DAILY VARIATION AND PREDICTING STOCK MARKET RETURNS FOR THE FRANKFURTER BÖRSE (STOCK MARKET)

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Abstract. In this article we test the random walk hypothesis in the German daily stock prices by means of a unit root test and the development of an ARIMA model for prediction. The results show that the time series of daily stock returns for a stratified random sample of German firms listed on the stock exchange of Frankfurt exhibit unit roots. Also, we find that one may predict changes in the returns to these listed stocks. These time series exhibit properties which are forecast able and provide the intelligent data analysts' methods to better predict the directive of individual stock returns for listed German firms. The results of this study, though different from most other studies of other stock markets, indicate the Frankfurt stock market behaves in similar ways to North American, other European and Asian markets previously studied in the same manner.

Keywords: unit root test, ARIMA models, prediction, stock returns, Frankfurt stock market.

1. Introduction

One issue often debated in financial time series analysis is whether holding period returns on a risky financial asset is a time series that contains predictability. Serial independence (no autocorrelation) is a requirement for the efficient market hypothesis in its weak form, i.e., the current stock return fully reflects all past financial information affecting stock prices. The precise formulation of an empirically refutable efficient market hypothesis must be model specific and historically the major portion of such analyses focused on the prediction of common stock return or indexes of common stock returns. Researchers broadly categorize these analyses of no predictability as the *random walk* explanation of stock prices. In the past, the evidence was mixed.

Lo and MacKinlay (1999) used variance-ratio test to infer that stock prices do not follow random walks for daily and weekly returns. However, they found no statistical evidence to refute the notion that monthly returns follow a random walk. In addition, research concluded that portfolio returns of the AMEX and

NYSE firms exhibit positive first order autocorrelation while security returns present negative first order autocorrelations as documented by Lo and Mackinlay (1999). The different signs for the autocorrelations between portfolios and stocks may be attributed to lead-lag positive autocorrelations across securities. Poterba and Summers found negative autocorrelation in monthly returns for a NYSE value-weighted index during the lengthy period, 1926 to 1985, whereas Lo and MacKinlay (1999) obtained positive autocorrelation in a value-weighted index formed by similar stock in a shorter period , 1962 to 1985.

Fama and French (1989) using regression and variance ratios, concluded that stock contain mean reversion. That is, autocorrelations become negative for two-year returns, reach minimum values for three to five-year returns and then decay towards zero. These findings may be associated with time varying on expected returns or investor overreaction or under reaction causing stock swings away from their fundamental values. These suggestions originate from numerous scientific researches. Lo and Mackin (1999) using a generalized form of rescaled range (R/S) statistic found no

evidence contradicting the *random walk* hypothesis. Unlike other, Caporale and Gil-Alana (2002) utilized annual data for United States returns and pointed out that their degree of predictability depends on the process followed by the error term.

Calendar or time effects do contradict the weak form of the efficient market hypothesis. The weak form refers to the notion that the market is efficient in past price and volume information and we do not have the knowledge to predict stock return and price movements accurately using historical information. If no systematic patterns exist, stock returns are time-invariant. By contrast, if variations in the time series of daily returns of securities markets exist, market inefficiency is present and investors may earn abnormal rates of return not in line with the degree of risk they undertake (Francis 1993). In addition, a large number of studies on predicting prices of traded securities confirm to some degree that patterns exist in stock market returns and prices. We know interest rates, dividend yields and a variety of macroeconomic variables exhibit clear business cycle patterns. The emerging literature concerning studies of United States (US) securities include Balvers *et al*. (1990), Breen *et al*. (1990), Campbell (1987), Fama and French (1989) and Pesaran and Timmermann (1995), and Granger (1992) provides an up to date survey of methods and results. Studies in other places (the United Kingdom) include Clare *et al*. (1994, 1995), Black and Fraser (1995) and Pesaran and Timmermann (2000). Last, Caporale and Gil-Alana (2002) pointed out that for US stock returns their degree of predictability depends on the process followed by the error term.

The expansion of time series analysis as a discipline permits one to analyze stock market returns in ways not heretofore explored. What is the predictability of the error term and is there predictability in daily stock market returns? Peculiar problems arise when daily patterns are present in stock return data and we know that stock returns possess patterns known as daily effects. For example, Kato (1990a) results suggested that there are patterns in stock returns in Japanese securities. He observed low Tuesday and high Wednesday returns within weekly prices. If a week did not have trading on a Friday, he would observe effects related to the Monday of the following week. The following Monday would have low returns indicating that transference of the pattern that would occur on the Friday if trading had occurred which it did not. A second study by Kato (1990b) found considerable anomalies on the Tokyo Stock Exchange (TSE), which is an organized exchange similar to the ones in North America.

Only a few studies focused on the investigation of time series components of equity prices and the predictability of these prices. Ray, Chen and Jarrett (1997) investigated a sample of 15 Japanese firms and found both permanent and temporary systematic components in individual time series of stock market prices of firms over a lengthy period of time. Moorkejee and Yu (1999) investigated the seasonality in stock returns of other Asian markets, i.e., Shanghai and Shenzen. They documented the seasonal patterns existing on these exchanges and the effects these factors have on risk in investing in securities listed on these exchanges. In addition they showed that risk in investing is related to the predictability of security returns. Rothlein and Jarrett (2002) investigated the existence of seasonality present in Japanese stock prices, which affect the prices of these securities. They documented the evidence of seasonality in the prices of 55 randomly selected Tokyo Stock Exchange firms over a lengthy period of 18 years (1975 through 1992). In addition, they indicated the accuracy of forecasts or predictions of these firms' prices are seriously decreased if one does not recognize the patterns in the time series.

Kubota and Takehara (2003) investigated whether the activity of financial firms creates value and/or risk to the economy within the asset pricing framework. They used stock return data from non-financial firms listed in the first section of the Tokyo Stock Exchange. Their value-weighted index which was solely composed of non-financial firms was augmented with the index of the firms from the financial sector. In turn, they estimated the multivariate asset pricing model with these two indices. We note that their procedure can simultaneously take into account the cross-holding phenomena among Japanese firms, especially between the financial sector and the non-financial sector. In conclusion, their financial sector model helps explain the return and risk structure of Japanese firms during the so-called "double-bubble" period indicating some predictability in closing prices of Japanese securities.

Jarrett and Kyper (2005) indicated how patterns in monthly stock prices have predictable patterns. This study differs in that we examine the predictable patterns in the closing daily prices of stock prices. It goes further than the study of Caporale and Gil-Alana (2002) noted before because it attempts to determine the patterns in daily prices of listed securities. Caporale and Gil-Alana (2002) did test for unit roots in the stock market though unlike this study, they test this hypothesis within fractionally integrated alternatives. Fractional differencing is generally employed to predict long-term rather than short-term properties of time

series. Finally, Jarrett and Kyper (2005) studied the predictability of daily returns to more than 50 firms listed on American Stock Exchanges and concluded that daily variation exists and is predictable.

2. Frankfurt stock market

We study the stock exchange of Frankfurt because it represents the pre-eminent financial market in the largest nation of Central and Western Europe playing an essential economic role between the larger financial markets in New York and Tokyo.

Frankfurter Wertpapierbörse (Frankfurt Stock Exchange) is one of the world's largest trading centres for securities. With a share in turnover of around 90 percent, it is the largest of the seven German stock exchanges. Deutsche Börse AG operates the Frankfurt Stock Exchange, an entity under public law. In this capacity it ensures the smooth functioning of exchange trading.

The Frankfurt Stock Exchange facilitates advanced electronic trading, settlement and information systems. Thus, it is able to meet the steadily growing requirements of cross-border trading. Besides traditional floor trading, it has in Xetra® one of the leading electronic trading platforms in the world. With the launch of Xetra in 1997, the Frankfurt Stock Exchange succeeded not only in strengthening its own competitive position. It also created attractive framework conditions for foreign investors and market participants.

The Frankfurt Stock Exchange is one of the biggest and most efficient exchange places in the world. It is owned and operated by Deutsche Börse, which also owns the European futures exchange Eurex and clearing company Clearstream.

The Frankfurt Stock Exchange has over 90 percent of turnover in the German market and a big share in the European market. Here the Frankfurt Stock Exchange floor trading loses, but in fast developing and expanding electronic trading (Xetra trading system) the FSE gains in European and international trade: partner-exchanges adopted the Xetra (trading system) (as the Vienna Stock Exchange in 1999, the Irish Stock Exchange in 2000 and the Budapest Stock Exchange in 2003); consolidation continues.

Mainly through Xetra, the German stock market was opened to foreign investors and market participants. About 47% of the 300 market participants in Frankfurt come from abroad.

3. Sample selection

We study the firms listed on the Frank Stock Exchange listed in the Appendix. The sample selection utilized a two-step process and is a stratified random sample of fifty firms. The five largest firms (in terms of capitalization) are selected for the first stratum. Second a simple random sample of 45 firms from all other firms is selected for the send stratum. This process yields a sample that better represents the population of all firms listed on the exchange than if we selected a simple random sample of firms. If that was done, it is like that most (if not all) of the largest firms would not have been selected for the sample. Such results would not represent the Frankfurt exchange. The data selected were the daily returns for the fifty firms sample and listed in the Appendix over a lengthy period (give dates). Note that the firms are also well known enough to eliminate any problems with start-up firms and problems associated with mergers and acquisitions. All data account for stock splits, stock dividends, which may dilute the usefulness of unadjusted information. We list the sampled firms in Figure 1 and all firms listed on the exchange may be found by writing to the author.

Those Firms Selected for the Sample:

Sample

ALTANA AG O.N.	HIGHLIGHT CMNCTS INH.SF 1
SYNAXON AG	SUNWAYS AG O.N.
INIT INNOVATION O.N.	SARTORIUS AG VZO O.N.
CONERGY AG O.N.	DR. HOENLE AG O.N.
AIG INTL REAL ESTATE	AIR BERLIN PLC EO -,25
SPARK NETWORKS REGS LS-01	HEIDELBERG.DRUCKMA.O.N.
TIPP24 AG NA O.N.	TAG TEGERNSEE IMMOB.
LHS AG INH.O.N.	TELEGATE AG O.N.
UTIMACO SAFEW.AG O.N.SVG	GOYELLOW MEDIA AG O.N.
TRIA IT-SOLUTIONS AG	WILEX AG O.N.
DT.TELEKOM AG NA	SINNERSCHRADER O.N.
PANKL RACING SYS	JETTER AG O.N.
BETA SYST.SOFTW.AG O.N.	THIEL LOGISTIK AG
DT.EFF,U,WECH,-BET,G,O.N.	DEUTSCHE WOHNEN AG NA
DYCKERHOFF ST O.N.	LUFTHANSA AG VNA O.N.
P U.I PER.U.INFO.AG O.N.	MASTERFLEX O.N.
TOMORROW FOCUS AG	FJH AG O.N.
DEUTSCHE BANK AG NA O.N.	PC-WARE INFOR.TECHNOLO.AG
HORNBACH-BAUMARKT O.N.	VIVACON AG O.N.
LEIFHEIT AG O.N.	VOSSLOH AG O.N.
PRAKTIKER BAU-U.H.HLDG ON	REALTECH AG O.N.
LUDW.BECK A.RATHAUSECK	GESCO AG O.N.
MTU AERO ENGINES NA O.N.	TECHNOTRANS AG O.N.
SURTECO AG	SOFTING AG O.N.
TIPTEL AG	EMPRISE AG

Fig. 1. List of sampled firms

4. Dickey-Fuller methodology and results

To determine if a daily pattern can be modeled for a sampled time series, we employ the Augmented Dickey-Fuller tests which we now illustrate (are applied in studies of Diebold and Kilian 2000, and Payne 2007); Let the earnings of a corporation be Y_t, the DF (Dick-

ey-Fuller) Unit Root Test is based on the following three regression forms:

1. Without Intercept and Trend $\Delta Y_t = \lambda Y_{t-1} + \mu_t$.
2. With Intercept $\Delta Y_t = \alpha + \lambda Y_{t-1} + \mu_t$.
3. With Intercept and Trend $\Delta Y_t = \alpha + \beta T + \lambda Y_{t-1} + \mu_t$.

The null hypothesis is H_0: $\lambda = 0$ (Unit Root), as opposed to H_1: $\lambda \neq 0$. In turn, the *decision* becomes as follows: If we reject H_0, unit root does not exist and if we <u>do not</u> reject H_0, the unit root exists. We run each regression separately and determine if the ADF (augmented Dickey-Fuller) statistic is greater than the critical value (i.e., MacKinnon, 1991, critical value for rejection of the hypothesis of a unit root), we do not reject H_0. Therefore, by noting rejecting , we conclude that the unit root exists.

Although this appears as a conventional *t*-test on the estimated γ, the *t*-statistic under the null hypothesis of a unit root does not have the conventional *t*-distribution. Dickey and Fuller (1979) showed that the distribution under the null hypothesis is not standard, and simulated the critical values for selected sample sizes. MacKinnon (1991) implemented a larger set of simulations than those tabulated by Dickey and Fuller. He estimates the response surface using the simulation results, permitting the calculation of Dickey-Fuller critical values for any sample size and for any number of variables on the right-hand side of the equation. (For details on other unit root tests see Maddala and Kim, 1999.)We report our results based on these MacKinnon critical values for the unit root test.

The *Augmented Dickey-Fuller* approach controls for higher than first order autocorrelation by adding lagged difference terms of the response variable *y* to the right-hand side of the regression model:

$$\Delta y_t = \mu + \gamma y_{t-1} + \tau_1 \Delta y_{t-1} + \tau_2 \Delta y_{t-2} + \ldots + \tau_{p-1} \Delta y_{t-p+1} + \epsilon_t.$$

This augmented specification is, in turn, used to test:

H_0: $\gamma = 0$ and H_a: $\gamma < 0$

in the regression model. An additional important result obtained by Fuller is that the asymptotic distribution of the *t*-statistic on γ is independent of the number of lagged first differences included in the augmented DF regression. Further, the parametric assumption that *y* follows an AR process restricts the use of the DF test, Said and Dickey (1984) demonstrate that the augmented DF test remains valid even when the time series is moving-average (MA), provided that enough lagged difference terms are augmented to the regression.

As noted before, we may choose a model with or without a constant term, and with or without a linear trend. The purpose of this analysis , however, is not to determine the precise model that best generates the time series of daily observation, but to consider whether a model could be built or not. If we find that the model has a unit root which can be expressed by an AR(autoregressive), MA(moving-average) or mixed ARMA process, we have shown that there is pattern in the daily observations and the notion that daily observations are completely random is nullified. Our results follow in Table 1.

Table 1. Augmented DF Test

	Panel A *Level*		
	Intercept	**Trend and Intercept**	**None**
Adidas	−1.569556	−2.458499	0.839952
AIG Int. Real	−1.039798	−1.985629	1.589802
Altana AG	**−2.619398	*-3.54161	0.048023
Beta systems	−1.176704	−2.610326	−1.484718
Continental	−1.580175	−2.732428	1.194389
Deutsche Bank	−1.521160	−1.739101	0.836502
Deutsche Telekom	−1.608121	−2.155087	−0.491161
DEWB AG	−1.850085	−2.331633	−0.687246
Dr. Hönle AG	*−3.374734*	*−3.483789	−0.110299
Dyckerhoff AG	−1.758164	−1.923295	0.513197
GESCO AG O.N.	−0.598059	−2.925209	1.751855
GOYELLOW MEDIA AG O.N.	−2.033674	−1.695701	**−1.750827
HEIDELBERG.DRUCKMA.O.N.	−1.838451	−2.023201	0.246552

	Intercept	Trend and Intercept	None
HIGHLIGHT CMNCTS INH.SF 1	–1.029777	–2.508406	1.019322
Hornbach Baumarkt	–1.363278	–2.207753	0.571716
Init- Innovation	–1.812347	–2.564911	0.857159
JETTER AG O.N.	–2.379826	–2.403940	0.297190
Leifheit	**–2.774254	**–3.215604	–0.350706
Ludwig Beck AG	0.445509	–1.725805	1.806192
LUFTHANSA AG VNA O.N.	–0.636884	–2.131460	1.519859
Man AG	0.632557	–2.171133	2.584371
MASTERFLEX O.N.	–1.974741	–2.668890	–0.621009
P&I Personal und Informatik AG	–1.311933	–2.817878	1.030875
Pankl Racing Systems AG	–0.690261	–2.321727	1.254700
PC-WARE INFOR.TECHNOLO.AG	-2.219865	–2.125737	0.476283
REALTECH AG O.N.	–0.924462	*–3.436559	1.206615
SARTORIUS AG VZO O.N.	–1.323465	*–3.520428	0.991449
SINNERSCHRADER O.N.	*–5.551903*	*–5.478712*	*-2.080436
SOFTING AG O.N.	–2.280631	–2.604125	0.588753
SUNWAYS AG O.N.	–2.095176	–2.289472	–0.383175
Surteco	–1.867181	–2.796199	0.428740
Synaxon AG	–1.558354	–2.596911	–0.946564
TAG TEGERNSEE IMMOB.	–2.172972	–2.323815	0.129071
TECHNOTRANS AG O.N.	–2.187816	–2.473739	0.303261
TELEGATE AG O.N.	–2.268474	–2.564434	0.425400
THIEL LOGISTIK AG	–1.679045	–2.321665	–1.227357
Tomorrow Focus AG	–1.932055	–2.957607	0.307173
Ultimaco	–1.995754	–0.970799	–0.024322
VIVACON AG O.N.	–1.910658	–1.329059	–0.247050
Volkswagen	3.964881	1.308427	5.509322
VOSSLOH AG O.N.	–0.557973	–1.853135	1.238489

Panel B
First Difference

	Intercept	Trend and Intercept	None
Adidas	–27.077670	-27.061460	-27.057520
AIG Int. Real	–31.050200	–31.041470	–30.920730
Altana AG	–26.537730	–26.527480	–26.551190
Beta systems	–28.419570	–28.402440	–28.389080
Continental	–30.542290	–30.546560	–30.449000
Deutsche Bank	–27.445070	–27.457000	–27.416570
Deutsche Telekom	–25.966590	–25.950600	–25.979990
DEWB AG	–27.758890	–27.750090	–27.776720
Dr. Hönle AG	–31.645350	–31.637290	–31.663890
Dyckerhoff AG	–29.930450	–29.924420	–29.924100
GESCO AG O.N.	–29.013470	–28.998750	–28.881340
GOYELLOW MEDIA AG O.N.	–26.463090	–26.490650	–26.452020

	Intercept	Trend and Intercept	None
HEIDELBERG.DRUCKMA.O.N.	–28.229540	–28.228020	–28.237830
HIGHLIGHT CMNCTS INH.SF 1	–29.702680	–29.684840	–29.650320
Hornbach Baumarkt	–27.115570	–27.097660	–27.108030
Init- Innovation	–29.531080	–29.524880	–29.524880
JETTER AG O.N.	–27.703070	–27.705850	–27.694280
Leifheit	–31.594830	–31.595970	–31.614180
Ludwig Beck AG	–32.470130	–32.532910	–32.332250
LUFTHANSA AG VNA O.N.	–29.491480	–29.472810	–29.383790
Man AG	–25.789790	–25.824690	–25.607040
MASTERFLEX O.N.	–28.719350	–28.704590	–28.732670
P&I Personal und Informatik AG	–29.214100	–29.200630	–29.148510
Pankl Racing Systems AG	–30.762930	–30.755820	–30.688920
PC-WARE INFOR.TECHNOLO.AG	–28.331740	–28.355210	–28.314870
REALTECH AG O.N.	–29.890080	–29.874500	–29.821930
SARTORIUS AG VZO O.N.	–31.602830	–31.597490	–31.513820
SINNERSCHRADER O.N.	–26.420210	–26.462380	–26.407130
SOFTING AG O.N.	–33.465060	–33.449560	–33.442730
SUNWAYS AG O.N.	–28.635320	–28.643890	–28.648740
Surteco	–30.366710	–30.352310	–30.362950
Synaxon AG	–28.165740	–28.153640	–28.166720
TAG TEGERNSEE IMMOB.	–31.137170	–31.152420	–31.147620
TECHNOTRANS AG O.N.	–29.977930	–30.002820	–29.977100
TELEGATE AG O.N.	–32.409300	–32.417860	–32.399200
THIEL LOGISTIK AG	–30.823500	–30.806010	–30.803430
Tomorrow Focus AG	–31.440640	–31.440350	–31.444440
Ultimaco	–29.034140	–29.136120	–29.036380
VIVACON AG O.N.	–28.512490	–28.573620	–28.518950
Volkswagen	–26.194410	–26.626890	–25.674630
VOSSLOH AG O.N.	–30.352390	–30.344190	–30.277110

In examining the data in Table 1, first examine Panel A which yields results of the augmented DF test for level series (no differencing). Note that we cannot reject the null hypothesis of no **unit root** at levels of 01.or .05 for most of the firms noted in the table. Only for a minority of firms could one reject the null hypothesis and conclude that no unit root exists. When one does not reject this null hypothesis, we conclude that a unit root is not likely to exist in the sampled time series. In Panel B of Table 1, we see the results for the first differenced time series. In general, once a difference is taken, we can reject the null hypothesis at levels of .01 or .05. Hence, the unit root is present in the sampled time series. When a unit root is present in the time series of data then it is possible to model the data. If we correctly model, then the data is predictable and no longer a completely random time series. We analyzed the augmented DF test discussed above for all the sampled time series of stock returns having about 250 observations per time series. The test statistics and significance levels at .001, .01 and .05 indicate that the unit root exists in the time series data. Noting the results of Table 1, we find that the financial times of German Stock Returns studied in our sample indicate that for the most part these times contain predictable properties. These properties may simply be a trend, an autoregressive (AR) property, a moving-average (MA) property or perhaps a mixed model that contains both AR and MA (ARMA) properties. Furthermore, it is likely that these time series have day-of-the-week properties, that is, Mondays are

different from Tuesdays which differ from Wednesdays or Thursdays or Fridays. Day of the week properties exhibit themselves as periodic cycles with a trading-day week interval. Previously, other studies indicated that the day of the week associates itself with stock returns (See Rothenberg *et al.* 1996; Greene 2003 and Jarrett and Kyper 2005). In the next section, we will indicate a model to include a specification for day-of-the-week variation.

5. Model specification and estimation

Once we determined that the model contains predictive properties (unit roots) , we use an ARIMA(1,1,1) × $(1,1,1)^5$ time series model to determine whether such a model will fit the data. Note this model contains one autoregressive term, one moving-average term difference by one period and a seasonal part having one seasonal autoregressive term and one seasonal moving-average term of order five. The order of five is chosen since we have five trading days in each week. In this way, we can begin to understand whether parsimonious ARIMA models are useful to predict the daily returns to firms listed in the sample. In statistics, an **autoregressive integrated moving-average (ARIMA)** model is a generalization of an autoregressive moving-average or (ARMA) model. These models are fitted to time series data either to better understand the data or to predict future points in the series. The model is generally referred to as an ARIMA (*p*, *d,* and *q*) model where

p, *d*, and *q* are integers greater than or equal to zero and refer to the order of the autoregressive, integrated, and moving average parts of the model respectively. Sometimes a seasonal effect is suspected in the model. For example, consider a model of daily road traffic volumes. Weekends clearly exhibit different behaviour from weekdays. In this case it is often considered better to use a SARIMA (seasonal ARIMA) model than to increase the order of the AR or MA parts of the model. In our study, the seasonal effect is the daily effect. We have five days of the weeks indicating the Mondays differ from other days of the week as do other days differ from each other. For a complete discussion of the mathematics of SARIMA models in applied finance and economics see Chan (2002, chapters 3 and 4).

The model specified in Table 2 did not fit all the time series, however, they do indicate that a parsimonious ARIMA models could fit most if not all the time series estimated. Furthermore, not shown, the results of the Portmonteau Q-statistic indicate for those models such as for firms …., the ARIMA $(1,1,1)(1,1,1)^5$ does adequately represent variation in the fitted model. [For details as to this important statistic for fitting ARIMA models see Chan 2002: 54–64, and 108–109.] Although, we do not finalize all predictive analysis, we do conclude that the sampled time series contain properties which can be modelled and would tend to indicate that these data are not completely random or completely random with a drift.

Table 2. Results of estimating The ARIMA model

Firm Name		AR 1	MA 1	SAR 5	SMA 5
Dr. Hönle AG	**coef.**	0.245	0.381	−0.005	0.982
	pvalue	0.295	0.087	0.897	0.000
Altana AG	**coef.**	0.583	0.727	0.079	0.988
	pvalue	0.000	0.000	0.040	0.000
Synaxon AG	**coef.**	−0.854	−0.846	−0.087	0.987
	pvalue	0.262	0.269	0.022	0.000
Init- Innovation	**coef.**	−0.558	−0.498	0.001	0.985
	pvalue	0.156	0.244	0.990	0.000
AIG Int. Real	**coef.**	0.721	0.832	0.092	0.984
	pvalue	0.000	0.000	0.018	0.000
Utimaco	**coef.**	0.446	0.505	0.008	0.984
	pvalue	0.299	0.222	0.823	0.000
Deutsche Telekom	**coef.**	0.208	0.136	0.014	0.988
	pvalue	0.661	0.778	0.700	0.000
Pankl Racing Systems AG	**coef.**	0.423	0.536	−0.037	0.983
	pvalue	0.067	0.013	0.326	0.000

Firm Name		AR 1	MA 1	SAR 5	SMA 5
DEWB AG	coef.	−0.716	−0.816	0.038	0.984
	pvalue	0.000	0.000	0.327	0.000
Dyckerhoff AG	coef.	0.406	0.490	0.006	0.982
	pvalue	0.206	0.109	0.882	0.000
P&I Personal und Informatik AG	coef.	0.062	0.113	−0.050	0.988
	pvalue	0.930	0.873	0.171	0.000
Tomorrow Focus AG	coef.	0.367	0.492	−0.009	0.975
	pvalue	0.104	0.020	0.814	0.000
Deutsche Bank	coef.	0.438	0.423	−0.068	0.981
	pvalue	0.816	0.823	0.065	0.000
Hornbach Baumarkt	coef.	0.595	0.559	0.025	0.977
	pvalue	0.324	0.368	0.518	0.000
Leifheit	coef.	−0.058	0.068	-0.089	0.982
	pvalue	0.841	0.814	0.013	0.000
Ludwig Beck AG	coef.	0.328	0.507	−0.047	0.981
	pvalue	0.043	0.001	0.205	0.000
Surteco	coef.	−0.181	−0.091	−0.044	0.981
	pvalue	0.638	0.816	0.220	0.000
HIGHLIGHT CMNCTS INH.SF 1	coef.	0.549	0.625	0.041	0.976
	pvalue	0.047	0.016	0.299	0.000
SUNWAYS AG O.N.	coef.	0.398	0.433	−0.001	0.981
	pvalue	0.605	0.566	0.985	0.000
SARTORIUS AG VZO O.N.	coef.	0.701	0.821	0.046	0.981
	pvalue	0.000	0.000	0.237	0.000
HEIDELBERG.DRUCKMA.O.N.	coef.	−0.568	−0.546	−0.032	0.982
	pvalue	0.588	0.609	0.406	0.000
TAG TEGERNSEE IMMOB.	coef.	0.130	0.252	−0.132	0.980
	pvalue	0.648	0.364	0.000	0.000
TELEGATE AG O.N.	coef.	−0.061	0.090	0.021	0.982
	pvalue	0.800	0.709	0.568	0.000
GOYELLOW MEDIA AG O.N.	coef.	0.322	0.263	−0.034	0.979
	pvalue	0.543	0.626	0.349	0.000
SINNERSCHRADER O.N.	coef.	−0.725	−0.821	−0.169	0.995
	pvalue	0.000	0.000	0.000	0.000
JETTER AG O.N.	coef.	0.569	0.556	−0.044	0.992
	pvalue	0.741	0.749	0.252	0.000
THIEL LOGISTIK AG	coef.	−0.003	0.099	0.012	0.980
	pvalue	0.994	0.779	0.735	0.000
LUFTHANSA AG VNA O.N.	coef.	−0.703	−0.656	−0.031	0.980
	pvalue	0.037	0.065	0.438	0.000
MASTERFLEX O.N.	coef.	0.355	0.392	−0.064	0.984
	pvalue	0.657	0.619	0.080	0.000

Firm Name		AR 1	MA 1	SAR 5	SMA 5
PC–WARE INFOR.TECHNOLO.AG	**coef.**	–0.769	–0.733	–0.040	0.987
	pvalue	0.028	0.046	0.315	0.000
VIVACON AG O.N.	**coef.**	–0.648	–0.616	0.039	0.993
	pvalue	0.282	0.321	0.330	0.000
VOSSLOH AG O.N.	**coef.**	0.743	0.843	0.038	0.978
	pvalue	0.000	0.000	0.324	0.000
REALTECH AG O.N.	**coef.**	0.087	0.190	–0.118	0.986
	pvalue	0.804	0.583	0.001	0.000
GESCO AG O.N.	**coef.**	–0.315	–0.271	–0.005	0.985
	pvalue	0.666	0.714	0.884	0.000
TECHNOTRANS AG O.N.	**coef.**	0.221	0.296	–0.017	0.987
	pvalue	0.613	0.489	0.645	0.000
SOFTING AG O.N.	**coef.**	0.039	0.225	–0.046	0.981
	pvalue	0.840	0.228	0.201	0.000
Continental	**coef.**	–0.462	–0.375	–0.098	0.991
	pvalue	0.139	0.251	0.008	0.000
Volkswagen	**coef.**	0.058	0.007	–0.033	0.983
	pvalue	0.935	0.993	0.385	0.000
Adidas	**coef.**	–0.423	–0.459	0.003	0.982
	pvalue	0.564	0.522	0.947	0.000
Man AG	**coef.**	–0.768	–0.854	–0.066	0.986
	pvalue	0.000	0.000	0.088	0.000
KEY:	AR 1	autoregressive order 1			
	MA 1	moving-average order 1			
	AR 5	autoregressive order 5			
	MA 5	moving-average order 5			
	coef.	coefficient			
	pvalue	probability of rejecting null hypothesis			

6. Conclusions

In this study, we document and present evidence that returns for a sample of lengthy times of firms listed on the Frankfurter Börse (Stock Market) contain properties that one can measure, model and use for prediction. With ample time to study the underlying mathematics of the processes that give rise to financial time series, forecasters can properly model and predict changes in the time series in the future. These results indicate that for the time period covered, sample chosen of listed firms on the Frankfurter Börse, the time series contain properties that are not random and do have daily effects. These results do not substantially differ from studies of other markets in the United States, Asia and Europe.

In addition, we should state again that the purpose of forecasting concerns out-of-sample wealth opportunities. Ascertaining in-sample wealth creating opportunities can be thought of as an application of "data mining." If you fit many models a few will randomly have high coefficients of determination and/or statistically significant model coefficients. Since we developed parsimonious (least costly) models, the wealth creating opportunities should be greater than transaction costs may include bid-ask spreads and commissions. If so, we have found real profitable trading opportunities.

Future studies should relate changes in the time series patterns of the German stock market with its Cointegration with other European stock exchanges as well as the large exchanges in the United States and Japan. Such studies will enable us to large additional reasons why there are predictable properties in stock returns. Although not easy to predict, we can understand the variation in stock returns by considerable study and time series data analysis.

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