INDIRECT MEASUREMENT OF THE MEAN MERIDIONAL CIRCULATION IN THE SOUTHERN HEMISPHERE

by

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Using IGY data analyses of Obasi (1963), the mean meridional circulations for the southern hemisphere are measured indirectly, using the method of characteristics. It is found that the slope of the characteristic curves is proportional to the ratio of the zonal vertical wind shear to the dynamic stability. The characteristic curves may be approximately identified with the curves of constant absolute angular momentum. The horizontal and vertical momentum convergences enter as forcing functions, in a manner similar to that of Kuo (1956), and Mintz and Lang (1955). Certain simplifying assumptions are made about the vertical convergences. The expression for $[\bar{\omega}]$, the vertical component of the circulation, strongly resembles the integrated form of the continuity equation, the difference being attributable entirely to the inclusion of the baroclinicity of the atmosphere. Both methods give similar overall $[\bar{\omega}]$ patterns, but there are differences in details, particularly in the winter season.

The calculations generally confirm Obasi's directly measured three cell pattern, but there are strong differences in cell intensity and position. Comparison with northern hemisphere indirect measurements of Mintz and Lang (1955) shows strong similarities in position and intensity, but with the much stronger southern hemisphere polar direct cell a marked exception.

Vertical eddy convergences of momentum in the "free" atmosphere are calculated using the directly measured mean meridional circulations. Large areas of <u>negative</u> eddy viscosity result, always in a region including the mid-latitude jet. Similar calculations for the northern hemisphere using data of Buch (1954) give the same general result. The negative signs do not appear to be implausible if large scale vertical eddies dominate at higher levels, but the magnitudes may be too large.

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ERRATA

Page 2 Line 7. "thought as" should read, "thought of as."

Page 5 Line 7. "other methods" should read, " other, 'indirect', methods".

Page 7 Line 14. " τ = wind stress" should read " τ = zonal wind stress"

Page 9 Line 14. "then represents" should read "G then represents."

Page 16 Line 2. "from the momentum convergence" should read "from the vertically integrated momentum convergence."

Page 20 Line 6. For "(1961)", read "(1960)".

Page 21 Line 10. For "(1961)", read "(1960)".

Page 22 After equation (20), add "where ν is the kinematic eddy viscosity".

Page 22 After last formula, insert, "where h is the vertical space coordinate.

Page 22, 23 Replace "$\frac{\partial T}{\partial z}$" by "$\frac{\partial T}{\partial h}$".

Page 26 Line 2. For "eddy transports", read "vertical eddy transports".

Line 7. For " $g\frac{[\overline{\tau}]}{\Delta p}$ " read " $-g\frac{[\overline{\tau}]}{\Delta p}$ ".

Page 29 Line 7. For "Isabele Cole", read "Isabelle Kole".

Page 48 Top Add reference:

Buch, H. 1954: Hemispheric wind conditions during the year 1950. Final Report, Part 2; M. I. T. General Circulation Project, Document No. AF 19-122-153.

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1. Introduction

The earliest modern theories of the generation of the zonal kinetic energy in the atmosphere (Hadley, 1735) stated in effect that the potential energy due to the meridional temperature gradient was released by a single symmetric toroidal cell of circulation comprising rising motion of relatively warm air near the equator, poleward flow at high levels, sinking motion near the poles, and equatorward flow near the ground. This kinetic energy of the mean meridional motion was thought as converted to kinetic energy of the zonal motion through the action of Coriolis torques.

The introduction in 1859 by Ferrel of the "indirect" cell in middle latitudes allowed relaxation of the earlier requirements that the winds near the surface at <u>all</u> latitudes be easterly. Though altered, then, the mean meridional circulation maintained its central position in the explanation of the mechanism of the general circulation.

In this century, however, following the suggestion by Jeffreys (1926, 1933) both theoretical and observational studies have shown (see Starr, 1958 for the most complete picture of what he calls this "New Outlook" on the general circulation) that the primary energy conversion process is the baroclinic instability associated with the traveling wave cyclones and anticyclones (the "transient eddies") and that in Starr's words "the net energy release by the meridional circulation is in all probability slightly negative". Furthermore, the momentum of the westerlies is maintained against frictional stresses primarily by these horizontal eddies, not the mean meridional circulation.

Finally, Kuo (1956) showed that for earth-atmosphere conditions the mean meridional circulations must be forced motions, and that the most important forcing terms involve derivatives of the eddy convergences of momentum and heat transports. It is evident, then, that the mean meridional circulations have lost the central role they once played. In the words of Starr (1954) "... the halcyon, free-wheeling days when investigators could propound mean meridional cells in a rough and ready manner to explain each feature of the mean zonal wind distribution, are irretrievably behind us".

Though less important, the mean meridional circulations are not, however, unimportant. They still comprise two components of the symmetric part of the mean wind field. Though they play a smaller role than the horizontal eddies in the energy conversion and momentum transport processes, a quantitative knowledge of them is necessary to complete the energy and momentum balances. They may also be of importance, primarily in the lower latitudes, in the global hydrological balance (see, e.g., Starr and Peixoto, 1963).

2. Methods of measurement

There are essentially two ways to measure the mean meridional circulation. The first is usually called "direct"; in the most systematic and objective of such studies, maps of the meridional component of the wind averaged for a season or a year are analyzed, and values are read at grid points around latitude circles to obtain zonal averages. Obasi (1963) has carried out the most systematic such study for the southern hemisphere. He divided the calender year 1958 into winter (April-September), and summer (January-March, October-December), using data collected during the IGY. Many studies, with varying degrees of objectivity and varying latitude ranges, have been made for the northern hemisphere, in particular by Buch (1954), Palmen (1955), Tucker (1957) and (1959), Palmen, Reihl, and Vuourela (1958), and Palmen and Vuourela (1963).

A serious fault, which cannot be escaped by any of these studies, however, is that errors in observation and analysis of horizontal meridional winds make the zonal averages very uncertain. This is because meridional winds vary approximately between $\pm$ 10 m/sec, while their zonal averages are of the order of 1 m sec^{-1}, or less*. This fact is a manifestation of the quasi-geostrophic nature of large scale atmospheric flow. The effect of these errors can be seen in the fact that meridional winds so calculated

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*With the necessary introduction of a graphical data reduction process to get the latitudinal averages, it appears to be almost impossible to make quantitative estimates of the probable error in these latitudinal values. Rather it seems that the best check on reliability of results is reproducibility of results using different analysts, or even different years of data, assuming there are not large yearly differences in these values.

often do not give, even in the grossest sense, zero mass transport across all latitude circles, a requirement which should hold very closely over a whole season or year. Spurious drift velocities of 50 cm sec^{-1} are not at all uncommon in such calculations (see, for example, Obasi 1963). For the same reason, the vertical velocities calculated from the continuity equation using such data are also quite uncertain.

For this reason, it is desirable to devise other methods of measuring the mean meridional circulations. The general approach is to infer them from more accurately measurable quantities. This is essentially the method of Kuo (1956) who solved for the forced mean meridional circulation, in the form of a stream function, as well as for the axially symmetric temperature and geopotential perturbations, utilizing derivatives of the eddy transports of heat and momentum and the diabatic heating as forcing functions.

It is considerably simpler, however, if one's task is to determine the meridional circulation already known to be forced, to make use of the fact that the mean zonal wind and the time and zonally averaged "transient eddy" transports of momentum are quantities whose measurement does not suffer the same fate as that of the mean meridional wind (Tucker 1960 not withstanding). Using the measured zonal wind dispenses with the need for computing the mean geopotential fields, since the geostrophic assumption is no longer made. It is no longer necessary to determine the temperature field either, and the meridional circulation may be found from a single second order nonhomogeneous partial differential equation for the stream function, with only the momentum transports as forcing terms.

As an alternative to the stream function approach, in this simple case, the horizontal and vertical components of the meridional circulation can be found directly by the use of the method of characteristics. This method has been used, for example, by Krishnamurti (1961) to determine the vertical motion field in a steady symmetric hurricane, given the radial velocity field; and by Eliassen and Hubert (1953) to compute the vertical motion field in a "blocking" situation.

This method has the advantage in this case of solving a first order, rather than a second order, partial differential equation, and it avoids the tedium of solution by relaxation techniques. Further, the solution for the vertical component bears strong resemblance to the ordinary integrated form of the continuity equation, the difference being due entirely to the inclusion of the baroclinicity of the atmosphere.

3. Notation

The following symbols, in most cases well known, will be used in the text that follows. Other notation, peculiar to this paper, will be defined in the text.

p = pressure, in mb

ϕ = latitude (from 0° to -90°)

λ = longitude

u = zonal wind (+ toward east)

v = meridional wind (+ toward north)

ω = $\frac{dp}{dt}$

Ω = angular velocity of the earth (7.29×10^{-5} sec^{-1})

f = Coriolis parameter = $2\Omega \sin\phi$

g = acceleration of gravity (980 cm/sec^2 throughout)

a = radius of the earth (6.37×10^6 meters)

τ = wind stress on a horizontal surface

χ = frictional force/unit mass

$z_1 = f - \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}[\bar{u}]\cos\phi$ = mean zonal vorticity = dynamic stability

ρ = density

T = temperature

Φ = geopotential

$\overline{(\ )} = \frac{1}{2t_1}\int_{t-t_1}^{t+t_1}(\ )\,d\xi$ = time average

$(\ )' = (\ ) - \overline{(\ )}$ = deviation from time average

$[(\ )] = \frac{1}{2\pi}\int_0^{2\pi}(\ )\,d\lambda$ = zonal average

$(\ )^* = (\ ) - [(\ )]$ = deviation from zonal average

4. Theoretical formulation

Using the notation defined in 3, and assuming hydrostatic equilibrium, we may write the zonal equation of motion and the equation of continuity in the (λ , ϕ , p , t) coordinate system as

$$\frac{\partial u}{\partial t} + \frac{2u}{a\cos\phi}\frac{\partial u}{\partial \lambda} + \frac{1}{a\cos\phi}\frac{\partial \Phi}{\partial \lambda} + \frac{1}{a\cos^2\phi}\frac{\partial}{\partial \phi}(uv\cos^2\phi) + \frac{\partial}{\partial p}(u\omega) - fv - \mathcal{X} = 0 \quad , \tag{1}$$

and

$$\frac{1}{a\cos\phi}\frac{\partial u}{\partial \lambda} + \frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}v\cos\phi + \frac{\partial \omega}{\partial p} = 0 \tag{2}$$

respectively.

Now, if we take the time and then the zonal averages of these equations and assume a steady state, i.e., $\frac{\partial \overline{X}}{\partial t} \equiv 0$, where X represents any of the dependent variables, then the first three terms in (1) and the first term in (2) vanish*, and we are left with

$$\frac{\partial}{\partial p}[\overline{u\omega}] - f[\overline{v}] + \frac{1}{a\cos^2\phi}\frac{\partial}{\partial \phi}[\overline{uv}]\cos^2\phi - [\overline{\mathcal{X}}] = 0 \tag{3}$$

and

$$\frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}[\overline{v}]\cos\phi + \frac{\partial[\overline{\omega}]}{\partial p} = 0 \tag{4}$$

*Here we are assuming that the effect of the mountain torque in the southern hemisphere is small, mainly because reliable information on its size is unavailable at present. However, it undoubtedly is smaller than in the northern hemisphere, due to the smaller number and extent of mountain ranges.

But we may write $[\overline{u\omega}]$ as

$$[\overline{u\omega}] = [\bar{u}][\bar{\omega}] + [\overline{u'\omega'}] + [\bar{u}^* \bar{\omega}^*] \tag{5}$$

Here the second term on the right represents the contribution of the "transient eddies", and the third term that of the "standing eddies". Then, using (5) and an analogous expression for $[\overline{uv}]$, and the relation

$$\frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}[\bar{u}][\bar{v}]\cos^2\phi = \frac{[\bar{v}]}{a\cos\phi}\frac{\partial}{\partial\phi}[\bar{u}]\cos\phi + \frac{[\bar{u}]}{a\cos\phi}\frac{\partial}{\partial\phi}[\bar{v}]\cos\phi$$

(3) becomes

$$[\bar{\omega}]\frac{\partial[\bar{u}]}{\partial p} - z_1[\bar{v}] + \frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}\left([\overline{u'v'}] + [\bar{u}^*\bar{v}^*]\right)\cos^2\phi + \frac{\partial}{\partial p}\left([\overline{u'\omega'}] + [\bar{u}^*\bar{\omega}^*]\right) - [\bar{X}] = 0 \tag{6}$$

In order to simplify notation, we shall denote collectively the last three terms on the left in (6) by G , and let G_H , G_V , and G_F denote them separately, in order. That is,

$$G = G_H + G_V + G_F$$

then represents the forcing by the eddy transports of momentum.

Then (6) becomes

$$[\bar{\omega}]\frac{\partial[\bar{u}]}{\partial p} - z_1[\bar{v}] + G = 0 \tag{7}$$

Repeating (4), to group the equations to be solved, we have

$$\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}[\bar{v}]\cos\phi + \frac{\partial}{\partial p}[\bar{\omega}] = 0 \tag{4}$$

Equations substantially the same as these in the (ϕ, z) coordinate system were written by Mintz and Lang (1955), but their method of solution was considerably different.

It is more convenient to solve for $[\bar{\omega}]$ first, by dividing (7) by z, operating on it with $\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\quad)\cos\phi$, and adding (7) to (4), giving

$$\frac{\partial[\bar{\omega}]}{\partial p} + \frac{1}{az}\frac{\partial[\bar{u}]}{\partial p}\frac{\partial[\bar{\omega}]}{\partial\phi} + \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\frac{1}{z}\frac{\partial[\bar{u}]}{\partial p}\cos\phi\right)[\bar{\omega}] = -\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\frac{G}{z}\cos\phi\right) \tag{8}$$

Note that (8) is a first order equation. It can be solved easily using the method of characteristics. Let us introduce the following simplifying notation, defining

$$S(p,\phi) \equiv \frac{1}{az}\frac{\partial[\bar{u}]}{\partial p}$$

$$J(p,\phi) \equiv \frac{1}{\cos\phi}\frac{\partial}{\partial\phi}(S\cos\phi)$$

and

$$K(p,\phi) \equiv -\frac{1}{\cos\phi}\frac{\partial}{\partial\phi}\left(\frac{G}{az}\cos\phi\right)$$

Then (8) becomes

$$\frac{\partial[\bar{\omega}]}{\partial p} + S\,\frac{\partial[\bar{\omega}]}{\partial \phi} + J(p,\phi)[\bar{\omega}] = K(p,\phi) \tag{9}$$

Here S is the slope of the characteristic curves in (p,ϕ) space, and we may write $\frac{d[\bar{\omega}]}{dp} = \frac{\partial[\bar{\omega}]}{\partial p} + S\,\frac{\partial[\bar{\omega}]}{\partial \phi}$, where $\frac{d[\bar{\omega}]}{dp}$ is a directional derivative taken along the characteristic curve.

Equation (9) now becomes

$$\frac{d[\bar{\omega}]}{dp} + J(p,\phi)[\bar{\omega}] = K(p,\phi) \tag{10}$$

The solution to the homogeneous part $(K \equiv 0)$ is simply

$$[\bar{\omega}] = [\bar{\omega}]_0\, e^{-\int_{p_0}^{p} J(p',\phi)\,dp'} \tag{11}$$

where it is understood that the integral is along a characteristic curve, not along a line of constant ϕ.

By variation of parameters, then, the solution to the nonhomogeneous equation becomes

$$[\bar{\omega}] = [\bar{\omega}]_0\, e^{-\int_{p_0}^{p} J\,dp'} + e^{-\int_{p_0}^{p} J\,dp'} \int_{p_0}^{p} e^{\int_{p_0}^{p'} J\,dp''} K(p',\phi)\,dp' \tag{12}$$

Now the characteristic curves, as will be shown (see Figure 1) are nearly vertical, so that it is quite natural to begin the integration at the "top" of the atmosphere, using the usual upper boundary condition that

$[\overline{\omega}]_{p=0} = [\overline{\omega}]_0 = 0$. Then (12) becomes

$$[\overline{\omega}] = e^{-\int_0^p J\,dp'} \int_0^p e^{\int_0^{p'} J\,dp''} K(p', \phi)\,dp' \tag{13}$$

Note that we are allowed to specify the value of $[\overline{\omega}]$ at only one point on each characteristic curve. That is, we cannot also fix $[\overline{\omega}]$ at the ground level. This is not a serious disadvantage, however, since the values of $K(p, \phi)$ should be such as to make $[\overline{\omega}]$ again small near the ground. This fact can be seen more clearly if we note that had we solved for $[\overline{V}]$ first from (7), neglecting the term $[\overline{\omega}] \frac{\partial [\overline{u}]}{\partial p}$, and then inferred $[\overline{\omega}]$ from the continuity equation (4), we would have obtained

$$[\overline{\omega}] = \int_0^p K(p', \phi)\,dp' = -\int_0^p \frac{1}{a\cos\phi} \frac{\partial}{\partial \phi} [\overline{V}] \cos\phi \; dp' \tag{14}$$

or, interchanging the integration and differentiation

$$[\overline{\omega}] = -\frac{1}{a\cos\phi} \frac{\partial}{\partial \phi} \left(\int_0^p [\overline{V}]\,dp \right) \cos\phi \tag{15}$$

The quite closely realized requirement that there be no mass transport across any latitude circle, i.e., $\int_0^{p_{Ground}} [\overline{V}]\,dp = 0$, then implies that $[\overline{\omega}]_{Ground} \approx 0$. Now (13) differs from (14) only in the added factors $e^{\pm\int J dp}$, and in that the integration is carried out along the characteristic curves. Since $0.5 < e^{\pm\int J dp} <$ 2.0 generally, and since the characteristic curves deviate no more than 4° from their 850 mb latitude, (13) should integrate giving results rather similar to (14), and therefore $[\bar{\omega}]$ from (13) should also tend to zero near the ground. The basic physical difference between (14) and (13) is that (13) includes the effect of the baroclinicity of the atmosphere on the calculation of $[\bar{\omega}]$.

Once $[\bar{\omega}]$ has been calculated from (13), $[\bar{V}]$ is found from (7), i.e.,

$$[\bar{V}] = \frac{G}{Z} + \frac{[\bar{\omega}]}{Z}\frac{\partial[\bar{u}]}{\partial p} \tag{16}$$

Equation (16) is essentially the expression first derived by Palmen (1955), and later used by several authors (Mintz and Lang (1955), Palmen, Riehl and Vuourela (1958), Dickinson (1962), and others).

5. Method of calculation and results

The slope of the characteristic curves is given by $S = \frac{1}{a z} \frac{\partial [\bar{u}]}{\partial p}$

It is easy to calculate the shape of the characteristic curves if we realize that these lines can be identified with lines of constant absolute angular momentum, m , given by

$$m = \Omega a^2 \cos^2\phi + a[\bar{u}] \cos\phi \tag{17}$$

as these lines have the same slope*.

The method used to calculate the position of these curves was to choose as a reference level the lowest pressure level where Obasi had calculated $[\bar{u}]$ (850 mb, except 700 mb for ϕ = 75° and 80° S), calculate the absolute angular momentum m_0 for that level, and then determine the latitude of all points with the same angular momentum.

Solving (17) for $\cos\phi$, we have

$$\cos\phi = \frac{-[\bar{u}] \pm \sqrt{4\Omega m_0}\sqrt{1 + \frac{[\bar{u}]^2}{4\Omega m_0}}}{2\Omega a} \tag{18}$$

Since $\cos\phi > 0$, we must choose the + sign. Approximating further by using the fact that $\frac{[\bar{u}]}{4\sqrt{\Omega m_0}} << 1$, we obtain

$$\cos\phi = \frac{2\sqrt{\Omega m_0} - [\bar{u}]}{2\Omega a} \tag{19}$$

*This fact was kindly pointed out to me by Professor N. Phillips.

Expression (19), strictly speaking, requires the mean zonal wind on the characteristic curve, the position of which we are trying to determine. However, since $|\phi - \phi_0| \leq 4^o$, the error made in using the mean zonal wind of the latitude ϕ_0 is only of the order of 10%, accurate enough for the purposes of this study. The characteristic curves so calculated are presented in Figure 1.

Now to calculate $[\overline{\omega}]$ and then $[\overline{v}]$; all of G_H , G_V , and G_F must be known. G_H is known fairly accurately from Obasi's data. However, very little is known about G_V , which represents the large scale vertical eddy momentum convergences, and G_F , which represents small scale eddies.* In the absence of such knowledge, we have made the assumption, for the sake of calculating a meridional circulation profile, that in the "free atmosphere", which we shall take to be above 850 mb,** there is no vertical transport of momentum by eddies of any scale. In the region below 850 mb, we will assume, to calculate reasonably representative values of $[\overline{\omega}]$, and of $[\overline{v}]$, that the frictional stress decreases linearly from the surface stress value to 0 at the top of the boundary layer.*** Then G_F , given by $G_F = g \frac{\partial[\overline{\tau}]}{\partial p}$, becomes $g \frac{[\overline{\tau}]}{\Delta p}$, where Δp is the pressure

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*Here "large scale" represents that scale of eddy measurable from a synoptic map. "Small scale" then refers to all smaller scales of eddies.

**Except near the south pole (67.5°, 72.5°, 77.5°) where, due to the elevation of the ice cap, we take the boundary layer to extend to 700 mb at 67.5°) and 600 mb at 72.5° and 77.5°.

***We also extrapolate Obasi's values of $[\overline{u}]$ and $[\overline{u'v'}]$ down into the surface layer from 850 mb, assuming the variation between 850 mb and 1000 mb is approximately the same as that between 700 mb and 850 mb.

depth of the boundary layer. The mean zonal surface stress $[\overline{\tau}]$ is calculated from the momentum convergence due to the transient eddies and the mean meridional motion, using the trapezoidal rule. (The stress due to the eddies above was calculated by Obasi using a somewhat different numerical integration procedure). Obasi's momentum transports due to the standing eddies (the term $[\overline{u}^* \overline{v}^*]$) were not used, because 1) they are much less important in the southern hemisphere in transporting momentum than either the transient eddies or the mean meridional motion, and 2) they are considerably harder to measure and therefore, much less reliable than the transient eddies. The mean meridional motion transports are those calculated first using the surface stress from the transient eddies alone to obtain an approximate set of values for $[\overline{v}]$ in the lowest layer, from which the mean meridional transports are calculated to obtain new surface stresses and new lowest layer values of $[\overline{v}]$. The surface stress values used appear in Table 1.

Finally, slight adjustments were made on the calculated results for $[\overline{v}]$ to ensure no mass transport across any latitude circle. These correcting "drift" velocities were never more than 4 cm sec^{-1}, only 10% of the typical drifts found from calculating $[\overline{v}]$ directly. The results of the calculations are presented in Tables 2 and 3, and Figures 2 and 4, with Figures 3 and 5 being Obasi's directly measured values for the corresponding season.* The $[\overline{\omega}]$ results from equations (13) and (14) are presented in

*Figures 3 and 5 include some relatively minor corrections to Obasi's (1963) original results.

Tables 4 and 5 and Figures 6 and 8, with $[\bar{\omega}]$ values calculated from Obasi's $[\bar{v}]$ via the continuity equation (4), presented for comparison in Tables 6 and 7, and Figures 7 and 9.

The results in Tables 4 and 5 indicate that the inclusion by (13) of the baroclinic effect generally makes a difference of 10 to 20%, and in some cases much larger differences in the $[\bar{\omega}]$ calculated at any one place, but the profiles of the two calculations are rather similar.

6. Comparison of direct and indirect measurements

The indirect measurements definitely confirm the 3 cell pattern of Obasi's observed values. In fact, the three-cell pattern in the indirect measurements is considerably sharper. The general equatorward shift of the cells with the change from summer to winter is also clear, though it is seen more strongly in the $[\overline{\omega}]$ patterns than in the $[\overline{v}]$ patterns, implying an accompanying change in shape of the cells. The shift generally ranges from 10 to 20° latitude.

The shift in intensities with season of the calculated and observed results are not the same, however. The winter indirectly measured circulations are generally somewhat stronger than their summer counterparts, particularly in the equatorial cell. The directly measured $[\overline{v}]$ also show a stronger equatorial cell in winter, but a weaker indirect mid-latitude cell, and a weaker polar cell. Such a result does not seem compatible with the decreased intensity of the zonal circulation, and probably reflects the uncertainty of directly measured $[\overline{v}]$.

Another general result is that the directly measured $[\overline{v}]$ indicates a stronger indirect mid-latitude cell at high levels, and a generally weaker direct equatorial cell than do the indirectly measured values. In the lower latitudes, at least, this may be a consequence of the fact that Obasi, in calculating the drift velocities to be subtracted off to give zero mass transport, assumed the velocities in the layer 850-1000 mb to be the same as those at 850 mb, which may be a considerable underestimate (see, for example, for the northern hemisphere, Palmen and Vuourela, 1963). Assuming larger velocities in this lower layer could have strengthened the equatorial cell, but a similar correction in mid-latitudes would have made the upper branch diverge even more from the indirectly measured values.

Another point of difference is that the zero lines between the upper and lower branches of the cells are at considerably lower levels in the indirect measurements than in the direct ones. This is very likely a consequence of our earlier assumption of no vertical eddy momentum transports of any scale above 850 mb.

7. Comparison with northern hemisphere results

Obasi (1963) has made comparisons of his directly calculated $[\bar{V}]$ values with northern hemisphere direct calculations of Buch (1954) and Tucker (1959), so such comparisons need not concern us further. Seasonal indirect measurements similar to ours have been done for the whole hemisphere for $[\bar{V}]$ and the vertical velocity by Mintz and Lang (1955), and for $[\bar{\omega}]$ by Murakami (1961). Calculations for the stratosphere alone have been done by Dickinson (1962) and, for one month periods, by Teweles (1963).

Before comparing, it should be noted that Mintz and Lang used only two month averages (Jan-Feb for winter, July-August for summer) which are the most winter and summer-like months, respectively, so that their winter circulation is probably more intense, and the summer less intense, than the corresponding 6 month averages would be.

Comparing the two hemispheres for the winter season, we see that the direct cells, and the indirect cells, are approximately the same intensity. Within each hemisphere, the equatorial direct cell is between two and three times the intensity of the mid-latitude indirect cell. In addition, within each hemisphere, the cell boundaries are between 30 and 35°, and 65° and 70°. The southern hemisphere, however, shows a very much stronger polar direct cell, and the southern hemisphere equatorial direct cell appears to extend further into the northern hemisphere than does Mintz and Lang's into the southern. These differences are in agreement with Obasi's general conclusion that the southern hemisphere circulation is the more intense.

In the summer season, the Northern Hemisphere mean meridional circulations are very much weaker. The poleward direct cell is missing entirely, and the equatorward cell is very disorganized. These differences are probably very strongly influenced by the fact that Mintz and Lang's values are for July and August only. The strong Southern Hemisphere polar cell even in the summer is probably primarily a manifestation of the "Katabatic wind", due to the very strong temperature contrast between the antarctic continent ice cap and the warmer seas around it.

Concerning the vertical component of the mean meridional circulation, Murakami (1961) inferred $[\bar{\omega}]$ for the Northern Hemisphere, for the entire year 1950, via the continuity equation. However, Murakami integrated from the bottom to the top of the atmosphere. Presumably due to the poorness of the $[\bar{v}]$ data, his $[\bar{\omega}]$s do not tend toward zero near the top. Consequently, the fact that his values above 700 mb are three or four times as large as ours is probably not significant. His regions of upward and downward motions are packed closer together than even the Southern Hemisphere summer, again indicative of the greater intensity of the Southern Hemisphere general circulation.

The stratospheric values are rather similar to those of Dickinson (1962) and Teweles (1963). In particular, Dickinson's (yearly averaged, also for the IGY) cell divisions are about 28° and 66°N, while ours are at 29° and 64°S.

8. Vertical eddy convergences of momentum

Even given that Obasi's $[\overline{V}]$ have considerable uncertainty, it is still interesting to calculate what kind of vertical eddy convergences of momentum are needed to make the indirectly measured meridional circulations agree exactly with them.

One way of doing this, when one lacks separate information on the large and small scale eddies, is to represent the effect of vertical eddies of <u>all</u> scales, in an empirical way, by an eddy viscosity proportionality to the time and zonally averaged vertical shear of the zonal wind. That is, the frictional stress may be written, in the pressure coordinate system, as

$$[\overline{\tau}] = -g\rho^2\nu\frac{\partial[\overline{u}]}{\partial p} \tag{20}$$

Now since the frictional stress is related to the frictional force per unit mass, $[\overline{X}]$, by the expression $[\overline{X}] = -g\frac{\partial[\overline{\tau}]}{\partial p}$, the latter may be written as

$$[\overline{X}] = \nu g^2\rho^2\left(\frac{\partial^2[\overline{u}]}{\partial p^2} + \frac{2}{\rho}\frac{\partial\rho}{\partial p}\frac{\partial[\overline{u}]}{\partial p}\right) \tag{21}$$

where the kinematic eddy viscosity ν has been assumed not a function of pressure. Now

$$\frac{\partial\rho}{\partial p} = \frac{\rho}{p}\left(1 + \frac{R}{g}\frac{\partial T}{\partial z}\right)$$

and if we take $\frac{\partial T}{\partial z} = -6.5^{\circ}K\ km^{-1}$ in the troposphere (taken here to be below 200 mb), and $\frac{\partial T}{\partial z} = 0$ in the stratosphere, and let $1 + \frac{R}{g}\frac{\partial T}{\partial z} = B$ we have

$$[\bar{\chi}] = \nu g^2 \rho^2 \left(\frac{\partial^2 [\bar{u}]}{\partial p^2} + \frac{2B}{p} \frac{\partial [\bar{u}]}{\partial p} \right) \qquad (22)$$

Here B then has the values 0.83 and 1.0 in the troposphere and stratosphere, respectively.

The two terms in parentheses in (22) are generally the same order of magnitude, with the curvature term the larger near the tropopause level.

Equation (22) may then be used to represent G_F and G_V together, since no separation of the large and small scale vertical eddies can be made. The eddy viscosity ν is then obtained from (16), according to the expression

$$\nu = \frac{z[\bar{v}] - [\bar{\omega}] \frac{\partial [\bar{u}]}{\partial p} - G_H}{R_F} \qquad (23)$$

where $R_F = G_F / \nu$. Here the $[\bar{v}]$ are Obasi's values, and the $[\bar{\omega}]$ are calculated via continuity from them. The actual numerical results were not considered to be very significant due to the uncertainty of the various terms in (23), so that only the signs of ν are presented, in figure 10. Similar calculations were made for the northern hemisphere using Buch's (1954) data, the signs for which are presented in figure 11.

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*Note that if $\rho\nu$, the dynamic eddy viscosity, had been held constant, $\frac{2B}{p}$ would be replaced by $\frac{B}{p}$.

The magnitudes of the eddy viscosity range from 10^4 to 10^7 cm^2 sec^{-1}, with the larger values generally at higher levels, but the striking feature of all these cases is that there are very large areas of all negative eddy viscosity, from 150 mb down to 600 or 775 mb, except near the poles. Generally positive values are found below these levels. Further, given the uncertainty in $[\overline{V}]$, the most reliable signs for ν come when both numerator and denominator of (23) have their largest values, which is generally the region when the negative values are found. In each case, the mid-latitude jet stream maximum is well within the negative region indicating a transport of momentum into the jet by the vertical eddies in the vicinity of the jet.

The above result is not implausible if we note that in the lowest layers of the atmosphere, where the details of orography become important, presumably the smaller eddies dominate, and momentum in the region of surface westerlies is transported down to the ground, as evidenced by the positive eddy viscosities there. At the higher levels in the troposphere, large scale upward motion is usually found to the east of troughs, where the zonal component of the wind is usually stronger (due to the "tilt" of these troughs). If these large scale vertical eddies dominate at these levels, negative eddy viscosities would result.

The eddy viscosities would, of course, be decreased in magnitude by closer agreement between Obasi's mean meridional circulations and ours. Indeed, Geostrophic scale theory does imply, and recent calculations by Starr and Dickinson (1963) indicate that vertical eddy stresses are probably smaller than those implied by our eddy viscosities. It is not at all certain, however, that closer agreement would make the signs all become positive. Therefore, to the degree that calculations of the eddy viscosity such as ours can be trusted, the usual assumption of a positive vertical eddy viscosity used in most numerical models of the general circulation is open to serious question. It should be remembered that there was a time when it was thought that the large scale horizontal eddies in the atmosphere had positive eddy viscosities, too.

9. Energy considerations

Earlier it was assumed that in the "free" atmosphere, there were no eddy transports of momentum of any scale; i.e., that part of the atmosphere was assumed to be frictionless. Therefore, all the frictional dissipation of kinetic energy of the zonal flow would take place in the surface layer. The amount of this dissipation is given by

$$E = \frac{2\pi a^2}{g} \int_0^{\pi/2} \int_{P_{Ground}}^{P_{Ground} - \Delta P} [\bar{u}][\bar{X}] \, dp \cos\phi \, d\phi \tag{25}$$

or since we assumed $[\bar{X}] = g \frac{[\bar{\tau}]}{\Delta P}$ in the surface layer,

$$E = 2\pi a^2 \int_0^{\pi/2} [\bar{u}] \Big|_{P_{Ground} - \frac{\Delta P}{2}} [\bar{\tau}] \cos\phi \, d\phi \tag{26}$$

The results are as follows:

E (winter) = 6.17×10^{20} ergs sec^{-1}

E (summer) = 4.02×10^{20} ergs sec^{-1}

It must be pointed out that these values have in them the uncertainty generated by the extrapolation used to estimate $[\bar{u}]$ in the surface layer.

The energy fed into the mean zonal flow by the horizontal eddies was found by Obasi to be 9.63×10^{20} and 9.72×10^{20} ergs sec^{-1} for the winter and summer, respectively. The difference of 3.55×10^{20} and 5.61×10^{20} respectively, must then, under our assumptions, be taken out of the mean flow by the indirect mean meridional cell, through the action of Coriolis torques. The energy taken out by this process would eventually be returned to zonal available potential energy. In the actual atmosphere, kinetic energy could also be taken out of the mean zonal flow by vertical eddies of all scales, the smaller scales eventually converting the energy into heat, and the larger scales probably converting it to eddy kinetic energy. Had Obasi's measured mean meridional circulations been more reliable, information on the relative importance of the mean meridional motions and the vertical eddies in the energy balance could have been obtained. We suspect from other evidence that the meridional motions do contribute the larger part. Starr (1959) found, from two years of data, that the ratio of the meridional cell conversion to the horizontal eddy energy input for the northern hemisphere was about 1/2, which would be approximately our ratio if all of the energy not dissipated by surface friction were taken out by the mean meridional motion. Further, calculations by Starr and Dickinson (1963) show that energy conversions by the vertical eddies from the mean zonal flow for two separate months were of opposite sign but approximately the same size, suggesting a small average value for a whole season. The uncertainty in accuracy of such calculations, however, must be kept clearly in mind, and these results should not be considered particularly conclusive, pending the outcome of further measurements.

10. Concluding remarks

Since this study did not have accurate direct information on either the mean meridional motion or the vertical eddies, their relative importance in the momentum balance could not be unequivocably determined. However, balance requires that the sum of their effects balance the transports by the horizontal eddies, so that both cannot be small relative to them. Numerical models generating statistics of the general circulation of an idealized atmosphere, such as that of Phillips (1956), cannot answer this question, as they are forced to assume the magnitude of the vertical eddy transports. Usually it is assumed that in the "free" atmosphere the vertical eddy stresses are at least one, and sometimes two, orders of magnitude less than their values at the ground level. This uncertainty in the relative contributions of the vertical eddies and mean meridional motions will not, therefore, be resolved until further, more accurate observational studies are made. If, for example, the mean meridional motions are found as average values for 5, or even more, summer and winter seasons, the statistical significance achieved may be great enough to calculate the vertical eddy stresses as a statistically significant residual. In this regard, the Planetary Circulations Project, in conjunction with the Travelers Research Center, is presently undertaking an extensive analysis of five years of meteorological data for the northern hemisphere. A resolution of this uncertainty may be a result.

ACKNOWLEDGEMENTS

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Table 1: Surface stress $[\overline{\tau}]$ from momentum convergences

units: dyne cm^{-2}

Latitude	Summer: transient eddies only	Summer: transient eddies and mean meridional motion	Winter: transient eddies only	Winter: transient eddies and mean meridional motion
77.5	1.39	1.35	2.00	1.89
72.5	0.93	0.97	1.91	1.88
67.5	-0.23	-0.10	0.75	0.83
62.5	-1.13	-1.00	-0.48	-0.36
57.5	-1.45	-1.36	-0.83	-0.70
52.5	-1.62	-1.57	-1.14	-1.07
47.5	-1.16	-1.17	-1.12	-0.99
42.5	-0.46	-0.50	-1.07	-1.03
37.5	-0.10	-0.17	-0.70	-0.82
32.5	0.12	0.02	-0.12	-0.27
23.5	0.30	0.20	0.09	-0.14
22.5	0.42	0.38	0.67	0.52
17.5	0.43	0.46	0.46	0.49
12.5	0.37	0.45	0.43	0.63
7.5	0.25	0.35	0.36	0.65
2.5	0.22		0.28	

Table 2: $[\overline{V}]$ - Winter

Units: cm sec^{-1}

Latitude	Drift	75mb	150	250	350	450	600	700	775	925
77.5	0	0	-9	-27	-36	-26	-15	129	—	—
72.5	4	-9	-14	-23	-29	-21	-17	123	—	—
67.5	2	-5	-7	-6	-11	-10	-7		27	—
62.5	0	-1	1	6	6	5	2		0	-19
57.5	0	1	8	14	13	8	3		1	-36
52.5	0	2	14	22	20	13	6		3	-57
47.5	1	3	12	19	21	14	7		5	-57
42.5	0	4	14	23	22	13	7		5	-64
37.5	-1	3	10	22	24	14	7		1	-59
32.5	0	1	0	4	6	6	5		4	-18
27.5	-3	0	-6	-8	-2	5	7		7	-10
22.5	3	-10	-28	-33	-23	-13	-2		2	74
17.5	1	-4	-31	-47	-29	-14	-5		4	85
12.5	2	-8	-37	-61	-35	-19	-17		-6	131
7.5	4	-5	-45	-89	-55	-33	-21		-16	173

Table 3: $[\overline{V}]$ - Summer

Units: cm sec^{-1}

Latitude	Drift	75mb	150	250	350	450	600	700	775	925
77.5	2	-3	-11	-21	-24	-19	-15	90	—	—
72.5	4	-4	-8	-10	-15	-16	-12	62	—	—
67.5	1	1	4	8	4	0	-5		-9	–
62.5	0	4	11	16	13	9	7		3	-47
57.5	1	7	16	21	16	11	9		7	-65
52.5	-1	6	19	26	20	12	9		11	-71
47.5	0	2	15	24	19	10	5		8	-59
42.5	0	0	9	18	13	6	2		1	-31
37.5	0	3	9	8	2	4	4		-1	-19
32.5	0	3	2	-6	-4	5	4		0	-1
27.5	0	1	-11	-19	-10	3	2		1	22
22.5	0	-6	-25	-30	-20	-5	0		2	51
17.5	0	-10	-31	-38	-23	-9	-5		1	75
12.5	-1	-1	-23	-39	-24	-22	-16		-4	96
7.5	1	12	6	-11	-19	-36	-28		-24	85

Table 4: $[\bar{\omega}]$ - Winter

Upper number: $[\bar{\omega}]$ from characteristic curves

Lower number: $[\bar{\omega}]$ from continuity

Units: 10^{-5} mb sec^{-1}

Latitude (°S)	75mb	150	250	350	450	600	775	925
77.5	0.4 0.4	1.1 1.2	1.4 1.5	1.1 1.2	0.6 0.7	-4.0 -3.9	—	—
72.5	0.1 0.1	0.2 0.2	-0.5 -0.5	-1.9 -1.9	-3.1 -3.0	-2.8 -3.2	—	—
67.5	-0.1 -0.1	-0.4 -0.5	-1.7 -2.0	-4.0 -4.6	-6.0 -7.1	-8.1 -9.4	-4.1 -5.6	—
62.5	-0.1 -0.1	-0.6 -0.7	-2.1 -2.2	-4.1 -4.1	-5.8 -5.9	-7.4 -7.6	-8.5 -8.6	-4.1 -5.3
57.5	-0.1 -0.1	-0.6 -0.7	-2.0 -2.2	-3.4 -3.8	-4.9 -5.0	-6.0 -6.0	-6.7 -6.5	-3.6 -3.5
52.5	-0.1 -0.1	-0.5 -0.5	-1.3 -1.2	-2.4 -2.1	-3.3 -3.1	-4.3 -4.1	-5.2 -4.9	-3.4 -3.0
47.5	-0.1 -0.1	-0.2 -0.3	-0.6 -0.6	-1.2 -1.1	-1.6 -1.5	-1.9 -1.8	-2.2 -1.9	-1.1 -0.6
42.5	0 0.1	0.2 0.2	0.4 0.1	0.3 -0.2	0.2 -0.3	0.2 0	0.5 0.7	1.8 2.3
37.5	0.1 0.1	0.5 0.5	1.9 1.9	3.4 3.7	3.9 5.1	4.3 6.1	4.5 6.6	2.0 4.5
32.5	0.2 0.1	1.0 0.8	3.3 3.1	5.7 6.3	6.8 8.6	7.5 10.1	7.4 10.4	4.1 7.4
27.5	0.2 0.3	1.2 1.4	3.9 4.2	6.5 7.3	8.2 9.5	9.5 11.3	9.6 11.9	2.8 5.2
22.5	0.1 0.1	0.8 0.9	3.3 3.5	5.8 6.1	7.5 7.9	9.2 9.5	10.0 10.3	3.2 3.5
17.5	0 0	0.3 0.5	2.1 2.4	4.6 4.3	6.1 5.1	7.9 6.4	9.7 8.2	6.0 4.4
12.5	-0.1 -0.1	-0.4 -0.6	1.3 0.6	4.2 3.7	5.8 5.5	7.6 7.0	9.8 8.8	2.9 2.1
7.5	-0.2 -0.3	-0.7 -1.1	0.9 0.6	4.5 5.0	6.9 7.9	9.2 9.2	11.3 10.4	4.1 3.0

Table 5: $[\bar{\omega}]$ - Summer

Upper number: $[\bar{\omega}]$ from characteristic curves

Lower number: $[\bar{\omega}]$ from continuity

Units: 10^{-5} mb sec^{-1}

Latitude	75mb	150	250	350	450	600	775	925
77.5	0	-0.3	-1.7	-3.5	-4.2	-4.6		
	0	-0.3	-1.7	-3.5	-4.3	-4.8		
72.5	-0.1	-0.6	-2.3	-4.5	-6.0	-4.3		
	-0.1	-0.6	-2.3	-4.4	-6.1	-4.5		
67.5	-0.2	-0.9	-2.6	-4.6	-6.5	-8.7	-5.1	
	-0.2	-0.9	-2.6	-4.8	-6.9	-9.3	-5.7	
62.5	-0.2	-0.8	-2.2	-3.6	-4.8	-6.3	-7.8	-4.0
	-0.2	-0.9	-2.3	-3.8	-5.2	-6.7	-8.3	-4.5
57.5	-0.1	-0.4	-1.5	-2.6	-3.3	-3.9	-4.8	-2.8
	-0.1	-0.6	-1.7	-2.9	-3.6	-4.2	-5.1	-3.1
52.5	0.1	0.1	-0.3	-1.0	-1.3	-1.3	-1.4	-0.9
	0.1	0	-0.6	-1.4	-1.8	-1.7	-1.6	-1.0
47.5	0.1	0.4	0.7	1.0	1.3	1.8	2.8	1.6
	0.1	0.4	0.6	0.7	1.0	1.7	2.7	1.7
42.5	0	0	0.9	2.1	2.9	3.2	4.0	2.4
	0	0.1	0.8	1.8	2.7	3.2	4.1	2.6
37.5	-0.1	0.1	1.6	3.2	3.9	3.9	3.9	2.0
	-0.1	-0.1	1.1	2.9	3.8	4.1	4.2	2.3
32.5	0	0.8	2.9	4.6	5.2	5.3	5.1	2.1
	0	0.4	2.4	4.2	4.9	5.3	5.5	2.7
27.5	0.2	1.4	3.7	5.4	6.4	7.1	7.3	3.4
	0.2	1.2	3.5	5.5	6.6	7.5	7.7	3.8
22.5	0.4	1.3	2.8	4.3	5.5	7.0	7.6	3.6
	0.4	1.6	3.7	5.3	6.3	7.4	7.8	3.8
17.5	0.1	-0.2	-0.5	0.1	1.3	3.8	6.0	3.2
	0.1	0.7	1.8	2.5	3.2	4.8	6.2	3.1
12.5	-0.6	-2.3	-5.2	-6.3	-5.3	-2.6	1.0	1.6
	-0.5	-1.6	-3.2	-4.3	-3.9	-1.7	1.5	1.5
7.5	-0.9	-3.2	-7.4	-9.7	-9.0	-6.6	-2.5	-0.1
	-0.9	-3.3	-7.6	-10.1	-9.7	-7.6	-3.4	-1.1

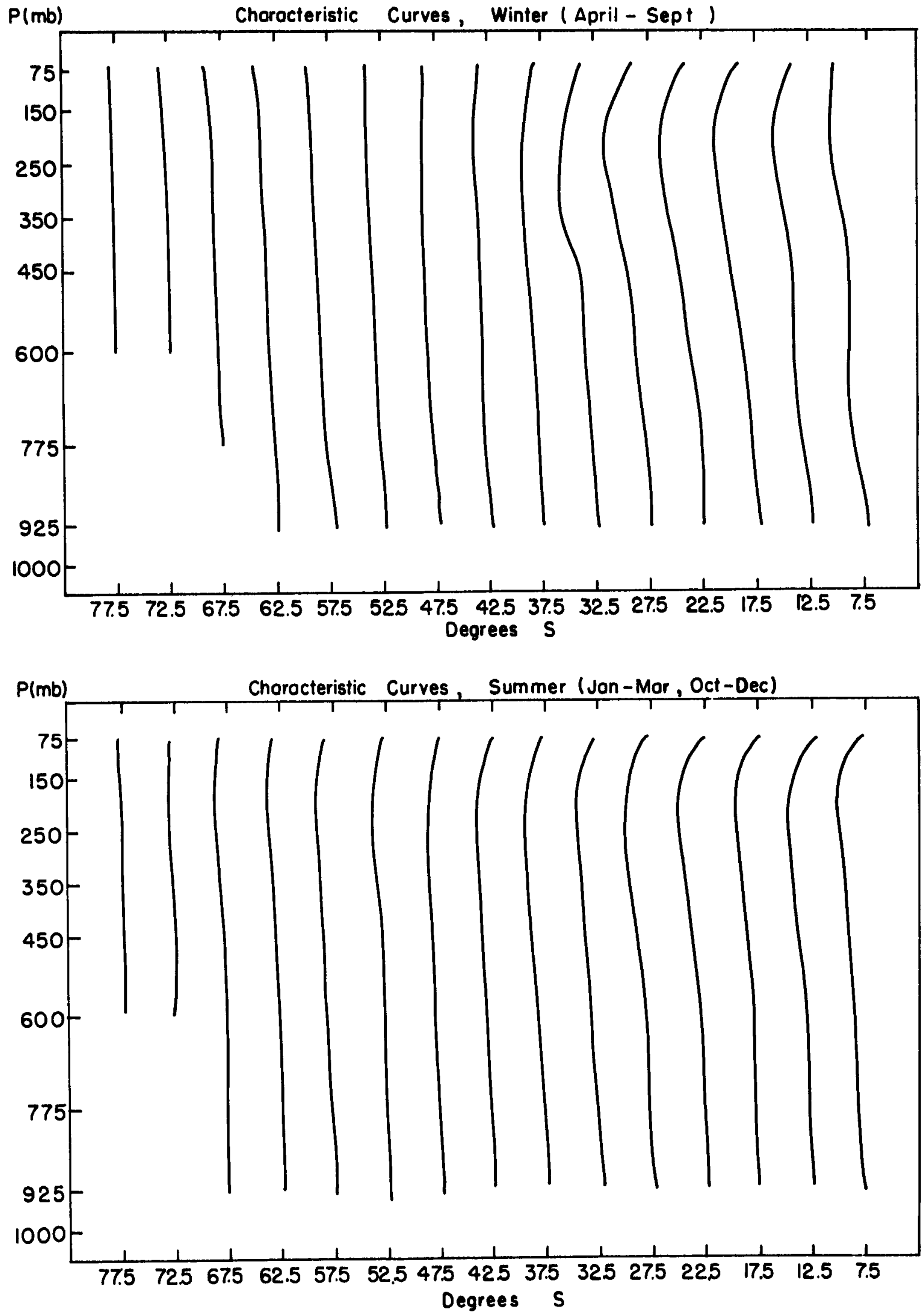
Table 6: $[\bar{\omega}]$ - Winter
from $[\bar{V}]$ of Obasi
Units: 10^{-5} mb sec^{-1}

Latitude	75mb	150	250	350	450	600	775	925
77.5	1.0	1.3	-0.5	-2.0	-3.1	-4.0	–	–
72.5	1.3	3.2	3.6	2.9	4.1	8.2	–	–
67.5	0.8	3.2	5.6	5.3	5.3	3.9	-2.6	–
62.5	1.7	5.4	10.5	15.4	19.4	23.1	24.6	–
57.5	0.3	1.1	2.3	3.1	2.6	-0.5	-2.3	-0.4
52.5	-0.4	-1.6	-3.6	-6.3	-9.3	-12.7	-10.7	-4.7
47.5	-0.7	-2.5	-5.2	-7.9	-10.7	-12.7	-9.9	-3.9
42.5	-0.9	-2.8	-5.4	-8.5	-11.4	-11.9	-7.2	-1.2
37.5	-1.1	-2.8	-5.2	-7.2	-7.3	-5.2	-1.8	0.3
32.5	1.5	2.6	0.8	-0.5	-0.4	0.5	0.9	-1.5
27.5	0.7	1.6	1.8	2.3	2.7	2.1	1.0	0.2
22.5	-0.1	1.2	5.8	10.3	12.5	12.2	7.5	2.8
17.5	0.2	2.0	6.8	10.9	13.6	16.4	12.9	5.0
12.5	0.5	2.1	6.3	10.4	12.5	14.7	12.2	3.7
7.5	0.2	0.5	1.5	2.8	2.3	-0.5	-2.8	-1.9
2.5	-0.6	-1.6	-2.7	-4.5	-7.4	-10.5	-8.7	-3.2

Table 7: $[\overline{\omega}]$ - Summer
from $[\overline{V}]$ of Obasi
Units: 10^{-5} mb sec^{-1}

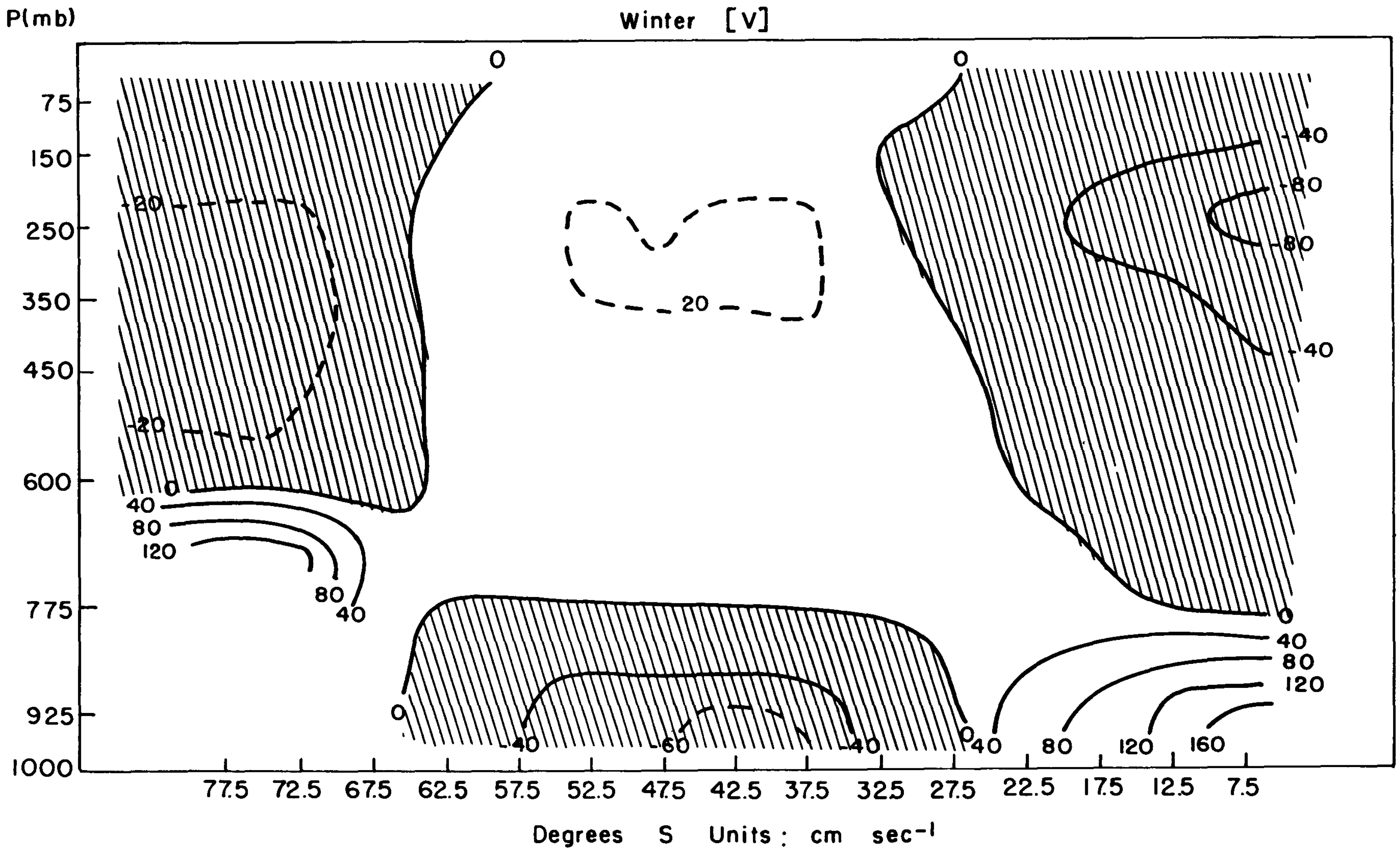
Latitude	75mb	150	250	350	450	600	775	925
77.5	0.3	0.7	1.3	0.7	-1.1	-1.5	—	—
72.5	-1.8	-5.0	-7.3	-7.5	-4.8	3.5	—	—
67.5	-3.0	-8.0	-11.5	-12.5	-11.4	-7.3	-1.8	—
62.5	-1.5	-4.7	-8.5	-10.8	-10.1	-6.9	-3.3	—
57.5	-0.9	-3.5	-8.2	-12.6	-14.1	-13.5	-11.9	-4.8
52.5	-0.5	-2.5	-7.1	-11.2	-12.4	-11.1	-7.9	-2.0
47.5	0.2	-0.1	-1.9	-3.7	-5.2	-5.6	-3.5	-2.3
42.5	-0.8	-1.9	-2.2	-1.9	-3.1	-5.7	-3.5	-0.5
37.5	0.1	0.5	1.6	1.6	1.4	3.3	4.3	1.3
32.5	0.3	1.6	3.2	3.5	2.2	-0.7	-2.0	-3.9
27.5	0.1	0.7	1.9	3.5	4.9	4.9	4.2	5.8
22.5	0.7	3.1	8.4	13.7	17.0	17.6	12.6	4.3
17.5	0.7	1.9	3.4	5.9	8.6	7.2	2.2	0.1
12.5	0.5	0.7	-0.5	-1.6	-1.8	-0.9	-0.6	-1.4
7.5	1.6	3.2	2.5	1.8	1.9	2.0	1.8	0.4
2.5	0.5	-0.2	-4.0	-7.1	-9.1	-9.6	-6.1	-1.9

Figure 1:



P(mb)
Characteristic Curves, Winter (April - Sept)
75
150
250
350
450
600
775
925
1000
77.5 72.5 67.5 62.5 57.5 52.5 47.5 42.5 37.5 32.5 27.5 22.5 17.5 12.5 7.5
Degrees S
P(mb)
Characteristic Curves, Summer (Jan-Mar, Oct-Dec)
75
150
250
350
450
600
775
925
1000
77.5 72.5 67.5 62.5 57.5 52.5 47.5 42.5 37.5 32.5 27.5 22.5 17.5 12.5 7.5
Degrees S

Figure 2:



Winter [V]
P(mb)
75
150
250
350
450
600
775
925
1000
0
-20
-40
-80
20
40
80
120
160
-60
77.5
72.5
67.5
62.5
57.5
52.5
47.5
42.5
37.5
32.5
27.5
22.5
17.5
12.5
7.5
Degrees S Units: cm sec^{-1}

Figure 3: $[\bar{V}]$ from Obasi

Winter

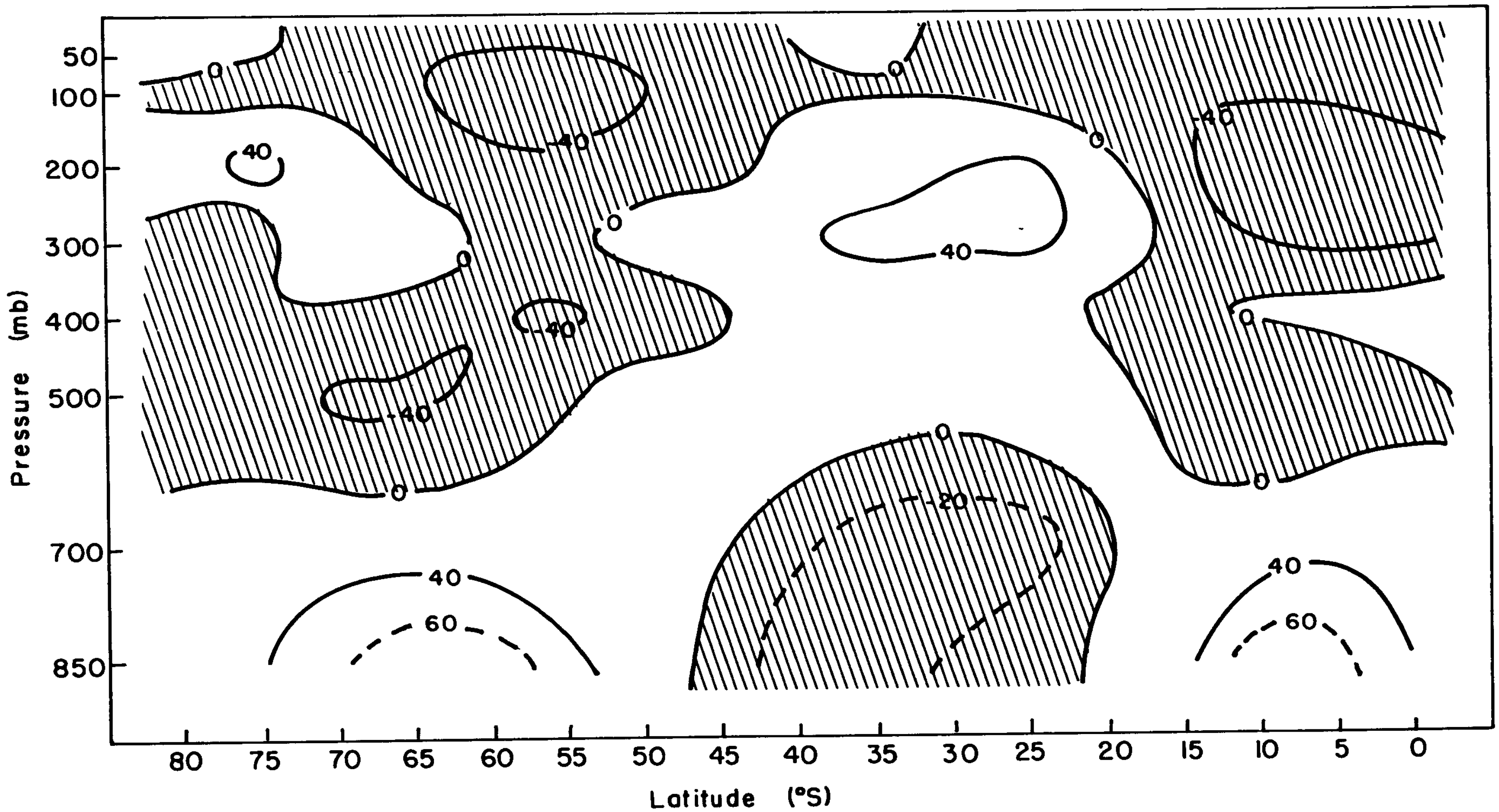


Figure 4:

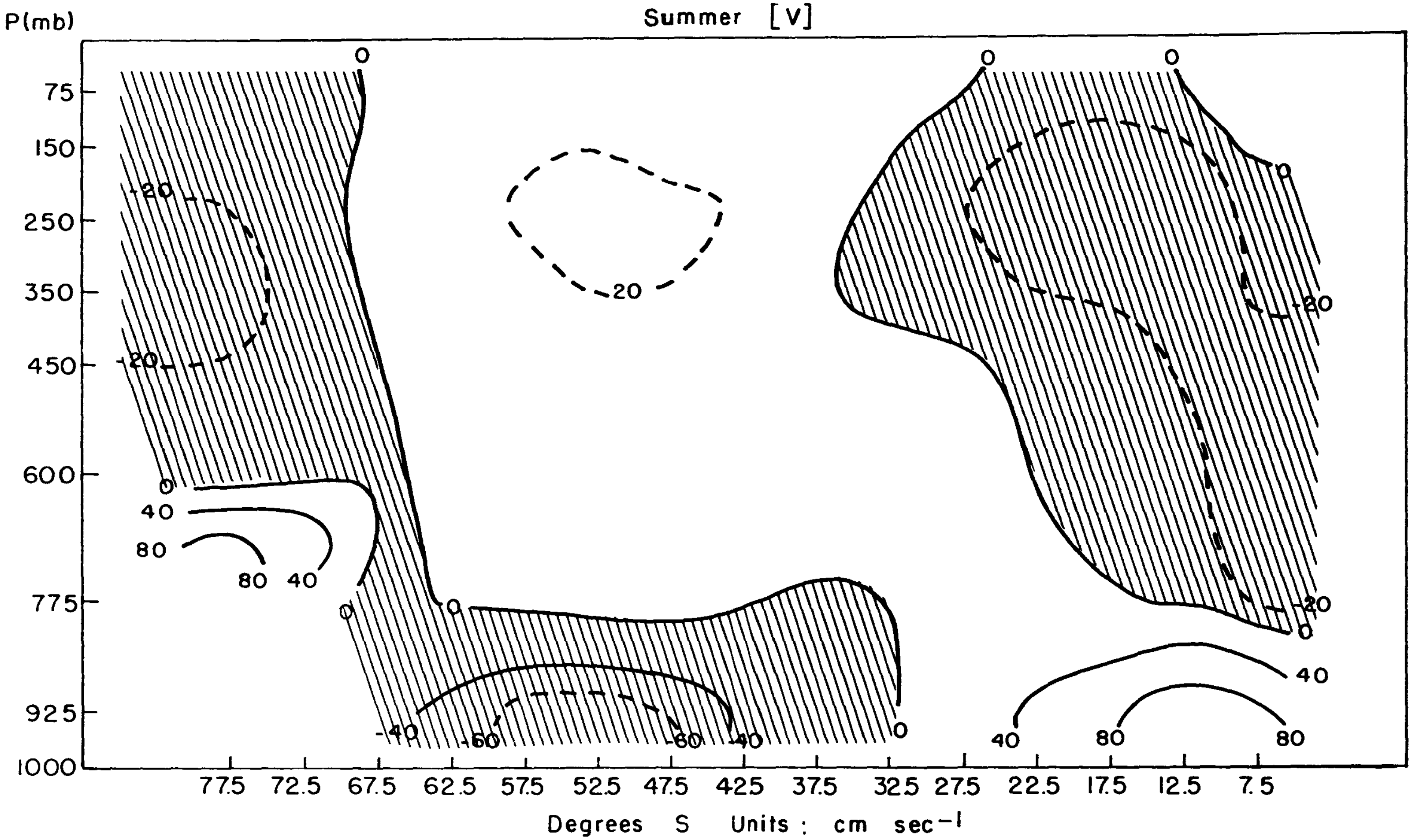

Summer [V]
P(mb)
75
150
250
350
450
600
775
925
1000
77.5 72.5 67.5 62.5 57.5 52.5 47.5 42.5 37.5 32.5 27.5 22.5 17.5 12.5 7.5
Degrees S Units : cm sec^{-1}

Figure 5: $[\bar{V}]$ from Obasi

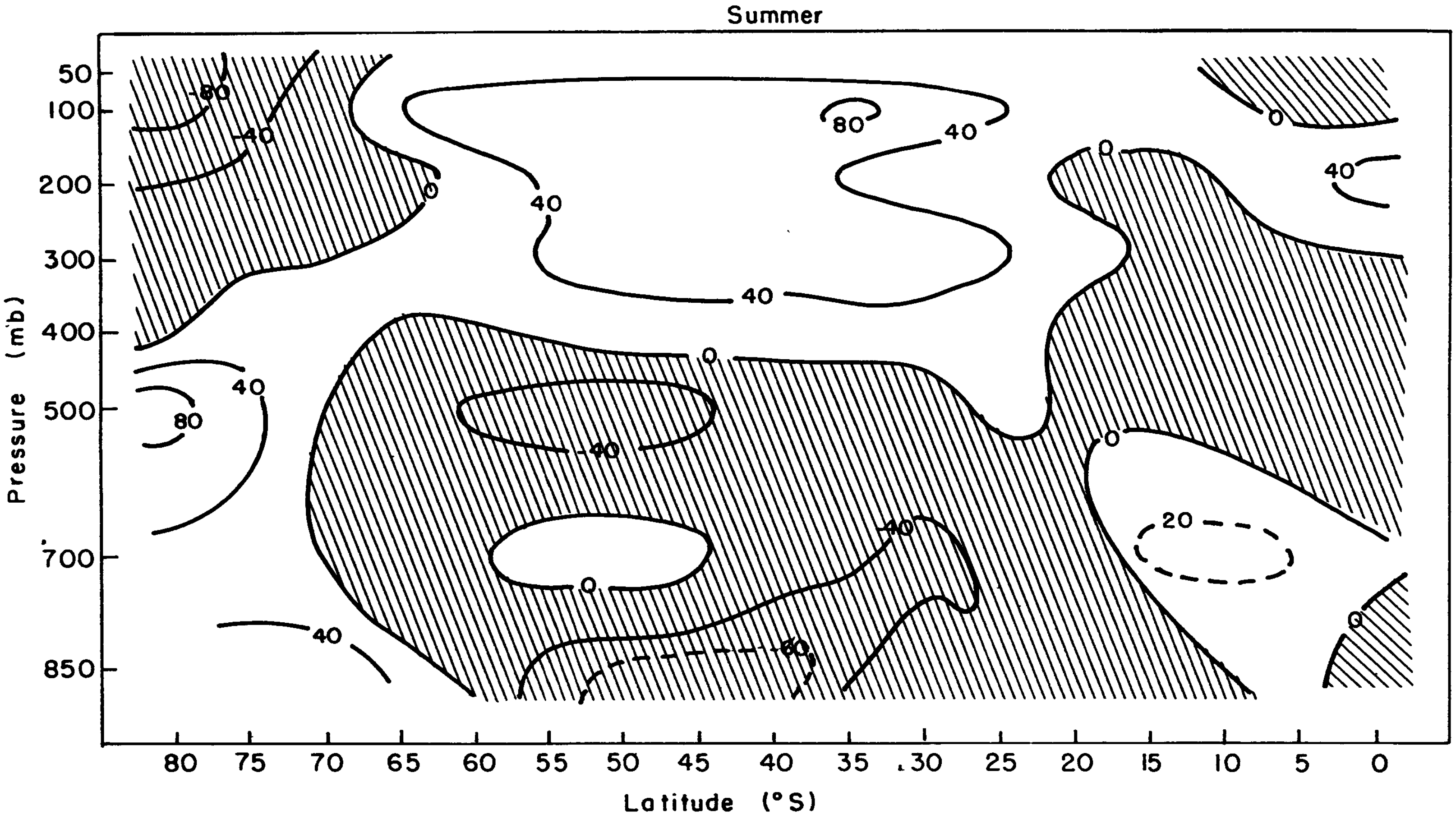


Figure 6:

Winter $[\bar{\omega}]$ From Characteristic Curves

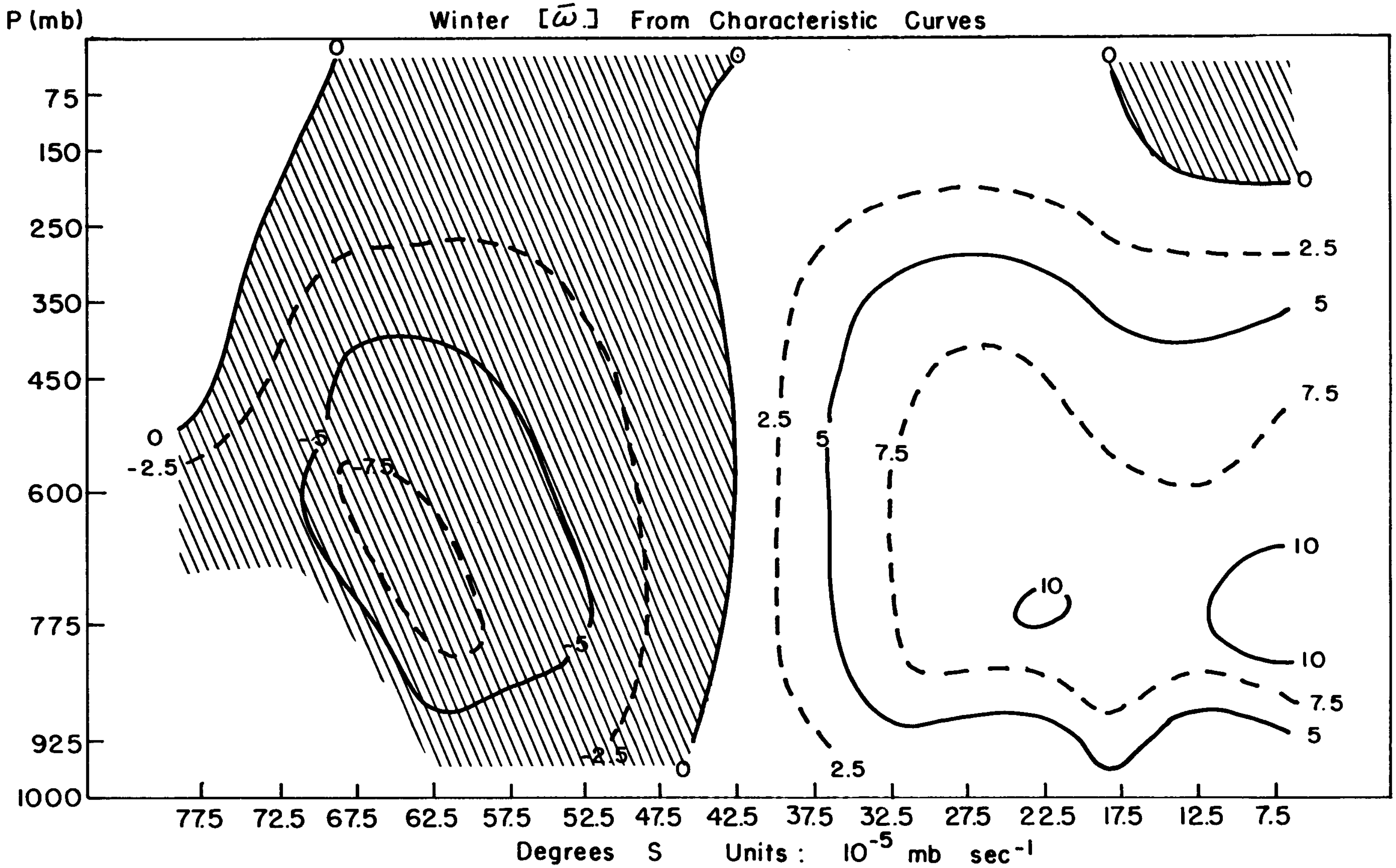


Figure 7:

Winter $[\bar{\omega}]$ From Directly Measured $[\bar{V}]$ (Obasi)

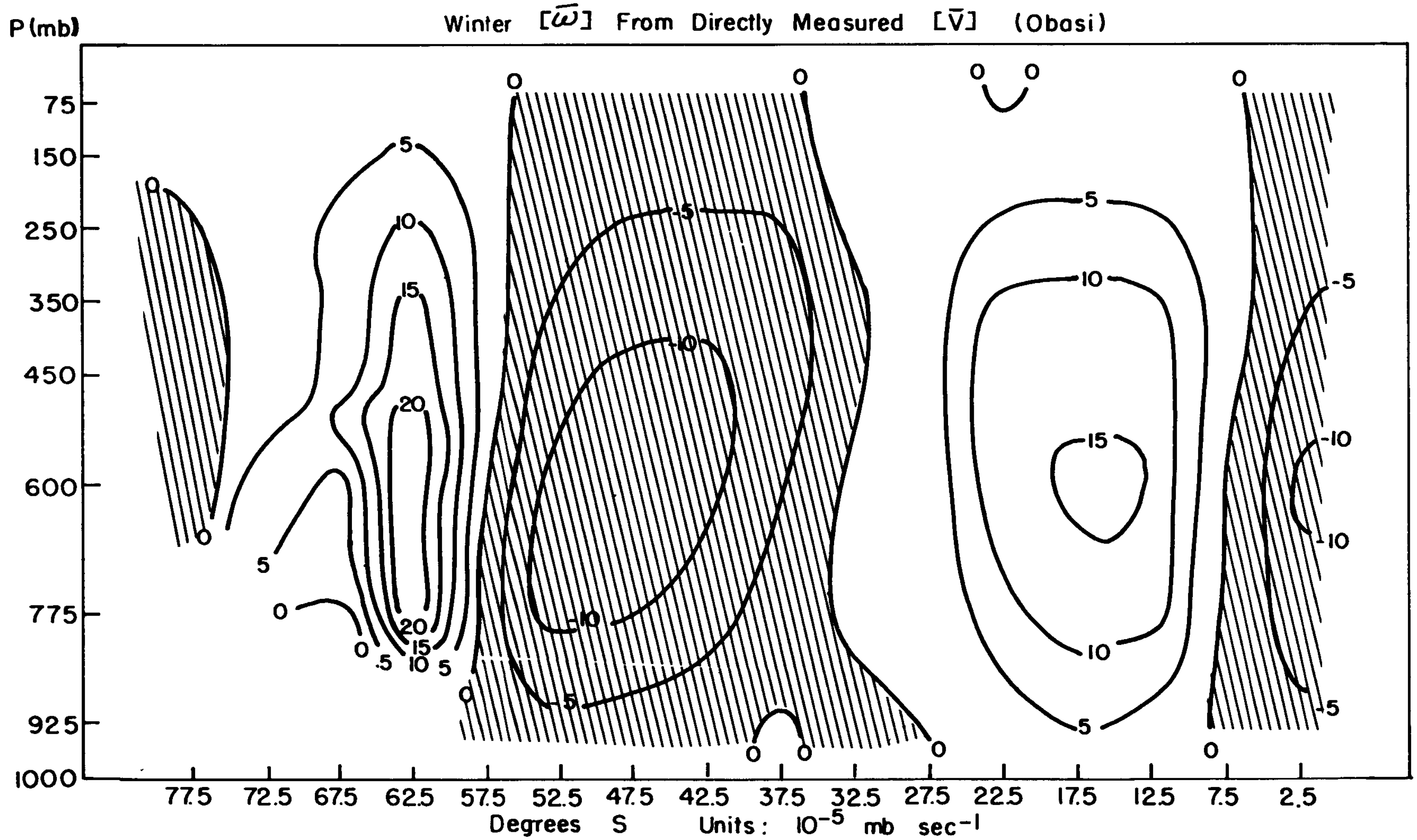


Figure 8:

Summer $[\overline{\omega}]$ From Characteristic Curves

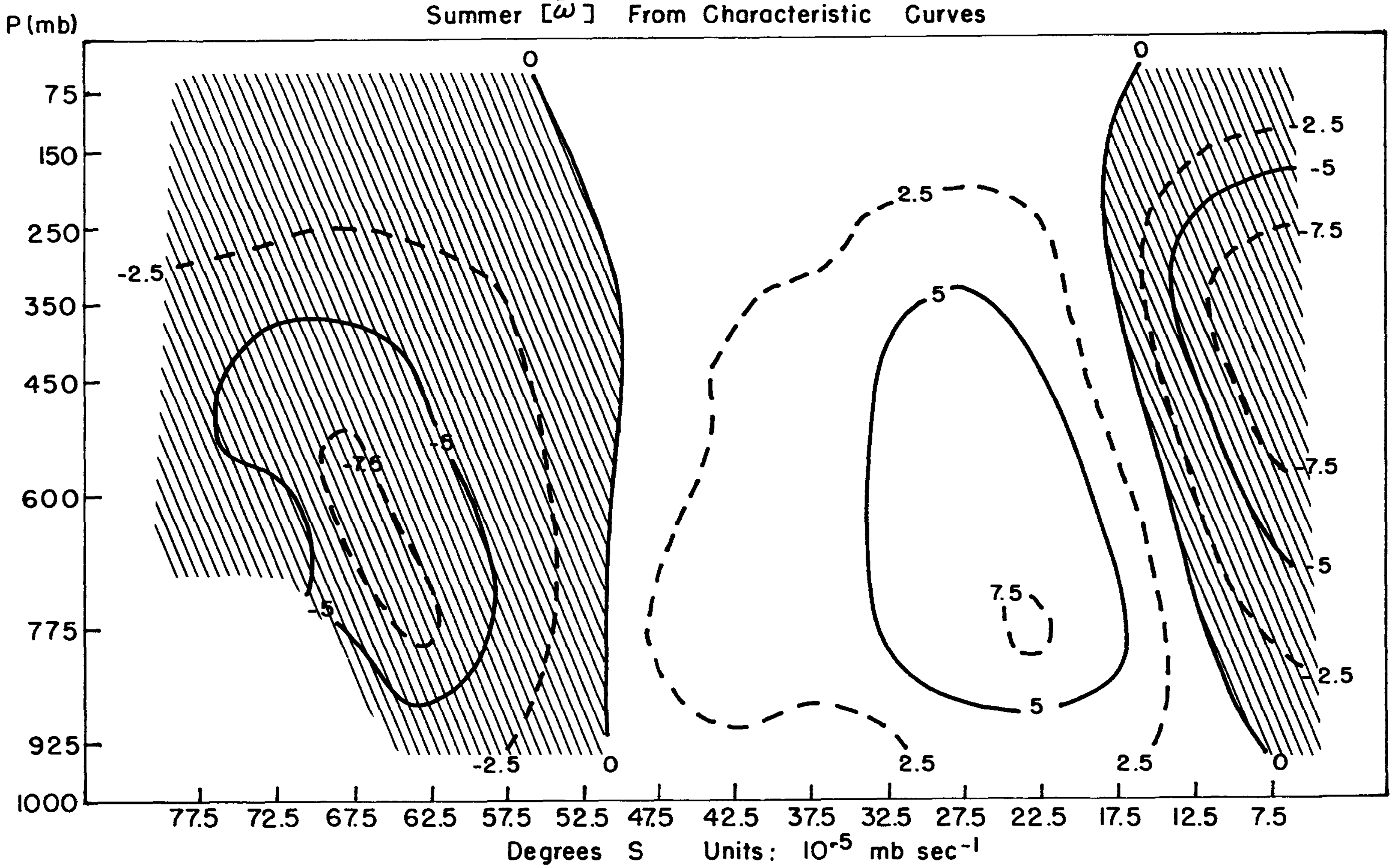


Figure 9:

Summer $[\bar{\omega}]$ From Directly Measured $[\bar{V}]$ (Obasi)

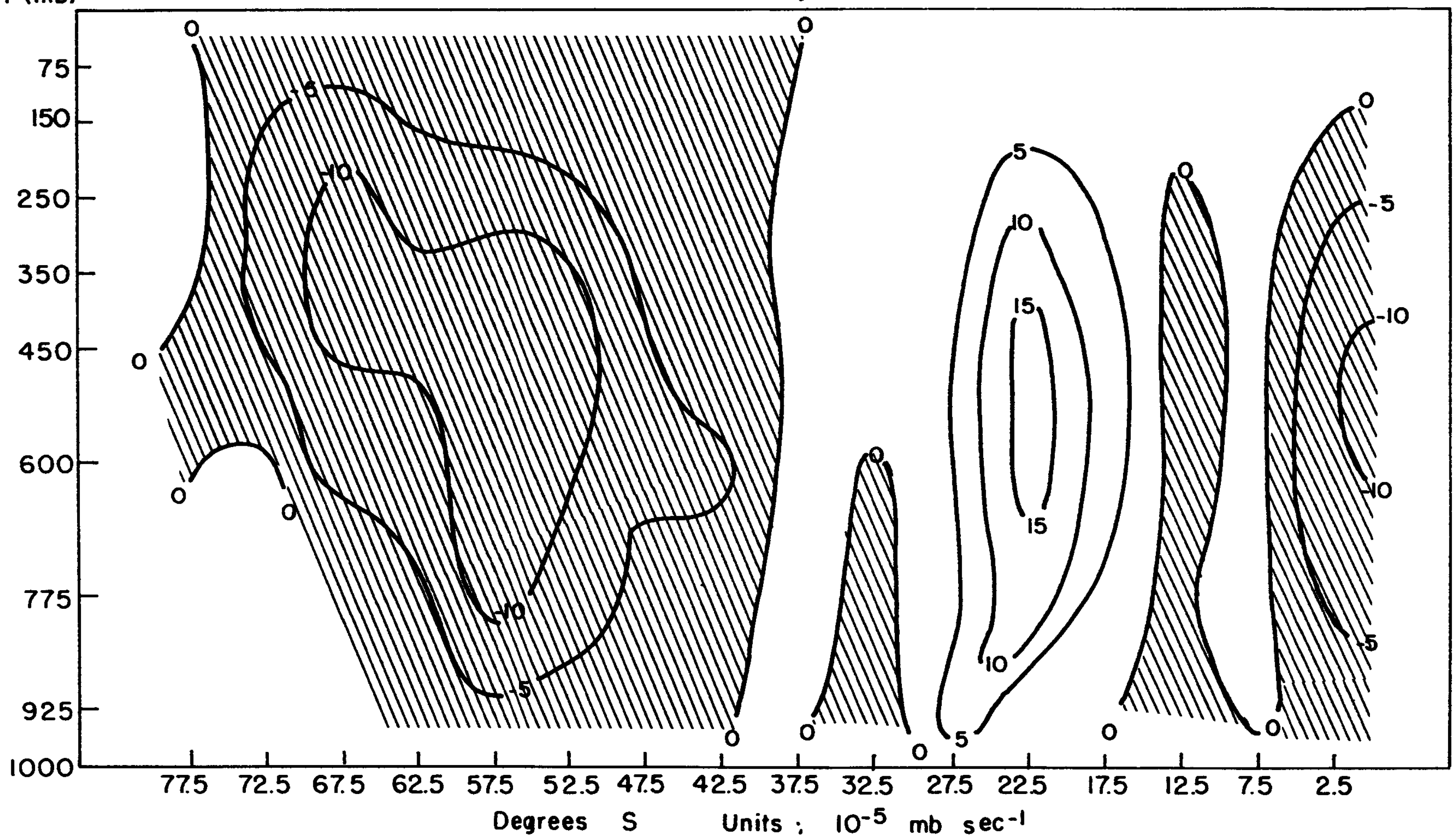
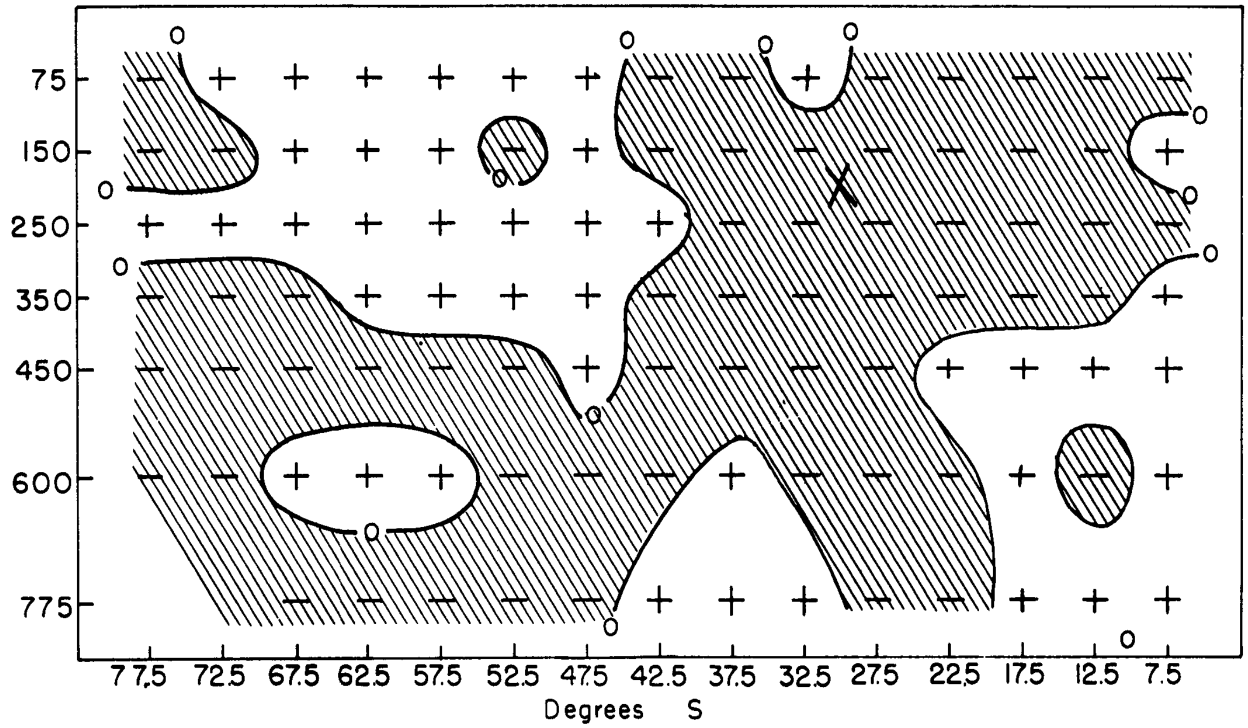
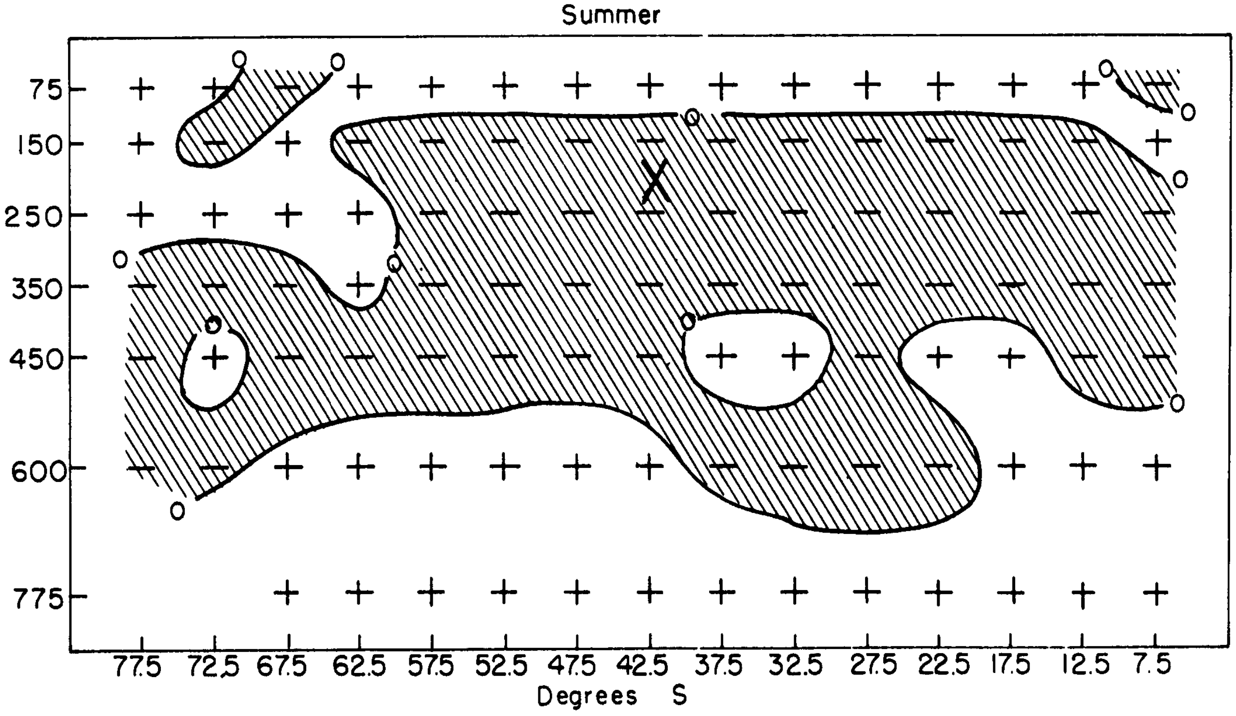


Figure 10

Winter

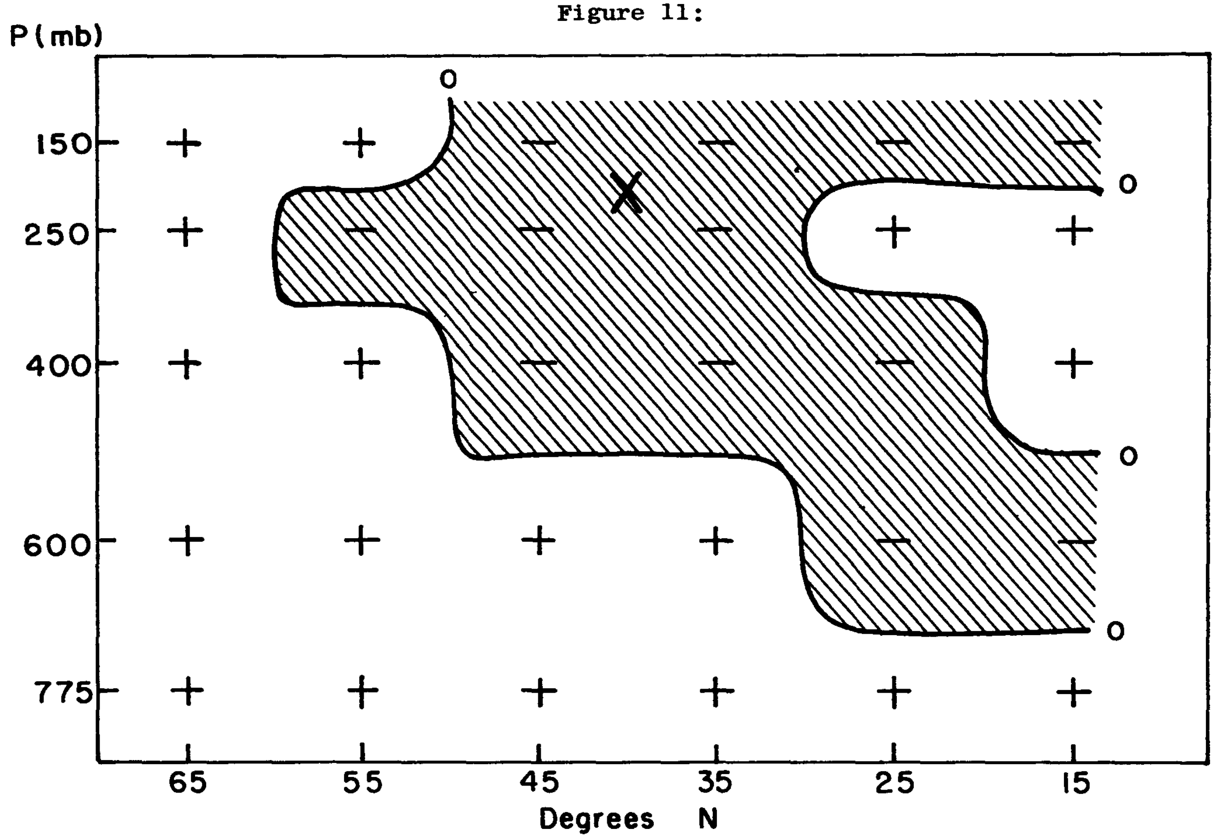


Summer



Eddy Viscosity Signs — X - Jet stream Maximum

Figure 11:



Sign of Eddy Viscosity, Northern Hemisphere

X — Jet Stream Maximum

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