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The measurement of the W mass at the LHC: shortcuts revisited

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Abstract

The claim that the W mass will be measured at the LHC with a precision of $\mathcal{O}(10)$ MeV is critically reviewed. It is argued that in order to achieve such precision, a considerably better knowledge of the u_v, d_v, s, c, and b structure functions of the proton than available today is needed. This will permit to assess with adequate precision the production characteristics of the W and Z bosons in the proton–proton collisions at the LHC, and their effect on the p_T spectra of charged leptons from W and Z decays. An experimental programme is suggested that will deliver the missing information. The core of this programme is a dedicated muon scattering experiment at the CERN SPS, with simultaneous measurements on hydrogen and deuterium targets.

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1 Relevance of the W mass

Whilst the Z mass is well measured to ±2.1 MeV [1], the W mass is measured at the Tevatron to ±48 MeV [2] and at LEP to ±33 MeV [3]. The W mass is a fundamental parameter of the Electroweak Standard Model. Although a precision of the W mass that matches the precision of the Z mass is experimentally not within reach, a much better precision than available today is desirable to make the most out of the relation between the W mass and the Fermi coupling constant G_{F}, the latter of which is also well measured with a relative precision of 1×10^{-5}.

We hold that in previous analyses of the precision of the W mass measurement that can be achieved at the LHC, shortcuts have been made that are not justified. Unless additional experimental information is provided, predictions that have been made so far are too optimistic.

Throughout this paper, we take it for granted that the W^+ and W^- masses are equal[1].

The experimental programme proposed in this paper will allow to test at the LHC the equality of the W^+ and W^- masses at the 10 MeV level, and to determine their average at the same level of precision.

2 Measuring the W mass at the LHC and at the Tevatron

The ATLAS [4] and CMS [5] Collaborations claim to determine the W mass with a precision of $\mathcal{O}(10)$ MeV from the p_{T} spectra of charged leptons from $W \rightarrow l\nu$ decays. Thereby, the comparison with $Z \rightarrow l^+l^-$ decays serves to 'calibrate' the relation between the p_{T} spectrum of decay leptons and the W mass.

We note that the scale gap between the p_{T} spectrum of decay leptons which is at the 40 GeV/c level and the wanted W mass precision which is at the 10 MeV level, amounts to a factor of 4000 (!).

The quantitative consequences of this large scale gap are highlighted by Fig. 1 which shows the change of the p_{T} spectrum of charged leptons from the decay $W \rightarrow l\nu$ by the inclusion of what PYTHIA predicts as p_{T} of W's at the LHC. Since the W mass is largely derived from the characteristics of the Jacobian peak, it is intuitively clear that an unusually precise understanding of the shape of the p_{T} spectrum is mandatory.

It is obvious that under such circumstances great care must be devoted to all sorts of effects that cause either the production characteristics of W and Z to be different, or the decay characteristics of $W \rightarrow l\nu$ and $Z \rightarrow l^+l^-$ to be different, or both. Either difference would lead to different p_{T} spectra of leptons from W decays and from the reference Z decays.

[1]The best experimental support of this assertion stems from a comparison of the measured μ^+ and μ^- lifetimes [1], which translates into an equality of W^+ and W^- masses at the 1.6 MeV level, a precision which is out of reach at the LHC.

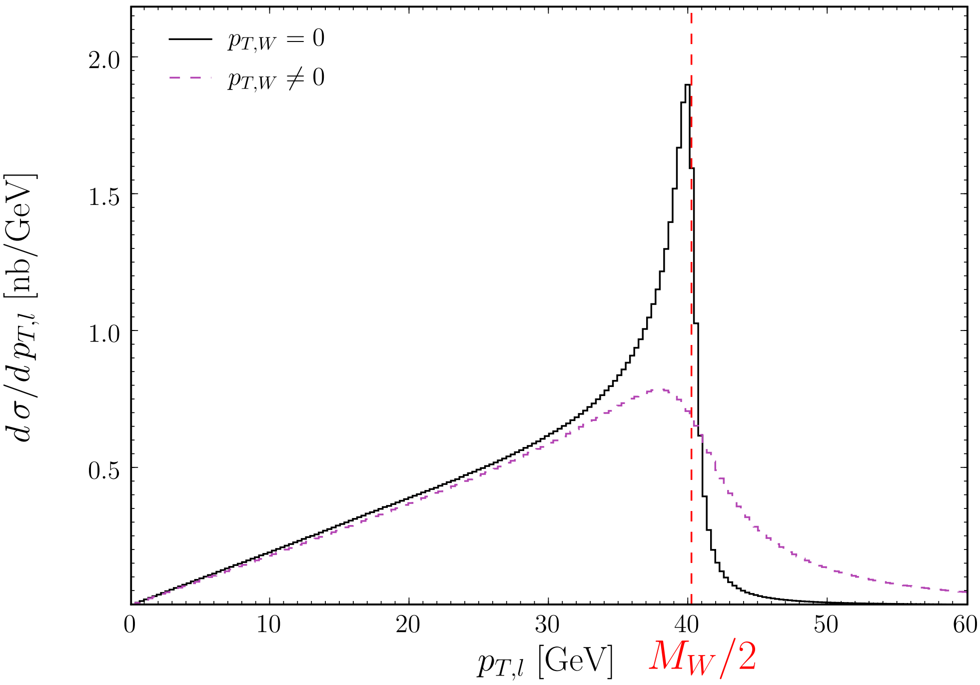

Figure 1: Simulation of the p_{T} spectrum of charged leptons from the decay $\mathrm{W} \rightarrow l\nu$; the full line is generated with zero p_{T} of the W's, the broken line represents the effect of a non-zero p_{T} at the level predicted by PYTHIA.

Table 1 recalls that quite different quark–antiquark pairs contribute to the production of W^+, W^-, and Z. In Fig. 2, taken from Ref. [6], the pertinent contributions to the W^+, W^-, and Z cross-sections are shown as a function of beam energy, and specifically for the the Tevatron and LHC energies.

Table 1: Quark-antiquark pairs that contribute to W^+, W^-, and Z production.

W^+	$\mathrm{u\bar{d} + u\bar{s} + u\bar{b} + c\bar{d} + c\bar{s}} + \cdots$
W^-	$\mathrm{d\bar{u} + d\bar{c} + s\bar{u} + s\bar{c}} + \cdots$
Z	$\mathrm{u\bar{u} + d\bar{d} + s\bar{s} + c\bar{c} + b\bar{b}} + \cdots$

As a consequence, in the production mechanisms of W^+, W^-, and Z, the following differences need to be carefully assessed:

- in the respective structure functions[2];

[2] We use the terms 'structure function' or 'pdf' (for 'parton distribution function') with the same meaning; throughout this paper, they refer to the proton.

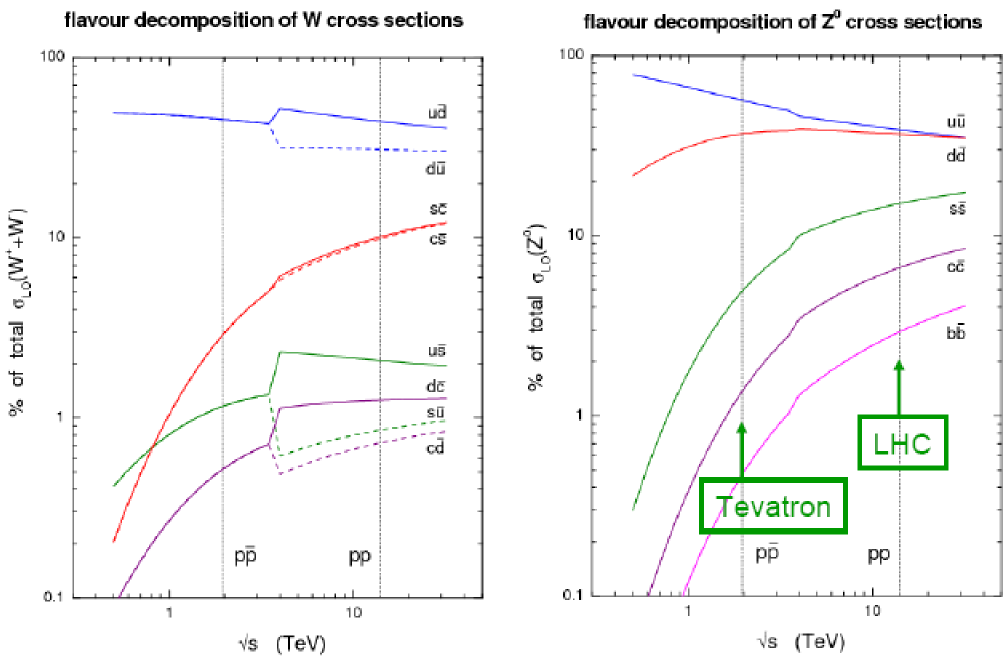

Figure 2: Contributions of different quark–antiquark annihilations to $W^{\pm}$ (left panel) and Z (right panel) production, as a function of the beam energy.

- in the respective weak quark coupling constants, and
- in the respective transverse momenta k_T.

Because the W mass is determined from the p_T spectrum of decay leptons, our interest focuses on the direction *perpendicular* to the beam.

Accordingly, first we discuss p_T's and the spin components of W^+, W^-, and Z, along the direction perpendicular to the beam. The respective non-zero spin components perpendicular to the beam direction constitute 'longitudinal' polarizations[3] of W^+, W^-, and Z. For several reasons, they are different from each other:

1. for the types of quarks that participate according to their weak coupling constants;
2. for the correlation of the Bjorken-x of the participating quarks and antiquarks with k_T (small x is correlated with large k_T);
3. for the correlation of the k_T with the hardness scale of the process (the W and Z masses are different); and

[3]In analogy to the longitudinal polarization vector of a virtual photon.

4. for the dependence of k_{T} on the quark type (heavier quarks have larger k_{T}).

Then, we recall that with respect to the boson spin direction, the angular distributions of decay leptons are different for W^+, W^-, and Z bosons: they reflect the V$-$A and V+A components in the boson–quark coupling. In the $\mathrm{W}^\pm$ rest frame, the pure V$-$A coupling leads to the following angular distribution of the charged-lepton emission amplitude:

$$w(\theta) \propto 1 \pm \cos\theta \,, \tag{1}$$

where θ denotes the angle between the direction of the spin vector and charged-lepton emission. In the Z rest frame, the quark-charge-specific mixture of V$-$A and V+A leads to the angular distribution

$$w(\theta) \propto 1 + \gamma \cos\theta \,, \tag{2}$$

where $\gamma < 1$.

The charged-lepton emission asymmetries with respect to the spin component perpendicular to the beam direction are modified by the Lorentz boost from the boson rest frame into the laboratory system.

It turns out that, on top of the genuine differences in the longitudinal polarizations of W and Z bosons, an important contribution to the differences in the p_{T} distribution of charged leptons from W and Z decay in the laboratory system, stems from the interference between transverse and longitudinal boson polarization amplitudes.

Finally, we note the joint effect of the quark structure functions and the quark k_{T} on the rapidity distribution of the charged leptons from boson decay. Since charged leptons with a rapidity $|\eta| > 2.5$ can hardly be measured, a limitation of the rapidity range impacts on the charged-lepton p_{T} distribution.

Altogether, from the different longitudinal polarizations of W^+, W^-, and Z, in conjunction with their different p_{T}'s, in conjunction with the different angular distributions, and in conjunction with the interplay with different structure functions of contributing quarks and antiquarks it follows that the p_{T} spectra of decay leptons from W^+, W^-, and Z decay, will be different from each other.

Next, we address the important differences of W^+, W^-, and Z production in pp collisions at the LHC, and in $\mathrm{p\bar{p}}$ collisions at the Tevatron.

In the pp collisions at the LHC, there is, for a given boson, inherent symmetry in the forward–backward production of charged leptons: at the polar angles θ and $\pi - \theta$, the rates and the momentum spectra are identical. However, the rates and the momentum spectra are different between W^+, W^-, and Z. We note in particular the difference in the rates and the momentum spectra of charged leptons from W^+ and W^- decays, which *a priori* renders a common analysis of leptons with positive and negative charge questionable.

In the $\mathrm{p\bar{p}}$ collisions at the Tevatron, there is a small forward–backward asymmetry in the production of charged leptons from Z decay, and a strong asymmetry from the decays of W^+ and W^-, since, e.g., W^+ are produced preferentially along the incoming proton direction. However, the rates and the momentum spectra of positive leptons from W^+ at the polar angle θ are exactly the same as the rates of negative leptons from W^- at the polar angle

$\pi - \theta$. The same holds when integrated over the same range of θ and $\pi - \theta$, respectively. As a consequence, a common analysis of leptons with positive and negative charge is justified.

Figure 3, taken from Ref. [7], illustrates the rapidity η and the p_{T} of $\mathrm{W}^{\pm}$ production, and Fig. 4, also taken from Ref. [7], shows the same for the respective decay leptons. For comparison, the characteristics of W production and decay in pp collisions are compared with those in $\mathrm{p\bar{p}}$ collisions.

The difference between pp and $\mathrm{p\bar{p}}$ collisions is rather striking. We also note that in pp collisions, at $|\eta| \sim 0$, the difference in the production of W^{+} and W^{-} is smallest. This is because in this region the contribution from the collision of sea quarks with sea quarks is largest.

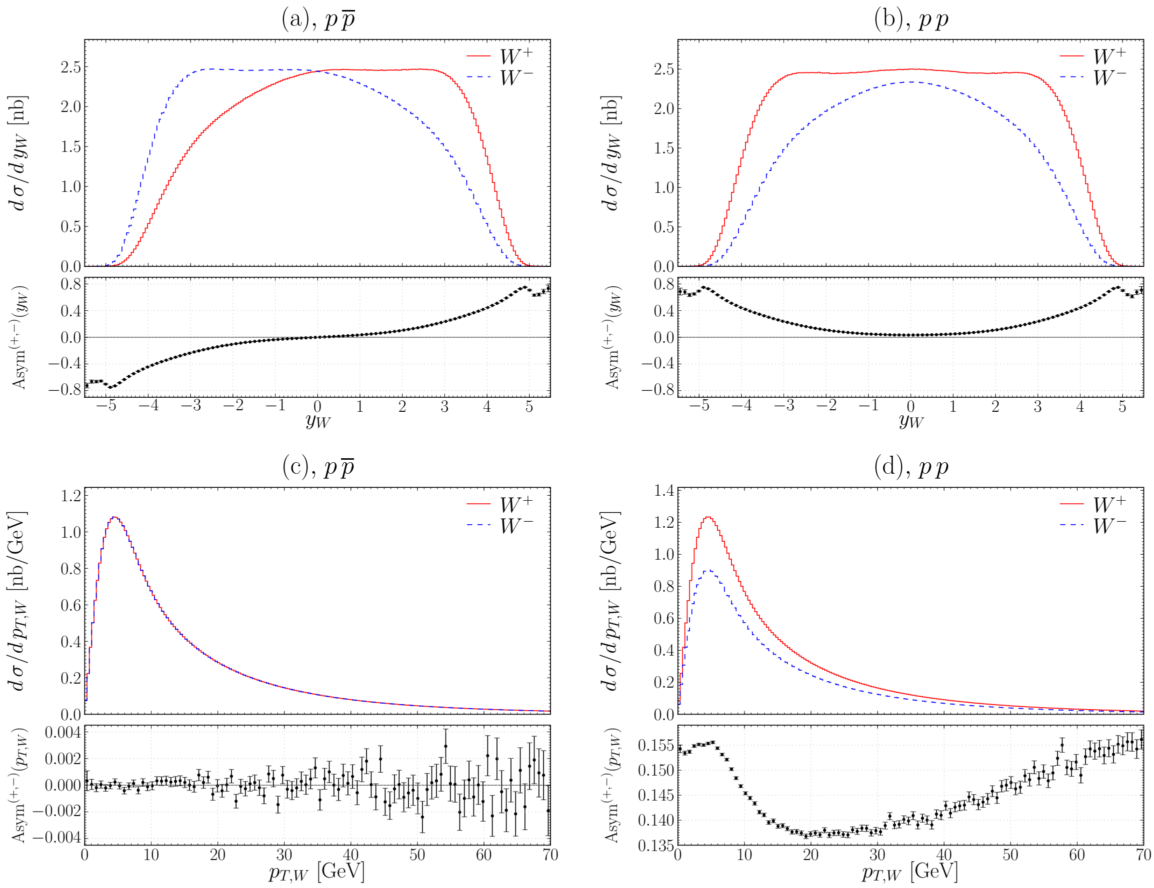

Figure 3: Rapidity η and the transverse momentum p_{T} of $\mathrm{W}^{\pm}$'s, in $\mathrm{p\bar{p}}$ collisions (left panels) and in pp collisons (right panels).

The common analysis of charged leptons from W^{+} and W^{-} is equivalent to a W decay with equal V−A and V+A decay amplitudes, which is close—because of Nature's choice of the electroweak mixing angle $\sin^2\theta_{\mathrm{w}}$—to the Z decay amplitudes.

At the Tevatron, there is a fortunate cancellation effect when one calibrates, without regard to the charge sign, charged lepton spectra from W decays with lepton spectra from Z decays. Further, at the Tevatron's energy the contribution of c quarks to Z production is small.

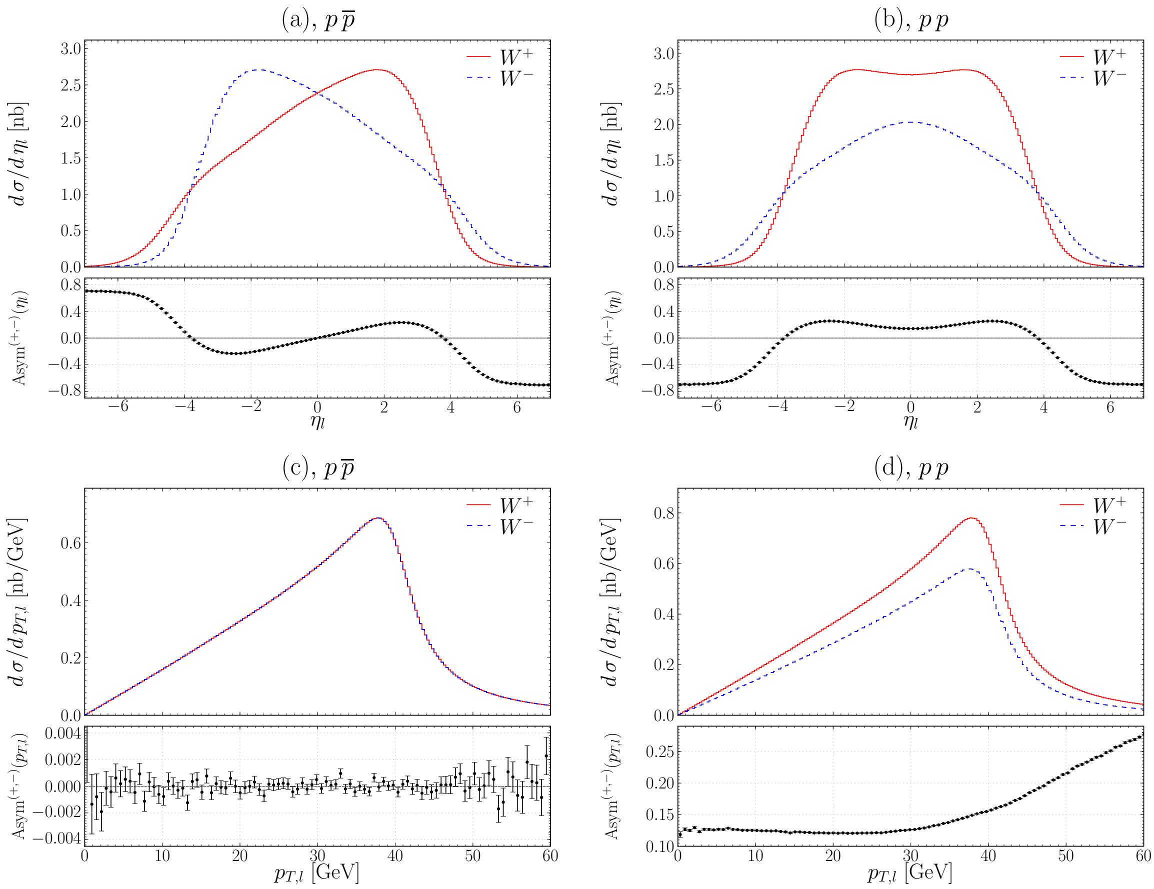

Figure 4: Rapidity η and the transverse momentum p_{T} of charged leptons from the decay of $\mathrm{W}^{\pm}$'s, in $\mathrm{p\bar{p}}$ collisions (left panels) and in pp collisons (right panels)

Summing up, the ensuing systematic error of the W mass at the Tevatron is not dominant and rather comparable with the statistical error.

At the LHC, because of the preponderance of W^+ over W^- in pp collisions, there is no cancellation at work.

If there were only sea quarks involved in the production of W's in pp collisions, symmetry in the production of W^+ and W^- would not be broken. Rather, symmetry is broken by valence quarks, more specifically through the difference of the $\mathrm{u_v}$ and $\mathrm{d_v}$ structure functions of the proton. It follows that the relative amount of light sea quarks to valence quarks, and the respective flavour compositions, must be known with sufficient precision. This is the effect from the 1st quark family.

The contribution of the c quarks to W and Z production at the Tevatron and LHC energies is compared in Fig. 2 and shows a distinct asymmetry: whilst c quarks are important for W production both at the Tevatron and even more so at the LHC, they are important for Z production only at the LHC. The partner of a c quark to form a W boson—an s quark—has different distributions in x and k_{T} than a c quark that is needed as partner to produce a Z boson. This is the effect of the 2nd quark family.

There is also an effect from the 3rd quark family, through the contribution of Z production from $b\bar{b}$ pairs[4].

As a consequence of the above, the analysis concept that is valid for $p\bar{p}$ collisions at the Tevatron, cannot *a priori* be used as template for pp collisons at the LHC. There, a more detailed analysis is warranted.

The importance of the intricacies of the production and decay mechanisms, and their effect on the p_T spectra of decay leptons of W^+, W^-, and Z, has been missed in the LHC physics studies made so far. We note that not a single study made a difference between charged leptons from W^+ and W^- decays. As a consequence, unrealistically small errors at or below the 10 MeV level were reported for the W mass measurement at the LHC.

In this paper, we estimate from existing experimental information the uncertainties in the production and decays of W^+, W^-, and Z, at the LHC and we argue that they are too large to be ignored if one wants to measure the W mass with a precision at the 10 MeV level. We will argue that better precision on specific aspects of the pdf's of quarks and antiquarks in the proton is needed than is available today. Last, but not least, we will discuss ways how to obtain experimentally the missing information.

The discussion in this paper applies *mutatis mutandis* also to the determination of the W mass from m_T spectra. The determination of m_T involves the reconstruction of the neutrino transverse momentum as missing transverse momentum which, however, leads to the question how large the systematic error of this measurement is. We consider it too large to be useful for the measurement of the W mass at the 10 MeV level.

3 Today's knowledge of the proton structure functions

The proton structure functions have been studied experimentally and theoretically for 40 years. As demonstrated by the efforts behind the CTEQ [8] and MSTW [9] sets of pdf's, the importance of the subject is widely recognized.

The current understanding of the proton pdf's is summarized in Fig. 5 which shows the MSTW–2008 set [6].

It is advocated and widely believed that the proton pdf's are precise enough not to pose a limitation for LHC data analysis. For example, the u_v and d_v distribution functions are claimed to be precise to 2% [6].

We consider the 2% precision as unrealistic and argue that the proton pdf's are not well enough determined for a precision determination of the W mass. This is because

1. the CTEQ and MSTW pdf's differ by much more than 2%, as shown in Fig. 6 taken from Ref. [9] although they stem largely from the same input data; and
2. the error estimation of the parameters of pdf fits is done in a way that tends to underestimate the real errors of fit parameters.

[4]For W production, the contribution from b quarks is negligible both at the Tevatron and at the LHC.

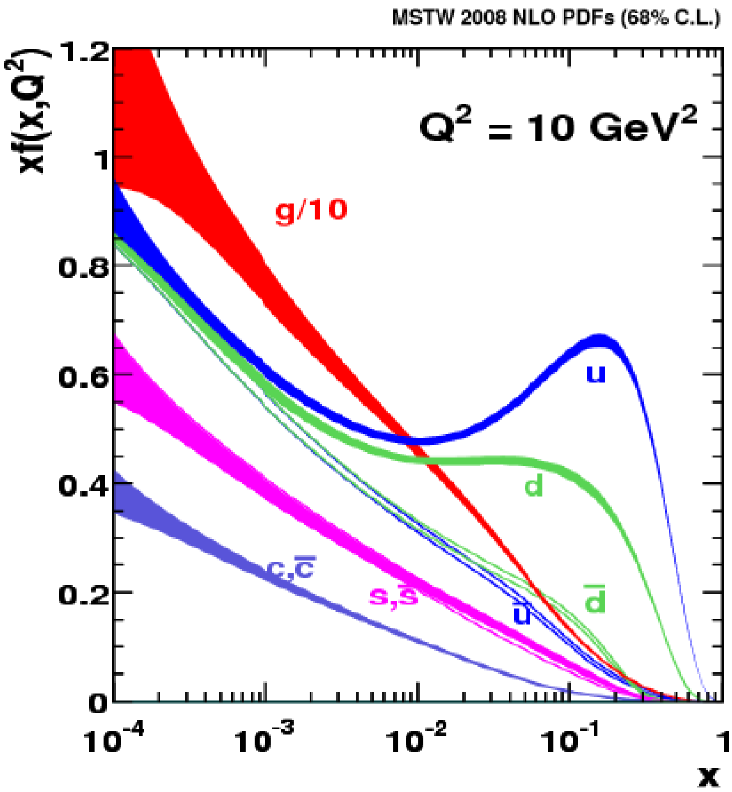

Figure 5: The MSTW–2008 parton distribution functions of the proton, at $Q^2 = 10$ GeV^2; the widths of the bands characterize the estimated uncertainty.

We consider that at present a 5% error of the u and d structure functions is more realistic.

We estimate the present uncertainty of the c structure function at the 10% level, see Fig. 7 taken from Ref. [9].

We estimate the present uncertainty of the b structure function at the 20% level, see Fig. 8 taken from Ref. [9].

We stress that this paper is not concerned with a general programme of improvement of the precision of the proton structure functions. Rather, this paper addresses a specific issue that is central to the precision with which the W mass can be measured at the LHC: a precision measurement of $F_2^{\rm n}/F_2^{\rm p}$, the ratio of the F_2 structure functions of the neutron and the proton, which constrains the information needed to reduce significantly the current uncertainty of the effects from the 1st quark family.

4 The uncertainty of the W mass at the LHC

In this Section, we assess quantitatively the uncertainty of the W mass at the LHC that results from realistic pdf and $k_{\rm T}$ uncertainties. Throughout our discussion, we use the

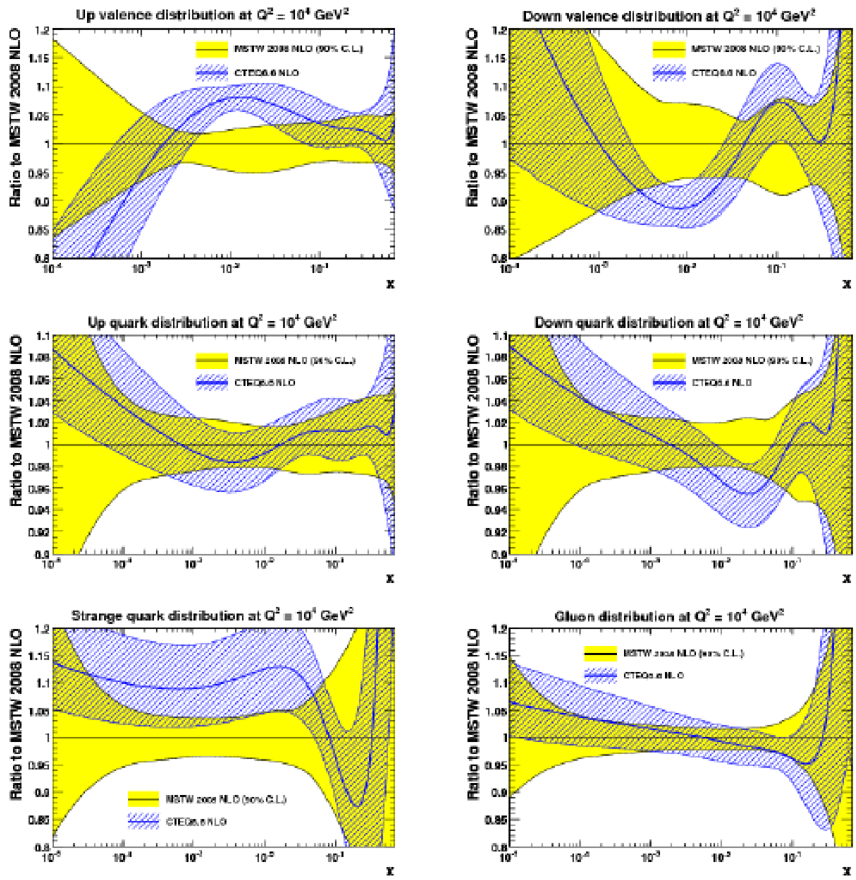

Figure 6: Comparison of the CTEQ6.6 and MSTW2008 (NLO) structure functions of the u, d, u_v, d_v, and s quarks, and gluons.

LHAPDF package [10] of pdf's, and PYTHIA 6.4 [11] for the modelling of the QCD/QED initial-state parton shower and its hadronization; in particular, the transverse momentum k_T of quarks and antiquarks is the one predicted by PYTHIA.

The tool for event generation is WINHAC 1.30 [12], a Monte Carlo generator for single W production in hadronic collisions, and leptonic decay. WINHAC includes also neutral-current processes with γ and Z bosons in the intermediate state. The novel feature of the WINHAC event generator is that it describes W and Z production and decay in terms of spin amplitudes [13]. The spin amplitudes involve, besides all possible spin configurations of the W and Z bosons, also the ones of the initial- and final-state fermions. The advantage of this approach is that one has explicit control over all spin states, and thus over transverse and longitudinal boson polarization and their interferences.

As an example detector, ATLAS was chosen. Charged leptons from W and Z decays are accepted with $p_T > 20$ GeV/c and $|\eta| < 2.5$. The number of events corresponds to an integrated luminosity of 10 fb^{-1}.

Both the electron- and muon decay channels of the bosons are considered.

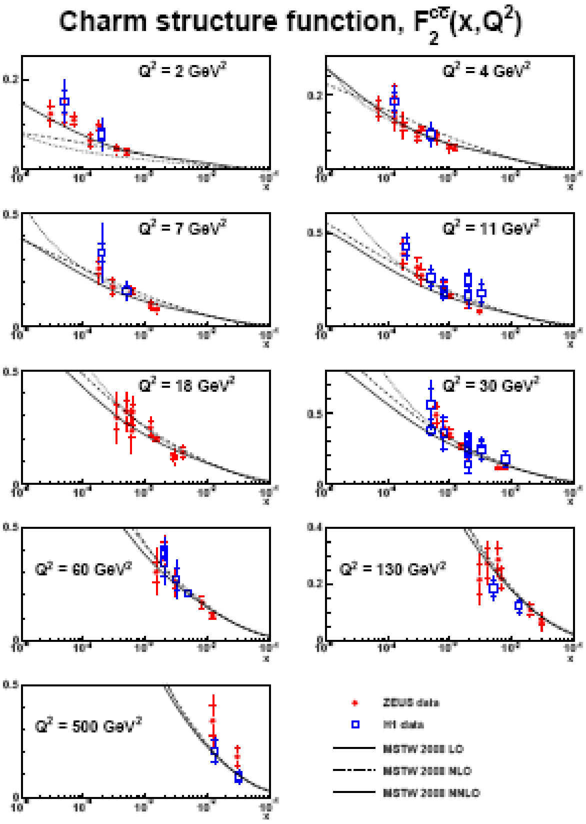

Figure 7: The MSTW2008 fit of the c quark structure function, at different values of Q^2.

The calibration of the p_{T} spectra of charged leptons from W decay by the ones from Z decay is a necessity. Since in pp collisions, as argued above, the spectra of positive and negative leptons must be analyzed separately, it is natural to make the same distinction also for the leptons from Z decay. Along this line of reasoning, we distinguish "Z^+" from "Z^-" lepton p_{T} spectra, in analogy to "W^+" and "W^-" lepton p_{T} spectra. This appears the more logical as a non-zero longitudinal Z polarization causes the p_{T} spectra of the positive and negative decay leptons to be different, for the charge-dependent correlation of the Z spin with the emission of charged decay leptons.

Accordingly, the analysis programme comprises the generation of W^+, W^-, Z^+, and Z^-, lepton p_{T} spectra, where, within the above-discussed error bands, various configurations of the relevant pdf's and k_{T}'s of quarks and antiquarks are selected.

Throughout our studies, a change of the pdf's of one quark is compensated by a change in the pdf of the other quark of the same family, so as to keep the rapidity distribution of Z bosons nearly invariant[5], and obtain independence from acceptance cuts. The rationale behind this concept is that non-compensating biases can eventually be pinned down at the LHC by high-statistics studies of the rapidity distribution of Z bosons, whereas compensating biases cannot. The exception is the 3rd quark family where this compensation is obviously

[5]The relative difference of the Z coupling to up-type and down-type quarks, the differences between $\bar{u}$ and $\bar{d}$, and between s and c structure functions, cause a small non-compensation which is of no importance for this analysis.

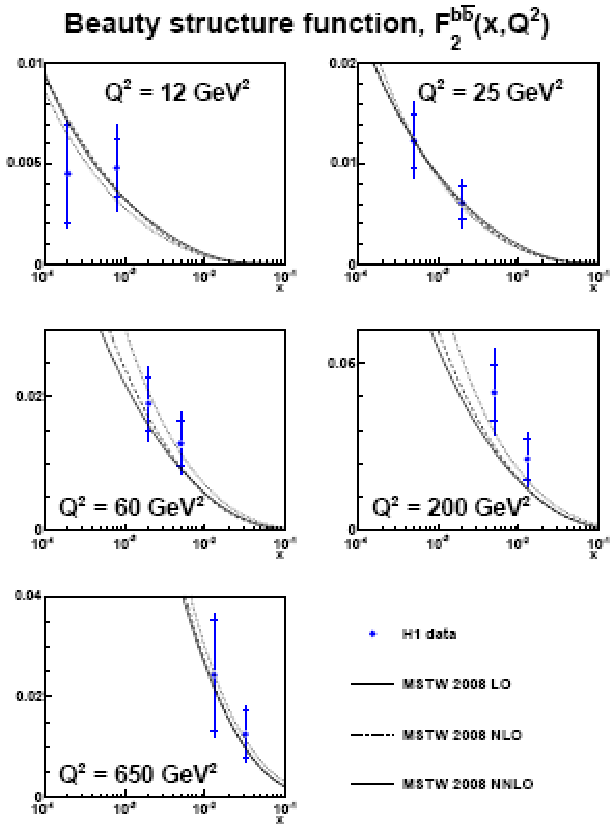

Figure 8: The MSTW2008 fit of the b quark structure function, at different values of Q^2.

not possible.

The Z^+ and Z^- lepton p_T spectra are corrected for QCD evolution from $Q^2 = M_Z^2$ to $Q^2 = M_W^2$.

From a fit of the Jacobian peaks in the p_T distributions, and imposing the known Z mass, the W^+ and W^- masses are determined.

For technical reasons, not M_{W^+} and M_{W^-} are separately determined but, equivalently, the average $(M_{W^+} + M_{W^-})/2$ and the difference $M_{W^+} - M_{W^-}$ of the masses.

Table 2 lists the biases of $(M_{W^+} + M_{W^-})/2$ and of $M_{W^+} - M_{W^-}$ caused by compensating shifts in the pdf's of the 1st quark family[6].

Table 3 lists the biases of $(M_{W^+} + M_{W^-})/2$ and of $M_{W^+} - M_{W^-}$ caused by compensating shifts in the pdf's of the 2nd quark family.

Table 4 lists the biases of $(M_{W^+} + M_{W^-})/2$ caused by shifts in the b quark structure function.

Evidently, with the present precision of the proton pdf's, there is no way to obtain a W mass with a precision at the 10 MeV level.

[6]The differences $M_{W^+} - M_{W^-}$ are taken from Ref. [7].

Table 2: Biases from the uncertainties in the 1st quark family.

	$\Delta[(M_{W^+} + M_{W^-})/2]$	$\Delta[(M_{W^+} - M_{W^-})]$
$u_v^{bias} = 1.05u_v$ $d_v^{bias} = d_v - 0.05u_v$	79 MeV	115 MeV
$u_v^{bias} = 0.95u_v$ $d_v^{bias} = d_v + 0.05u_v$	−64 MeV	−139 MeV

Table 3: Biases from the uncertainties in the 2nd quark family.

	$\Delta[(M_{W^+} + M_{W^-})/2]$	$\Delta[(M_{W^+} - M_{W^-})]$
$c^{bias} = 0.9c$ $s^{bias} = s + 0.1c$	148 MeV	17 MeV
$c^{bias} = 1.1c$ $s^{bias} = s - 0.1c$	−111 MeV	−11 MeV

5 Can better experimental information on the proton structure functions be expected?

There is much discussion about the contributions on the proton structure function that came from HERA, and that still are expected from final analyses of HERA data.

With regard to the structure functions of the 1st quark family, there is a limitation that comes from the fact that HERA provided collisions of electrons with protons but not, for comparison, with deuterons. This renders impossible, with NC interactions, to separate the u and d structure functions of the proton, which is necessary input for a high-precision measurement of the W mass at the LHC.

In principle, though, selecting CC interactions of protons with electrons and positrons, respectively, would permit the separation between u and d structure functions. In practice, the achieved luminosities are not sufficient for a precision measurement.

With regard to the c and b structure functions, the precision obtained at HERA is limited by large acceptance corrections and small statistics, with no improvement possible anymore.

Also, the present and the possible future experimental programme at the Jefferson Laboratory cannot improve the knowledge of the proton structure functions at the virtuality scales of W and Z production. This is because only a fraction of the pertinent deep-inelastic scattering data—where the higher twists and target mass corrections can be neglected—lends itself to the classical QCD fits of pdf's[7]. At the Jefferson Laboratory where $W^2_{max} = 11\ \mathrm{GeV}^2/c^4$, the relevant kinematical region is beyond reach.

[7]For example, in the MSTW set of QCD fits, only data are used that satisfy the condition $W^2 > 15\ \mathrm{GeV}^2/c^4$ on the squared hadronic mass [14].

Table 4: Biases from the uncertainties in the 3rd quark family.

	$\Delta[(M_{W^+} + M_{W^-})/2]$
$b^{bias} = 1.2$ b	42 MeV
$b^{bias} = 0.8$ b	−39 MeV

If, as planned, the electron beam momentum at the Jefferson Laboratory will be increased to 12 GeV/c, the boundary of the useful region will be crossed but only barely so[8].

6 Eliminating experimental problems in the p_T spectra of decay leptons

Before we discus ways toward a high-precision measurement of the W mass at the LHC, we need to address briefly the problems arising from experimental biases in the p_T measurement.

Experimental biases may arise from:

- a sagitta bias in the measurement of muon trajectories;
- a non-linearity in the calorimetric energy measurement of electrons;
- variations of lepton detection efficiency;
- biases in lepton identification, and
- variations in lepton reconstruction efficiency.

In the pp collisions at the LHC, symmetry in particle production for the transformation $\theta \rightarrow \pi+\theta$, and ϕ symmetry, is guaranteed. Exploiting these spatial symmetries, the equality of the p_T spectra (after correction of resolution effects) of electrons and muons with the same charge sign, and the constraint that opposite-charge lepton pairs with different momenta must reconstruct the same Z mass, customary measurement biases such as variations in trigger efficiency, differential and integral scale non-linearity, and an error in the scale calibration, can be eliminated. It is of paramount importance that this can be done *independently* for leptons with positive and negative charge, and hence without prejudice to the mass of W^+, W^-, and Z.

The precision with which the differential and integral linearity of the p_T spectrum of electrons and muons, and the absolute scale, can be ascertained, depends only on the accumulated statistics of Z decays[9].

[8] The useful data would have inelasticity $y > 0.75$ where resonant photo-production processes are dominant and where QED radiative corrections are large.

[9] One fb^{-1} corresponds to some 10^6 leptonic Z decays.

In the following discussion, we assume that the p_{T} spectra of decay leptons are free of measurement biases.

Biases from electron and muon identification depend on the detector design and the analysis software, and can hardly be discussed in generic terms. Without underestimating these problems, we assume in the following that also these biases are kept under control and do not discuss them any further.

7 Quantifying the missing information

In pp collisions at the LHC, there are seven pdf's that contribute to W and Z production: two valence quark pdf's (u_{v} and d_{v}), and five sea quark pdf's (u, d, c, s, b), all of them functions of x, k_{T}, at $Q^2 = M_{\mathrm{W}}^2 (M_{\mathrm{Z}}^2)$.

The available experimental information are four measured lepton spectra, in (p_{T}, η) bins, from W^+, W^-, Z^+ and Z^- decays.

There remain $3 = 7 - 4$ degrees of freedom which remain unconstrained.

For these three degrees of freedom, we have chosen the following pdf's and combinations of pdf's, respectively:

1. $u_{\mathrm{v}} - d_{\mathrm{v}}$ for the 1st quark family;
2. $s - c$ for the 2nd family; and
3. b for the 3rd family.

The reasons behind the choices of differences of pdf's within the same quark family are the following. First, as discussed above, the difference lends itself to the implementation of compensation (one pdf within one family moves up while the other pdf moves down while leaving the Z rapidity distribution essentially unchanged). Second, the QCD evolution of non-singlet pdf's is smaller and hence less prone to uncertainty than the QCD evolution of singlet pdf's.

8 Ways forward: elegance versus realism

As discussed above, there is no sufficiently precise information on the structure functions of quarks and antiquarks in the 1st, 2nd, and 3rd family.

In order to cut through this dilemma, we assume for the moment that there is infinite precision of the structure functions of the 1st family (we will return to this point later on). In this case, the W and Z production characteristics can be unfolded from precision measurements of the η and p_{T} spectra of charged leptons from $\mathrm{W}^\pm$ and $\mathrm{Z}^\pm$ decays, and the problem is solved. Therefore, we have reduced the overall problem to understanding the structure functions of the 1st quark family.

An elegant and technically feasible way to solve the 1st family problem is to circumvent it. Evidently, if the u_v and d_v structure functions of the proton were equal, there would be no problem. Precisely that would *de facto* be achieved if the LHC would collide deuterons with deuterons rather than protons with protons.

However, we consider that LHC running with deuterons is not realistic in the foreseeable future.

The other, and—so we believe—the only realistic option is a significant reduction of the uncertainties of the u_v, d_v, u_s and d_s structure functions of the proton. This would permit sufficient precision in the W and Z polarizations and in the analysis of p_T spectra of decay leptons. This latter option calls for a dedicated muon scattering experiment that provides a missing experimental constraint for the unambiguous unfolding of the valence and sea sectors of light quarks at the LHC[10]. Without this additional constraint the unfolding of the u_v, d_v, u_s and d_s structure functions will remain ambiguous at the LHC [15].

9 Measured observables in muon scattering and their use

The observables in the muon scattering experiment are the simultaneously measured differential cross-sections $d^2\sigma/dQ^2dx$ on protons and on deuterium nuclei.

The measured differential cross-sections must be corrected first for electroweak radiative effects which are dominated by the emission of hard photons from the incoming and the outgoing muon. The principal difficulties are to understand the relative radiative tails arising from elastic and quasi-elastic scattering, and to take into account corrections that depend on the target length.

As for differences of the electroweak radiative corrections between a free proton and a nucleon bound in the deuterium nucleus, these effects are small enough that they can be safely neglected.

Next, nuclear effects must be corrected for:

1. Fermi motion in the deuterium nucleus;
2. off-shell corrections for nucleons bound in the deuterium nucleus;
3. shadowing in the deuterium nucleus.

After the nuclear corrections, the ratio of deep inelastic muon scattering cross-sections for a free neutron and a free proton are obtained.

[10]This constraint from muon scattering is complementary to those that will be provided by precision measurements of the forward-backward asymmetry of charged leptons from Z-boson decays and of the charge asymmetry of leptons coming from W-bosons decays.

The interpretation of this ratio in terms of partonic distributions brings additional sources of uncertainties[11]:

1. higher-twist contributions which alter the QCD evolution of the proton and of the neutron structure functions; and
2. the ratio of the absorption cross-sections of longitudinally and transversely polarized photons, R.

A thorough estimation of the uncertainties of all above corrections will eventually be needed but is beyond the scope of this Letter of Intent. Here, we limit ourselves to pointing out that important progress has been made since the last measurements of structure functions on stationary targets.

Progress in the understanding of electroweak radiative corrections was driven by the HERA programme (see e.g. Ref. [16] and references cited therein), and we note that the errors of cross-sections from uncertainties of electroweak radiative corrections are below 1%. They can be reduced further by suitable cuts on the hadronic activity.

As for uncertainties from nuclear corrections, we start with the $x > 0.1$ region where shadowing corrections can be neglected. In Fig. 9, taken from Ref. [17], the sizes of the nuclear corrections are shown. These corrections are due to (i) Fermi motion and nucleon binding effects (FMB), (ii) target mass corrections (TMC), and (iii) off-shell corrections. The nuclear corrections are at the 2% level—except for the very large x domain which does not play a role in the W-boson mass measurement at the LHC. The difference between the shaded area and the full line permits an estimation of the uncertainty of these corrections. The latter is considerably reduced if one avoids the region of small mass W of the hadronic system. In addition we consider to control experimentally these corrections, we come back to this below.

In the shadowing region the nuclear correction may reach 5% as shown in Fig. 10, taken from Ref. [19]. This plot illustrates the size of the shadowing correction according to various models: (i) by Badelek and Kwiecinski [20]; (ii) by Melnitchouk and Thomas [21]; and (iii) by Barone *et al.* [22]. The corrections are calculated in terms of $\delta/F_2 = (F_2^{\mathrm{n}}/F_2^{\mathrm{p}})_{\mathrm{free}} - F_2^{\mathrm{n}}/F_2^{\mathrm{p}})$ and refer to the Q^2 and x of data from the NMC measurement, and hence are relevant for this Letter of Intent.

Owing to the wealth of data from HERA, shadowing corrections can be now calculated with considerably better precision than at the time of the NMC measurement.

The higher twist contributions to the structure functions of the proton and the neutron are also known now with better precision than in the past (see, e.g., Ref. [17]). Their size can be controlled using high-precision data from SLAC and Jefferson Laboratory. For the CERN muon beam energy, their contribution is small and can be reduced further by a suitable cut of the invariant mass W of the hadronic system. Besides, we propose to use at least two beam energies to access the same Q^2, x region at different hadronic masses, with a view to getting experimental control of this contribution.

[11] Note that we deal with with ratios between neutron and proton, with no intention to understand the absolute magnitude of corrections.

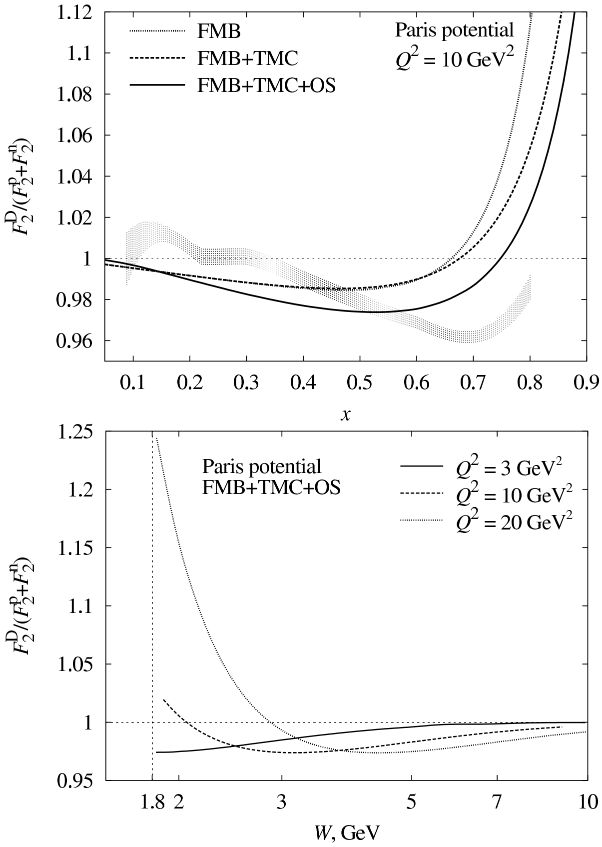

Figure 9: The ratio of the deuterium structure function to the sum of the structure functions of free nucleons calculated in different approximations. In the upper panel this ratio is shown as a function of x for fixed Q^2: Fermi motion and binding effects (dotted line), Fermi motion and binding effects and target mass corrections (dashed line); Fermi motion, binding ffects, target mass and off-shell corrections (solid line), according to calculations in Ref. [17]; for comparison, the shaded area in the upper panel shows the prediction of another model, discussed in Ref. [18]; in the lower panel the ratio of the deuterium structure function to the sum of the structure functions of free nucleons is shown as a function of the hadronic mass W for various Q^2.

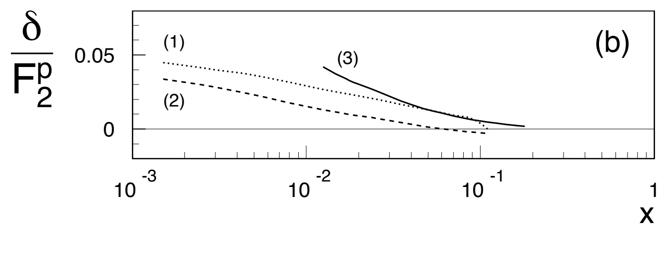

Figure 10: The size of the shadowing correction in terms of $\delta/F_2 = (F_2^n/F_2^p)_{\rm free} - F_2^n/F_2^p)$, according to various models; for details, see the text.

The proposed measurement has been performed before by the New Muon Collaboration (NMC) at CERN [19, 23], using simultaneously targets of liquid hydrogen and deuterium. They published a measurement of the proton and deuteron structure functions, $F_2^{\mu\mathrm{p}}$ and $F_2^{\mu\mathrm{d}}$, in the kinematic range $0.002 < x < 0.60$ and $0.5 < Q^2 < 75$ $(\mathrm{GeV}/c)^2$, with small systematic errors, and statistical errors of typically 1–3%. We note that the aim of this Letter of Intent is the improvement of the statisticsl recision while retaining the systematic precision that has been already achieved.

10 Requirements for the muon scattering experiment

At CERN, the only viable option for performing the proposed muon scattering experiment, is utilizing the existing muon beam in the North Area and the existing COMPASS detector.

In order to achieve high statistics, the driving consideration is the highest possible beam intensity.

With a view to assessing effects arising from a non-zero $R = \sigma_\mathrm{L}/\sigma_\mathrm{T}$, and from higher-twist contributions on top of the logarithmic QCD evolution of structure functions, we propose to take data at two (possibly three) beam momenta which are as different as possible, say +80 GeV/c and +160 GeV/c.

At these beam momenta, an intensity of 2×10^8 μ^+ per 4.8 s spill can be routinely achieved. The muon momentum bite is $\Delta p/p \sim 3\%$, the incident muon angle has an angular divergence of ≤ 0.8 mrad (r.m.s.), and a transverse position spread of ~ 8 mm (r.m.s.).

For comparison, the NMC experiment [19, 23] utilized $\mu+$ beams of 90, 120, 200, and 280 GeV/c momentum.

To achieve the wanted precision, it is imperative to take data concurrently on a liquid hydrogen and a liquid deuterium target. Pending an optimization of the target section, we take—tentatively—a target length of 12 m, segmented in the downstream direction as follows: 4 m H_2, 2 m D_2, 4 m H_2, 2 m D_2. The target is a cylinder made of mylar, with a radius of 5 cm. At the end of the target, the multiple scattering angle for 80 GeV/c is with $\sigma \simeq 0.2$ mrad small compared with the beam divergence. The displacement due to multiple scattering is with $\sigma \simeq 1.5$ mm small compared with the transverse position spread of the beam and with the target radius.

Ignoring small corrections like target mass correction, non-zero R, and QCD evolution, the double-differential cross-section for muon scattering on the proton is given by

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}x} = \frac{4\pi\alpha^2}{Q^4} \cdot \frac{F_2^{\mu\mathrm{p}}}{x} \cdot \frac{1 + (1-y)^2}{2} \; 1/(\mathrm{GeV}/c)^4 \quad (3)$$

$$= 0.389 \cdot \frac{4\pi\alpha^2}{Q^4} \cdot \frac{F_2^{\mu\mathrm{p}}}{x} \cdot \frac{1 + (1-y)^2}{2} \; \mathrm{mbarn}/(\mathrm{GeV}/c)^2 \;, \quad (4)$$

and analogously for the deuteron.

With 2×10^8 muons per spill, a spill cycle time of 14.4 s, and 3 net months of data taking with an efficiency of 70%, the total number of μ^+ on target is 7.6×10^{13}. We divide this number equally between an 80 GeV/c and a 160 GeV/c beam.

Table 5 gives the number of μ^+ scatterings with $1 < Q^2 < 100$ GeV/c^2 and $0.1 < x < 0.8$, for a momentum of the outgoing muon above 15 GeV/c and a scattering angle $\theta > 5$ mrad, with a fiducial length of the hydrogen and deuterium targets of 7.2 m and 3.8 m, respectively[12]. We stress that the event numbers are strongly dependent on Q^2: they fall by a factor of 10^4 from $Q^2 = 1$ (GeV/c)^2 to $Q^2 = 100$ (GeV/c)^2.

Table 5: Number of μ^+ scatterings with $1 < Q^2 < 100$ GeV/c^2 and $0.1 < x < 0.8$.

80 GeV/c on hydrogen	1.3×10^8
80 GeV/c on deuterium	1.3×10^8
160 GeV/c on hydrogen	1.4×10^8
160 GeV/c on deuterium	1.4×10^8

For comparison, we estimate the equivalent number of μ^+ scatterings in the NMC experiment [19, 23] on the hydrogen targets to be around 1×10^6 per beam setting, i.e., considerably lower.

Figure 11 shows the distributions of μ^+ scatterings in the hydrogen target in the kinematic variables Q^2 and x for the 80 GeV/c and 160 GeV/c μ^+ beams, respectively.

Table 6 gives the event rates dN/dQ^2dx at selected points in the (Q^2,x) plane, for the scattering of 160 GeV/c μ^+ in the hydrogen target (event migration due to finite experimental resolution is ignored).

Table 6: Event rates dN/dQ^2dx [(GeV/c)$^{-2}$] at selected points in the (Q^2 [(GeV/c)^2],x) plane, for the scattering of 160 GeV/c μ^+ in the hydrogen target.

	$x = 0.10$	$x = 0.30$	$x = 0.50$	$x = 0.70$
$Q^2 = 1$	1.1×10^9	2.9×10^8	8.1×10^7	1.5×10^7
$Q^2 = 3$	1.2×10^8	3.1×10^7	8.9×10^6	1.7×10^6
$Q^2 = 10$	8.5×10^6	2.6×10^6	7.7×10^5	1.5×10^5
$Q^2 = 30$		2.3×10^5	7.5×10^4	1.5×10^4
$Q^2 = 100$			4.6×10^3	9.8×10^2

[12]When calculating these event numbers, no z-dependent acceptance and no resolution effects have been taken into account.

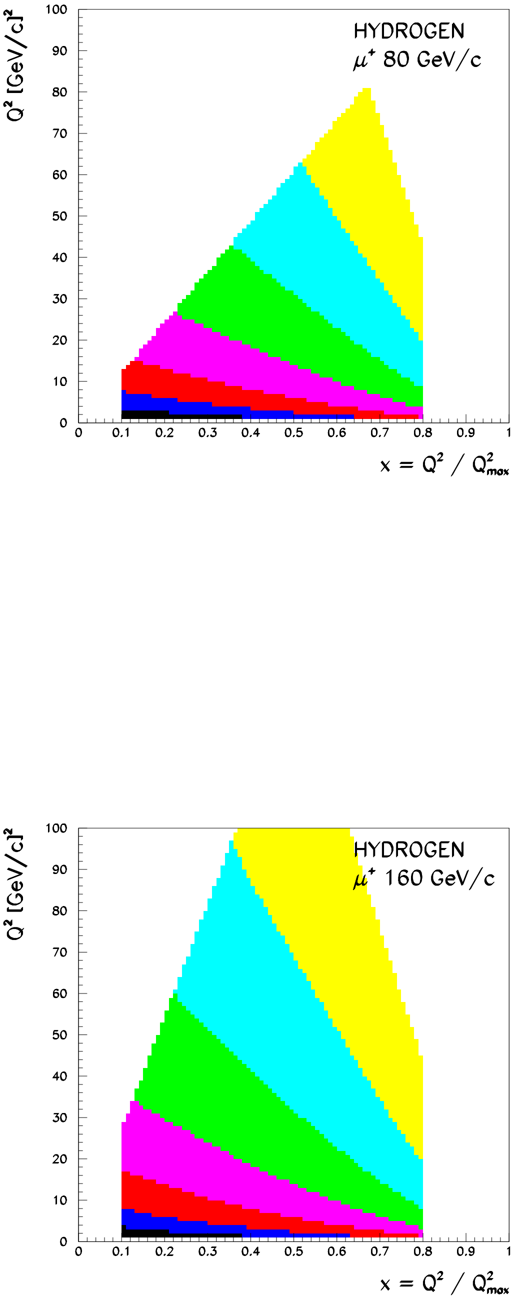

Figure 11: Distribution of muon scatterings as a function of Q^2 and x, in hydrogen for an 80 GeV/c (upper panel) and a 160 GeV/c (lower panel) μ^+ beam (logarithmic colour/shading scale).

11 Performing the experiment with the COMPASS detector

The COMPASS detector [24] has been in operation at CERN since 2002, and has been upgraded in 2006. It is proposed to use this detector for the proposed experiment without major changes.

The estimated experimental resolutions at selected points in the (Q^2 [(GeV/c)^2],x) plane, for the scattering of 160 GeV/c μ^+ in the hydrogen target, are given in Table 7 for the variable Q^2, and in Table 8 for the variable x. The following resolutions of directly measured quantities have been used for these estimations: $\Delta p/p = 0.01$ for the incoming muon momentum, $\Delta p/p = 0.012$ for the scattered muon momentum, and $\Delta\theta = 0.1$ mrad for the polar angle of the scattered muon.

Table 7: Resolutions [(GeV/c)^2] of the variable Q^2 at selected points in the (Q^2 [(GeV/c)^2],x) plane, for the scattering of 160 GeV/c μ^+ in the hydrogen target.

	$x = 0.10$	$x = 0.30$	$x = 0.50$	$x = 0.70$
$Q^2 = 1$	0.3 (25.2%)	0.3 (25.5%)	0.3 (25.6%)	0.3 (25.6%)
$Q^2 = 3$	0.4 (14.1%)	0.4 (14.6%)	0.4 (14.7%)	0.4 (14.8%)
$Q^2 = 10$	0.7 (6.8%)	0.8 (7.8%)	0.8 (8.0%)	0.8 (8.1%)
$Q^2 = 30$		1.2 (4.1%)	1.3 (4.5%)	1.4 (4.6%)
$Q^2 = 100$			2.1 (2.1%)	2.4 (2.4%)

Table 8: Resolutions of the variable x at selected points in the (Q^2 [(GeV/c)^2],x) plane, for the scattering of 160 GeV/c μ^+ in the hydrogen target.

	$x = 0.10$	$x = 0.30$	$x = 0.50$	$x = 0.70$
$Q^2 = 1$	0.053	0.427		
$Q^2 = 3$	0.021	0.15	0.40	0.77
$Q^2 = 10$	0.008	0.046	0.12	0.23
$Q^2 = 30$		0.017	0.042	0.078
$Q^2 = 100$			0.012	0.023

The only significant change of the COMPASS detector configuration is motivated by the need of a longer target than can be presently accommodated. For the complexity of the detector proper, it is unrealistic to think of a move of the detector downstream of the target. Rather, a compression of the last part of the muon beamline just upstream of the target, by several metres, is envisaged.

Further, it would be of interest to integrate into the downstream end of the deuterium target, for some 60 cm, a detector that would identify and measure recoil protons from the scattering of muons on deuterons. On the one hand, this would permit an experimental check of the

Fermi motion corrections that enter the comparison of muon scatterong in hydrogen and in deuterium. On the other hand, this would permit to get first experience with the virtual Compton scattering programme of the COMPASS Collaboration.

The recoil proton detector would have to measure the specific ionization $\mathrm{d}E/\mathrm{d}x$ with a view to proton identification, and the proton momentum vector. Perhaps concentric layers of thin silicon strip detectors immersed in the cryogenic liquid of the deuterium target would do the job.

12 Summary and Outlook

The W mass being a central parameter of the Electroweak Standard Model, every effort should be made to measure it with the best possible precision at the LHC, that is to better than 10 MeV. Unless the current knowledge on the proton structure functions, especially u_v and d_v, is significantly improved, this goal cannot be achieved. A dedicated muon scattering experiment with simultaneous measurement on hydrogen and deuterium targets would deliver the missing information.

It is proposed to carry out such an experiment with the COMPASS detector at CERN. Ideally, data taking should take place in 2011. Discussions with the COMPASS Collaboration are under way.

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