticity, $\partial \zeta / \partial t$, are:

$-J(\psi, \zeta + f)$, the vorticity advection term,

$-k\zeta$, the vorticity damping term, and

$F(x,y)$, the vorticity forcing term.

The numerical method for integrating the vorticity equation involves making finite difference approximations to the partial differential equation. The Jacobian is replaced by the finite difference Arakawa Jacobian (Arakawa, 1966) which conserves both the mean squared vorticity and kinetic energy.

The stream function used in evaluating the Jacobian is obtained by solving Poisson's equation, $\nabla^2 \psi = \zeta$, using a numerical procedure for inverting del-square devised by Lorenz (1976). The numerical time integration is carried forward using the N-Cycle scheme (Lorenz, 1971) with N=8. The finite differencing grid is rectangular with 32 by 32 points. Periodic boundaries are used at all four sides.

We now rewrite (1) in dimensionless variables using τ , which will be related to the integration time step, and L , the grid-point separation.

$$\frac{\partial \zeta_1}{\partial t_1} = - J_1(\psi_1, \zeta_1 + f_1) - k_1 \zeta_1 + F_1(x,y) \qquad (2)$$

where

$$\zeta_1 = \tau \zeta$$
$$t_1 = t/\tau$$
$$J_1 = L^2 J$$
$$\psi_1 = \tau/L^2 \, \psi$$
$$k_1 = \tau k$$
$$F_1 = \tau^2 F$$
$$f_1 = \tau f$$

The values of τ and L will be used later to make a rough comparison between the model scale of motion and the atmospheric scale of motion.

In this model, the forcing function, F(x,y), was chosen as a periodic function of y (latitude), independent of x (longitude). Specifically, F(x,y) = Acos(3y), where A is the specified amplitude of the forcing. In discrete form, the forcing function is $F_{i,j} = A\cos\left(\frac{3\pi}{16} i\right)$.

III. NUMERICAL EXPERIMENT

The model in finite difference form was programmed to run on an IBM 370/95, with all floating point values computed to 16 decimal places. The amplitude of the forcing was set to 0.25, the integration time step was 0.5τ, and the Coriolis parameter was taken as constant.

Initially all grid-point values of the vorticity and streamfunction were set to zero. Since the forcing had a zero mean, the vorticity had a zero mean throughout the integration as a result of the initial conditions.

During the integration, the intensity of the flow increased initially due to the forcing. A steady state was soon reached when the forcing and damping balanced. At this time, the vorticity field was identical in form to the forcing field and the stream field reflected the purely zonal flow which had become established. The direction of the flow alternated with the same period as the forcing.

Once the steady state had been achieved, perturbations were introduced in the vorticity field. These perturbations were made at six randomly chosen points. The perturbation consisted of adding 0.1 at three of the points and subtracting 0.1 at three of the points, keeping the mean vorticity equal to zero. The size of the perturbation was approximate- ten percent of the rms vorticity of the model.

Immediately following the perturbation, the amplitude of the local disturbances grew until the shape of the flow pattern was drastically altered from its previous form. The integration was carried out for over 20,000 time steps, corresponding to a model time of 10,000τ. A typical stream field is shown in Figure 1.

IV. ATMOSPHERIC AND MODEL SCALE COMPARISON

To establish a time scale comparison between the atmosphere and the model we examine the ratio of the typical wavelength to the typical velocity in both cases.

For the atmosphere, the typical wavelength is on the order of 6000 km and the typical wind speed is approximately 20 $msec^{-1}$. Thus the atmospheric time scale is approximately $t_a \approx \frac{6000 \text{ km}}{20 \text{ msec}^{-1}} \sim 3.5$ days.

For the model, the typical wavelength is on the order of $16 L$. The typical velocity is determined from the mean kinetic energy, which is calculated periodically during the integration. The mean kinetic energy per unit mass is approximately $0.13\, L^2/\tau^2$. Then the root mean square velocity is approximately $0.5\, L/\tau$. Thus, the time scale of the model is $t_m \approx \frac{16 L}{0.5\, L/\tau} \sim 32\tau$.

If we take $\tau \sim 0.1$ days, we can consider our results as roughly comparable to atmospheric scales of motion. Making this correspondence between atmospheric and model time scales means our integration was carried out for about 1000 "days". Henceforth our reference to elapsed time in the model will be in days with the understanding that the correspondence between the model time and real time is based on the above scale comparison.

V. RESULTS

Following the perturbation, a flow pattern became established consistent with a stream function whose largest term is of the form: Bcos2xcos3y. Throughout the integration, the gross features of the flow pattern could be described by a stream function of the form:

$$\psi(x,y,t) = B(t)\cos(nx + \phi_n(t))\cos(3y + \alpha(t)) + E(t)\cos 3y \quad (3)$$

with n = 1 or 2,

and where $B(t)$ and $E(t)$ are time varying amplitudes, $\phi_n(t)$ is the phase angle associated with zonal wavenumber n, and $\alpha(t)$ is the meridional phase angle.

One of the noteworthy results of this investigation was the relative immobility of an established pattern for extended periods of time, even though the forcing varied only with latitude and presented no preferred longitude. What little mobility a pattern did exhibit was mainly in the zonal direction. That is, in (3) the phase angle, $\phi_n(t)$, showed much more variation than $\alpha(t)$, which varied only slightly from zero.

It was seen as convenient to describe the pattern mobility in terms of the phase angle $\phi_n(t)$ alone. This was accomplished by a harmonic analysis of the meridional averaged stream function. By doing so, the y-dependence of (3) was removed; however, the loss in describing the pattern mobility was considered minimal due to its relatively small meridional motion. Figures numbered 5 through 10 show the time series of the phase and amplitude of zonal wavenumbers 1 and 2, which were dominant, and wavenumber 3, which is presented for comparison.

In Figures 5, 6, and 7, a phase angle which is constant for a long period of time is indicative of zonal immobility of the pattern if the associated wavenumber has a dominant amplitude. The length of time that a pattern was stationary seemed to vary aperiodically.

The phase of wavenumber 1 (Figure 5) exhibits a stationary behavior for approximately 100 days, between Day 420 and 520. An examination of the amplitudes of the three wavenumbers shown in Figures 8, 9, and 10 shows that during this time wavenumber 1 had the dominant amplitude. Thus, the flow pattern during this time was characterized by a stationary zonal wavenumber 1. Figure 2 illustrates the stream field at a time when wavenumber 1 is dominant.

Wavenumber 2 exhibited the greatest tendency to persist without motion and to characterize the flow pattern. The amplitude of wavenumber 2 usually dominated wavenumbers 1 and 3. Figure 3 shows the stream field for a pattern dominated by wavenumber 2. The tendency for the phase of the dominate wavenumber to be constant for a considerable time and then suddenly change and become nearly constant again is shown in Figure 6, between Day 770 and 1050. The rapid shift of the pattern coincides with a sudden drop in the amplitude of the dominant wavenumber. Figure 4 shows the stream field during a transition in phase of the dominant wavenumber for wavenumber 2.

Wavenumber 3, as well as larger wavenumbers, exhibited amplitudes which were generally less in magnitude than wavenumber 1 or 2. The time series of the phase of wavenumber 3 shows no tendency for prolonged immobility of this wave.

An examination of the mean kinetic energy of the zonal component as a function of time (Figure 12) reveals a marked increase associated with a transition from one stationary pattern to the next. During a transition, as shown in Figure 4, the zonal variations in the pattern become ill-defined and the flow nearly returns to its initial, unperturbed state of purely zonal flow. Within a few time steps, a new stationary pattern has been established, coinciding with a noticeable decrease in the kinetic energy of the zonal component.

From the harmonic analysis of the meridional averaged stream function, we can conclude that the zonal wavenumber possessing the greatest amplitude has a tendency to remain stationary while those of lesser amplitude undergo nonperiodic variations in phase and amplitude. Wavenumber 2 most often characterized the pattern and exhibited a strong tendency for prolonged immobility throughout the integration. There was only one occurrence where wavenumber 1 was dominant and characterized the pattern.

Two types of transitions were noted. One was the phase transition in the dominant zonal wavenumber. The other was a transition in dominant wavenumber from 2 to 1 and back to wavenumber 2. Transitions in phase were rapid and the time between occurrences appeared aperiodic. Transitions in dominant wavenumber were less rapid and occurred only twice during the integration.

To further examine the system for possible periodicities, it is convenient to look at a quantity that characterizes the entire system. The assumption is that if periodically the system returns to the same state, it would be revealed by such a quantity. The mean kinetic energy was taken to be an appropriate quantity for this purpose. A time series of the mean kinetic energy was computed. Figures 11, 12, and 13 show the results of this calculation, displaying the mean total kinetic energy, mean kinetic energy of the zonal component, and the mean kinetic energy of the meridional component respectively.

A casual inspection of these time series reveals no obvious periodicity. A spectral analysis of the mean kinetic energy of the zonal component based on 900 consecutive days using a discrete Fourier transform, revealed a continuous spectrum within the frequency resolution limits (0.5 day^{-1}).

A measure of linear predictability one step ahead (cf.

Lorenz, 1973b) is the ratio of the a posteriori variance, represented by the geometric mean of the spectral density function, to the a priori variance, represented by the arithmetic mean. For no error in prediction, this ratio is zero. In this case, the arithmetic and geometric means of the spectral density function of the time series of the mean kinetic energy of the zonal component are 7.9×10^{-7} and 3.2×10^{-11}, respectively. The ratio is of order 10^{-5}, which indicates only a small error in predicting the mean kinetic energy of the zonal component one day ahead. However, if the series is indeed nonperiodic, and not just apparently so from the sample, the imperfect prediction one day ahead implies it would be linearly unpredictable at extended range.

VI. DISCUSSION

Kinematically, the flow represented by the model bore some resemblance to the atmosphere. The zonal flow alternated direction in latitudinal bands similar to the easterlies and westerlies in the atmosphere. Zonal waves developed in the flow, producing circulation centers with cyclonic and anticyclonic fluid motions (in the atmospheric sense). The absence of variation in the Coriolis force prevented Rossby waves from developing.

There are four observed features of the flow which are relevant to its predictability: (1) The presence of persistent stationary patterns, (2) patterns which can be separated according to the dominant zonal wavenumber into regimes, (3) rapid transitions which occur between regimes with phase transitions occurring within a regime, and (4) no observed periodicity in the flow.

The mechanics of the observed persistent patterns can be explained to some extent analytically. If the stream function has the form

$$\psi = C\cos(nx + \phi)\cos(my + \alpha) \qquad (4)$$

with n and m integers, the vorticity advection, represented by the Jacobian, $J(\psi, \zeta)$, is identically zero. It can be seen that (4) with m=3 is a special case of (3) with $\mathcal{E}(t)$ =0. When the stream field is described by (4), the local change in vorticity given by (1) is a result of the forcing and damping alone. Thus, when $\mathcal{E}(t)$, the amplitude of the zonal flow, is near zero, there will be little tendency for alterations in the shape of the pattern. The amplitude of the pattern, represented by $B(t)$ in (3) will decrease with time due to the damping. The forcing will tend to increase the amplitude of the zonal flow, $\mathcal{E}(t)$ in (3). When $\mathcal{E}(t)$ is no longer small in comparison with $B(t)$, (4) no longer applies and the nonlinear advection term becomes nonzero.

The sequence of events in the transition from one mode to another is the following. The features of the established pattern deteriorate slowly while the intensity of the zonal flow increases. The strength of the zonal flow is limited by its instability with respect to perturbations by the existing small scale motions. A new pattern is then established with different phase or dominant wavenumber.

A preference for establishing patterns with zonal wavenumbers 1 or 2 and not higher wavenumbers was observed. The large number of grid points would tend to partly eliminate the truncation of higher wavenumbers as the source of this phenomena. The preference seems to be inherent in the flow and not a result of restricted spacial resolution.

It would appear that the selection of the dominant wavenumber following a transition is a stochastic process. A random nonlinear interaction between motions of various scales at the time of transition may determine the dominant wavenumber. Such a process may be not unlike the atmosphere. This concept will be discussed further.

The large scale features of the flow pattern exhibited two degrees of freedom zonally, the dominant wavenumber and phase. Meridionally, the forcing "locked-in" the pattern giving zero degrees of freedom. Essentially, the problem of predicting the flow pattern reduced to predicting the dominant wavenumber and phase. This is similar to predicting the dominant Rossby wavenumber and the location of the troughs and ridges in the atmosphere.

The persistent, stationary patterns may be analogous to "blocking patterns" found in the atmospheric motions. Once identified, they may be successfully used in a persistence forecast. An important physical difference between the form of the forcing in the model and in the atmosphere should be noted. In the model, there is no longitudinal variation in the forcing to favor a stationary pattern. Whereas, in the atmosphere, considerable longitudinal variation exists due to topography and thermal effects which may contribute to immobility of the pattern in a "blocking" situation.

As previously noted, the flow contained two regimes of motion, even though the forcing was invariant throughout the integration. The regimes are identified by the dominant zonal wavenumber. The observed transitions between regimes are a good example of an almost-intransitive system as defined by Lorenz (1968). In such a system, a single solution will exhibit different statistics within different segments of a long time span. Clearly, the statistics of the model differ when wavenumber 1 was dominant, Day 420 to 520, from a 100 day period when wavenumber 2 was dominant.

Almost-intransitivity in a hydrodynamic system has important implications in determing long term statistics, or climate in the atmospheric case. In the model, with a sample of nearly a thousand days, only for about one hundred consecutive days was the flow in the wavenumber 1 regime. If the transition to this regime is considered stochastic, it is fortunate that it was observed, even in this lengthy sample. There may be regimes in the atmosphere which are likewise relatively short in duration and infrequent in appearance. Such regimes may easily be excluded in climatic statistics through sampling inadequacies, such as insufficient record length. So even without a forcing change, a significant climatic change might be noted when the atmosphere enters a regime that has not been previously observed and accounted for in climatic statistics.

The lack of observed periodicity in the flow has definite consequences relating to its predictability. The theory of predictability for nonperiodic flow has been presented by Lorenz (1963a; 1963b). Nonperiodic flow is linearly nondeterministic if the logarithm of the spectrum is integrable.

If we consider the kinetic energy of the zonal component as a fundamental measure of the state of the system, then on the basis of Figure 14 we can conclude that the flow is intrinsically linearly nondeterministic. While predictable with errors at short range, linearly nondeterministic, nonperiodic flow is unpredictable by linear means at extended range.

Furthermore, since the numerical solution found in this integration is nonperiodic, it is unstable. A slightly different perturbation would yield a very different numerical solution. From a different viewpoint, if this solution was to be determined from measurements of some real fluid system, an error in measuring the state would lead to a completely different solution. Thus, accurate dynamical prediction at extended range is dependent on error free measurements, which are practically unobtainable.

VII. MODE TRANSITIONS AND PREDICTABILITY

The changes in regime or in phase within a regime may be collectively referred to as mode transitions. Within a particular mode the statistics of the system are relatively invariant, but differ significantly from one mode to another.

The modes observed in the flow may be considered quasi-stable. The stability of the modes was indicated by their persistence even though perturbed by random small scale motions which were present in the flow. Changes in the stability occurred with time as a result of the damping, forcing and nonlinear interactions.

At some time a mode appeared to become very unstable and the flow entered a transition period. It seems likely that during transition, the nonlinear interactions between various scales of motion determine the regime and phase that evolve. It seems possible, though not explicitly investigated, that the small scales may be equally important as the large scales during transition in determining the state that evolves. From this investigation, there appears to be little evidence that the large scale motions by themselves determine the outcome of the transition.

The average lifetime of a particular phase of a regime was about 75 days. The lifetime of a particular regime was about 100 days for wavenumber 1. Wavenumber 2 persisted about 400 days on the first occasion and at least 500 days on the second. During a particular regime, a persistence forecast could be quite accurate in prediction of the dominant wavenumber, though the accuracy of the phase prediction would deteriorate more rapidly. Prediction of the following mode during a transition does not appear possible unless the state of the system is specified without error.

Thus, there seems to be varying degrees of predictability depending on the state of the flow. When a transition has just occurred, it is possible to predict the regime for a considerable time span. However, the exact length of time that the regime will exist is not apparently predictable if measurements are not exact. When the flow is entering a transition, there is essentially no predictability of the ensuing states if there are measurement errors.

This situation can be described in terms of a phase space where the points in the space correspond to states of the flow (cf. Lorenz, 1963a). The space can be partitioned into regions corresponding to the degree of stability of the flow. In our case, the exhibited flow is presumed to be inherently unstable due to its nonperiodicity. When the flow is in a persistent regime, it would correspond to a region of quasi-stability, i.e., low instability. A transition would correspond to entering a region of high instability.

The degree of instability is measured by the growth rate of small perturbations made in the flow. In the quasi-stable region, the statistics of neighboring trajectories diverge slowly. During transitions, the statistics of neighboring trajectories diverge rapidly. This would also be an indication of the growth rate of measurement errors in a numerical prediction of the flow. Trajectories in phase space enter either region nonperiodically.

The indeterminacy of the state of the system due to measurement errors translates into an uncertainty as to which trajectory the system is following. This indeterminacy would eventually result in a large departure in the predicted state of the system from that observed. What is important to note is that the rate at which the observed and predicted states diverge is determined by the degree of instability, i.e., the region occupied in phase space.

In this situation, a thorough knowledge of the large scale motions may be sufficient to predict the flow at considerable range with acceptable error during a quasi-stable mode. The lack of information about or inability to faithfully represent small scale motions renders the prediction scheme useless in predicting the length of the mode's existance and the evolution of the flow after entering a transition. Given that measurement errors will occur, the accuracy of a prediction scheme, in this case, is a function of the state of the system. When in certain states, such as those following a transition, the system is highly predictable. In other states, such as those associated with transitions, predictability is nonexistant.

Such a realization does not bode well for prediction at ranges which span one or more transitions. The integration of the governing equation of this model yielded two regimes, the atmosphere, presumably, possesses many more. An accurate prediction of the evolution of a system, such as modelled here, requires the successful prediction of transition processes. It is possible that successful prediction of transition processes, in this model or in the atmosphere, requires an accurate knowledge of small scale motions. In practical application, such as for the atmosphere, a model which faithfully represents the small scale motions requires observations on a small scale, numerous grid points, and excessively long computation time. This may not be feasible.

Thus, if a transition process similar to the one described here exists in the atmosphere, and if the atmosphere possesses almost-intransitivity, it would seem that prediction of the state or a statistic of the atmosphere at extended range would not be practically possible by direct integration of the governing equations.

Should the atmosphere possess a trajectory in phase space with varying instability, it would be valuable to catagorize the atmospheric states according to their stability. This would allow a determination at the time of observation of the degree of predictability that exists at that time. Forecasts could be made to cover the indicated predictability limit.

VIII. CONCLUSION

In this investigation, we have used a model of two-dimensional incompressible flow, which kinematically resembles some of the large scale features of the atmosphere, to examine extended range predictability. A sufficiently fine grid mesh was chosen so as to minimize the effects of truncation in the simulation.

After perturbing the initial zonal flow, we found with fixed forcing that the flow exhibited almost-intransitive behavior. There were two identifiable regimes within the span of the integration. They were found to be highly persistent. Within the regimes, the pattern movement was zonal at irregular intervals. Transitions occurred rapidly between regimes and between zonal phase of the pattern.

The tendency for persistence in a model possessing almost-intransitivity was suggested by Lorenz (1976). This simple model provides a vivid example of such behavior. This work provides further evidence that models which contain motions resembling those of the atmosphere possess almost-intransitivity. It does not, nor perhaps may any model which deals with a continuous system as a set of discrete points, establish the existance of almost-intransitivity of the atmosphere. However, it does suggest the possibility.

An examination of the time variations of the mean kinetic energy of the zonal component revealed no periodicities. This was taken as an indication that the system itself was nonperiodic. Examination of the motions of the patterns and the period between transitions likewise evidenced no periodicities. From a power spectrum analysis of the time variations in the mean kinetic energy of the zonal component, together with the assumption of nonperiodicity, it is concluded that the flow is unpredictable by linear means at extended range. Since nonperiodic flow is not uniformly stable (cf. Lorenz, 1963a), it is unpredictable at extended range by any method if errors exist in the measurement of the initial state.

In referring to "extended range", it must be understood to span one or more transitions. Certain features, such as dominant zonal wavenumber were highly predictable for ranges of 100 days and more due to persistence. However, no feature of the flow was seen as practically predictable through a mode transition.

It is suggested here that nonperiodic, almost-intransitive fluid motion may possess variable range predictability, depending on the state of the system. The critical factor in the range of predictability is the relative stability of the phase space trajectory with respect to small perturbations, which are in effect measurement errors. From this investigation, the trajectory stability is seen as variable along the trajectory.

Whether active on seasonal or climatic time scales, the existence in the atmospheric motions of a regime and transition mechanism similar to that suggested here would have a profound effect on extended range predictability. At a time when the atmosphere is "choosing" between various possible regimes, the measurements must be most precise, if not exact, to determine the subsequent behavior. Between transitions, the regime motions might well be predictable with less accurate measurements owing to its decreased instability compared to transition periods. However, the determination of the lifetime of a regime a priori might, like transition predictions, rely on perfect or near perfect specification of the atmospheric state.

Lorenz (1963b) found that two neighboring solutions of nonperiodic flow must eventually diverge until no resemblance between the two can be detected. What is suggested here is that the difference between neighboring solutions may increase almost discontinuously in conjunction with the occurrence of transitions and much more slowly in between. A prediction scheme which utilizes this information would endeavor to determine the error growth rate for the observed state of the system. Predictions could then be made at a range consistent with the stability.

This investigation indicates the difficulty of extended range prediction of the instantaneous state of an almost-intransitive system when knowledge of the initial state is imperfect. It is suggested that this is due partly to the transition mechanism. Furthermore, prediction of mean values of the system at extended range would seem no easier than prediction of the regime itself, since the statistics of each regime would be considerably different for an almost-intransitive system.

FIGURES

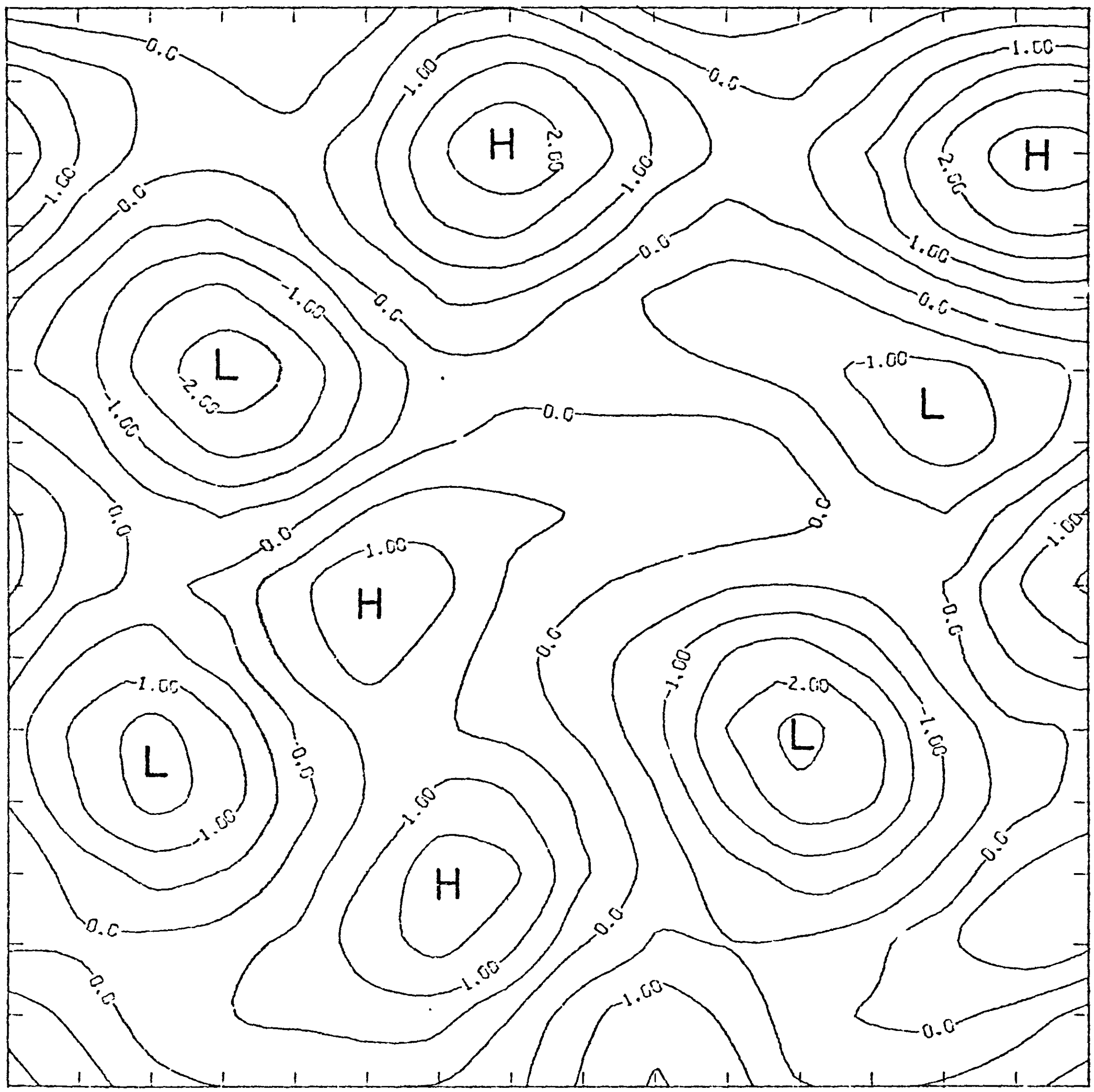

Figure 1. Typical stream field after perturbation. Day 885. Contours in dimensionless units.

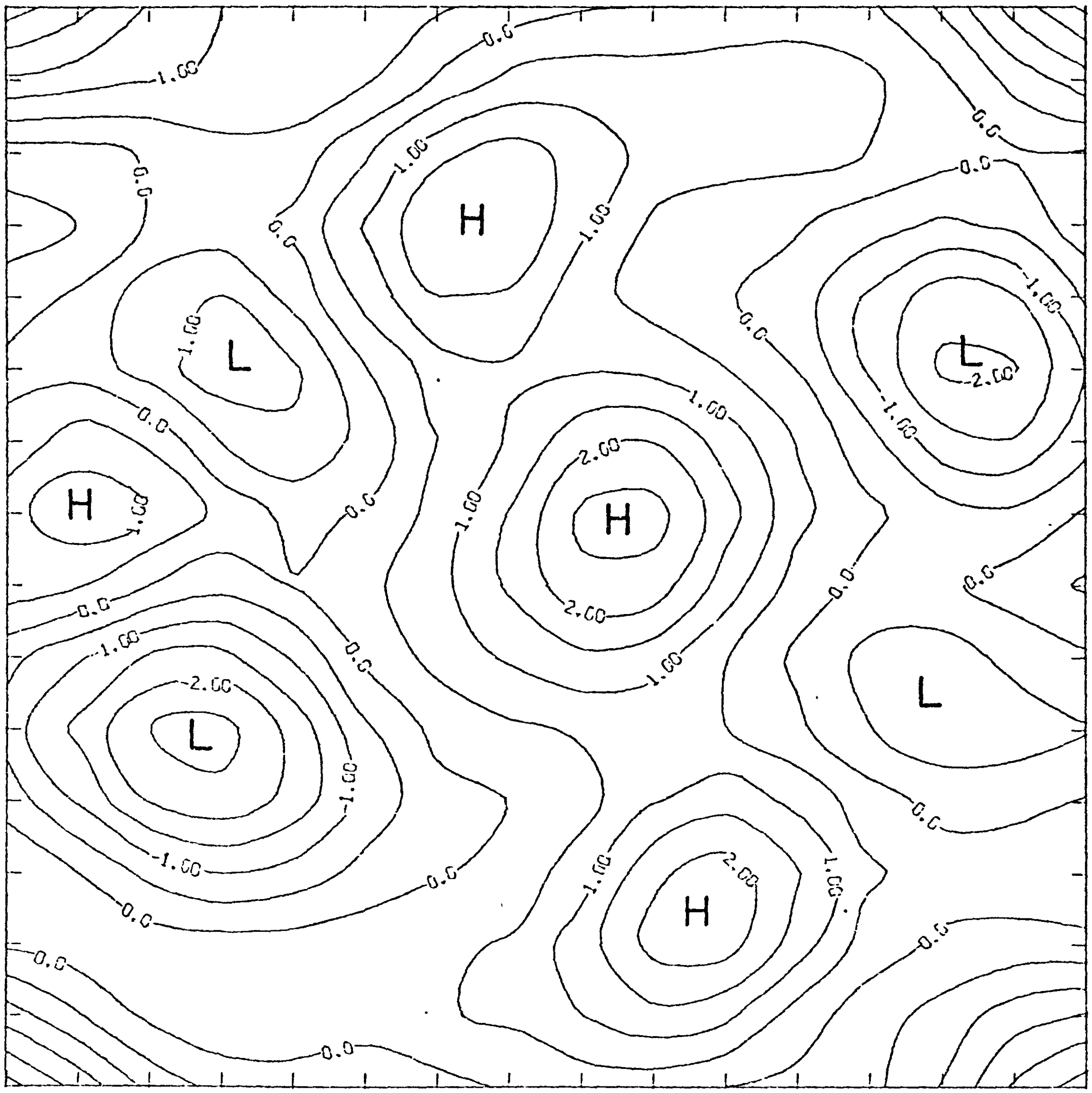

Figure 2. Stream field, Day 495, showing the regime with zonal wave-number 1 dominant. Contours in dimensionless units.

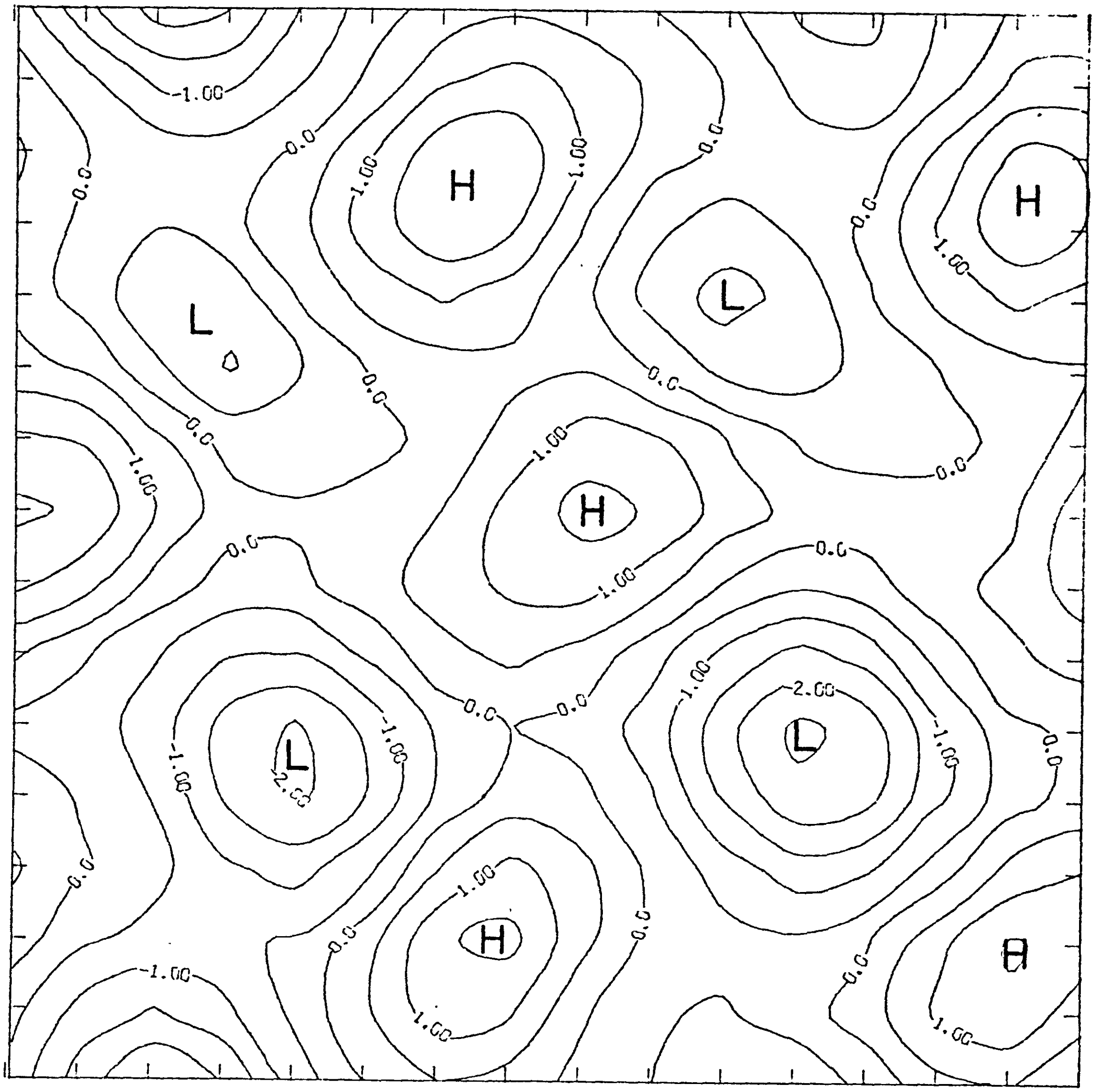

Figure 3. Stream field, Day 65, showing the regime with zonal wavenumber 2 dominant. Contours in dimensionless units.

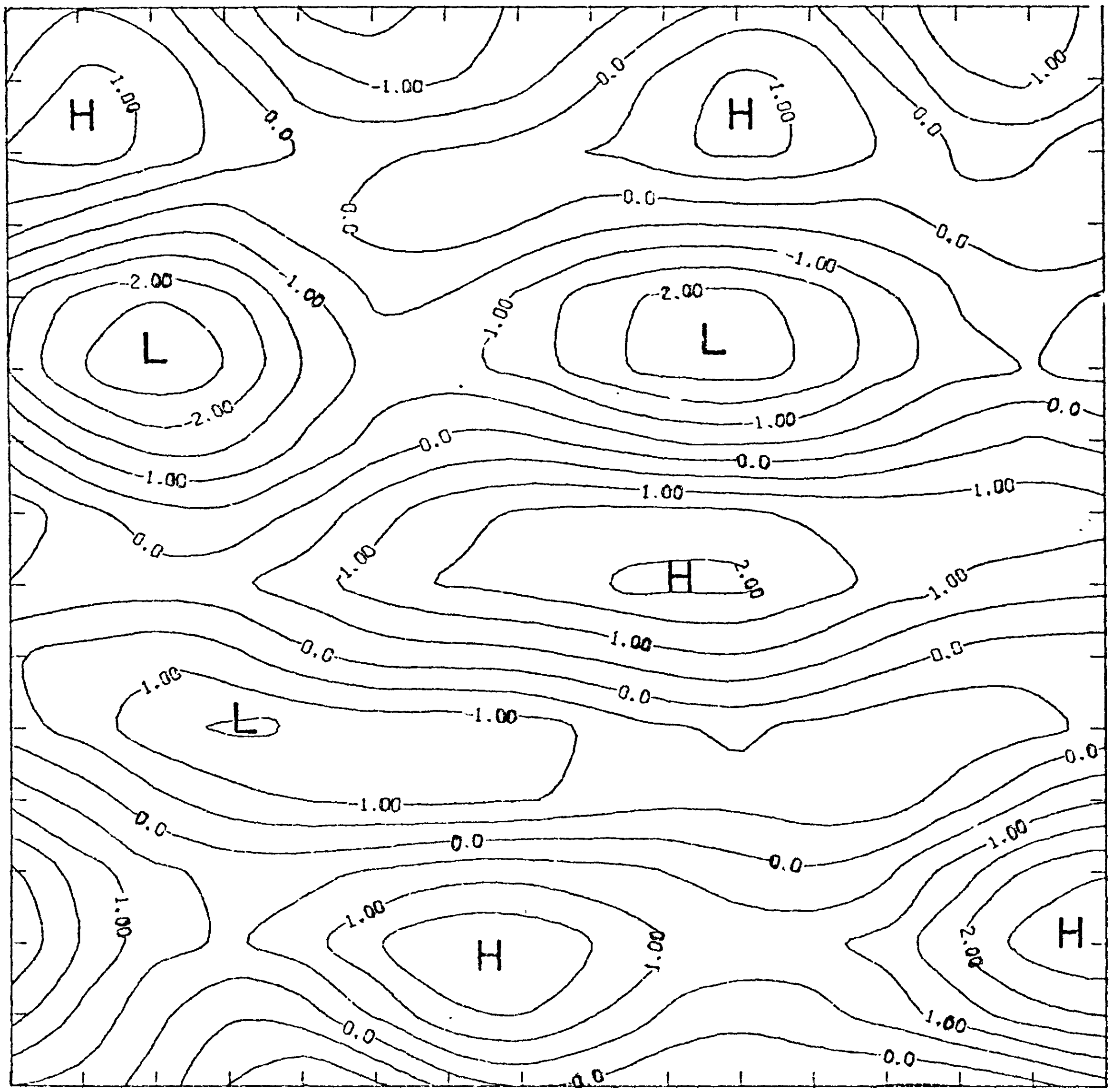

Figure 4. Stream field, Day 125, showing the strong zonal flow during a phase transition. Contours in dimensionless units.

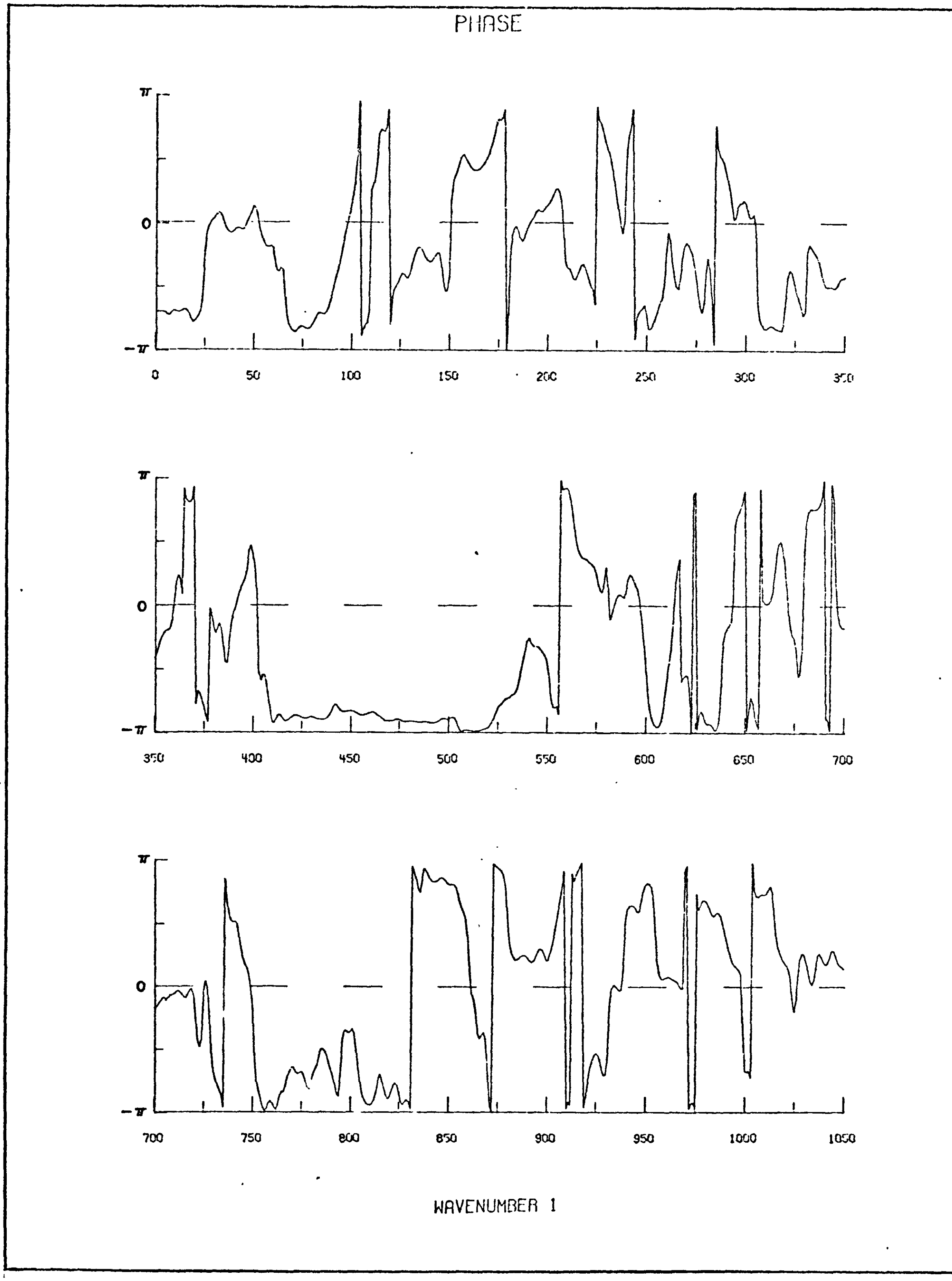

Figure 5. Phase angle, ϕ_1, as a function of time, in days, for zonal wavenumber 1.

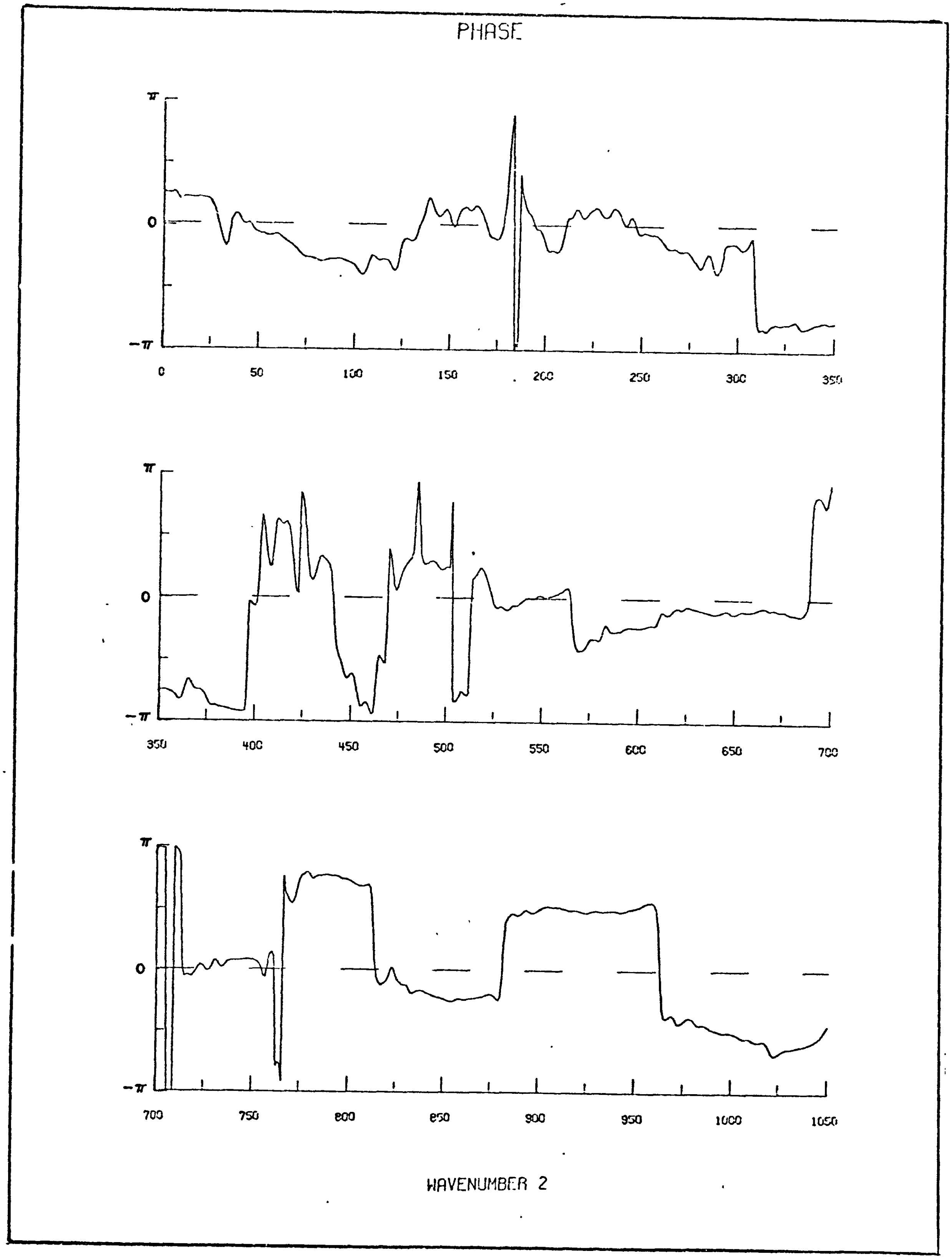

Figure 6. Phase angle, ϕ_2, as a function of time, in days, for zonal wavenumber 2.

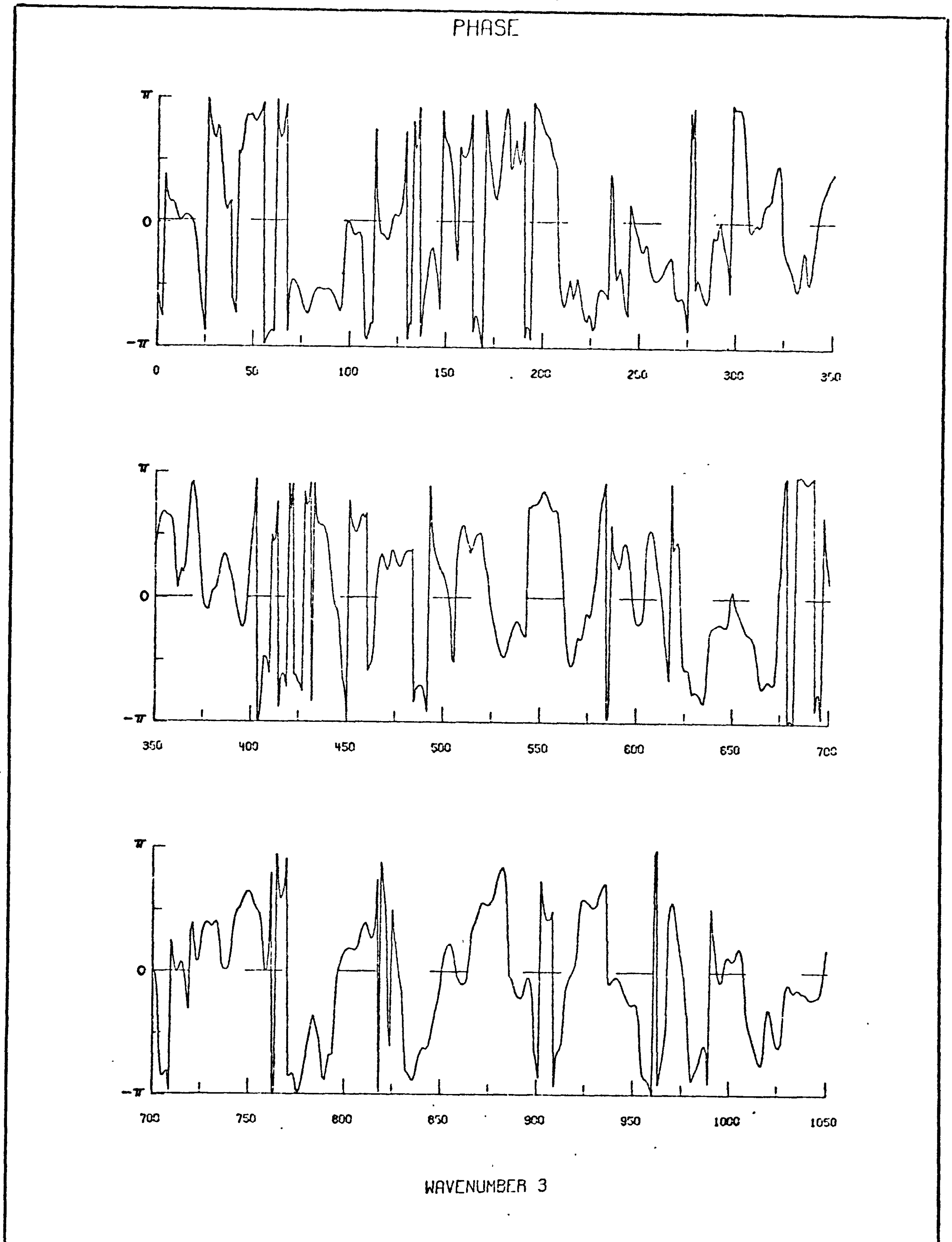

Figure 7. Phase angle, ϕ_3, as a function of time, in days, for wavenumber 3.

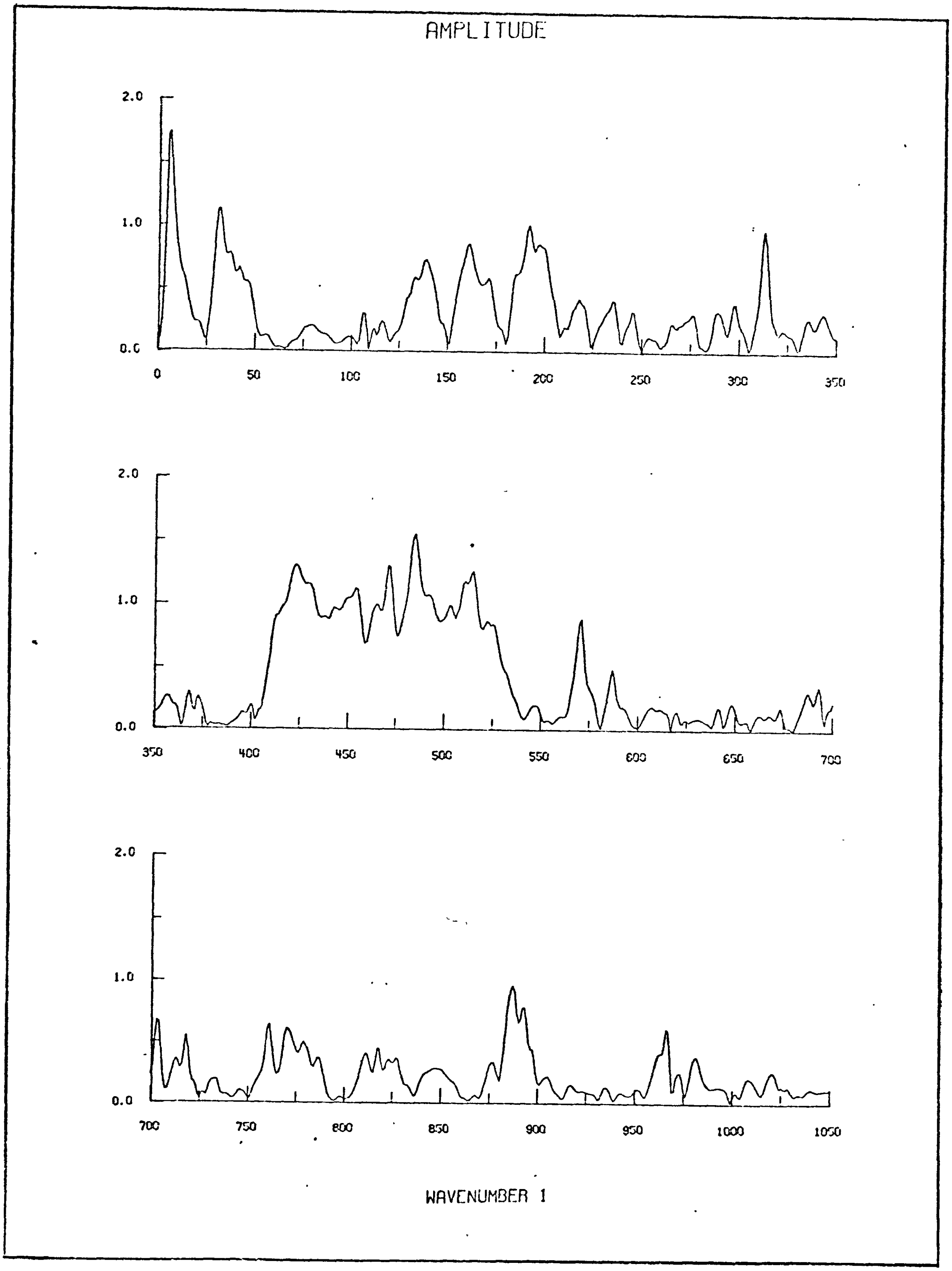

Figure 8. Amplitude time series for zonal wavenumber 1. Amplitude in dimensionless units, time in days.

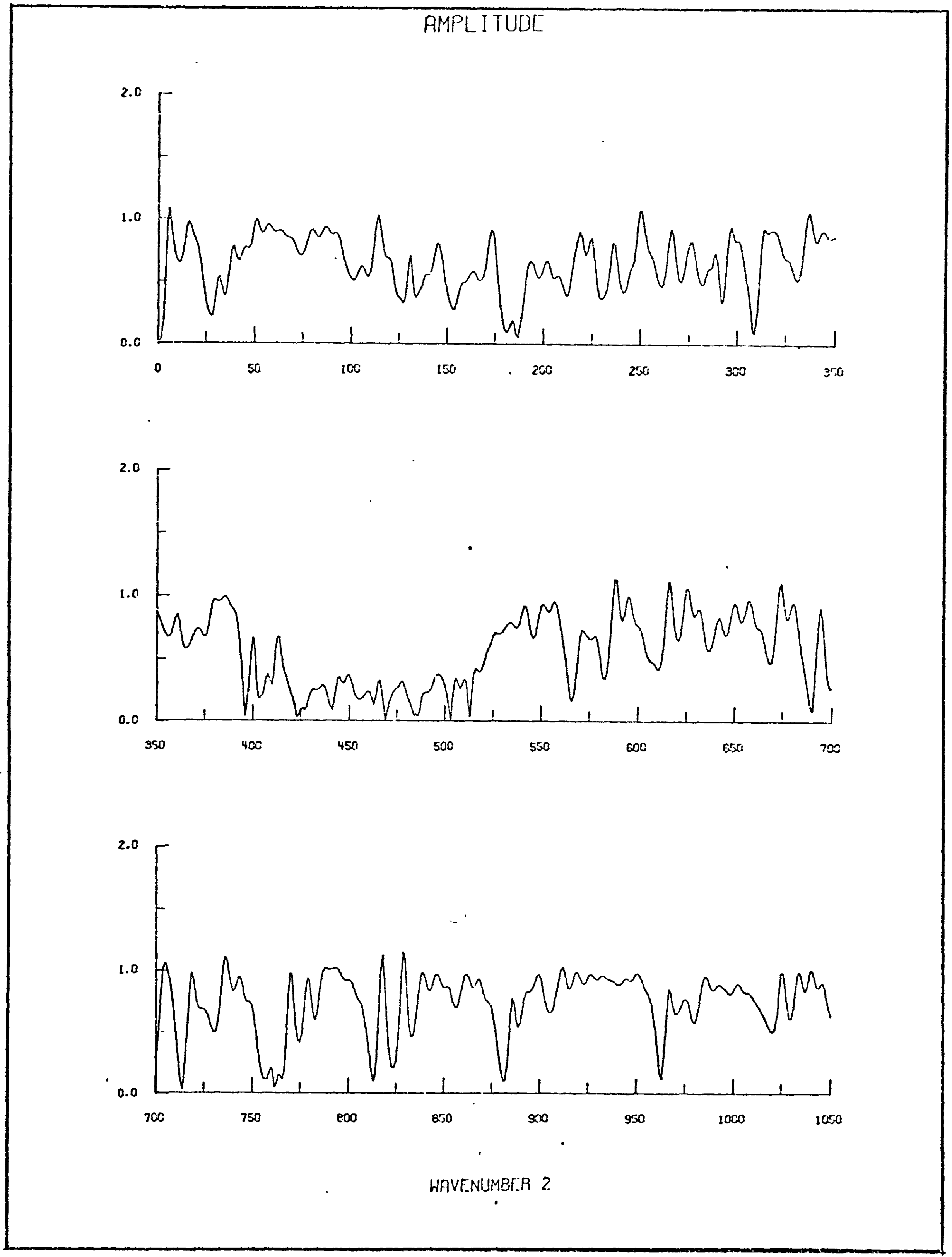

Figure 9. Amplitude time series for zonal wavenumber 2. Amplitude in dimensionless units, time in days.

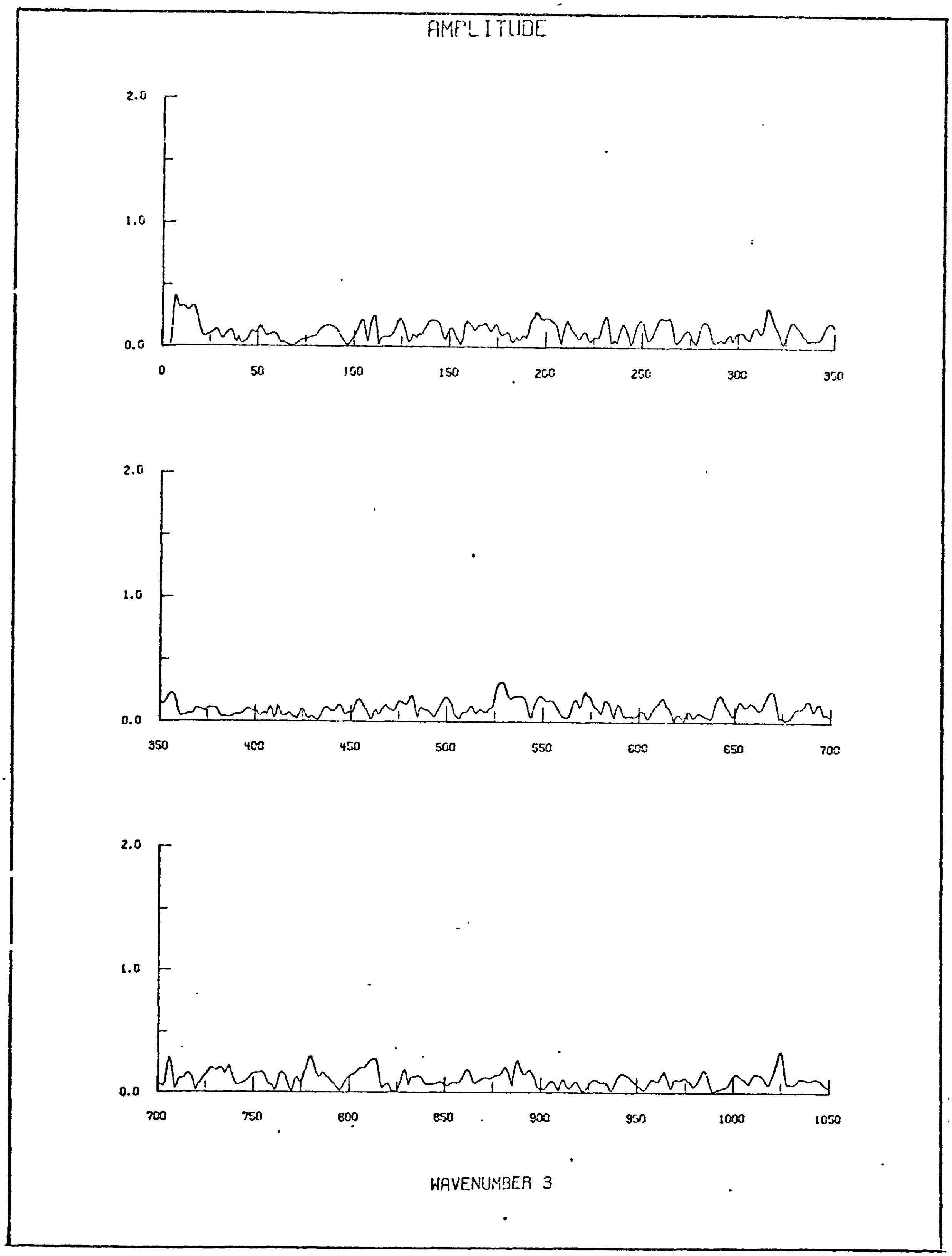

Figure 10. Amplitude time series for zonal wavenumber 3. Amplitude in dimensionless units, time in days.

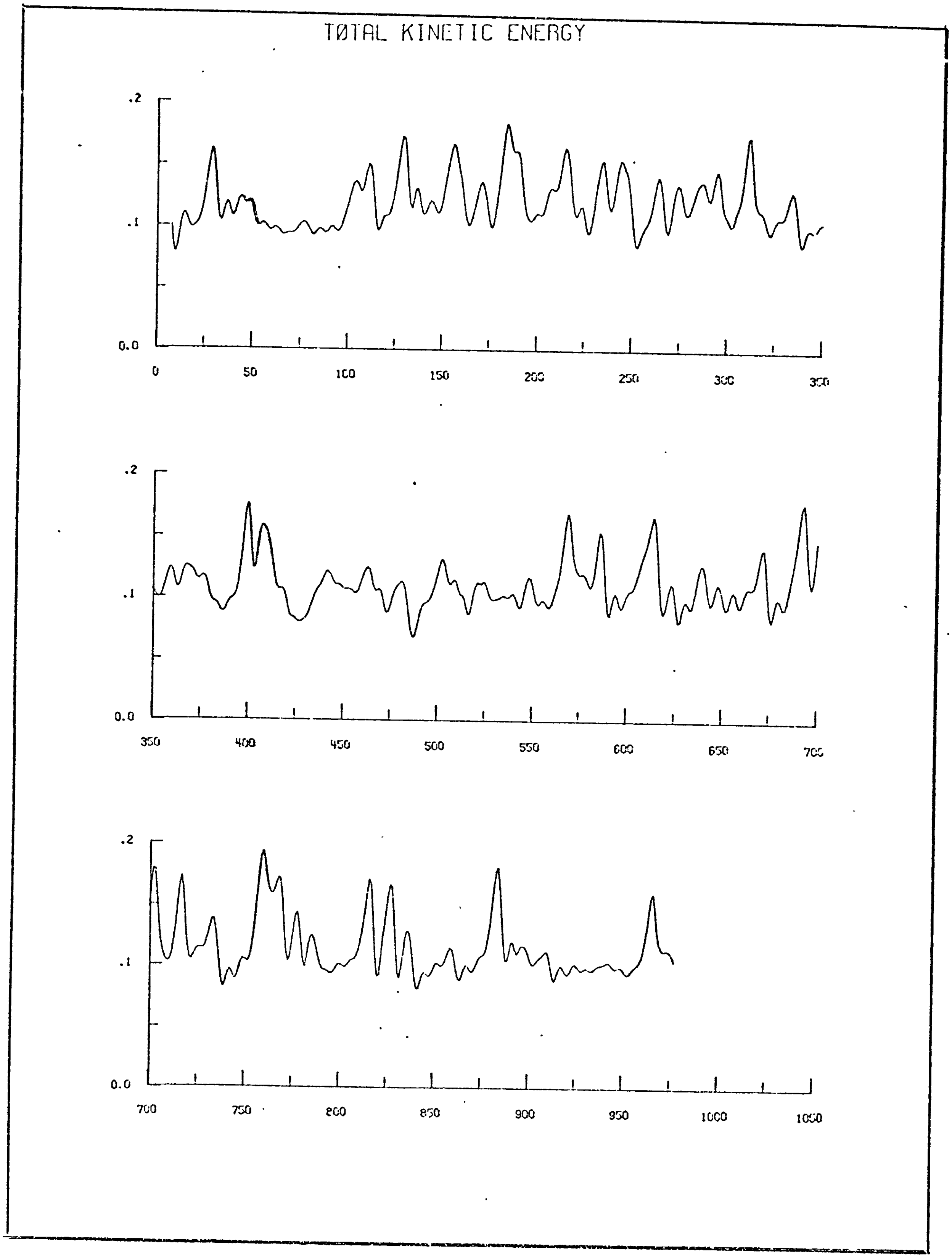

Figure 11. Time series of the mean total kinetic energy per mass. Kinetic energy in dimensionless units, time in days.

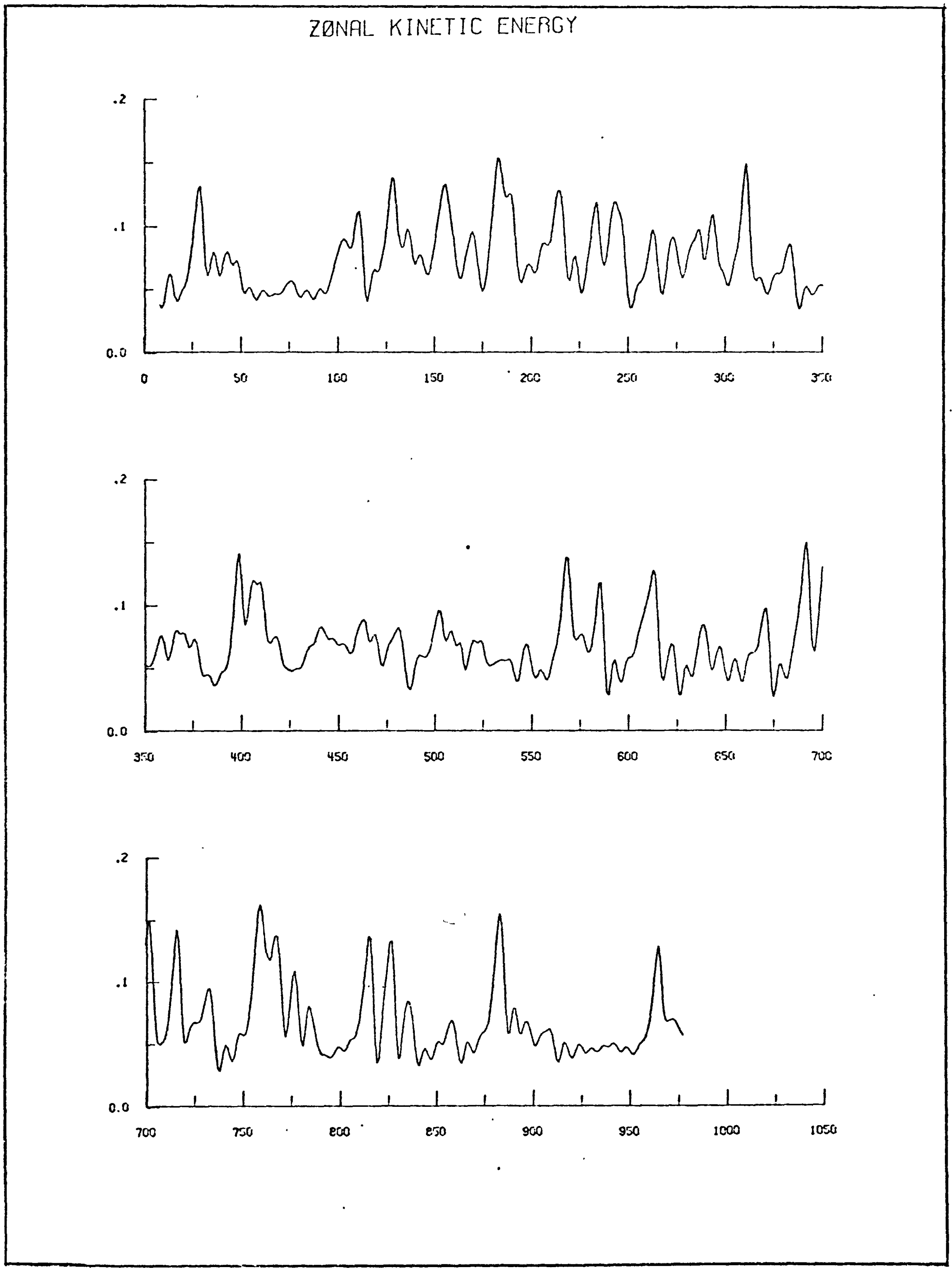

Figure 12. Time series of the mean zonal kinetic energy per mass. Kinetic energy in dimensionless units, time in days.

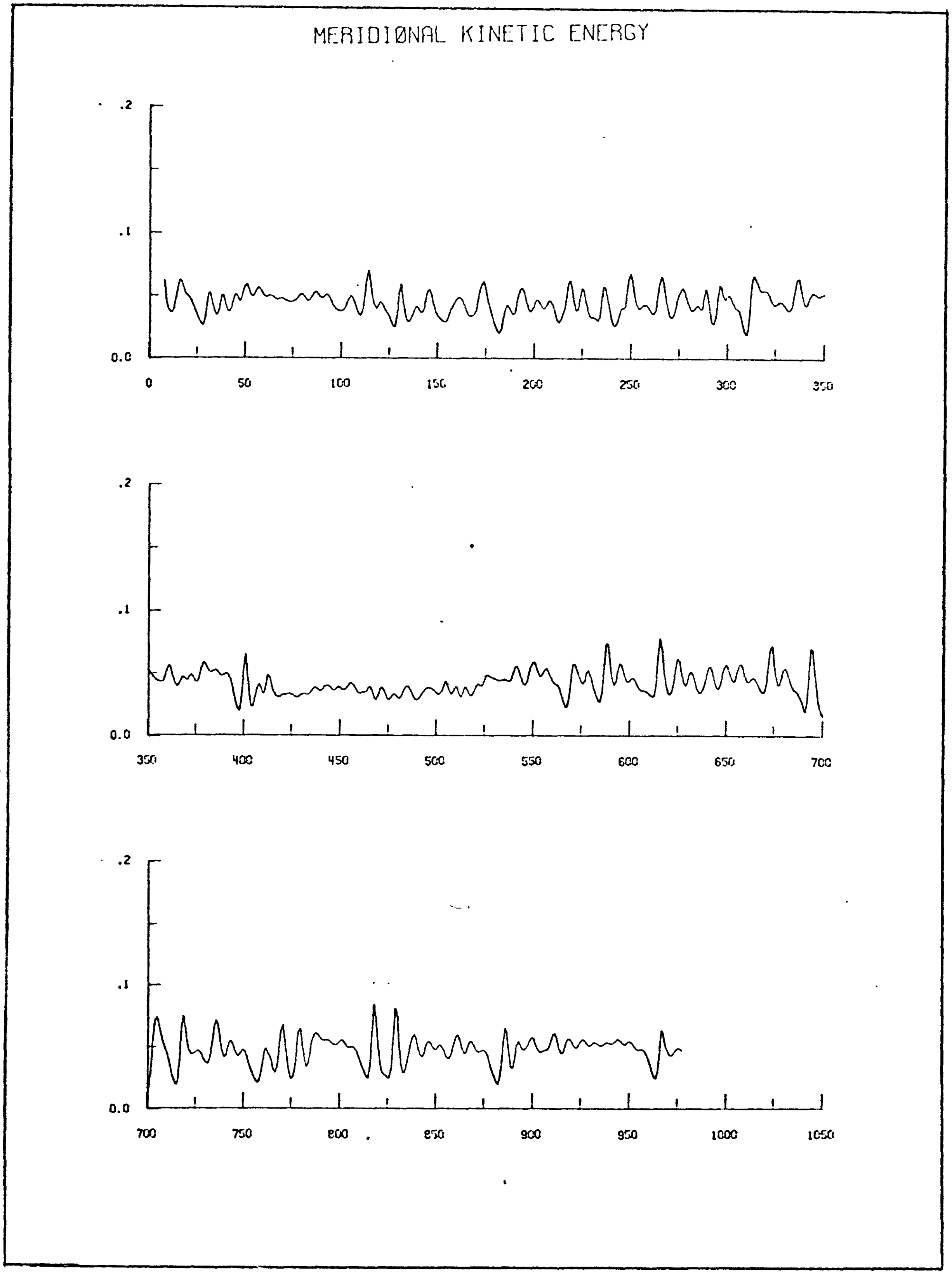

Figure 13. Time series of the mean meridional kinetic energy per mass. Kinetic energy in dimensionless units, time in days.

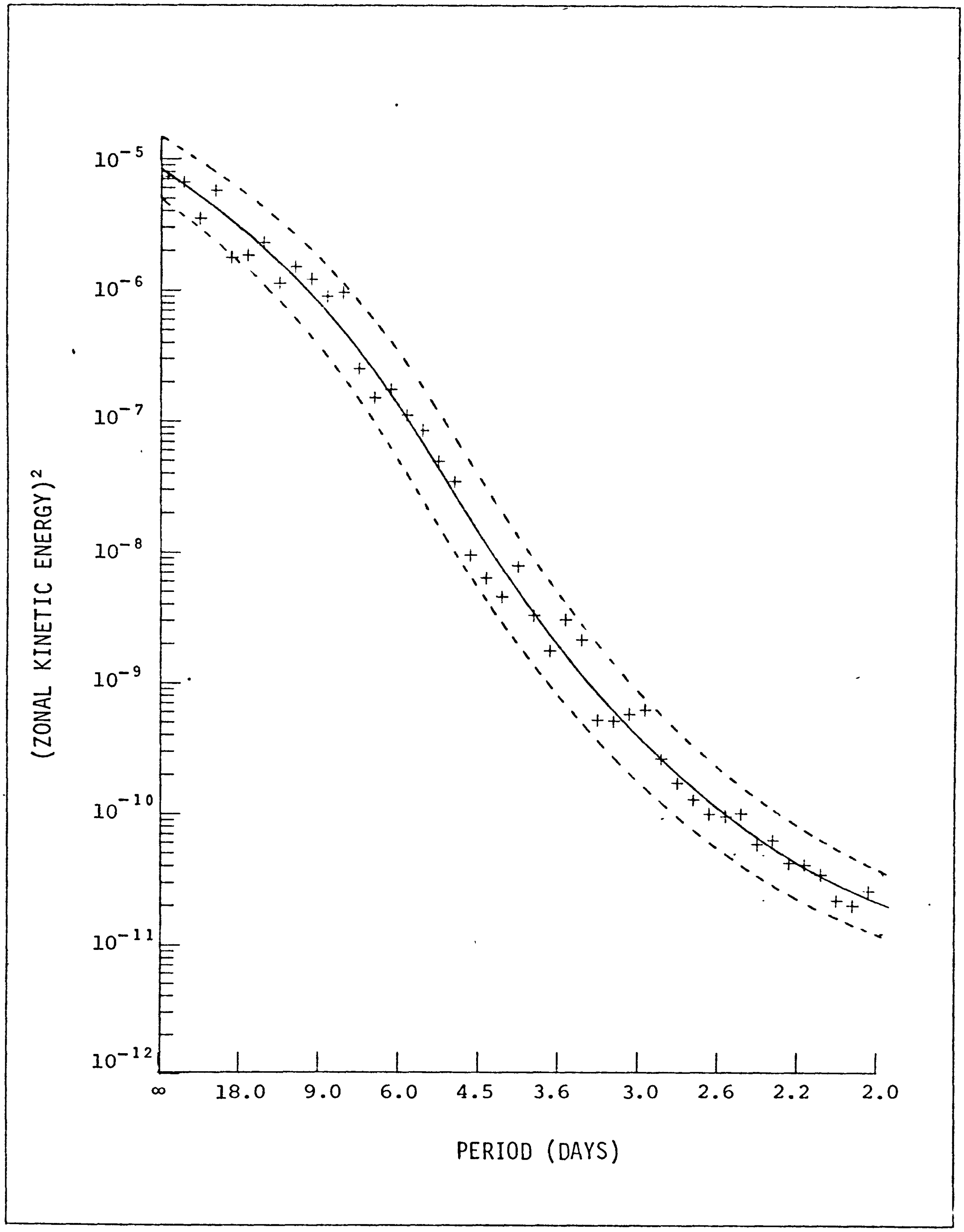

Figure 14. Power spectrum of time variations in the mean kinetic energy of the zonal component of the flow. Dashed lines indicate 90% confidence limits.

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