A STATISTICAL ANALYSIS OF THE VOLCANIC ACTIVITY AT STROMBOLI, ITALY

BY

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TABLE OF CONTENTS

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ABSTRACT

Four hundred eruptions of 3 active vents within the summit crater of the volcano Stromboli were recorded during 43.5 hours. The main vent (1) typically erupts pasty molten bombs in a 2 to 5 second burst with a mean frequency of 10.7 minutes. A second vent (2) normally fires a 1 second cannon-like burst of molten material averaging every 54 minutes. A third vent (3) predominantly erupts hot luminous gas for 2 minutes every 20.7 minutes on the average. Distributions of intereruption periods for all three vents imply that at a given time the eruption probability is independent of elapsed time from the previous eruption. Regression techniques applied to the intereruption periods at each vent together with parameters of activity at other vents (number of eruptions, time since last eruption) reveal a striking linear correlation of the activity at Vents 2 and 3 with the activity at Vent 1 (correlation coefficients were +0.986 and +0.87 for Vents 2 and 3 respectively). Physically this means the number of eruptions at Vent 1 is a far better indicator of the time to the next event at Vents 2 and 3 than any other parameter such as previous intereruption period, eruption duration, or time since the last eruption.

INTRODUCTION

Volcanoes pose a threat and a question. They are the dynamic representatives of a geologic process that has provided the earth with the crust, oceans, and atmosphere we know today. How they work is clearly an important question. In such parts of the world as Japan, Indonesia, and Central America, volcanoes border dangerously close to densely populated areas. When they will choose to erupt again is a serious threat.

A key to understanding the volcanic process and developing a capability to anticipate volcanic eruptions lies in monitoring. The U. S. Geological Survey has maintained a long term monitoring station at Kilauea in Hawaii over the past 40 years. Detailed work there has shed light on relationships between magmatic reservoirs at depth and volcanic activity at the surface.

This study is based on field data acquired during a short-term monitoring experiment at an active volcano. The purpose of this field experiment is to measure physical parameters describing the material flow at the vent-surface interface of a volcano in a state of eruption. The temporal analysis reported in this study supports the general experiment and will ultimately extend the implications of 'point results' for single eruptions over longer periods of recurring activity (see Chouet, 1973).

DEFINITION OF THE PROBLEM AND APPROACH

The purpose of this study is to determine the ability to predict eruptions for individual vents at the summit of the volcano Stromboli on the basis of oberved times between eruptions. The approach is statistical and employs a compilation of repose periods observed by the author in September, 1971. At the outset it is clear that a more meaningful approach to developing a prediction capability would be to dynamically monitor physical parameters such as ground tilt, seismicity, temperature, etc. which could then be fit into an integrated model of the physical process driving the eruptive mechanism. Unfortunately, few such models exist. Furthermore, even where a consistently demonstrated relationship between surface volcanism and conditions at depth is qualitatively understood, the time resolution of the resulting model in anticipating an eruption is not always significantly enhanced.

The models that will be investigated here will be strictly empirical. They are, of necessity, based on past behaviour and can be no more accurate or representative of the volcanic process than was the observed data employed in their formulation.

I. BACKGROUND

1. STROMBOLI

Stromboli is a member of a chain of volcanic islands and seamounts situated to the north of Sicily in the Mediterranean Sea. This chain is known as the Lipari Islands. Stromboli itself is a massive stratovolcano rising 2750 meters above the adjacent seafloor, while only 920 meters protrude above sea level. The surface expression of the island is approximately square, measuring 3.5 kilometers on a side. Human habitation is confined to two ledges formed by successive lava flows which lay near sea level on the flanks of the cone.

Stromboli lies in a corner of the Mediterranean referred to as the Tyrrhenian Sea. The Tyrrhenian Sea roughly resembles a right triangle, bound to the south by Sicily, to the west by Sardinia and Corsica, with the western coast of Italy approximating the triangle's hypotenuse (see Figure 1-1). A concise review of the regional setting of the Tyrrhenian Sea is given by Ryan et. al. (1971). In brief, the seafloor is overlain by thin sedimentary deposits and observed heat flow is the highest in the Mediterranean. Local magnetic anomalies are aligned parallel to rifting troughs in the crustal floor. Seismicity in the Mediterranean Basin predominantly occurs in the Tyrrhenian and Aegean Seas. Thus, the seafloor is interpreted as a young, extensional feature by Ryan et. al (1971). To the south, fault plane solutions suggest that the North African plate is underthrusting Europe at a very slow rate (McKenzie, 1970). However the tectonics of the area is complicated significantly by the

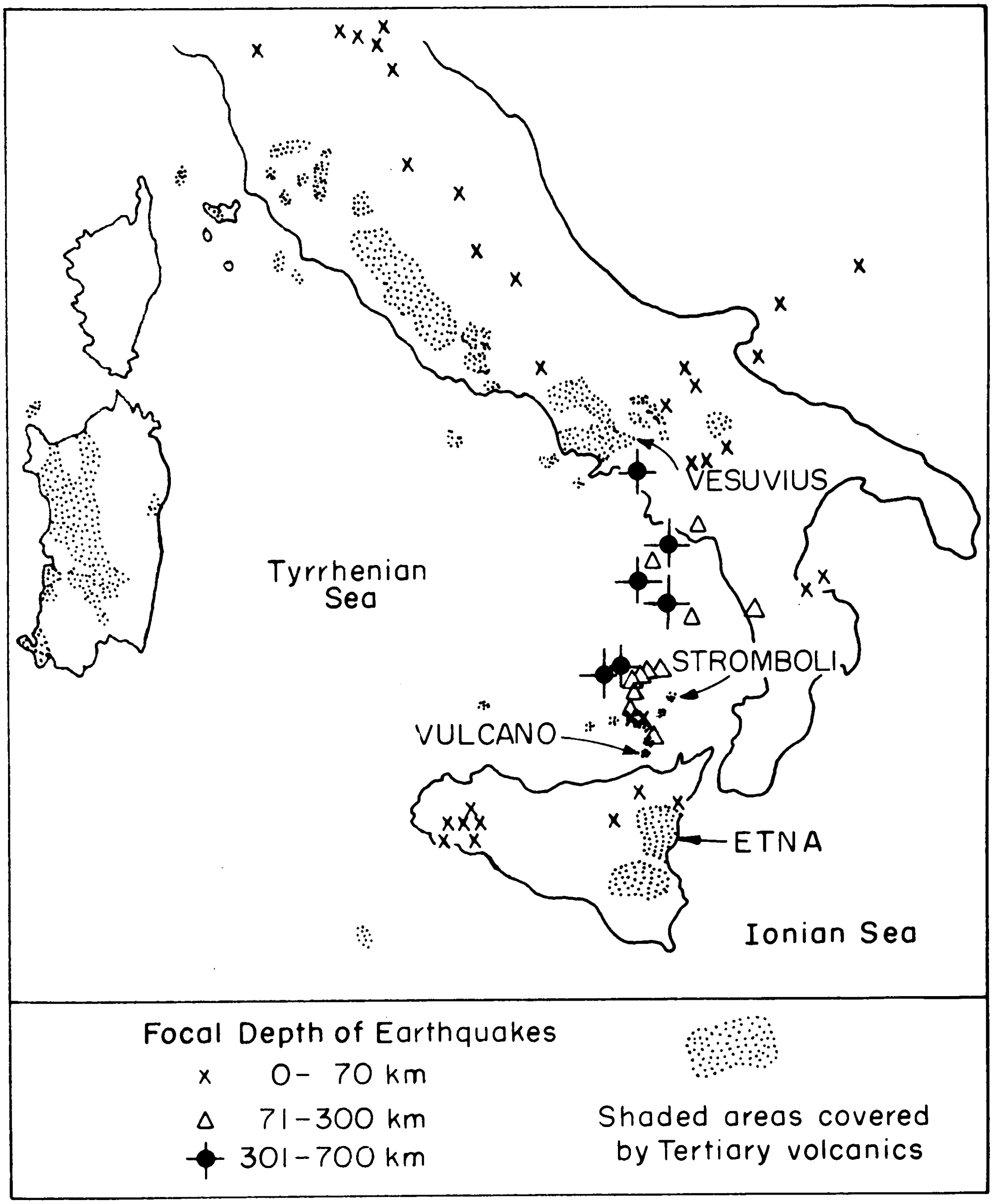


Figure 1-1 Regional setting of Stromboli

existence of remnant Alpine orogeny which trends through the Appenine Mountains of Italy into the Atlas Mountains of northern Africa. It is interesting to note that the Lipari Islands lie along a line connecting the more famous volcanoes Vesuvius to the north and Etna to the south. This line may represent a young tensional crack opened by the compressive forces operating a depth along the southern Sicilian coastline.

Currently Stromboli is producing olivine and pyroxene bearing basalts. Earlier lavas are successively less mafic and more siliceous. The deepest exposures on the island are near the summit and they reveal the existence of pyroxene bearing andesites.

Volcanic activity at Stromboli was recorded by the Greeks more than 2400 years ago. Recent historical activity can largely be categorized by two modes: one of continuous explosive activity at the summit, the other involving major eruptions affecting the entire volcanic cone. The remarkable persistence of the former mode throughout the twentieth century has led volcanologists to coin the term 'Stromvolian activity' (Rittman, 1962, Bullard, 1954). The term is used to describe a steady series of moderately explosive eruptions which eject discreet lava fragments as bombs or scoria to lateral distances very much smaller than the diameter of the volcano. This mode of volcanism contributes little to the areal growth of the volcano. Another unusual feature of Stromboli's steady-state volcanic mode is the configuration of the active summit vents. Throughout this century activity at the summit has been confined to a broad elliptical crater which measured 300 meters by 100 meters in September, 1971 (see Bullard, 1954, and Perret, 1907). Though the actual number of vents within the crater emitting gases or lava is variable, the activity of two stable vents within the summit crater at its western and eastern edges has been documented over the past 20 years.

The alternate mode of volcanic activity affecting the entire cone has involved combinations of severe explosive outbursts at the summit, the extrusion of aerial and submarine lava flows, and the generation of tsunamis. Major eruptions of this sort were recorded in 1906, 1907, 1915, 1919, 1930, 1936, 1937, 1954, and 1959 (Imbo, 1964, Johnston and Mauk, 1972).

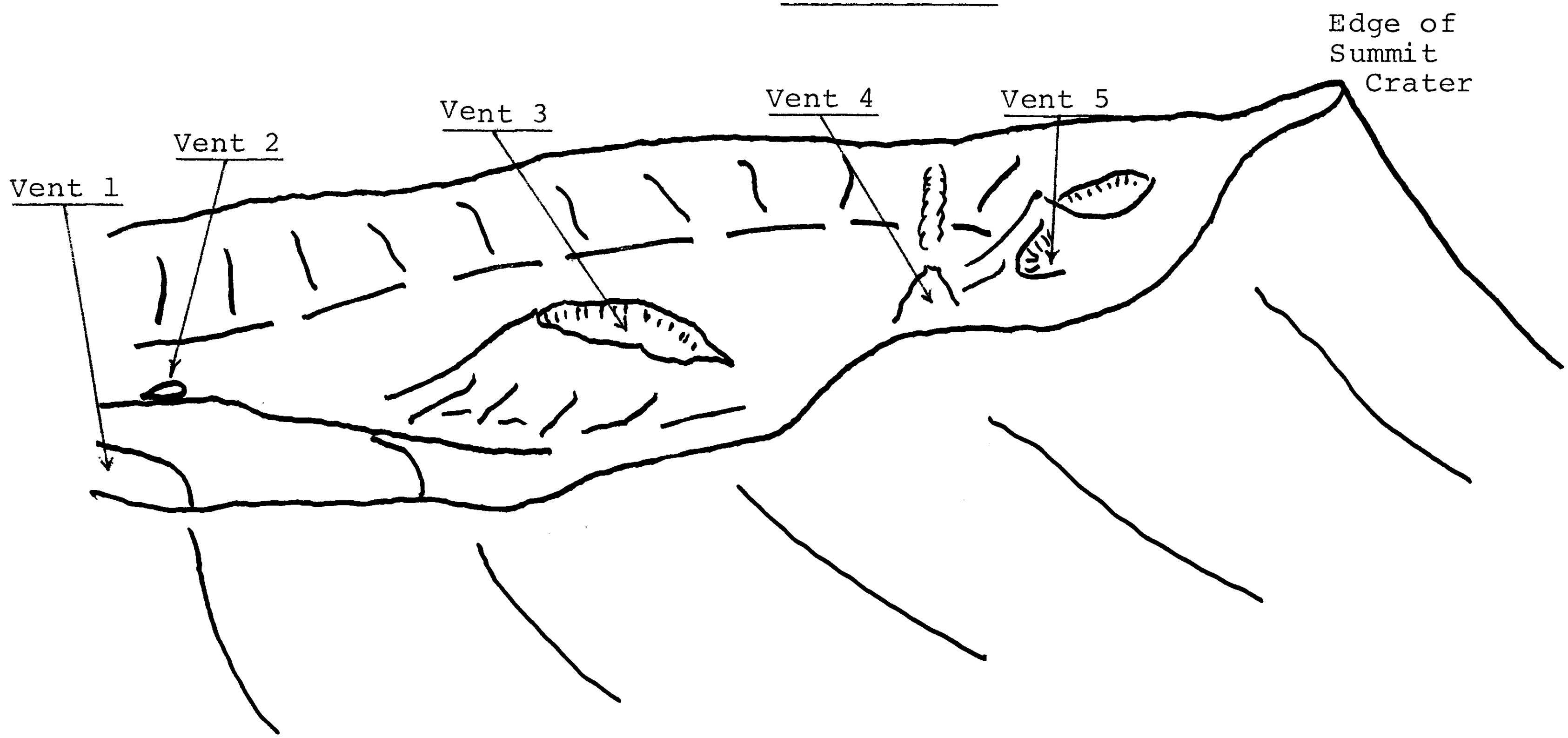
The morphology of the summit crater in September 1971 is shown in Figure 1-2. Gas and lava appearred at the surface at five active vents. These vents are characterized below by shape, product, and style of eruption.

Vent 1. Physically this vent lay at the center of a gentle depression about 30 meters in diameter. The vent itself was roughly triangular with its longest dimension on the order of 1 meter. A typical eruption consisted of a single pulsed explosion which hurled molten lava in the form of bombs, lapilli, and ash into the air and lasted for a period of a few seconds. It erupted more frequently than any other vent.

Vent 2. This vent lay at the northern edge of the large crater containing all the vents. It was canted in such a fashion that ejected material was projected to the north away from all observation stations. It was nicknamed 'Big Bertha' as a typical eruption consisted of a cannon-like blast. This vent appeared to be highly unpredictable in the field.

Figure 1-2A Summit Crater at Stromboli

Figure 1-2B Schematic sketch map of the summit crater at Stromboli



Vent 3. This vent was slightly elliptical in shape and proved to be the site of intense fumarolic activity. No emission of lava was observed at this vent though it was obscured a major portion of the time by copious cloud-like emanations which appearred to be steam.

Vent 4. A small cone-shaped spatter mound which stood approximately 3 meters above the crater floor was labelled Vent 4. Typically eruptions would commence with a muffled, relatively weak explosion followed by a whining or roaring noise not unlike a jet engine which would continue for one to three minutes. In the field it was referred to as 'the jet'. Lava would be ejected at the very inception of activity. After 15 - 20 seconds molten material appearred only intermittently, the bulk of the fluid being 'erupted' was gaseous. The gas appearred blue near the mouth of the spatter cone.

Vent 5. This vent was approximately circular, forming a crater lying on the eastern inside flank of the summit crater. Characteristically it would explode in a multiple pulsed manner, ejecting molten material feebly in a ENE direction. The first pulse announcing the eruption was typically the strongest. Usually an eruption at Vent 5 was followed immediately by an eruption at Vent 4. Upon initiation of activity at Vent 4 the activity at Vent 5 would decay rapidly.

2. DATA ACQUISITION

The Experiment

A chronologic record of eruptions within the summit crater of Stromboli was compiled during September 5 - 7, 1971. Actual monitoring occurred over three discontinuous periods of observation totalling approximately 45 hours. A time log was kept using an ordinary man's wrist watch with a sweep second hand. Observations were made from Station B on a ridge overlooking the entire summit crater, Station C on the western rim of the summit crater, and Station F on the eastern rim of the summit crater.

The position of a station generally biased the quality of observations of the individual vents. Station C permitted detailed monitoring of Vent 1 while Station F provided better 'seeing conditions' of the activity at Vents 4 and 5. Copious emanations of steam, sulphur dioxide and other fumarolic gases would occassionally obscure both the visual and acoustic effects of an eruption occurring at the opposite end of the summit crater. This was particularly true with regard to observing Vents 4 and 5 while at Station B. As a result of such clouding conditions the activity at Vent 2 was primarily recorded on the basis of the acoustic report of a given eruption. It should also be pointed out that ovservation periods are preferentially oriented about night time hours when sighting conditions were optimal.

Real time variation in the summit activity was commented upon by several members of the field team. Specifically, a consistent increase in eruption frequency at Vent 2 and an increase in the eruption duration at Vent 3 were remarked upon while in the field.

A subjective estimation of error bounds in clocking a specific time is +2 seconds. Thus for clocking the beginning and end of an eruption the error bound increases to +4 seconds. However, I noticed that some sort of learning process tended to attenuate the error in recording the time at the end of an eruption. The eruption style of each active vent was distinct enough to permit anticipation of the conclusion of an eruption fairly accurately (± 1 second).

The major consequence of the above error estimation is to undermine confidence in the eruption durations reported for Vent 1 which are typically on the order of 5 seconds. Error in other measurements should cancel over the large number of observations made.

Detailed observations at Station F near Vents 4 and 5 were limited to the last period of recording. From a distance only eruptions at Vent 4 were detectable due to the long duration of the audio signal accompanying the actual eruption. Observations at Station F, however, were complicated by a different problem. There existed a marked difficulty in distinguishing between background 'sloshing' activity at Vent 5 and an actual 'eruption' when weak transient behaviour failed to be followed by an eruption at Vent 4. For these reasons Vents 4 and 5 are combined in the analysis presented here. I consider their combined activity as the manifestation of a single 'event' driven by conditions at some shallow depth. In other words, it is assumed that Vent 5 never erupts unaccompanied by Vent 4. In this study Vents 4 and 5 were combined and given the designation Vent 3, replacing the field designation of Vent 3 which was a relatively large fumarolic crater. This change was made to facilitate computation.

A summary of the observational data for each observation period is given in Table 2-1.

TABLE 2-1 - SUMMARY OF OBSERVATIONAL DATA*

Observation Period	Vent 1	Vent 2	Vent 3
Period 1 Station B 9/5/71 Duration: 12.28 hr.	N = 70 $\overline{E}$ = 5.71 $\overline{D}$ = 4.80	N = 5 $\overline{E}$ = 0.41 $\overline{D}$ = 1.0	N = 36 $\overline{E}$ = 2.94 $\overline{D}$ = 29.6
Period 2 Station C 9/6/71 - 9/7/71 Duration: 19.32 hr.	N = 106 $\overline{E}$ = 5.48 $\overline{D}$ = 5.09	N = 17 $\overline{E}$ = 0.88 $\overline{D}$ = 1.0	N = 59 $\overline{E}$ = 3.06 $\overline{D}$ = 201.0
Period 3 Station F 9/7/71 - 9/8/71 Duration: 11.87 hr.	N = 58 $\overline{E}$ = 4.89 $\overline{D}$ = 4.07	N = 15 $\overline{E}$ = 1.27 $\overline{D}$ = 1.0	N = 34 $\overline{E}$ = 2.87 $\overline{D}$ = 315.0
Average for all periods combined	$\overline{E}$ = 5.57	$\overline{E}$ = 1.11	$\overline{E}$ = 3.98

*Legend

N - number of eruptions observed

$\overline{E}$ - average eruption rate (eruptions/hr)

$\overline{D}$ - average eruption duration (second/eruption)

3. PREVIOUS WORK

Earlier statistical analyses of volcanic repose periods have differed substantially in scope and purpose from the present study. Previous work has focussed upon repose periods on the order of 10^0 - 10^2 years which characterize intervals of dormancy between catastrophic re-awakenings of major eruptive centers. In this study repose periods are on the order of 10^3 seconds (i.e. 10^{-4} years). Furthermore, they characterize a recurrent geyser-like process whose nature may be fundamentally distinct from the volcanic mechanism which releases pressures of the magnitude witnessed in Krakatoan or Vesuvian types of eruptions.

The pioneering work of Wickman (1963, 1965a, 1965b, 1965c, 1965d, 1965e) in the last decade has essentially laid the groundwork for all other studies. His approach involves the manipulation of three straightforward functions which are defined by the distribution of repose periods recorded for a specific volcano. It will be useful to review these functions.

The most common representation of this sort of empirical data is a histogram plot or frequency distribution. The frequency distribution is simply the number of eruptions observed which were preceded by repose periods of length t, for values of $t > 0$. Normalizing the frequency observed by the total number of observations yields an estimate of the probability of a present period of repose persisting for some time interval t. This group of probabilities for periods of different lengths is termed the probability density function (PDF) and is represented here as f(t). The cumulative probability distribution, F(t), is simply the integral of the PDF

$$F(t) = \int_0^t f(t)\ dt \qquad (3.1)$$

The probability of an eruption within some time interval t following the last eruption can be determined directly from the cumulative frequency distribution F(t). The maximum likelihood of an eruption at a time t following the last eruption will occur at the time coinciding with the peak (if any) in the PDT, f(t).

An eruption rate function $\phi(t)$ can be defined as the limit of the ratio of the probability of an eruption in the time interval h to that time interval, as h becomes very small:

$$\phi(t) = \lim_{h \to 0} \left| \frac{(f(t+h) - f(t))}{h} \right| \quad 3.2$$

This can be shown to be equivalent to the following (see Cox, 1962).

$$\phi(t) = \frac{f(t)}{1 - F(t)} \quad 3.3$$

It should be apparent that the functions f(t), F(t), and $\phi(t)$ are not independent. Formulation of f(t) on the basis of empirical observations completely determines the form of F(t) and $\phi(t)$.

Essentially Wickman's analysis consists of 1) re-expressing the empirical frequency distribution in the form of a logarithmic survivor function, $\log_e S(t)$, in order to 2) calculate the eruption rate function $\phi(t)$. It is easily demonstrated that

$$\phi(t) = - \frac{d}{dx} (\log_e S(t)) \quad 3.4$$

The survivor function S(t) represents the number of observed repose periods of length greater than time t. For example, the value of the survivor function for the median intereruption period in a given sample of observations would be half the total number of repose periods observed (N_{total}). Because the empirical frequency distribution (such that $S(t) = N_{total} - F(t)$) will also be rather 'lumpy'. In calculating $\phi(t)$ from $\log_e S(t)$, Wickman fits a smooth curve to $\log_e S(t)$ by a least squares method.

Knowledge of $\phi(t)$ provides insight into the relative changes in the activity of the volcano over long periods of time. It can be used to detect 'acceleration' or 'deccaleration' in eruption occurrence. Determination of no trend in $\phi(t)$, or a constant rate, is indicative of a simple Poisson process in which the time to the next eruption is completely independent of all preceding repose intervals.

Wickman applied this formal analysis on a regional scale to volcanoes in the East Indian Ocean and Japan and concluded that eruption rates differed too greatly among neighboring volcanoes within individual regions to permit any inference of regional contrasts. In a general analysis of a variety of volcanoes he concluded that historical activity recorded for Etna in Italy, Mauna Loa in Hawaii, and Popocatepétl in Mexico were in reasonable accordance with a simple Poisson model.

Problems in this technique arise mainly in the quantity and quality of data. There has been little consensus over the last five hundred years as to what constitutes a major eruption. Clearly Vesuvian-type eruptions which take life and destroy property will leave a signature on historical records. Other phenomena such as fumarolic activity and severe ground motions commonly occur in the vicinity of large volcanic piles. Variations in the intensity of these phenomena fail to necessarily reflect a major change in eruptive activity for a particular volcano. The modern definition of 'an eruption' is still far from being a uniform concept. In fact, recent documentation of volcanic activity has significantly increased with the creation of the Smithsonian Institution Center for Short-Lived Phenomena. This presents a problem in combining recent and historical records.

Problems in the data will effect the curve-fitting routine which produces an exact expression for the logarithmic survivor function prior to the calculation of $\phi(t)$. Presence of a trend in the eruption rate will clearly be sensitive to the fit for the 'tail' of the log S(t) distribution. This tail represents the number of observations of long repose periods. Typically these longer repose periods represent

Historically older data when only catastrophic activity was recorded. This implies an undesirable tendency for the results to depend strongly upon the least reliable repose periods in the empirical data.

Wickman appears to be the only previous investigator who has attempted to relate the results of a statistical analysis of repose periods to physical processes producing the observed volcanic activity. He has proposed that the subsurface movement of magma can be modelled as a pumping phenomena in which material is transported from a primary magma chamber at depth to a secondary near-surface chamber (1965e). In applying this model to volcanoes with typical repose periods on the order of 10^0 - 10^1 years, he hypothesized an eruption rate for each reservoir and a 'death rate' for the near-surface chamber which reflected emptying and/or cooling. On a smaller time scale, similar to the observations analyzed here, Wickman has proposed a different model for recurrent volcanic activity (1963). His data for this second model is taken from a record of the April, 1960 eruption of Northeast Crater, Mount Etna. This eruption was Strombolian in character, with repose periods in the range of 0 - 200 seconds. In this case Wickman hypothesizes another multi-staged process consisting of 1) an ejection stage of actual eruption, 2) a mixing stage during which lava fragments from the preceding eruption falling back into the vent are re-melted and 3) essentially a waiting stage during which the lava column is capable of erupting. The observable repose periods represent the simple addition of the length of time the process spends in stages 2 and 3.

Clearly, the two stochastic models describe phenomena which are operating at significantly different time scales. Though both are based on ad hoc assumptions concerning the nature of the volcanic process, each provides an interesting and convenient framework in which the results of the statistical studies can be viewed. Unfortunately, there exists no means of substantiating these specific points of view.

A somewhat different approach to the same type of volcanologic data has been taken by Reyment (1969). He purposely selects volcanoes which have a relatively well documented eruption chronolgy and then scans the resulting sets of repose periods for trends in the empirically observed rate of eruption, dependence within individual series of repose periods, and correspondence with Poisson-like probability densities. Initially Reyment generates a series of repose periods by simulating a series of eruptions at a 'perfectly Poisson' volcano. This simply means that the resulting density distribution is described by the Poisson PDF

$$f(t) = \frac{\nu^t e^{-\nu}}{t!} \qquad 3.5$$

where μ is a parameter characterizing the distribution. A special property of a Poisson density is that μ equals both the mean and the standard deviation of the distribution. The simulated set of repose periods is subjected to a comprehensive group of statistical tests alongside empirical repose series as a 'control case' for comparison. Reyment's results for Japanese and Indonesian volcanoes are consistent with those of Wickman.

Clearly Reyment's analysis reflects a higher degree of statistical sophistication than Wickman's earlier work. However, the results remain qualitatively the same: Reyment is principally concerned with characterizing the process underlying the empirical distribution as Poisson or non-Poisson.

To the author's knowledge, the first statistical study specifically directed towards predicting future volcanic activity has been presented by Thorlaksson (1968). The object of his study was to forecast the probability of a major eruption at the Katla and Hekla volcanoes in Iceland. This requires a knowledge of the functions f(t) and F(t). The PDF can be constructed in a straightforward manner from the empirical frequency distribution or some smooth curve fit to this distribution. Thorlaksson, however, developed a different approach. He hypothesized a relationship between the ratio of the mean, m, and the standard deviation, s, of the PDF and the eruption rate function $\phi(t)$. As pointed out previously, formulation of the PDF defines the form of the eruption rate function $\phi(t)$. Thorlaksson reasoned that the mean and standard deviation of the PDF were sensitive constraints on the shape of the PDF. Therefore, he assummed the ratio s/m of the PDF would be an accurate indicator of the form of the eruption rate function. Specification of $\phi(t)$ will then explicitly define the form of the PDF. Constants in the resulting f(t) are, in turn, determined by m and s. The categorization procedure he developed is summarized in Figure 3-1.

In support of this technique Thorlaksson must adopt certain ad hoc relationships between the s/m ratio and the nature of the eruption rate function. To proceed from these assumptions to infer the original PDF is a highly convoluted procedure. Though the proposed relationships are not drastically counterintuitive, they also are not empirically verifiable. In this method the exact nature of the PDF is not examined on the basis of the empirical frequency distribution, rather it is inferred as a function of m and s. These two parameters are not always

FIGURE 3-1 THORLAKSSON'S CATEGORIZATION PROCEDURE

Category	$\frac{s}{m}$	$\phi(t)$	f(t), PDF	Additional Equations
I	$\frac{s}{m} > 1$	$\frac{a}{1 + bt}$	$a(1 + bt)^{-(a/b+1)}$	$a = \frac{2s^2}{m(m^2+s^2)}$ $b = \frac{s - m}{m(m + s)}$
II	$\frac{s}{m} = 1$	a	$a \exp(-at)$	$a = \frac{1}{m}$
III	$0.5 < \frac{s}{m} < 1$	$a + bt$	$(a + bt) \exp(-(at + \frac{b}{2} t^2))$	a and b found by iteration
IV	$\frac{s}{m} = 0.5$	bt	$bt \exp(- \frac{b}{2} t^2)$	$b = \frac{\pi}{2m}$
V	$\frac{s}{m} < 0.5$	$b(t - a)$ for $t \geq a$	$b(t - a) \exp(- \frac{b}{2} (t - a)^2)$	$a = m - \frac{s}{0.5}$ representing a 'loading time' $b = \frac{4 - \pi}{2s}$

representative of the data from which they are computed. For example, if the underlying distribution were bimodal the s/m estimator of $\phi(t)$ and thus f(t) would be seriously in error.

The only apparent advantage of Thorlaksson's method would be in formulating a PDF for a small sample of repose periods where the accuracy of a fitted curve to the raw data would be poor while s and m may be known with a greater degree of confidence. However, he fails to demonstrate the magnitude of such an effect to various sample sizes and underlying distributions. As was the case with Wickman, the prime constraint in manipulating f(t) and $\phi(t)$ is the amount of data available for specific volcanoes. Forexample, in applying the above anslysis to Hekla in Iceland, Thorlaksson employs 13 repose periods to forecast a 7% probability of eruption between 1966 and 1976. In fact, a major explosive eruption occurred in May, 1970.

In an allied field Schlien and Toksoz (1970) have examined the distribution of times between earthquakes. Such periods are termed recurrence times. In their work the recognition of the occurrence of a family of aftershocks associated with single major events suggests generalizing the simple Poisson representation of the recurrence times to include finite probabilities for more than one event occurring at the same instant of time. In other words, the generalized model admits the possibility of superposition of one series of aftershocks on some part of a previous or subsequent series. This more generalized Poisson formulation provides a significantly better fit to the empirical distribution.

II. ANALYSIS

4. CHARACTERIZATION OF THE EMPIRICAL DATA

The sequence of repose periods recorded during the three observation periods are combined in frequency histograms in Figures 4-1, 4-2, and 4-3. Aggregation of the data is justified due to the great length of the individual observation periods relative to the repose periods at the individual vents.

In the field the two distribution parameters of immediate interest were the mean and standard deviation of the sampled intereruption periods. Both parameters can be maintained by simple calculation while actually in the field. The mean represents a zero-order estimate of the current repose period at a specific vent. The ratio of the mean to the standard deviation can be roughly regarded as a measure of signal to noise in the data and thus a measure of the confidence to be placed in employing the sample distribution's average value as an estimator of the time to the next eruption.

Additional intuition can be gained by carefully examing the repose period distributions. In fact, experience has shown that mere knowledge of a sample's mean and standard deviation can disguise the true nature of the distribution for certain special cases such as an underlying bimodal population (Rinehart, 1969). Therefore the higher order moments of the individual distributions were calculated and are summarized in Table 4-1. These parameters are commonly compared with typical values for the summetric 'bell-shaped' curve of the normal or Gaussian distribution in order to quantitatively assess the shape of

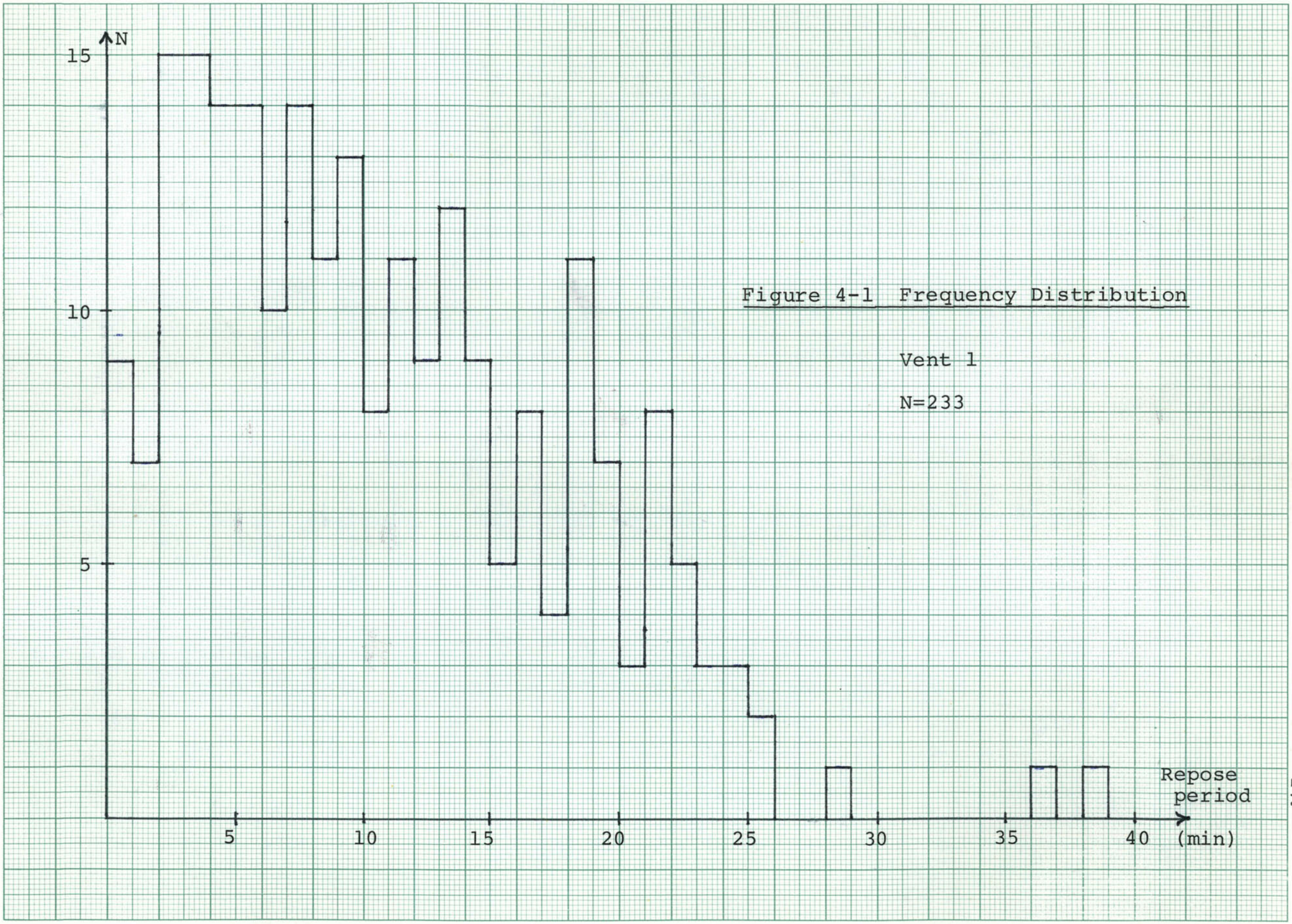


Figure 4-1 Frequency Distribution

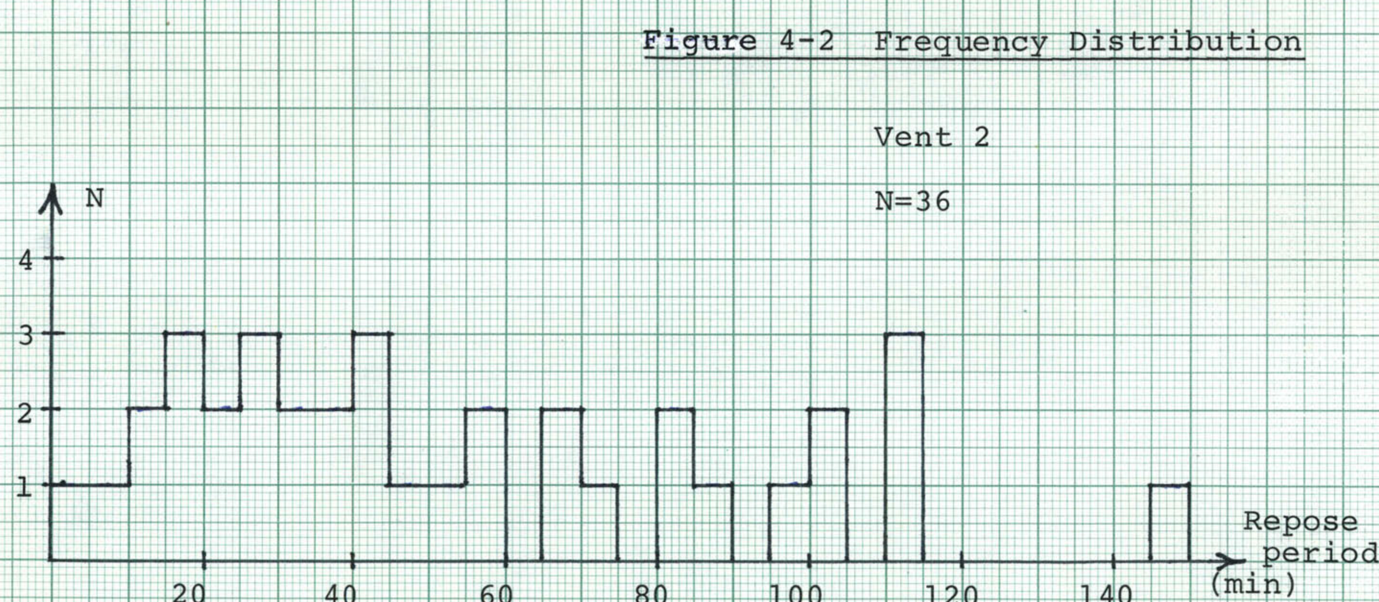


Figure 4-2 Frequency Distribution
Vent 2
N=36
N
4
3
2
1
20
40
60
80
100
120
140
Repose period (min)

25.

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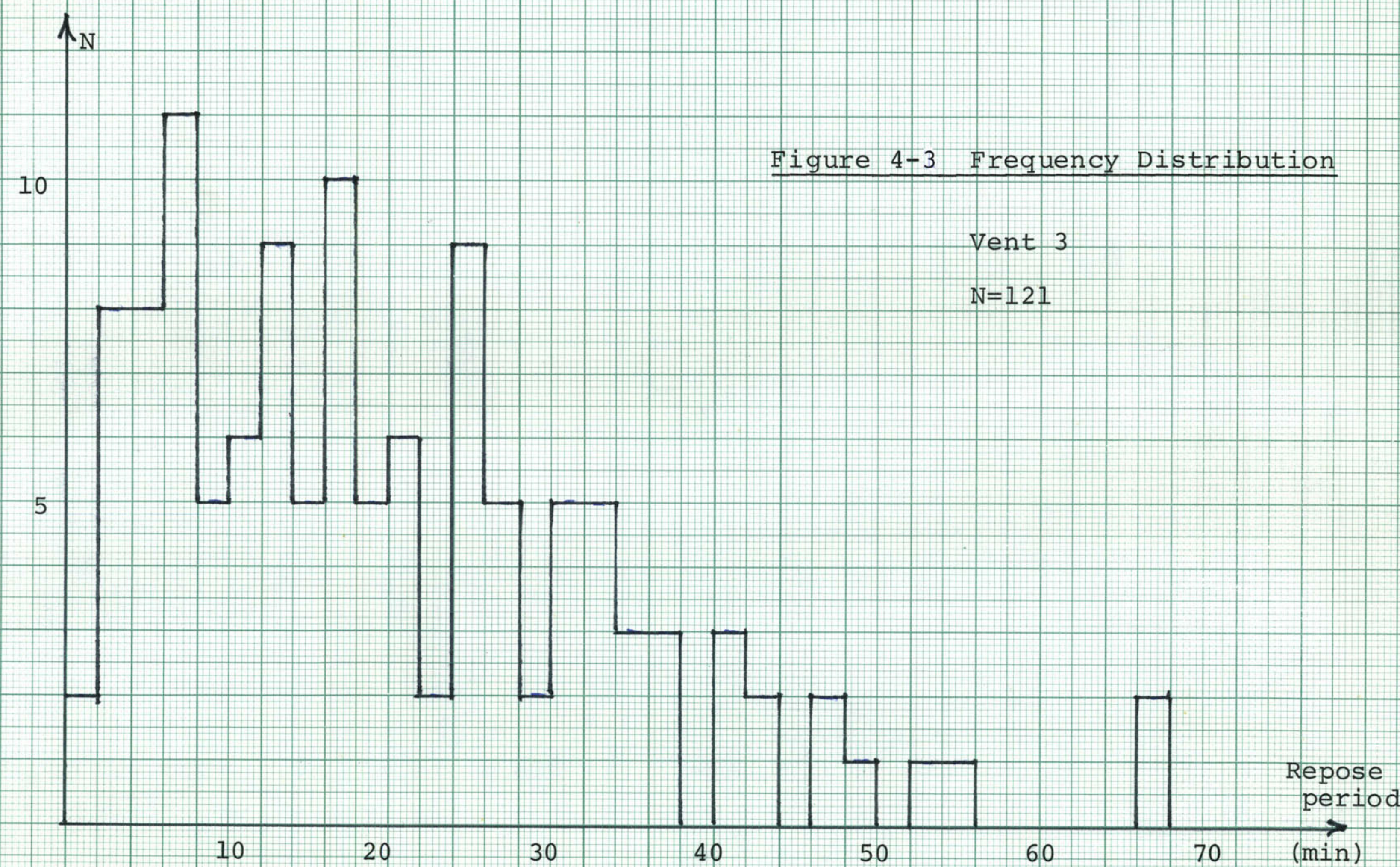

Figure 4-3 Frequency Distribution

TABLE 4-1 FREQUENCY DISTRIBUTION SHAPE COEFFICIENTS

Frequency distribution at	N	m_1 Avr. Repose Period (sec.)	$\sim m_2^{1/2}$ Standard Deviation (sec.)	a_3 Coefficient of skewness	a_4 Coefficient of kurtosis
Vent 1	233	645.15	427.56	1.46	2.48
Vent 2	36	3240.11	2179.38	1.45	2.35
Vent 3	121	1204.39	837.44	2.26	2.78
'Perfect' Gaussian				0.0	3.0

the sample distribution. The k-th moment of the data sample, m_k, is defined as

$$m_k = \frac{1}{N} \sum_{i=1}^{N} (X_i - \overline{X})^k \qquad 4.1$$

where

N represents the number of samples or data points

X_i represents the value of the i-th sample

$\overline{X}$ represents the mean value of the sample of X's

To permit a direct comparison of the sample distribution with a typically Gaussian distribution, the moment is normalized by the factor $1/(m_2^{k/2})$. This results in a dimensionless coefficient

$$a_k = \frac{m_k}{(m_2)^{k/2}} \qquad 4.2$$

which can be employed in contrasting moments of order three and greater.

The quantity a_3 represnts a coefficient of skewness which gauges bias in the sample distribution towards values greater or less than the distribution mean. The quantity a_4 measures the degree of peakedness in the distribution, or its kurtosis. Alternatively this statistic can be conceptually considered a measure of the distribution's peakedness relative to the size and length of the distribution 'tails' (i.e. extremities). This is a more sensitive gauge of the degree to which the sample mean represents the entire sample distribution than simply determining the ratio of the mean to the standard deviation.

Positive values of the skewness coefficient indicate a bias in the sample distribution for values greater than the mean. Equivalently the skewness coefficient discriminates between the longer 'tail' of the distribution. A positive a_3 simply indicates that the distribution has a longer tail to the right of the sample mean (i.e. towards higher values). The frequency distribution at each vent is positively skewed. In contrast, a Gaussian distribution is completely unskewed with $a_3 = 0.0$. The frequency distribution at Vent 3 exhibited the strongest bias towards repose periods greater than the sample mean.

Values of a_4, the kurtosis coefficient, greater than 3.0 are characteristic of distributions with sharp, prominent peaks. Such distributions are termed leptokurtic. Values of a_4 less than 3.0 describe platykurtic distributions which are relatively flat-topped, or alternatively marked by no strong peak. The normal distribution which is not dominated by an excessive peak nor by a flat top is referred to as mesokurtic (i.e. $a_4 = 3.0$). The frequency distribution at each vent is platykurtic. The sample distribution at Vent 2 demonstrated the strongest tendency to be flat-topped while the distribution at Vent 3 approaches a Gaussian peakedness more closely than the other two empirical distributions.

<u>Chi-squared test</u>

Another standard technique that is helpful is interpreting the sample distributions is to test the conformity of the sample distributions with common distributions such as the Gaussian and the Poisson whose properties are well known. A successful fit of the sample with the general form of a known distribution permits the investigator to draw some general inference regarding the nature of the underlying process.

The standard distributions which are relevant to this study are the normal or Gaussian, the Poisson, and the Gamma density distributions.

The mathematical representations of these distribution forms is summarized in Table 4-2 (see Krumbein and Graybill, 1965, Benjamin and Cornell, 1970, or Remington and Shork, 1970). The method of employing a test statistic in determining goodness-of-fit is reviewed in Appendix A. The statistical test employed here is the common chi-squared test.

The chi-squared statistic is a measure of the departure of the observed data in lumped histogram form from the hypothesized distribution. It is given by the expression

$$\psi^2 = \sum_{i=0}^{R-1} \frac{(n_i - Np_i)^2}{Np_i} \qquad 4.3$$

where

- R represents the total number of intervals employed in the test
- n_i represents the number of 'observed points' in interval i
- N represents the total number of data points
- p_i represents the average probability of an event in the i-th interval predicted by the hypothesized distribution
- Np_i represents the expected number of events in the i-th interval based on the hypothetical distribution

The test statistic is distributed with R-1-d degrees of freedom where d equals the number of parameters estimated from the sample in formulating the hypothesized density function. For example, the mean and variance are estimated in representing the normal distribution, thus d equals 2.

Remington and Shork (1970) cite work done by Cochran (1954) on potential instabilities in applying the chi-squared goodness-of-fit test to a frequency distribution. A major problem is encountered in using the statistic for intervals where the expected number of occurrences (Np_i), which occurs in the denominator of the summing expression,

TABLE 4-2 COMMON PROBABILITY DENSITY FUNCTIONS

Gaussian	$p(x) = \frac{1}{s\sqrt{2\pi}} \exp(-(x - m)^2/2s^2)$	where m = mean of the sample distribution s = standard deviation
Poisson	$p(x) = \frac{\nu^x e^{-\nu}}{x!}$	$\nu = m$
Gamma r = 0	$p(t) = \lambda e^{-\lambda t}$	$\lambda = \frac{1}{m}$
Gamma r = 1	$p(t) = \lambda^2 te^{-\lambda t}$	$\lambda = \frac{1}{m}$

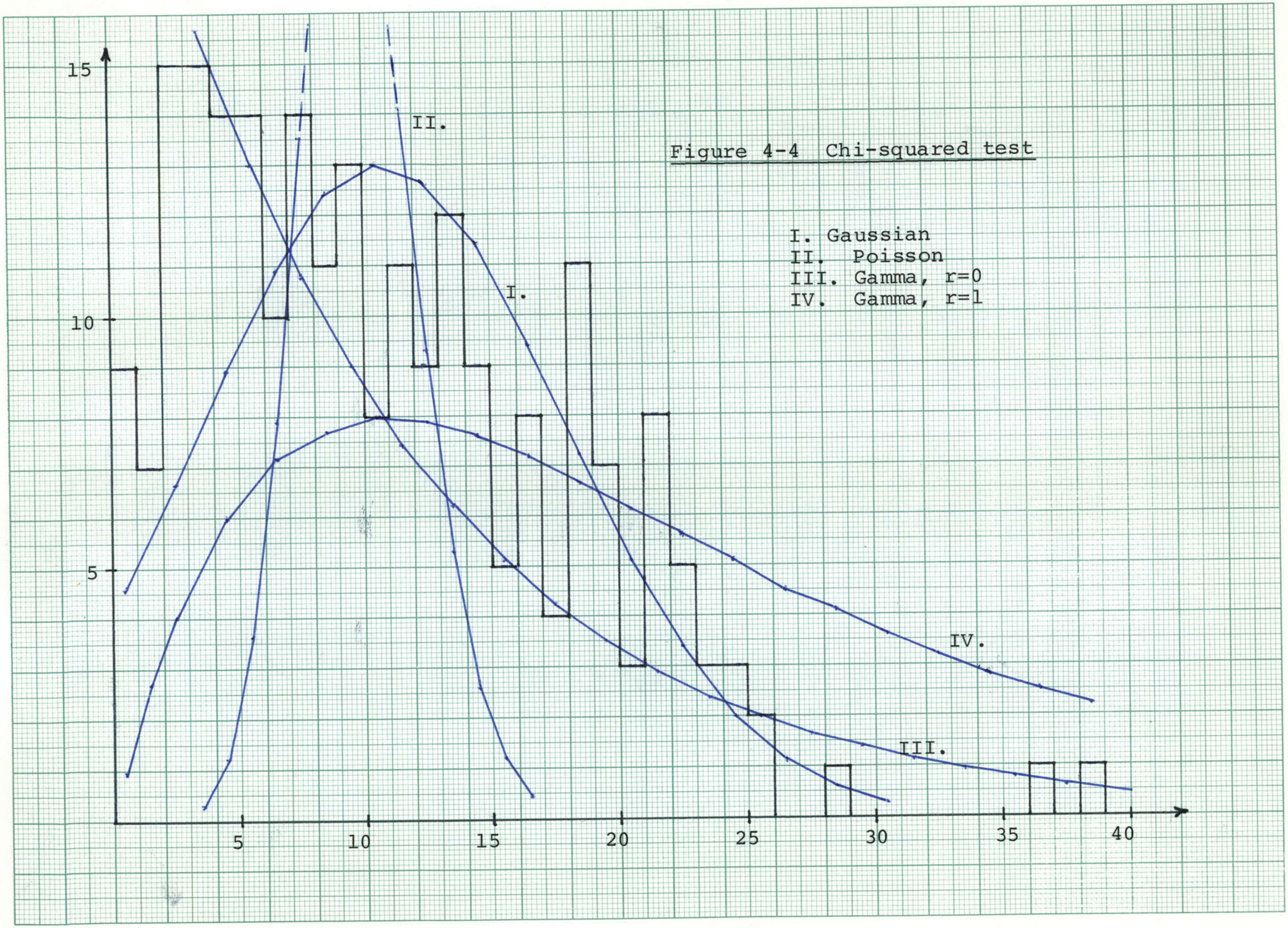

Figure 4-4 Chi-squared test

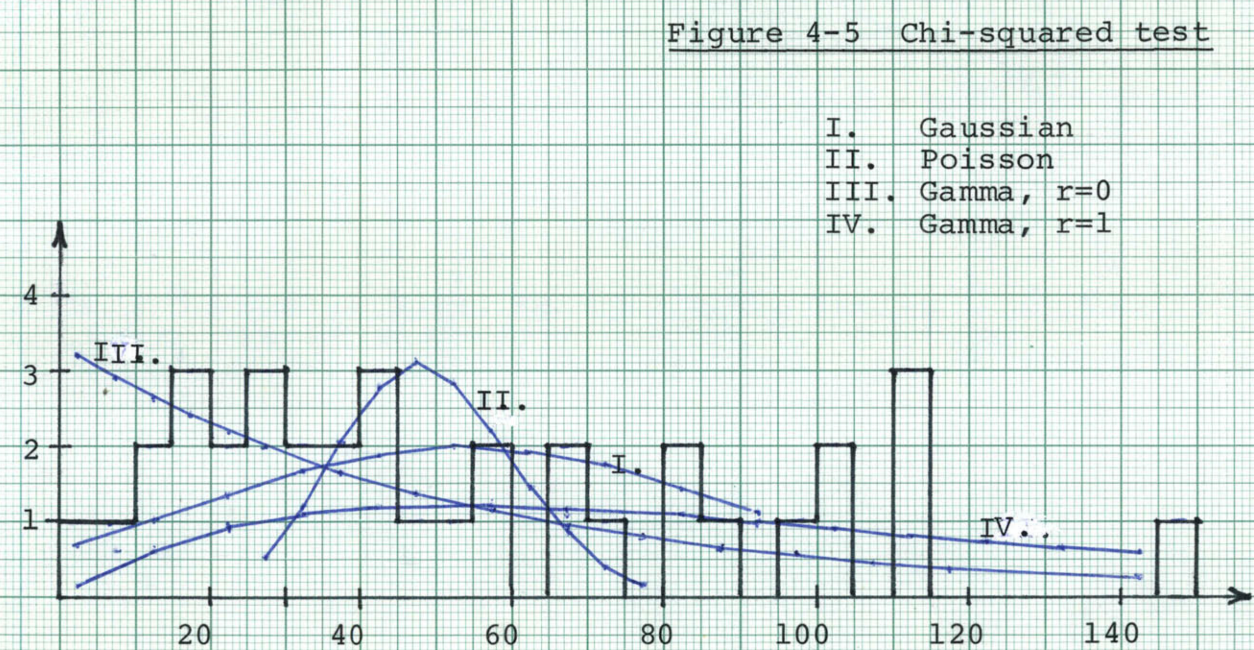
Figure 4-5 Chi-squared test
I. Gaussian
II. Poisson
III. Gamma, r=0
IV. Gamma, r=1
4
3
2
1
20
40
60
80
100
120
140
III.
II.
I.
IV.

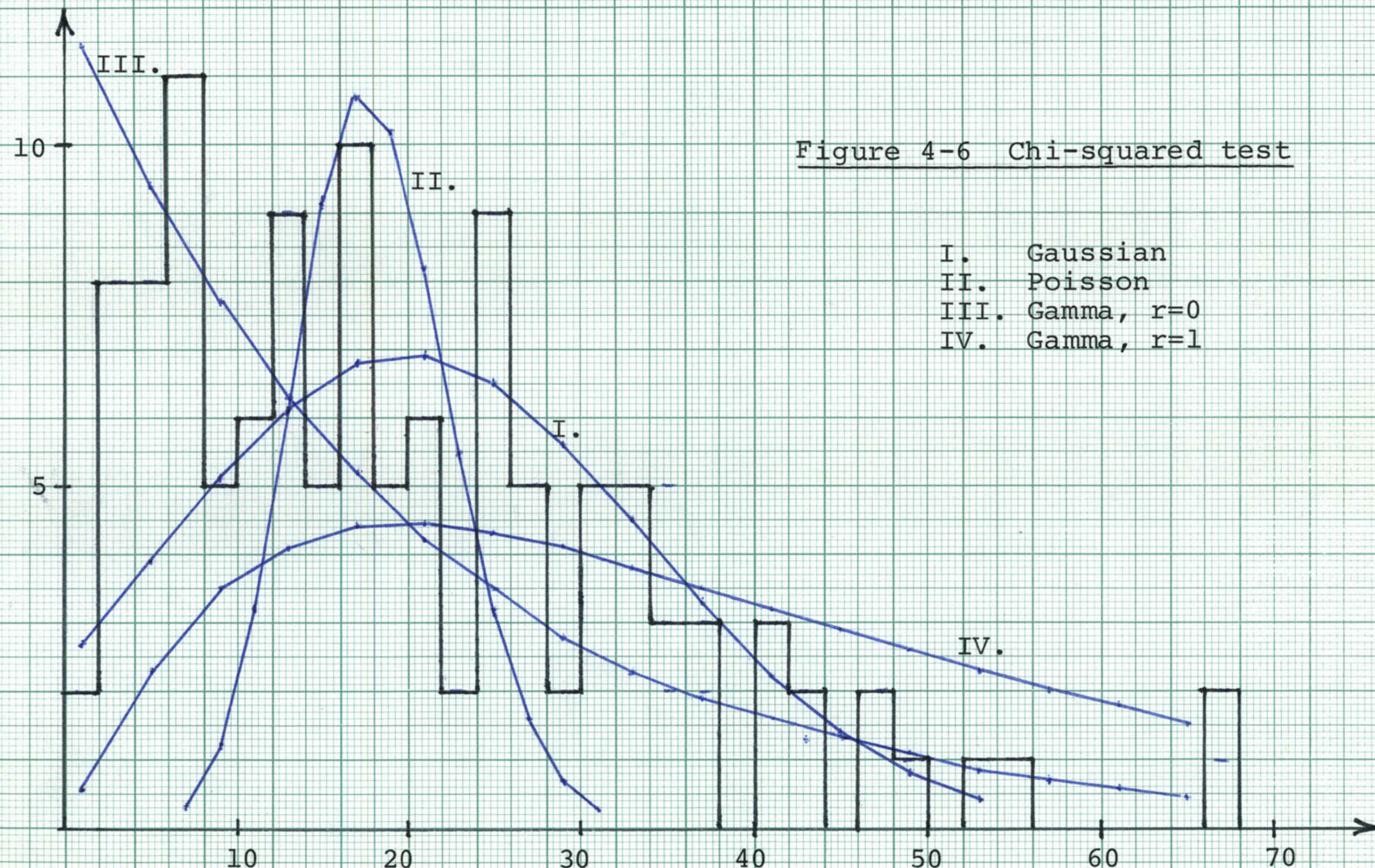

Figure 4-6 Chi-squared test

becomes less than one. Individual terms for which this is the case become very large. Cochran suggests, as a rule of thumb, that only intervals in which Np_i is greater than 1.0 be employed in the test. This restriction was adopted in the test performed here.

Results

A tabulation of test statistics appears in Table 4-3. The results are reported at the 99.9% confidence level. Values of the statistic for which the hypothesized form of the frequency distribution cannot be rejected are indicated by X's.

Vent 1. Application of the chi-squared test to the repose period distribution at Vent 1 results in a rejection of the Poisson hypothesis for the nature of the sample density. Neither the hypothesis of a Gaussian or Gamma density (with r=0) characterization can be rejected.

Vent 2. The amount of data available for Vent 2 is too meager to permit the chi-squared test to discrimate between a Poisson, Gaussian, or Gamma representation of the sample distribution. None of these hypothesized forms can be rejected.

Vent 3. As at Vent 1, application of the chi-squared test to the repose period data for Vent 3 leads to a rejection of a Poisson form for the empirical frequency distribution. The test fails to discriminate between a Gaussian or Gamma (with r=0) characterization. Neither of these hypothesized forms can be rejected. This is the same ambiguity which existed at Vent 1.

TABLE 4-3 RESULTS OF CHI-SQUARED TEST

Vent	Distribution	ψ^2	(DOF)	$\psi^2/(DOF)$	Acceptance region at 99% confidence level with (DOF)	Cannot be rejected
1	Gaussian	45.0	(24)	1.87	0.310 - 2.23 (24)	X
	Poisson	210.54	(11)	19.1	0.144 - 3.01 (11)	X
	Gamma r = 0	66.25	(31)	2.14	0.360 - 2.07 (30)	X?
	Gamma r = 1	221.60	(47)	4.72	0.469 - 1.79 (50)	
2	Gaussian	12.5	(11)	1.14	0.144 - 3.01 (11)	X
	Poisson	6.23	(5)	1.24	0.032 -4.42 (5)	X
	Gamma r = 0	6.93	(11)	0.63	0.144 - 3.01 (11)	X
	Gamma r = 1	28.70	(16)	1.79	0.221 - 2.58 (16)	X
3	Gaussian	37.44	(23)	1.63	0.291 - 2.30 (22)	X
	Poisson	42.08	(9)	4.67	0.108 - 3.30 (0)	
	Gamma r = 0	39.47	(23)	1.72	0.310 - 2.23 (24)	X
	Gamma r = 1	115.34	(36)	3.21	0.394 - 1.98 (35)	

Implications

The conclusions which can be drawn from the chi-squared test are tenuous at best. It is interesting to note that the ambiguity in the test results for the empirical sample of repose periods observed at Vent 1 is identical to the ambiguity in the outcome of the test when applied to the data for Vent 3. In both cases there exists an inability to discriminate between a Gaussian or Gamma density (with r=0) representation of the observed frequency distribution. At Vent 2 insufficient data renders the test meaningless.

The properties of the Gaussian distribution are well known. It is a summetrical distribution which implies no bias towards values greater or less than the distribution mean, m. The standard, s, of the Gaussian distribution can be used to characterize the nature of the grouping which occurs within the distribution: 68.2% of recorded values can be expected to fall in the region $m \pm s$, 95.5% fall in the region $m \pm 2s$. The mode (value occurring the most frequently), the median (value at the 50-th percentile of the cumulative distribution), and the mean are identical in the normal population density. A measure of the skewness and kurtosis of the empirical distributions at Vents 1 and 3 demonstrate a lack of conformity with the symmetry present in a true Gaussian or normal curve. The chi-squared test suggests that a Gaussian representation may not be a bad approximation of the true distribution.

The Gamma density has not been extensively used in geology. However, when the free parameter, r, for this distribution is set equal to zero, the density function simply reduces to an exponential distribution.

Benjamin and Cornell (1970) have shown that the distribution of interarrival times in a Poisson process are distributed exponentially. (In this study, eruptions would represent events in a Poisson process while intereruption periods would correspond to interarrival times.) It has been pointed out that previous studies of volcanic repose periods have uniformly identified the timing of eruptions at a variety of volcanoes as Poisson-like processes. Although the frequency distributions at Vents 1 and 3 cannot be fit by a Poisson density, the hypothesis that a random Poisson process is responsible for the observed volcanic activity cannot be rejected because of the conformity of the repose period distributions with an exponential distribution. What intermediating factors may be modifying the process such that the empirical distribution mean is not equal to the standard deviation at these two vents are not known. (Recall that in the Poisson density function the parameter $\nu = m = S$.)

All of the hypothesized distributions employed here in the chi-squared test represent random processes. A feature common to all these densities is the independence of successive values of the measured variable, which in this case is the length of an individual repose period. This means that the sequence of past repose periods at a specific vent is not related to the length of the current intereruption period. This implies that the best estimate of a current repose period is the average repose period observed at that vent. Therefore, though the properties of the empirical densities have been determined in detail, the basic ability to predict a current repose period has not been significantly increased over the capability which existed in the field.

5. RELATIONSHIPS BETWEEN VENTS: MARKOV PROCESS MODEL

The Model

Many natural phenomena can be characterized as multistage processes. A river, for example, could be modelled as a three state phonomenon which can be classified as being in either a 'low', 'normal', or 'flood' stage. For some phenomena the present state serves as a strong indicator of behaviour in the immediate future. In the case of the river, intuition suggests that a flood stage a time t would most likely be followed by a flood or normal stage at time t + dt, for small dt. One would rarely expect a river in flood stage to make a direct transition to a 'low' stage. In a sense, past events can be said to influence future events without actually exerting causal control. Such a process can generally be termed a Markov process (Harbaugh and Bonham-Carter, 1970).

More rigorously, a Markov process can be defined as one in which the probability of being in a given state at a given time can be estimated by knowing the immediately preceding state of states. The probability of the process passing from some state s_1 at time t_1 to state s_2 at time t_2 is termed a transition probability.

Characterization of the repose period distributions as being consistent with a random process implies that the current intereruption period is completely independent of all past periods at the individual vent. Invoking a Markov model for the sequence of 'activity stages' observed for the volcanic process at Stromboli extends the earlier consideration to include the influence that an eruption at one of the vents will have on the probability of eruption at some other vent.

An illustrative example of a Markov process in a geological context is the cyclothem phenomenon found in massive sedimentary deposits situated in the American Midwest (see Carr, 1966, Krumbein, 1968, 1967, Harbaugh and Bonham-Carter, 1970). In the cyclothem deposit a basic sequence of lithologies is repeated, with minor variations, thruout a unit with extensive lateral and vertical dimensions. A Markov model can be employed with great success in describing dependency relationships in such sequence of 'events', in this case lithologies. Specifically Krumbein (1967) has reported the results of strata identification within the Chester formation of the Illinois basin. Based on 309 equal thickness observations, he estimates a transition probability matrix for the three basic lithologies contributing to the formation: sandstone, shale, and limestone (see Table 5-1).

The observations are collected in a 'tally matrix' made up of i rows and j columns. The i, j-th element of the tally matrix represents the number of observed transitions from state i to state j. Calculation of the transition probability matrix simply involves dividing each row element in the tally matrix by its respective row sum. This insures that the transition to some other state from state i will equal 1.0.

In the above example the model assumes a system with discrete states (i.e. lithologies) which can be observed at discrete times (i.e. at regular sampling intervals). Note that the diagonal element is the largest element in each row of the transition probability matrix in Table 5-1. This insures that Krumbein's sampling interval is small enough to yield a meaningful matrix. A larger sampling interval would miss tha transition to thin strata altogether leading to a bogus set of probabilities. The regular sampling interval also permits the straightforward formulation of the transition matrix by conceptualizing the

TABLE 5-1 CYCLOTHEM EXAMPLE OF A MARKOV MODEL

Tally Matrix

		A	B	C	Row Totals
sandstone	A	58	18	2	78
shale	B	15	86	39	140
limestone	C	5	35	51	91
					309 = N

Transition Probability Matrix

		A	B	C
sandstone	A	0.74	0.23	0.03
shale	B	0.11	0.61	0.28
limestone	C	0.06	0.38	0.56

transition from one lithology to the next as occuring at 'discrete time intervals' which are best observed in this case as discrete thickness intervals. Krumbein does not want to imply that the sedimentation rate was constant over the long time period represented by the Chester Formation. Rather in characterizing the observed sequential depositional process in a 'discrete time' Markovian model he interpretes the regular sampling procedure as the "ticking of a conceptual Markovian clock".

In Table 5-1 the transition probability matrix represents the complete set of conditional probabilities describing the process being modelled. The rows of this matrix sum to 1.0. The matrix is square and is composed of non-negative elements that are not greater than 1.0. A matrix with these properties is termed a stochastic matrix. The actual probabilities are assumed not to vary as a function of time (i.e. sampling interval in the above example).

For the case of the summit activity at Stromboli, the volcanic process can be characterized by four states, namely eruption at any of the three active vents, and a repose state of no explosive activity. To include the activity at Vent 2 which erupts instantaneously, the sampling interval can be no greater than one second. A review of the summary of observational data (see Table 2-1) demonstrates that the average amount of time during which some vent was erupting varied between approximately three and ten per cent of the total period of observation for different observation periods. An attempt at characterizing the volcanic process as a discrete time Markov process with a one second sampling interval will yield a transition matrix which is dominated by the transition to the non-eruptive state from any other condition. In addition, the activity at Vent 2 will be negligible in such a model. (The probability of a transition from inactivity to eruption at Vent 2 would be on the order of 10^{-4}). At the same time, the choice of a larger time interval will discriminate against recognition of eruptive activity at Vent 2 and Vent 1.

Formulating a Markov model in this manner offers little insight into the eruption phenomenon. It is already well recognized that most of the time no eruption is taking place. A more powerful use of the Markov model is to examine the sequence in which the vents erupt. In this way the model becomes a means of probing for any consistent relationship between an eruption at a given vent (such as Vent 1), and the probability that the following eruption will occur at some specific vent (such as Vent 3).

The i x j tally matrix for a Markov model constructed in this fashion will record the number of times an eruption at Vent i is followed by an eruption at Vent j. In effect, each eruption represents a unit advance of the 'Markovian clock'. In a sense, the time scale used in the previous examples will be replaced by an 'eruption scale' on which individual eruptions are uniformly spaced at an arbitrary sampling interval of 1. This corresponds to uniform sampling intervals with the dimensions of thickness in the cyclothem problem and uniform sampling intervals in units of time for the river problem. Conditional probabilities in the resulting transition matrix will describe the influence of past behaviour on <u>where</u> the next eruption is likely to occur rather than <u>when</u> it probably will occur. The results for the three periods of observation are presented in Table 5-2.

TABLE 5-2 TRANSITION PROBABILITY MATRICES FOR OBSERVED ERUPTION SEQUENCES

Observation Period 1 N = 114

		A	B	C
Vent 1	A	0.549	0.070	0.380
Vent 2	B	0.667	0.000	0.333
Vent 3	C	0.756	0.027	0.216

Test statistic $-2 \log_e \lambda = 5.40$

Observation Period 2 N = 195

		A	B	C
Vent 1	A	0.504	0.103	0.393
Vent 2	B	0.667	0.056	0.278
Vent 3	C	0.767	0.083	0.150

Test statistic $-2 \log_e \lambda = 13.66$

Observation Period 3 N = 111

		A	B	C
Vent 1	A	0.373	0.186	0.441
Vent 2	B	0.938	0.000	0.063
Vent 3	C	0.639	0.111	0.250

Test statistic $-2 \log_e \lambda = 21.43$

Test for Markov Property

Not all transition probability matrices exhibit the Markov property. Consider the following game. Suppose six marbles are placed in a black box. A player draws a marble and returns it to the box. At the start of the game the box is filled with three red marbles, two white marbles, and one blue marble. A probability transition matrix describing the game probabilities is given as follows.

		A	B	C
red	A	1/2	1/3	1/6
white	B	1/2	1/3	1/6
blue	C	1/2	1/3	1/6

Clearly the outcome of the previous draw has no effect on the next draw once the marble chosen is returned to the box. The process has no 'memory'; it is not a Markov process.

There exists an explicit statistical test for the Markov property (see Harbaugh and Bonham, 1970). The null hypothesis tested is that the events considered are independent while the alternative hypothesis is that they are dependent. The test statistic λ is

$$\lambda = \prod_{i=1}^{m} \prod_{j=1}^{m} \left(\frac{p_j}{p_{ij}} \right)^{n_{ij}} \qquad 5.1$$

where

p_{ij} = the probability in cell i, j of the transition probability matrix

p_j = the marginal probability for the j-th column of the transition probability matrix $= \left\{ \left(\sum_{j=1}^{m} n_{ij} \right) / N \right\}$

n_{ij} = the transition frequency total of observed transitions from i to j recorded in the tally matrix

m = the total number of transitions observed

The quantity $- 2\log_e \lambda$ is known to be distributed in conformity with a chi-squared distribution with $(m - 1)^2$ degrees of freedom. A more convenient expression for the test quantity is

$$- 2 \log_e \lambda = 2 \sum_{i=1}^{m} \sum_{j=1}^{m} n_{ij} \log_e \left(\frac{p_{ij}}{p_j}\right) \qquad 5.2$$

The number of degrees of freedom in the case of 3 states is $(3 - 1)^2 = 4$.

The results of the test statistic computation are included in Table 5-2 for each observation period. At the 99.9% confidence level the null hypothesis of independence cannot be rejected for values of the test statistic in the range 0.064 - 20.00. This means that for observation periods 1 ($- 2 \log_e \lambda = 5.40$) and 2($-2 \log_e \lambda = 13.66$) the null hypothesis, that eruptions in the sequence are independent of the preceding event, cannot be rejected. Therefore Markov model of dependency is invalid. Curiously, the value of the statistic for observation period 3 ($- 2 \log_e \lambda = 21.43$) is consistent with the expectations of the Markov process.

It is not possible to pinpoint the cause of the unique conformity demonstrated by the eruption sequence recorded during observation period 3(OP3) with the Markov model on the basis of the data available here. It is only possible to conclude that the model is a sometimes valid description of Strombolian volcanic activity.

Test for Stationarity

Substantial variation in the value of the statistic employed in testing the hypothesis of independence of events in the eruption sequences is observed from one observation period to the next. This suggests that the conditional probabilities in the transition matrices may be varying on the 'eruption scale' adopted here. A basic assumption of the discrete Markov model is that the conditional probabilities remain constant. Failure of the OP3 eruption sequence to exhibit this property will invalidate the hypothesized Markov model.

By segmenting an eruption sequence it is possible to determine the degree of variation which exists between the component subdivisions and the whole sequence. A standard statistical test permits calculation of the significance of such variations (see Harbaugh and Bonham-Carter, 1970). The test applied only to observation period 3 on the basis of its consistency with the discrete Markov model.

The null hypothesis in this test is that the postulated Markov process is stationary; the alternate hypothesis is that it is unstationary. The test statistic λ' is

$$\lambda' = \prod_{t=1}^{T} \prod_{i=1}^{m} \prod_{j=1}^{m} \left| \frac{p_{ij}}{p_{ij}(t)} \right|^{n_{ij}(t)} \qquad 5.3$$

where

T = the number of subdivisions

$n_{ij}(t)$ = the frequency tally for transitions from state i to state j in the t-th subdivision

$p_{ij}(t)$ = the probability in cell i, j of the transition probability matrix for the t-th subdivision

The quantity $-2 \log_e \lambda'$ is distributed as chi-squared with (T-1) (m(m-1)) degrees of freedom. Re-expressed in computational form the test quantity is

$$-2 \log_e \lambda' = 2 \sum_{t=1}^{T} \sum_{i=1}^{m} \sum_{j=1}^{m} n_{ij}(t) \log_e \left| \frac{p_{ij}(t)}{p_{ij}} \right| \qquad 5.4$$

For 3 system states the number of degrees of freedom equals 6(T-1).

TABLE 5-3 STATIONARITY TEST ON OP3 ERUPTION SEQUENCE

Number in Subdivision	$-2\log_e\lambda'$	(DOF)*	$-2\log_e\lambda'/DOF$	Acceptance Region at 99.9% Confidence Level
10	56.69	(60)	0.946	0.506 - 1.71 (60)
15	26.56	(36)	0.737	0.394 - 1.98 (35)
20	20.28	(24)	0.845	0.310 - 2.23 (24)
25	11.31	(18)	0.630	0.247 - 2.47 (18)
30	15.13	(12)	1.261	0.161 - 2.90 (12)
35	12.94	(12)	1.079	0.161 - 2.90 (12)
40	11.13	(6)	1.858	0.050 - 4.02 (6)
45	8.77	(6)	1.444	0.050 - 4.02 (6)
50	6.79	(6)	1.130	0.050 - 4.02 (6)

* (DOF) ≡ degrees of freedom

The test was performed by initially breaking the sequence of events for OP3 into groups of 10 eruptions. A tally matrix and a transition probability matrix was then computed for each group. The statistic was calculated, and a new interval including five more eruptions was defined. The original sequence was then segmented into groups of 15 eruptions, transition matrices are calculated, the statistic is computed and so on. The results for each grouping is shown in Table 5-3.

The results indicate that the process is 'well behaved' during OP3. The null hypothesis of stationary cannot be rejected at the 99.9% confidence level for any subdivision of the observed eruption sequence ranging in length from 10 - 50 eruptions.

Test for Higher Order Dependence

The Markov model employed thus far in characterizing the sequence of eruptions documented for the three summit vents at Stromboli has only investigated the influence of the immediately preceding eruption on the probable location of the next event. Because the conditional probabilities depend only on a single preceding state, the model process is one of single-dependence and because the preceding state is the immediately preceding state the model process is termed first-order.

At this point it may prove useful to generalize the model inferred for OP3 to include double-dependence effects. This requires introducing a second variable in addition to the location of the immediately preceding event. The second variable will be permitted to range over various 'eruption intervals' into past behaviour. The number of intervals is referred to as the step length. For example, consider a sequence of e_1, e_2, e_3, e_4, e_5, ... A double-dependence, second-order model will estimate the transition probabilities to one of the system states s_i for the e_6 event on the basis of the states occupied during e_5 and e_4. Similarly a double-dependence, fourth-order model will estimate transition probabilities on the basis of the states occupied at e_5 and e_2. The order of the model reflects the size of the step length being used in sampling past behaviour.

By increasing the level of dependence to two prior states occupied by the system, the two dimensional square array representation of the transition probability matrix will expand to a three dimensional cubical array. The probability p_{ijk} indicates the probability of entering state k, given the two prior states of i and j somewhere in the past history of the phenomena.

To ascertain the significance of the double-dependence model versus the single dependence model, a new test statistic is introduced. To be able to compare the two directly, however, it is necessary to keep the j-th state employed in the double-dependence model fixed to the immediately preceding event as in the single dependence model. The null hypothesis to be tested is that there is no significant double-dependence memory effect versus the alternative hypothesis that a double dependence effect is present in the sequence.

The test statistic is

$$\lambda'' = \prod_{i=1}^{m} \prod_{j=1}^{m} \prod_{k=1}^{m} \left(\frac{p_{jk}}{p_{ijk}} \right)^{n_{ijk}} \qquad 5.5$$

where the quantity $-2 \log_e \lambda''$ is distributed as chi-squared with $m(m-1)^2$ degrees of freedom. In the present case the number of degrees of freedom equals $3(3-1)^2 = 12$. Again this test will only be applied to the eruption sequence recorded for observation period 3. The acceptance region at the 99.9% confidence level for a chi-square distributed variable with 12 DOF

TABLE 5-4 DOUBLE DEPENDENCE MARKOV MODEL FOR OP3 ERUPTION

Model order	$-2 \log_e \lambda''$	Model order	$-2 \log_e \lambda$
2	219.83	26	175.17
3	221.78	27	172.06
4	217.28	28	172.71
5	216.70	29	173.16
6	213.55	30	168.01
7	215.31	31	165.39
8	209.68	32	162.12
9	207.91	33	161.66
10	204.12	34	158.21
11	207.50	35	159.29
12	203.57	36	156.66
13	201.24	37	150.94
14	196.64	38	151.67
15	197.41	39	151.98
16	195.85	40	146.56
17	196.22	41	146.42
18	189.41	42	142.15
19	189.80	43	143.08
20	188.28	44	140.00
21	183.64	45	138.58
22	185.28	46	139.59
23	179.10	47	136.44
24	180.38	48	133.34
25	179.58	49	129.48
		50	128.07

TABLE 5-1 DOUBLE DEPENDENCE, SECOND ORDER MARKOV MODEL FOR ERUPTION SEQUENCE OBSERVED DURING OP3

Eruption Sequence
state i → state j → state k

k = 3

	j = 1	2	3
i = 1	0.429	0.091	0.308
2	0.467	0.000	0.000
3	0.435	0.000	0.111

k = 2

	j = 1	2	3
i = 1	0.190	0.000	0.077
2	0.067	0.000	0.000
3	0.261	0.000	0.222

k = 1

	j = 1	2	3
i = 1	0.381	0.909	0.615
2	0.467	0.000	1.000
3	0.304	1.000	0.667

is comprised of values of the test quantity V lying in the range 1.93 - 34.80 test quantity.

The results of the test for double-dependence are presented in Table 5-4. The numerical values show that the null hypothesis can be rejected at the 99.9% confidence level, and that a strong double dependence seems to exist in the empirical data. The general decrease in the value of the test quantity is due to the forced shortening of the data sequence with the investigation of higher order dependence. For example, in testing for tenth-order dependence the original data sequence can only be employed for events following e_{10} in the sequence e_1, e_2, 11111, e_n.

Although this test supports the hypothesis of double dependence, it also reveals an inability to discriminate between the higher orders of dependence. Keeping in mind the foreshortening of the input data sequence previously described, there appears to be no criterion for judging what sized step length yields the most accurate transition matrix. Thus, for the sake of simplicity and on the basis of the test result this study will assume a second-order, double dependence to be the most accurate. The second-order, double-dependence matrix for OP3 is shown in Figure 5-1.

This same inability to discriminate discourages generalizing the Markov model to higher levels of dependence.

Summary of Markov Results

The appearance of a significant 'memory effect' between an eruption at a specific vent and the last vent to erupt occurs sporadically in the observed eruption chronology. Only one period of observation (OP3) out of three appreciably demonstrated the existence of such a property, also known as the Markov property. A hypothesized Markov process for OP3 was also able to pass a statistical test for stationarity within the observed eruption sequence.

The principal outcome of the test for the Markov property is the suggestion that activity at a specific vent may be influenced by eruptions elsewhere in the summit crater. This implies a potential for increasing prediction accuracy in forecasting eruptions over the simple expectation of the average repose periods. The principal drawback of the Markov model approach as employed here lies in creating an 'eruption scale' in order to analyze the sequence of eruption events. In forfeiting a time scale the model necessarily forfeits the ability to quantitatively predict the time to the next eruption.

6. RELATIONSHIPS AMONG THE VENTS: LINEAR REGRESSION

The Model

Regression is a common data evaluation technique which basically expands the least squares fit approach to two dimensional data of the form Y = f(X) to include an unlimited number of independent variables. A linear regression model postulates a linear relationship of the form

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 . . . . + \beta_k X_k + e \qquad 6.1$$

where Y is some observable random quantity of interest (i.e. the'dependent'variable), X_1, X_2, ..., X_k are observable quantities (i.e. the 'independent' variables), β_0, β_1, β_2... β_k are unknown coefficients, and e is the error between the expected value of Y, E(Y), and the observed value where

$$E(Y) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_k X_k \qquad 6.2$$

The estimation of the unknown coefficients is performed by minimizing the sum of the squared error e^2. If $L = \Sigma e^2$ then for n sets of data of the form $Y = f(X_1, X_2, ..., X_k$

$$L = \sum_{j=1}^{n} e^2 = \sum_{j=1}^{n} (Y_j - \beta_0 - \sum_{i=1}^{k} \beta_i X_i)^2 \qquad 6.3$$

Minimizing L amounts to differentiating the above equation with respect to each unknown coefficient and setting the resulting set of partial differential equations equal to zero. This forms a set of k+1 equations in k+1 unknowns, namely β_0, β_1, ...β_k which can be solved in a straightforward manner by matrix methods. Computer programs designed to determine β_0, β_1, ...β_k are available on most FORTRAN compilers as part of a standard scientific library of subroutines while the theoretical development of the procedure is developed in detail in many places (see Graybill and Krumbein, 1965). A full reiteration of the technique will not be presented here.

Characterization of the volcanic process as a Markov process has suggested that for some periods of observation the sequence of eruptions is not completely random. Unfortunately the Markov approach developed here is unsuited to the task of predicting, even approximately, the time to the next eruption at a specific vent. Alternatively the regression approach is a tool which can be used in probing for more complicated relationships between a single intereruption period and past repose periods, eruption durations, and the level of activity at all the other vents. The following symbols were adopted in applying the method.

P_1' - represents the repose period between the last two eruptions at Vent 1 (seconds)

P_1'' - represents the repose period immediately prior to P_1' (sec)

D_1' - represents the duration of the last eruption at Vent 1 (sec)

N_3 - represents the number of eruptions occuring at Vent 3. In using N_3 to describe the activity at Vent 1, N_3 will represent the number of eruptions occurring at Vent 3 since the last eruption at Vent 1.

L_3 - represents the amount of time since the last eruption at Vent 3 (sec)

Generally a variable subscript refers to a specific vent and the superscript refers to the order in the past sequence of values of the specific parameter.

Meaning of Results: Correlation

An evaluation of the degree to which the empirical data conform to the hypothesized linear equation is provided by the correlation coefficient r. This coefficient measures the ratio of the variability in the predicted Y values to the variability in the raw distribution. If the variance is defined in sum-of-squares (SS) notation, the variance in the raw distribution can be described by

$$SS_y = \sum_{j=1}^{n} (y_i - \bar{y})^2 \qquad 6.4$$

where $\bar{y}$ is the distribution mean. Another measure of the variance in an empirical distribution is the second moment of the distribution, m_2 with respect to the mean

$$m_2 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{N} = \frac{SS_y}{N} \qquad 6.5$$

where N represents the total number of y values sampled.

The variance in the predicted value of Y when the linear equation is employed is simply a sum of the error.

$$SS_{reg} = \sum_{j=1}^{n} (y_i - \beta_0 - \sum_{i=1}^{k} \beta_i X_{ij}) \qquad 6.6$$

The correlation coefficient is then defined as

$$r^2 = \frac{SS_y - SS_{reg}}{SS_y} \qquad 6.7$$

The variance SS_y is a measure of how well $\bar{Y}$ represents the Y distribution, while SS_{reg} is a measure of the 'residual' variation which remains in the empirical Y data when the estimated linear equation is used to predict Y. Clearly if SS_{reg} is equal to SS_y, the regression equation can be no better a predictor than the original distribution mean and the correlation coefficient equals zero. As SS_{reg} becomes small relative to SS_y, the error in predicted Y values will be smaller than the spread in the original data and r increases to a value not greater than 1.0.

Though r is a convenient correlation parameter, the fraction of the variance in the original sample of Y's which is 'accounted for' or 'explained by' the regression equation is actually r^2, This can alternately be expressed as a $100r^2$ % increase in accuracy in anticpating on the basis of the X_i parameters involved in the correlation. This does not imply that all predicted values will be more accurate by the given percentage, but rather that the use of the equation with the data employed in the regression procedure would yield an average increase in accuracy of $100r^2$ % over simply anticipating the mean repose period.

It should be noted that the correlation coefficient is purely a statistical yardstick which makes no assumption as to the cause-and-effect relationship between the X_i variables and Y. Inferring a dependency relationship between the two variables solely on the basis of a significant correlation cannot be justified. This method alone is incapable of interpreting a natural process. In the same vein it is important to note that the dimensions of the equation coefficients may fail to represent anything meaningful in the physical world.

One-to-One Correlation

The aim of this study is ultimately to predict the actual time to the next eruption at a specific vent. This is equivalent to estimating the value of the current repose period at that vent P_v, and simply subtracting the time since the last eruption there, L_v. In selecting relevant parameters for a linear equation model it is useful to construct a correlation coefficient matrix for all the data available as input. Such a matrix displays the one-to-one correlation which exists between all input parameters. It follows that the matrix will be symmetric and that all diagonal elements will equal 1.0. This diagonal property is the result of the 'perfect' correlation between each parameter and itself.

A simple procedure for scanning potential variables involves reducing in input data matrix , X, to a 'deviation matrix', Z (see Graybill and Krumbein, 1965). This is done as follows. Initially the data is organized in an i x j matrix (X), where i equals the number of sample points and j equals the number of parameters supplied. Every i-th eruption can be characterized by a certain set of values for the j parameters being used in the data matrix X. The transformation is accomplished by an element-by-element operation with

$$Z_{ij} = \frac{x_{ij} - \bar{x}_j}{S_j} \qquad 6.8$$

where

x_{ij} = k om data set i value of parameter

x_j = average of parameter j over all input data (i.e. a column average)

s_j = standard deviation in parameter j calculated over the entire data matrix

z_{ij} = element i, j of the deviation matrix.

The correlation coefficient matrix R is obtained by left multiplying the deviation matrix by its transpose and multiplying the result of the scalar 1/(N-1)

$$R = \frac{1}{N - 1} Z^T Z \qquad 6.9$$

where

N = the total number of data points

Z^T - represents the transpose of the deviation matrix

The results of this computation are shown in Tables 6-1, 6-2, and 6-3. They are summarized in the next section. An advantage in calculating correlation in this manner is that the sign of the coefficient is preserved. A positive sign indicates that Y increases with an increase

TABLE 6-1 CORRELATION COEFFICIENT MATRIX R

	P_1	P_1'	N_2	N_3	D_1	D_1'	D_3	D_3'	L_3	
P_1	1.0000	-0.2318	0.2336	0.5274	0.0523	0.2298	0.0043	-0.0229	0.0224	-0.0736
P_1'		1.0000	0.0244	-0.0943	-0.0132	0.0598	-0.0407	0.0030	-0.0074	-0.0091
N_2			1.0000	0.0314	0.0900	0.1660	0.2129	0.1471	-0.3327	0.0933
N_3				1.0000	0.1208	0.2276	-0.0335	-0.0402	-0.0225	-0.5653
D_1					1.0000	0.4619	-0.1691	-0.0722	-0.2152	-0.1614
D_1'						1.0000	-0.1135	-0.1159	-0.1802	-0.1720
D_3							1.0000	0.5755	-0.3095	0.1470
D_3'								1.0000	-0.3087	0.0238
L_2									1.0000	-0.0108
L_3										1.0000

Vent 1 N = 233

TABLE 6-2 CORRELATION COEFFICIENT MATRIX R

	P_2	P'_2	P''_2	D_3	D'_3	N_1	L_1	N_3	L_3
P_2	1.000	-0.1094	-0.1240	-0.2585	-0.2352	0.9823	0.1956	0.9371	0.3805
P'_2		1.0000	-0.189	-0.2759	-0.2079	-0.1065	-0.2339	0.0095	-0.3703
P''_2			1.0000	-0.1639	-0.1823	-0.0711	-0.2376	-0.1690	-0.0927
D_3				1.0000	0.4107	-0.2801	0.1972	-0.2615	-0.0672
D'_3					1.0000	-0.2293	0.0565	-0.2973	0.0470
N_1						1.0000	0.1340	0.9067	0.4371
L_1							1.0000	0.1902	0.0234
N_3								1.0000	0.1182
L_3									1.0000

Vent 2 N = 31

TABLE 6-3 CORRELATION COEFFICIENT MATRIX R

	P_3	P'_3	P''_3	D_3	D'_3	D''_3	N_1	L_1	N_2	L_2
P_3	1.0000	-0.1452	-0.0576	-0.0596	0.2313	0.0007	0.8705	0.0294	0.3789	0.0187
P'_3		1.0000	-0.1354	-0.0539	-0.0368	0.2413	-0.1629	0.1056	-0.1883	0.0515
P''_3			1.0000	0.0322	-0.0300	-0.0319	0.0421	-0.2818	-0.0053	0.0825
D_3				1.0000	0.5602	0.2961	-0.0695	0.0610	0.1448	-0.2812
D'_3					1.0000	0.5434	0.2341	-0.1474	0.3580	-0.3322
D''_3						1.0000	-0.0032	-0.0194	0.1658	-0.3438
N_1							1.0000	-0.2398	0.3222	0.0295
L_1								1.0000	-0.1655	0.1488
N_2									1.0000	-0.3895
L_2										1.0000

Vent 3 N = 122

in X while a negative sign implies a decrease in Y with a positive change in X. This data scanning technique reveals a wider variety of correlations than obtained by testing P_v against other specific parameters.

Regression Results

Significant results from the correlation matrix R are reviewed below for each vent. The data for the input matrix employed in calculating R was drawn from all three observation periods. The general level of correlation appeared to warrant extending the regression analysis to no more than five parameters.

Vent 1 N = 233

The parameter exhibiting the strongest correlation with the intereruption period at Vent 1 was the number of events occurring at Vent 3 (r = +0.5274) and with the duration of the last event at Vent 2 (r = + 0.2336) and with the duration of the last event at Vent 1 itself (r = +0.2298) were exhibited. The period to the next eruption at Vent 1 was negatively correlated with the last period (r = -0.2318). A relatively strong positive correlation between the duration of an event at 1 and the duration of the prior eruption was also noted (r = +0.4619). Parameters selected for regression analysis included P_1', N_2, N_3, D_1', and L_3.

Vent 2 N = 25

The intereruption period at Vent 2 was surprisingly well correlated with the number of events at Vent 1 (r = 0.9860) and the number of events at Vent 3 (r = 0.9413). A weaker correlation with the length of time since the last eruption at Vent 3 (r = +04541) and the time since the last eruption at Vent 1 (r = +0.2304) was also exhibited. The time to the next eruption was negatively correlated with both the previous eruption period (r = -0.1094) and with the duration of the last event at Vent 3 (r = -0.2352). Parameters selected for regression analysis include P'_2, N_1, L_1, N_3, and L_3.

Vent 3 N = 122

The parameter demonstrating the strongest correlation with the period between eruptions at Vent 3 was the number of events occurring at Vent 1 (r = +0.8705). This was accompanied by a weaker correlation with the number of events occurring at Vent 2 (r = +0.3789). As was the case at Vent 1 and Vent 2 a negative correlation was observed with the prior repose period (r = -0.1452) while in contrast a positive correlation with the duration of the last event at Vent 3 (r = +0.2313). Parameters selected for regression analysis included P'_3, D'_3, N_1, L_1, and N_2.

The results of the regression analysis are presented in Tables 6-4, 6-5, 6-6 and 6-7. Results are reported for each observation period and for the combined data from all three periods. Inspection of the tables shows that significant variations (typically ± 0.10) in correlation (r) occur from one observation period to the next for the same combination of independent variables. However, the relative correlation of one grouping of variables to another remains fairly constant across observation periods. Thus the absolute changes are interpreted as reflecting variations in the number of 'data sets' employed for the different observation periods.

TABLE 6-4 MULTIPLE REGRESSION CORRELATION COEFFICIENT, r — VENT 1

Observation Period	1+2+3	1	2	3
	N = 233	N = 60	N = 112	N = 56
P'_1	0.2318	0.2909	0.2586	0.1941
N_2	0.2336	0.2266	0.3232	0.8871
N_3	0.5274	0.5399	0.4602	0.6353
D_1	0.2298	0.1512	0.3415	0.4305
L_3	0.0736	0.0844	0.9342	0.8588
P'_1, N_2	0.3332	0.3706	0.3929	0.2374
P'_1, N_3	0.5582	0.5725	0.5082	0.6542
P'_1, D_1	0.3367	0.3476	0.4232	0.4655
P'_1, L_3	0.2439	0.3025	0.2808	0.2108
N_2, N_3	0.5703	0.5732	0.5323	0.6615
N_2, D'_1	0.3035	0.2393	0.4455	0.4313
N_2, L_3	0.2525	0.2486	0.3398	0.1347
N_3, D'_1	0.5393	0.5413	0.5208	0.6891
N_3, L_3	0.5935	0.6050	0.5095	0.7096
D'_1, L_3	0.2324	0.1612	0.3419	0.4480
P'_1, N_2, N_3	0.6008	0.6054	0.5649	0.6918
P'_1, N_2, D'_1	0.3925	0.3886	0.4966	0.4658
P'_1, N_2, L_3	0.3475	0.3841	0.4101	0.2591
P'_1, N_3, D'_1	0.5728	0.5771	0.5633	0.7052
P'_1, N_3, L_3	0.6154	0.6249	0.5424	0.7261
P'_1, D'_1, L_3	0.3384	0.3506	0.4246	0.4804
N_2, N_3, D'_1	0.5757	0.5740	0.5763	0.6969
N_2, N_3, L_3	0.6208	0.6248	0.5633	0.7264
N_2, D'_1, L_3	0.3093	0.2544	0.4468	0.4481
N_3, D'_1, L_3	0.6071	0.6084	0.5772	0.7324
P'_1, N_2, N_3, D'_1	0.6081	0.6055	0.6069	0.7199
P'_1, N_2, N_3, L_3	0.6285	0.6248	0.6150	0.7398
P'_1, N_2, D'_1, L_3	0.6314	0.6315	0.6046	0.7475
P'_1, N_3, D'_1, L_3	0.3971	0.3957	0.4990	0.4817
N_2, N_3, D'_1, L_3	0.6436	0.6457	0.5872	0.7517
P'_1, N_2, N_3, D'_1, L_3	0.6531	0.6467	0.6363	0.7615

TABLE 6-5 REGRESSION CORRELATION COEFFICIENT, r

Observation Period	1+2+3	1	3
	N = 25	N = 15	N = 13
P'_2	0.1145	0.2103	0.1983
N_1	0.9860	0.9741	0.9294
L_1	0.2304	0.0701	0.3270
N_3	0.9413	0.7727	0.8118
L_3	0.4541	0.5036	0.0184
P'_2, N_1	0.9860	0.9749	0.9295
P'_2, L_1	0.2357	0.2176	0.3363
P'_2, N_3	0.9508	0.8156	0.8123
P'_2, L_3	0.4565	0.5040	0.2050
N_1, L_1	0.9884	0.9839	0.9294
N_1, N_3	0.9907	0.9834	0.9323
N_1, L_3	0.9868	0.9850	0.9298
L_1, N_3	0.9424	0.7789	0.8377
L_1, L_3	0.4916	0.5333	0.3272
N_3, L_3	0.9786	0.9415	0.9044
P'_2, L_1, N_3	0.9885	0.9854	0.9295
P'_2, N_1, N_3	0.9914	0.9834	0.9326
P'_2, N_1, L_3	0.9871	0.9850	0.9298
P'_2, L_1, N_3	0.9508	0.8195	0.8448
P'_2, L_1, L_3	0.5024	0.5334	0.3387
P'_2, N_3, L_3	0.9794	0.9435	0.9074
N_1, L_1, N_3	0.9924	0.9928	0.9325
N_1, L_1, L_3	0.9893	0.9912	0.9298
N_1, N_3, L_3	0.9916	0.9851	0.9405
L_1, N_3, L_3	0.9791	0.9649	0.9300
P'_2, N_1, L_1, N_3	0.9926	0.9928	0.9331
P'_2, N_1, L_1, L_3	0.9929	0.9928	0.9441
P'_2, N_1, N_3, L_3	0.9796	0.9653	0.9300
P'_2, L_1, N_3, L_3	0.9920	0.9851	0.9406
N_1, L_1, N_3, L_3	0.9893	0.9913	0.9298
P'_2, N_1, L_1, N_3, L_3	0.9930	0.9928	0.9441

TABLE 6-6 MULTIPLE REGRESSION CORRELATION COEFFICIENT, r VENT 3

Observation Period	1+2+3	1	2	3
	N = 122	N = 32	N = 56	N = 33
P'_3	0.1452	0.2632	0.0773	0.1362
D'_3	0.2313	0.5350	0.3222	0.2732
N_1	0.8706	0.8667	0.8457	0.9325
L_1	0.0294	0.0355	0.0922	0.0612
N_2	0.3789	0.1778	0.3487	0.5960
P'_3, D'_3	0.2688	0.5513	0.3223	0.3310
P'_3, N_1	0.8706	0.8671	0.8464	0.9354
P'_3, L_1	0.1520	0.2944	0.1167	0.1419
P'_3, N_2	0.3863	0.2746	0.3487	0.6092
D'_3, N_1	0.8710	0.8707	0.8457	0.9325
D'_3, L_1	0.2401	0.5487	0.3680	0.2732
D'_3, N_2	0.3925	0.5529	0.4257	0.6016
N_1, L_1	0.9045	0.8937	0.9004	0.9468
N_1, N_2	0.8767	0.8702	0.8567	0.9329
L_1, N_2	0.3902	0.1834	0.3766	0.6126
P'_3, D'_3, N_1	0.8710	0.8708	0.8466	0.9358
P'_3, D'_3, L_1	0.2800	0.5775	0.3695	0.3331
P'_3, D'_3, N_2	0.4003	0.5594	0.4287	0.6117
P'_3, N_1, L_1	0.9047	0.8985	0.9005	0.9484
P'_3, N_1, N_2	0.8768	0.8702	0.8567	0.9362
P'_3, L_1, N_2	0.3989	0.3017	0.3767	0.6311
D'_3, N_1, L_1	0.9060	0.8988	0.9016	0.9470
D'_3, N_1, N_2	0.8767	0.8740	0.8573	0.9329
D'_3, L_1, N_2	0.4060	0.5677	0.4711	0.6174
N_1, L_1, N_2	0.9135	0.8977	0.9150	0.9469
P'_3, D'_3, N_1, L_1	0.9062	0.9026	0.9016	0.9484
P'_3, D'_3, N_1, N_2	0.9136	0.9027	0.9153	0.9470
P'_3, D'_3, L_1, N_2	0.9135	0.9000	0.9154	0.9484
P'_3, N_1, L_1, N_2	0.4152	0.5829	0.4783	0.6325
D'_3, N_1, L_1, N_2	0.8768	0.8742	0.8574	0.9363
P'_3, D'_3, N_1, L_1, N_2	0.9136	0.9043	0.9158	0.9484

TABLE 6-7 EQUATION COEFFICIENTS

Vent 1 $P_1 = \beta_0 + \beta_1 P_1' + \beta_2 N_2 + \beta_3 N_3 + \beta_4 D_1' + \beta_5 L_3$

Observation Period	1+2+3	1	2	3
	N = 233	N = 60	N = 112	N = 56
β_0	245.69	295.90	144.28	103.17
β_1	- 0.18	- 0.18	-0.17	- 0.19
β_2	191.76	241.57	202.32	159.60
β_3	390.33	449.43	292.20	439.40
β_4	23.10	6.11	56.40	57.62
β_5	0.18	0.21	0.13	0.18

Vent 2 $P_1 = \beta_0 + \beta_1 P_2' + \beta_2 N_1 + \beta_3 L_1 + \beta_4 N_3 + \beta_5 L_3$

Observation Period	1+2+3	1	3
	N = 25	N = 15	N = 13
β_0	-637.51	-485.99	-410.32
β_1	- 0.03	0.01	0.01
β_2	422.82	495.74	363.11
β_3	0.62	1.16	- 0.48
β_4	433.04	219.12	611.27
β_5	0.35	0.09	0.66

Vent 3 $P_3 + \beta_0 + \beta_1 P_3' + \beta_2 D_3' + \beta_3 N_1 + \beta_4 L_1 + \beta_5 N_2$

Observation Period	1+2+3	1	2	3
	N = 122	N = 32	N = 56	N = 33
β_0	- 39.97	137.98	-214.29	-102.09
β_1	- 0.00	-0.06	0.03	0.06
β_2	0.34	5.02	1.00	0.06
β_3	466.82	406.42	446.86	582.85
β_4	0.63	0.47	0.80	0.49
β_5	206.56	101.89	259.93	1.61

The correlation coefficient computed for a linear equation with several terms is more precisely referred to as a multiple correlation coefficient. Simply grouping parameters X_1, X_2, and X_3 will not result in a linear equation with a multiple correlation coefficient equal to the sum of the one-to-one correlations r_1, r_2, and r_3. The reason for this is that the parameters used in the equations as independent variables are not independent of one another. This is demonstrated in the correlation coefficient matrix R. For example, in the R matrix computed for Vent 3 the one-to-one correlation for $P_3 = f(N_1)$ is r = 0.8706 while for $P_3 = f(D'_3)$, r = 0.2313. However when the variables N_1 and D'_3 are combined in $P_3 = f(N_1, D'_3)$ the multiple correlation coefficient equals 0.8710. Inspection of the correlation coefficient matrix reveals that the correlation between N_1 and D'_3 was r = +0.2346. Thus because N_1 and D'_3 were weakly correlated, the addition of the parameters D'_3 to the estimation equation for Y failed to increase the average prediction accuracy significantly. In a sense, the 'information' added was for the most part redundant.

This effect dominates the tradeoff between increasing accuracy and carrying additional terms in the algebra of the actual equations. Specifically, consider the Vents 1 and 2. The current repose period at Vent1, P_1, is best correlated on a one-to-one basis with the number of events at Vent3, N_3, with r = 0.5275. Incorporating the additional variables P'_1, N_2, D'_1, and L_3 into the linear equation increases r to 0.6531. The addition of each term corresponding to a specific parameter results in an incremental increase in the average ability to predict the current repose period at Vent 1. Alternatively to predict the current repose period at Vent 1. Alternatively the best singly correlated parameter for P_2 is N_1, with r = 0.9860. The addition of four other parameters, P_2', L_1, N_3, and L_3, serves to increase r by less than one percent (to r = 0.9930). Thus the ability to predict the current repose period at Vent 2 fails to increase appreciably with the inclusion of other variables. The behaviour of the linear model at Vent 3 is essentially the same as that at Vent 2, with r = 0.8706 for $P_3 = f(N_1)$ increasing to r = 0.9136 for $P_3 = f(P_3', D_3', N_1, L_1, N_2)$.

Summary

The most significant discovery of the regression model is that the time to the next eruption at both Vents 2 and 3 is best correlated singly with the number of events occurring at Vent 1. Furthermore, the increase in estimation accuracy with the consideration of other parameters proved to be marginal at best. The overall capability of the linear model to accurately anticipate intereruption periods was the greatest at Vent 2 (r = 0.9939), intermediate at Vent 3 (r = 0.9136), and poorest at Vent 1 (r = 0.6531).

It is important to recall that $100r^2$% represents the increase in prediction accuracy relative to the variance in the empirical sample of repose periods with respect to the sample mean. In terms of absolute time consider the difference between the standard deviation in the raw data and the standard deviation in the group of anticipated repose periods (i.e. $P_v = F(X_1, X_2, \ldots, X_k)$) shown in Table 6-8. It has already been pointed out that the variance in the anticipated distribution is a measure of the error between predicted and observed repose periods. On the other hand the variance in the empirical distribution is a measure

TABLE 6-8 REGRESSION RESULTS IN ABSOLUTE TIME

$E(P_v)$ = expected value of the current repose period at Vent v

	Standard deviation in observed distribution of P_v	Standard deviation in anticipated distribution of P_v
	$E(P_v) = \bar{P}_v$	$E(P_v) = f(X_i, 111, X_k)$
Vent 1	427 sec	324 sec
Vent 2	2179	260
Vent 3	837	344

the degree to which the sample mean represents the whole range of observed values of P_v. The most dramatic reduction in the variabilty of P_v occurs in employing the linear model to Vent 2. At Vent 2 the value of a standard devistion is reduced by an order of magnitude and it becomes more 'predictable' in an absolute sense than either Vents 1 or 3.

This is completely contrary to field intuition. In the field the 'regularity' of eruption was predicted largely on the relative eruption frequency observed for the three vents. Vent 1 erupted the most frequently and hence was expected to fire regularly every 15 - 20 minutes. Vent 2 erupted the least frequently and, thus, no attempt to anticipate an eruption at Vent 2 was made while actually monitoring in the field.

The regression results generally corroborated teh implications of the random nature of the repose period distributions regarding the independence of the time to the next eruption at any single vent from immediate past behaviour at that vent. Correlations of P_v with repose periods in the immediate past (P'_v, P''_v, P'''_v) were generally less than r = 0.25. This means that only 6% of the variance in the repose periods can be accounted for by the immediately preceding repose period at a specific vent.

It has already been pointed out that the correlation coefficient for a particular linear equation relating some variable Y to a set of variables X_1, X_2, ... X_k is a simple measure of the average accuracy in the ability to predict Y within the same group of data employed in formulating the linear equation. This means that the linear regression model is not a dynamic model. Its ability to forecast future behaviour is grounded solely on the tendencies exhibited by the data provided as input. On the other hand the regression method represents a powerful quantitative technique. The dynamic evolution of the volcanic process may be reflected in the time variation of the equation coefficients. However, considerably longer monitoring periods would be required in ascertaining the efficacy of the regression approach in detecting such changes.

III. SUMMARY

7. OVERVIEW AND IMPLICATIONS

In attempting to build an empirical predictive capability that can be used to forecast eruptions, this study has considered three models. The first 'model' was a characterization of the observed frequency distribution of repose periods in terms of common frequency distributions whose properties are well known. Basically this type of model is useful in describing how representative the mean repose period is in relation to the entire distribution. The second model, a Markov model, probed the observed eruption sequence for consistent relationships between an eruption occurring in the recent past. Such a relationship is termed a 'memory effect' in a Markov process. The third model, a linear model, attempted to integrate the behaviour of all the active vents in predicting the time to the next eruption. Variables reflecting past repose periods, eruption durations, number of eruptions and time since a previous eruption were employed in the forecasting equation.

The Markov model is distinguished from the other two in that its ability to anticipate an eruption is spatial in nature. The characterization and linear models are temporal and can be employed in quantitatively anticipating the length of current repose periods. The linear model has been shown to be a consistently more accurate predictor. (Specific results are summarized in Chapters 4, 5, and 6.)

There is strong temptation to interprete the results of the linear regression technique in terms of the physical relationships between the vents. Wickman (1965e) has yielded to the same sort of temptation at the conclusion of his characterization analysis. Although such interpretations can be provocative and stimulate further research, they are no substitute for observed physical parameters in modelling the volcanic process. Hawaiian studies of the geophysical (Eaton, 1966) and geochemical (Murata and Richter, 1966, Wright and Fiske, 1971) relationships which exist within a sequence of eruptions provide an insight which can be extrapolated in understanding future volcanic behaviour. Alternatively empirical studies of repose periods are strongly bound to past behaviour. Eruption prediction based solely on past experience cannot anticipate a perturbation at depth which results in a major reorientation of the volcanic activity at the surface.

Regression analysis of volcanic activity is a statistical technique which has not been investigated in the past. The Strombolican setting, in which three vents erupt several times per hour, represents a unique opportunity to compile enough data to make the results of such a rechnique statistically significant. In addition, the process of data acquisition consists essentially of maintaining an observer with a clock at the volcano. There is no major equipment expenditure as is the case in establishing and operating a geophysical station. By implication, areas of the world which are characterized by high volcano densities and low GNP's, such as Central America and the Southwest Pacific, might find the regression approach to eruption prediction a cost effective short term investment.

It has been suggested earlier that application of the regression technique to long term monitoring may reveal gross changes in the levels of correlation which may reflect real changes in the nature of the driving mechanism operating at depth. The ability of such correlation changes to forewarn of a major realignment of volcanic activity in a manner analogous to shifts in seismic activity in and around volcanoes remains to be proven.

In conclusion, the most accurate statistical model for predicting current volcanic repose periods at Stromboli in September, 1971 was a linear model based upon a regression technique. The results of such a statistical model place temporal constraints on any physical model which attempts to predict eruptive behaviour on the basis of observed physical parameters. Longer termed statistical analysis of observed activity in conjunction with geophysical and geochemical monitoring may be a useful tool in anticipating major changes in volcanic behaviour.

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APPENDIX A STATISTICAL HYPOTHESIS TESTING

Hypothesis testing can be thought of as an example of decision theory in practice (see Remington and Shork, 1970). A statistical test is performed when an investigator wishes to determine some specific property of a group of sample data. The decision problem must be formulated in a binary manner which admits to a yes-or-no answer.

The hypothesis to be tested is termed the null hypothesis (or tested hypothesis). The other possible outcome of the test is termed the alternative hypothesis. The procedure followed is to 1) calculate some test statistic from the sampled data which is known to be distributed as a random variable in conformity with some standard statistical distribution; 2) calculate the degree of freedom present in the test; and 3) specify an acceptable level of significance in the test. The level of significance is equivalent to the probability of rejecting the null hypothesis when it is, in fact, true. If the level of significance, α, is set to 0.01, the investigator is willing to accept a 1% chance of incorrectly rejecting the tested hypothesis when it is valid. A knowledge of the degree of freedom present in the test is required in employing common tabulations of the standard distribution characterizing the test statistic.

The level of significance determines an acceptance region and critical region within the standard statistical distribution. For a two-sided test the critical region in which the null hypothesis will be rejected consists of values of the test statistic at each extreme of the standard distribution. Alternatively, the critical zone for a one-sided test would consist of extreme values of the test statistic in one direction only.

An example of a one-sided test arises in the case of a coin tossing game. Suppose there were a game in which each participant chose 'heads' or 'tails' for some group of throws, say for 10 tosses. Once either 'heads' or 'tails' is chosen each participant must wager at least one dollar on each of the 10 throws. One cannot switch from 'heads' to 'tails' or vice versa once a group of tosses starts. Suppose you have chosen 'heads' in such a game. The only bias in the coin which costs you money is a possible 'tails' bias. In this sense you are not concerned by a 'heads' bias since it costs you nothing. A physical analogy might be in measuring the average concentration of some infectious germ in a person's blood. Only when the average concentration exceeded some critical concentration would there be cause for alarm.

The level of significance, α, is more commonly translated into a confidence level which equals $1 - \alpha/2$ for the two-sided test, and simply $1 - \alpha$ for the one-sided test. In determining the degree of conformity of the sample distributions with well known distribution densities, all tests will be two-sided.