

A search for deviations from the inverse square law of gravity at nm range using a pulsed neutron beam

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Abstract. Recently published results and ongoing experimental efforts to search for deviations from the inverse square law of gravity at the nanometer length scale using slow neutron scattering from the noble gases are discussed. Using the pulsed slow neutron beamline BL05 at the Materials and Life Sciences Facility at J-PARC, we measured the neutron momentum transfer (q) dependence of the differential scattering cross section for the noble gases He, Ne, Ar, Kr, and Xe. By comparing to the distributions obtained using pseudo-experimental Monte Carlo simulations and forming ratios between Xe and He, we placed an upper bound on the strength of a new interaction as a function of interaction length λ which improved upon previous results in the region $\lambda < 0.1$ nm, and remains competitive in the larger λ region. Additionally we describe how we are using our technique to extract relative values of the total neutron scattering cross sections of the noble gases, as well as how we plan to measure the neutron-electron scattering length using the NOVA instrument on BL21 at J-PARC.

1. Constraints on Yukawa-like gravity

The force of gravity is confirmed to follow an inverse-square law (ISL) famously known as “Newton’s Law of Universal Gravitation” and has been verified experimentally down to distances of less than 1 mm [1]. However at shorter distances the precise functional form of the gravitational interaction is very poorly constrained. Many alternative theories of gravity possess an additional force term resulting from the exchange of massive quanta and therefore extend only to short ranges. Other theories which try to explain why gravity is so weak compared to the other interactions of Nature also produce short-distance modifications to the theory. The various speculations which can lead to such modifications has been reviewed recently [2].

For weakly-coupled interactions between nucleons and electrons in the nonrelativistic limit, a number of parametrizations of new possible interactions have been developed which are largely model-independent under the assumption that the new interaction is local [3,4]. These parametrizations typically produce ISL violations with a Yukawa-like exponential falloff with distance multiplied by some power of $1/r$, where r is the distance between the nucleons or electrons.

Experiments using neutrons are sensitive to ISL deviations at submicron scales and, due to their

net electrical neutrality, are not limited by large electromagnetic background effects. For the case of neutron – noble gas scattering, the largest contribution to the measured signal originates from the strong nuclear force and results in an isotropic neutron scattering distribution, while the much weaker ($<10^{-3}$) interaction between the neutron charge density and the atomic electric field of the noble gas atom results in an asymmetric scattering distribution. Similarly, in the presence of a Yukawa-like deviation from the ISL, an asymmetric scattering distribution would also be induced although its functional form would be distinct from the electric field contribution thus allowing for its contribution to be separated in analysis.

We summarize the results from a dedicated experiment which probed the ISL at the nanometer length scale by analyzing the momentum transfer (q) dependence (or, equivalently, the energy and angular dependence) of neutron-noble gas scattering measurements performed on a cold neutron beamline with a peak velocity of 1534 m/s (see Fig. 3, and published in [5]).

1.1. Methodology

In general, the nonrelativistic limit of a quantum field theory describing the exchange of a single massive boson generates a potential of Yukawa form in position space. We may treat this interaction as a perturbation to the Newtonian force so that the potential $V(r)$ between masses

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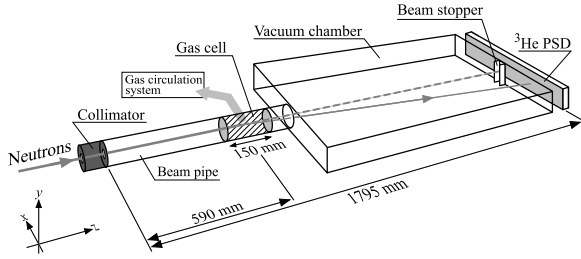


Figure 1. Layout of our experiment as mounted on BL05 at the MLF facility at J-PARC. Image taken from [5].

of m and M can be described as

$$V(r) = -G_N \frac{mM}{r} (1 + \alpha e^{-r/\lambda}), \quad (1)$$

where r is distance between the masses, α parametrizes the strength of the short-range interaction relative to gravity, λ is the Compton wavelength of the exchange boson (or, equivalently, the interaction length scale), and G_N is the gravitational constant. The contribution to the total neutron – noble gas scattering length due to the Yukawa-like interaction is computed simply in the first Born approximation as the Fourier transformation of the second term in Eq. (1) into the q -plane and is given as

$$b_Y(q) = \alpha \left(\frac{2G_N m_n^2 M_A}{\hbar^2} \right) \frac{1}{\lambda^{-2} + q^2}, \quad (2)$$

where M_A is the mass of the atom, m_n is the neutron mass, and \hbar is the reduced Planck's constant. The momentum transfer, q , is given in terms of the neutron wavelength and scattering angle θ measured with respect to the incident neutron path as

$$q = \frac{4\pi}{\lambda_n} \sin\left(\frac{\theta}{2}\right). \quad (3)$$

We constructed a simple scattering apparatus whose essential components consist of a gas cell, an evacuated scattering chamber (also called the “vacuum chamber”) and a ^3He position sensitive detector (PSD). The layout is shown in Fig. 1, and the apparatus functionality and components are described in detail in [5].

During our run in late 2016 on BL05 the beam power to MLF was an average of 150 kW (1.1×10^6 incident neutrons/sec). Before taking gas scattering data we performed a cross-sectional scan of the incident beam area using a $1 \times 1 \text{ mm}^2$ collimator formed from two sets of neutron absorbing B_4C plates. By recording the data as a function of slit position in the x - y plane, a two-dimensional intensity distribution was obtained. This data was used to produce the plots shown in Figs. 3 and 2.

We recorded 2.5×10^6 scattering events for evacuated cell, 2.8×10^6 for He gas-filled cell, and 1.1×10^7 scattering events for the Xe gas-filled cell. Each event was recorded with time of flight and position information which allowed us to construct the scattered intensity $I(q)$.

Gas purity was maintained using flow – controlled circulation through a rare gas purifier whose outlet gas purity is rated to be less than 0.01 ppm for H_2O , N_2 , O_2 or hydrocarbons, to ensure there are no measurable q -dependent effects from neutron – hydrocarbon scattering. The region of q used for analysis was chosen to be

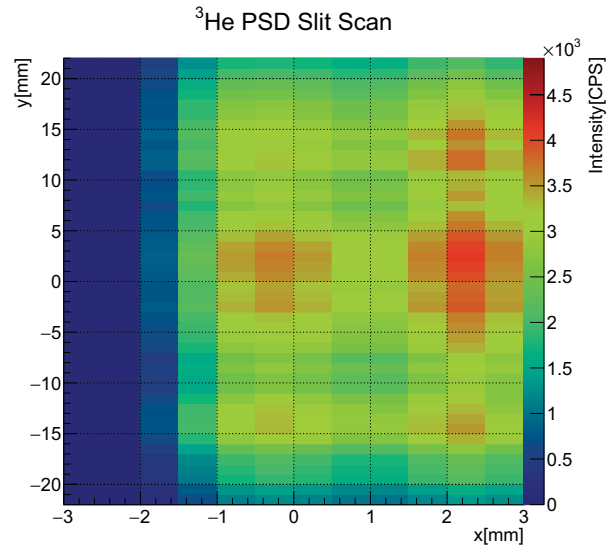


Figure 2. Beam intensity profile during 150 kW proton beam power operation on BL05 at MLF.

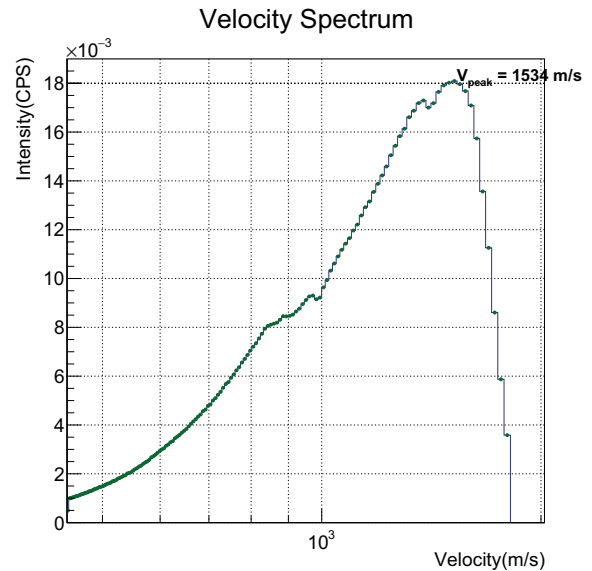


Figure 3. Velocity spectrum obtained from beam scan data.

between 1 and 4 nm^{-1} , in order to remove effects from the beam stopper at low q and possible diffraction effects at the larger q .

1.2. Results

The complete theoretical expression for $I(q)$ normalized by the solid angle is given to sufficient accuracy for neutron scattering from non – interacting gases in for example [6,7]. When one includes the effects from the interatomic pair potential between gas atoms, small diffraction effects appear [8]. These effects are negligible at our present statistical precision however they must be considered when additional data is analyzed to extract the total scattering cross section (c.f. Sect. 2).

We developed a Monte Carlo simulation within the ROOT analysis framework to reproduce our experimental data using the measured velocity spectra and incident flux map as input. Neutrons were generated in a loop and propagated to and from scattering centers using standard

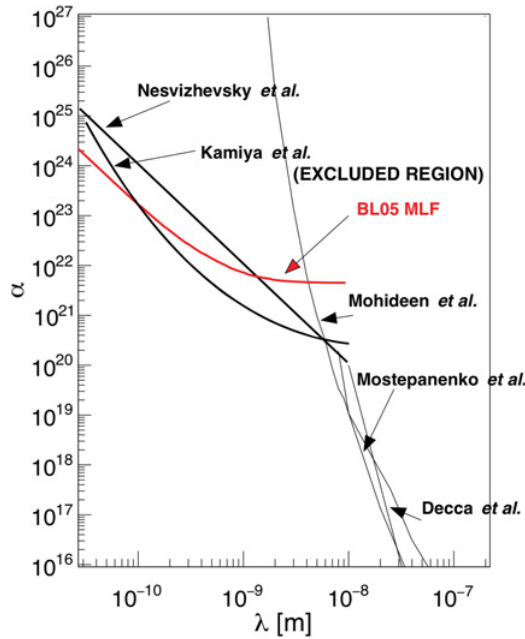


Figure 4. Constraints on Yukawa-like gravity parameter α as a function of interaction length λ by comparing measured q spectra to simulated data. Image taken from [5].

kinematic relations and the velocity dependent mean free paths. At the scattering centers angular distribution and energy transfer was computed using the known structure factor derivable analytically for the case of the ideal gas, as well as the neutron-electron interaction relying upon precision x-ray scattering measurements of the atomic form factors. Neutrons that made it into the virtual detection region were registered and intensity plots as a function of momentum transfer q were obtained.

The experimental $I(q)$ data was normalized by the simulated data to remove the known q -dependent effects, leaving a residual spectra which was fit with a function including the possible Yukawa-like effect. The fit returned values of the strength of the Yukawa-like effect, α , as a function of the interaction length λ which were consistent with zero and their upper bound was plotted at the 95% confidence level in Fig. 4.

1.3. Discussion

Although our constraint is consistent with zero, we did not achieve the statistical sensitivity we originally hoped for over the interaction range of λ between 10^{-11} and 10^{-8} m. The primary reason for this is that we encountered significant small angle scattering from our LiF beamstop, which contaminated our lower q (and therefore larger λ) data. We have since redesigned a more narrow beamstop which also includes a thick inner Cd layer. We expect that with the new beamstop, our sensitivity will increase in the larger λ region and thus be very competitive with the leading constraints placed by Kamiya et al. [9].

At the level of statistical sensitivity of the q -dependent ratios of total count rates at our current values of $\sim 1\%$ used to produce Fig. 4, the uncertainties in the total scattering cross section of the noble gases in the literature, typically $\sim 0.1\%$ only produce a negligible shift in the fitting function used to constrain the $b_Y(q)$ contribution to the scattering intensity. However if future statistical

Table 1. Measured count rate per gas atom normalized by beam power is given in the second column. Values relative to Ne data are given in the third and fourth columns for experimental and simulated data, respectively. Errors are statistical only, and do not include variation in pressure and temperature during runs. Simulated data does not yet include effects from interatomic pair potential which may affect the results at the 1% level.

Gas	$\dot{N}[\text{MW a s}]^{-1}$	$\dot{N}/\dot{N}_{\text{Ne}}$	$(\dot{N}/\dot{N}_{\text{Ne}})_{\text{SIM}}$	$\sigma_{\text{SIM}}[\text{bn}]$
He	1.218(3)	0.596(7)	0.67	1.34
Ne	2.042(6)	–	–	2.63
Ar	0.483(3)	0.237(7)	0.23	0.68
Kr	4.869(8)	2.384(9)	2.36	7.68
Xe	2.698(7)	1.321(9)	1.28	4.41

precision in the q -dependent measurement is achieved which is comparable to, or exceeds, the literature values of relative uncertainty in the scattering cross sections, or if more accurate measurements are made on the cross sections which disagree with previously assumed values significantly, the constraint on the strength of the Yukawa-like interaction could be altered and should be considered.

2. Cross section measurements

During the acquisition of our data on BL05, we noticed a slightly lower count rate from what was expected given the existing values of the scattering cross section $\sigma(^4\text{He})$ in the modern neutron interferometry literature [10] and used in our simulations, but consistent with older values obtained from neutron transmission [11] and refraction [12]. This discrepancy has been mentioned recently [13] and can amount to as much as a 10% difference in the cross section.

In placing our limits on Yukawa-like interactions, the application of thermal effects of the gas sample in our simulation reproduced the raw q spectrum to sufficient accuracy so that a limit on interactions leading to q -dependent amplitudes could be made, and any small shift of the q -independent nuclear component was negligible at our published sensitivity. However in order to place more stringent constraints in the future it is crucial that we know with more precision the value of $\sigma(^4\text{He})$. Therefore we opted to pursue a more rigorous study of the ^4He cross section during a future beam cycle, by taking more ^4He data as well as the remaining noble gases which could be used as a control.

We acquired high purity gas canisters of the natural noble gases He, Ne, Ar, Kr, and Xe, and took q -dependent scattering measurements of each gas separately (plus evacuated cell) using the scattering and gas handling apparatus described in Sect. 1.1. Since our method necessarily relies on forming the ratio of two gases to remove q -dependent systematics, we are unable to make an absolute measurements of the total cross sections, but only relative measurements. Following [6], we chose Ne as our control sample as it still currently has the most precise and consistent cross section values in the literature ([11, 14, 15]).

The total count rate for each gas normalized by the number density as measured indirectly via independent pressure and temperature measurements is listed in Table 1. The vacuum cell data was subtracted, and the

errors added in quadrature. The shutter closed background data has not been subtracted yet, however this effect was small and should contribute $\ll 1\%$ to the measured cross section. The errors listed are statistical only and do not yet account for the inaccuracies of the temperature and pressure readings, which when combined will affect the measured count rates by less than 1%.

The count rate alone is not enough to infer the cross sections of the gases as there are contributions from the thermal motion of the gas atoms, the atomic electric field, and the so called pair correlation function resulting from the interatomic potential experience by the gas atoms. The q -dependence of these effects are distinct and can be studied via MC simulation as was done to our data producing the Yukawa-limit in Sect. 1.2. However our MC simulation does not yet include effects from the interatomic pair potential of the gas sample, which may affect the results at the 1% level in our q region of $\sim 0.5 \text{ nm}^{-1}$ to $\sim 5 \text{ nm}^{-1}$, which is slightly more broad than the region used in placing the limits on Yukawa – like interactions. Nevertheless we can make an approximation by neglecting these effects and comparing the ratio of the count rate of a gas species with respect to Ne (Table 1, column 3) to the respective ratio of the simulated values without including interatomic pair potential effects (Table 1, column 4). The values used as inputs for the bound coherent scattering cross sections are given in the last column of Table 1. It would appear that measured He count rate is significantly lower than predicted by simulation as compared to the other gas species, and consistent with the older literature results from neutron transmission.

We also note that recent measurements carried out at the Neutron Interferometry and Optics Facility (NIOFa) at the NIST Center for Neutron Research produced preliminary data [16] suggesting a range of values between $\sigma(^4\text{He}) = 1.1286(18) \text{ bn}$ and $1.2342(12) \text{ bn}$ for the coherent scattering length of ^4He , consistent with estimates based off of our measurements. However their errors are statistical only as the systematic error analysis for their work is not yet complete at the time of this writing.

3. Neutron–electron scattering

The internal structure of the neutron is often overlooked in low energy neutron scattering experiments where the interaction is dominated by an effective strong nuclear potential which results in an isotropic elastic scattering distribution. However, many neutron scattering experiments perform high precision measurements of the momentum transfer (q) distribution which may be sensitive to effects resulting from the much weaker ($\sim 10^{-4}$) interaction between the internal quark structure of the neutron and the atomic electric field of the scattering centers, even at low energies. It is therefore crucial that the size and functional form of this interaction is well understood to prevent a systematic error in these measurements.

In Sects. 1.2 and 2, we implicitly assumed a value of the neutron electron scattering amplitude $b_{ne} = -1.32(4) \times 10^{-3} \text{ fm}$ [17]. The value of b_{ne} has been sought through measurement for over 50 years, yet there still remains some disagreement among the various methodologies (Fig. 5) which although consistent with

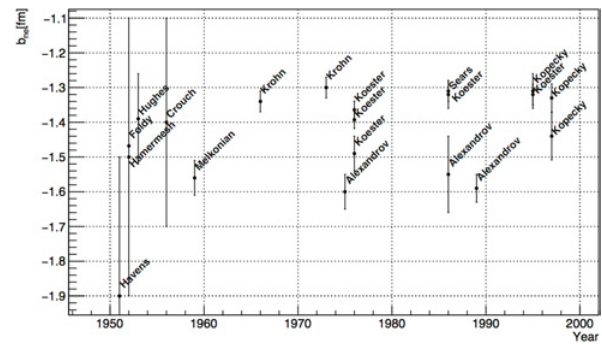


Figure 5. Measurements of b_{ne} over time. Data points taken from [18] and include data from transmission, diffraction, reflectometry, and scattering experiments.

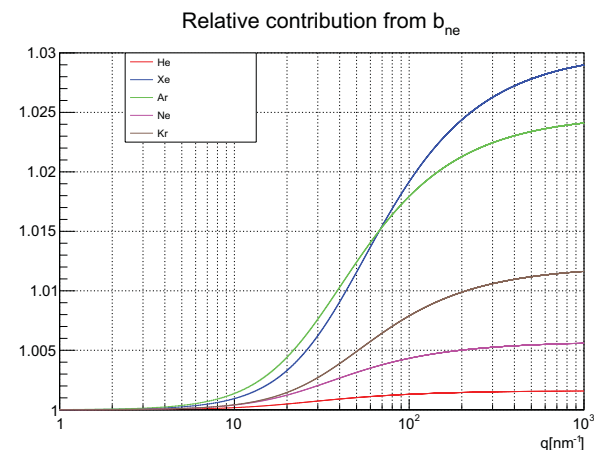


Figure 6. Theoretical relative contribution from the b_{ne} term in the neutron-noble gas differential cross section over the q spectrum measured on NOVA at BL21 at J-PARC.

the theoretically predicted range based on the ratio of the proton to neutron charge radius [18,19], does not provide a precise enough result to give any further insight into the nucleon structure. This is most likely due to the fact that within the standard methods of neutron transmission, diffraction, or reflectometry, there exists relatively large corrections due to effects which are either not fully understood theoretically or rely heavily on other experimentally determined parameters. It is therefore attractive to use a method with fewer, smaller corrections. One of the most precise methods of measuring b_{ne} used neutron scattering from noble gases [6], however it has since been pointed out that diffraction effects were overlooked in their analysis and may lead to a non-negligible effect in the value of b_{ne} [20].

We will apply our recently developed comparative differential scattering analysis to neutron-noble gas scattering measured over the wider range of q values and gas pressures (below 10 atm) offered at the NOVA instrument on BL21 at J-PARC. By forming the ratio of two gases we can extract b_{ne} by a simple fitting procedure to the known q distributions (including interatomic pair potential effects) convolved with the experimental conditions. This procedure is distinct from [6] as we are sensitive to a wide range of scattering angles, and therefore q , as opposed to only two scattering angles. This method has already been suggested in recent literature however a dedicated experiment has not yet been performed. Rather a reanalysis of existing experimental data was published [21]

and it was shown that this method is capable of measuring b_{ne} to an uncertainty of as little as 2% provided one has measured the product of the neutron spectrum with the detector efficiency to sufficient accuracy, something which has not been done in many recent neutron – noble gas scattering experiments performed to deduce interatomic pair potentials of the gas atoms. We plan to begin data taking on BL21 in early 2019 using Xe and Kr as the two gases from which we form a ratio. We have chosen these gases as they have the highest cross sections and masses of the noble gases, giving increased statistics and a decrease in the correction for thermal motion (which scales with the inverse of the atomic mass), respectively. The difference in q -dependence due to their respective atomic form factors (See Fig. 6) will then be easier to identify

4. Conclusion

Using the pulsed slow neutron beamline BL05 at the Materials and Life Sciences Facility at J-PARC, we measured the neutron momentum transfer (q) dependence of the differential scattering cross section for the noble gases He, Ne, Ar, Kr, and Xe. By comparing to the distributions obtained for Xe and He, we placed an upper bound on the strength of a new interaction as a function of interaction length λ in the sub – 10 nm length scale. Following this analysis we performed scattering measurements on the remaining noble gases and plan to use this data to extract their relative total scattering cross sections. Our preliminary data suggests that the value for the total scattering cross section for He is in disagreement with existing interferometry data, but consistent with older data based on transmission and reflectometry. Final results including a more rigorous treatment of systematic effects are forthcoming. Finally we described a neutron – noble gas scattering method which we plan to use on the NOVA scattering instrument at J-PARC in order to extract the neutron-electron scattering length. The beam time for this work is planned for early 2019.

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