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HULL-WHITE'S VALUE AT RISK MODEL: CASE STUDY OF BALTIC EQUITIES MARKET

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Abstract. Analysis of the applicability of the Hull and White (FHS) model on the Baltic equities market has not been the subject of significant research, especially not in the context of meeting the Basel Committee backtesting rules. The paper discusses the applicability of different variants of this model, in order to answer the question whether any variants (and which of them) of the model can be used in these markets in the context of the Basel II and III standards. The survey results show that 1) there isn't an optimal variant of this model, but that risk managers have to keep in mind stylized facts of financial returns when they specify the FHS model; 2) according to different criteria of the validity of the model (Basel II and III standards) different variants of models are differently ranked, which suggests that selection of a suitable model implies the use of a large number of different criteria, the model validity and loss function, especially those who take care of the size of tail loss and ES.

Keywords: value at risk, historical simulation, Hull-White model, volatility, GARCH, emerging markets, market risk.

JEL Classification: G24, C22, C52, C53.

Introduction

Although there is a widespread agreement about the use of VaR as the general measure of market risk and the economic loss that banks and other financial institutions may suffer due to exposure to the market risk (Radivojevic *et al.* 2016a), there is no agreement on the best model to estimate VaR. All VaR models are based on assumptions which represent a compromise between the efficiency of implementation, on the one hand, and the statistical precision of market risk estimates on the other. The validity of VaR estimation depends on a degree of compatibility between the characteristics of the real market and the assumptions on which the models are based. Essentially, there are two different groups of VaR models: parametric and nonparametric VaR models. The first one is based on the assumption that returns follow a theoretical distribution, while the second is based on the idea that the movement of the risk factors in previous periods contains all the necessary information to estimate VaR. A common feature of these models is their inability to be simultaneously effective in capturing kurtosis and fat tails and strong time-varying volatility, and to be easy to implement at the same time. The most famous a nonparametric model is the standard historical simulation model (further marked as the HS). More specifically, all nonparametric and semiparametric models are derived from the HS model. The HS model is the simplest and according to the research conducted by Perignon and Smith (2010), the most commonly used VaR model in banks and other financial institutions (about 73%).

Since the choice of the optimal model represents the decision between the efficiency of implementation and the statistic reliability of the model, in that context the HS model represents the optimal choice for the estimation of market risk when fulfilling assumptions on which the model is based. However, when assumptions of the model are not satisfied, its VaR estimates are inadequate and, at best, it can only provide unconditional coverage of market risk. The main drawback of the HS model is its inability to capture time-varying volatility, i.e. conditional heteroscedasticity, because it is based on the assumption of stationarity, (based) on identical and independent return distribution (IID). The only source of dynamism in this model stems from the usage of the moving window approach. However, in reality it is an insufficient source of conditionality (Zikovic, Prohaska 2010). In order to eliminate this drawback, many authors have been working on its improvement. These efforts have resulted in a number of models. Some of the most famous models derived from the HS model are: the BRW model proposed by Boudoukh *et al.* (1998), filtered historical simulation proposed by Barone-Adesi *et al.* (1998), the Mirrored historical simulation (MHS) proposed by Holton (1998), the Hybrid historical simulation model (HHS) proposed by Zikovic (2010), Dynamic historical simulation (DHS) proposed by Bee (2012), new Hybrid historical simulation model proposed by Radivojevic *et al.* (2016a), Bootstrap historical simulation proposed by Radivojevic *et al.* (2017), etc. The common feature of these models is that they do not behave as it would be expected based on the procedures behind the models (see Stancic *et al.* 2013); they are computationally very intensive because they require a relatively large number of parameters that cannot be solved in a closed, analytical form and can result in negative values, where both problems have a negative influence on the maximum estimated likelihood. On the other hand, the numerous empirical researches (Zikovic 2007; Diamandis *et al.* 2011; Şener *et al.* 2012; Rossignolo *et al.* 2012, 2013; Cui *et al.* 2013; Louzis *et al.* 2014; Del Brio *et al.* 2014) show that filtered historical simulation which was proposed by Hull and White (further marked as FHS) performs better than the commonly used and the most popular VaR models at both markets (developed and emerging) in the context of meeting the backtesting rules of the Basel Committee. Theoretically, the FHS model represents improvement of the HS model, concerning the ability to capture time-varying volatility (conditional heteroscedasticity), without a significant increase in computational complexity. Therefore, in terms of trade-off between efficiency of implementation and statistical precision, the model presents an adequate solution for the estimation of market risk.

The possibility of achieving higher risk premiums compared to the EU average (Eurostat 2016) and low correlation coefficients with developed financial markets of OECD (Deltuvaitė 2016; Maneschiöld 2006; Nielsson 2007; Nikkinen *et al.* 2012) have turned financial markets of the Baltic countries into interesting investment alternatives in the process of international diversification of investments. During the last decade these financial markets have gone through major changes. Financial markets have been liberalized and there is an increasing number of foreign financial institutions operating there. The size of financial markets has grown significantly (especially in the period before the economic crisis, 2008 to 2011) with a clear domination of banks; their share in total financial institutions assets exceeds 80%. The rapid growth of the banking sector was driven to large extent by external capital flows from foreign parent banks to their Baltic subsidiaries. Entrance of foreign banks in the Baltic markets through privatization, greenfield investment or establishment of subsidiaries has increased competition in the sector (Darškuvienė *et al.* 2014). During economic crisis the Baltic countries managed to avoid large capital outflows, since than even steeply declining economies did not discourage international investors, which is best evidenced by the Net international investment position indicator. Further, they did not devalue their currencies, they controlled their public finances in the prudent way and this laid an attractive ground for foreigners to invest in the stock markets after the economic crisis.

Integration into NASDAQ OMX, i.e. the creation of a common Baltic equities market with harmonized trading rules, integration of infrastructures and market practices, the unique trading system, joint trading lists, single membership, trading and settlement currency (from 2015, when Lithuania joined in Eurozone) (Jacikevičius, Raos 2014), as well as the change of the government's fiscal policy and borrowing money have increased the demand for traded securities. Today, the Baltic countries show a positive investment outlook, where Latvia is ranked as the 9th most-promising emerging market, while Estonia takes the 2nd place and Lithuania is in the 4th place in the most-promising emerging markets (Khan 2015).

When European Commission adopted the Capital Adequacy Directive – CAD3, the Basel Accord has become obligatory for all the EU countries. This is how the banks operating in the Baltic countries were given the opportunity to estimate the capital adequacy for covering market risks, using the VaR model. When investing in emerging markets, it is customary for the banks to use the models for estimating the market risk same as in developed financial markets. This means that they assume that these markets behave in the same or the similar way as the developed financial markets of EU member states. However, the specific characteristics of Baltic equities market, such as high volatility and the presence of volatility clusters, limit the applicability of VaR models. When we take into consideration the adequacy of the FHS model, in terms of effective implementation and accuracy of assessment models on one side and the fact that its applicability has not been the subject of significant research in Baltic equities market, on the other, we can define the aim of the paper: to examine its applicability on capital markets of these countries, in terms of meeting the backtesting rules of Basel Committee.

The paper is organized as follows: Section 1 contains an overview of empirical research which deals with applicability of the FHS model at emerging markets. Section 2 presents a theoretical background of the FHS model. Section 3 provides a brief description of the analyzed data and the used methodology. In the Section 4 there are presented, analyzed and discussed the backtesting results. The Monte Carlo testing technique was used for verification of the backtesting results. The final section summarizes the conclusions.

1. Literature review

Although there is an abundance of research papers that examine performances of the various HS model, a small number of these papers deal with the FHS model, especially in the emerging markets and frontier markets. The most important studies are conducted by Rossignolo *et al.* (2012, 2013), Louzis *et al.* (2014), Zikovic (2010) and Radivojevic *et al.* (2016b).

The first research regarding the applicability of the model was carried out by its creators. Their results confirm theoretical superiority of the model compared to other models of historical simulation. In a sample of twelve exchange rates in the period between 1988 and 1997, and 5 stock exchange indexes of shares in the period between 1988 and 1998, Hull and White (1998) proved that the procedure they had suggested assured significantly better assessment compared to the HS model and the time-weighted model.

Similar ideas concerning the FHS returns were presented by Barone-Adesi *et al.* (1998). Results of their research show that the FHS model represents an efficient choice for measuring portfolio market risk which contains financial derivatives. The results suggest that the model produces valid VaR assessments when the assessment is done for shorter holding periods and that it shows tendency to produce conservative assessments when they are conducted for longer holding periods (over 10 days). Results of the simulations done by Pritsker (2001) lead to a different conclusion. However, he warns that research results should be taken with caution when making conclusions about the model. He gives two reasons for this. The first reason lies in the fact that models for testing the validity of VaR models have little power to detect incorrect models because they are based on the determined number of exceedances. The second reason lies in the fact that model performances are under the influence of the mistakes made while calculating returns, specifications of the GARCH model, etc.

Rossignolo *et al.* (2012) studied the applicability of a wide range of different VaR models at six emerging markets (Brazil, Hungary, India, Czech Republic, Indonesia and Malaysia) and at four frontier markets (Argentina, Lithuania, Tunisia and Croatia). Among other models, they have studied the performance of the FHS model with GARCH and EGARCH specifications with Normal and Student-t distributions. They found that the model delivers marginal improvements only in emerging markets. Rossignolo *et al.* (2013), also, examined the performance of this model at the stock market of Portugal, Ireland, Greece and Spain, and found that the EGARCH technique brings no significant advantage over GARCH. Moreover, the density assumption exerts dominance over the particular specification: GARCH-t and EGARCH-t improve the performance of their Normal counterparts, therefore making the usage of heavy-tailed distributions are unavoidable. Their results suggest that regulators should discourage or prohibit usage of traditional VaR approaches, because they perform quite poorly in emerging markets in terms of meeting the backtesting rules of the Basel Committee. Louzis *et al.* (2014) tested the VaR forecasting performance of the FHS model by employing four commonly used asset classes, i.e. equities, foreign exchange, fixed income securities and commodities. For each of these categories is used a representative security. They choose the benchmark S&P500 stock index for equities, the EUR/USD exchange rate for FOREX, the 10 year Treasury-Note future for fixed income securities and Gold future prices for commodities. They point out that for the 10 year T-Note and Gold future the GJR-GARCH-FHS along with Realized GARCH-FHS and the GJR-GARCH-FHS/EVT along with Asym. HAR-FHS/EVT are the best models.

Radivojevic *et al.* (2016b) studied the performances of a semi-parametric VaR models at emerging markets of South-Eastern European countries during crisis. They found that the FHS model performs quite poorly at these markets.

According to our knowledge there are almost no researches which related to testing of applicability of VaR models on Baltic equities market. The exceptions are researches conducted by Zikovic (2010) and Jurgilas (2012). Zikovic (2010) studied widespread VaR models, such as the HS, the VCV and the RiskMetrics model. He found that these models do not capture the dynamics of the return series from the Baltic countries. He did not study applicability of the FHS model. Jurgilas (2012) examined performances of seventeen VaR models in the stock markets of the Baltic countries and found that APARCH (1, 1) VaR model with Student's t and skewed Student's t distribution assumption, as well as EVT VaR models with fat-tailed GARCH filter, were able to produce accurate VaR forecasts for Baltic stock markets during high volatility period between 2008 and 2011. He states that unconditional VaR models failed most of the time due to volatility clustering present in historical stock index returns. Also, he did not study applicability of the FHS model. According to our knowledge only Rossignolo *et al.* (2012) tested applicability of the FHS model at the Lithuania stock market. For that purpose they used OMX index. The research was conducted for a period of time between 2000 and 2007.

2. Theoretical basis of the FHS model

In order to resolve the main drawbacks of the HS model, Hull and White (1998) came up with an idea to combine the HS model with parametric VaR models. What they noticed is that when distribution of market risk factor returns is scaled using estimated volatility, it is often approximately stationary (Hull, White 1998). This suggests that the HS model can be improved by taking into account changes in volatility which occurred during the period covered by historical data. If current volatility of basic market risk factor returns is high, it can be expected that tomorrow's portfolio returns will be large due to the tendency of volatility clusters. However, if historical volatility is lower compared to the current volatility, historical returns will underestimate returns that are to be expected tomorrow (in the future?). The situation is reverse in the case when historical volatility is larger than current volatility. This is why historical returns, for each risk factor, need to be adjusted to the ratio between current and historical volatility.

While presenting these ideas Hull and White (1998) were actually promoting the new VaR model, the basis of which is the HS model. Basically, the model is a combination of the HS model and the parametric VaR model, based on GARCH, i.e. EWMA approach for evaluation of conditional volatility. So this model is, actually, a semiparametric VaR model.

The main idea on which the model is based is that the GARCH(p,q) model or the EWMA model is issued in combination with the HS, in order to adjust historical returns to volatility changes that have happened recently. Volatility-weighted returns are gained by the application of the following formula, under the assumption that the probability distribution for $r_{t,i}/\sigma_{t,i}$ is stationary:

$$r_t^* = \sigma_T \frac{r_t}{\sigma_t}, \tag{1}$$

where: r_t^* – volatility weighted return, r_t – historical return, σ_T – predicted volatility at a certain moment (T) obtained in the future by the GARCH(p,q) /EWMAmodel, σ_t – assessment of volatility at a certain moment (t) obtained by the GARCH(p,q) / EWMA model.

Assessment of volatility by the GARCH(p,q) model is done by the following form:

$$\sigma_{t+1}^2 = \omega + \sum_{i=1}^{p} \alpha_i r_{t+1-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t+1-j}^2, \tag{2}$$

or by the application of the EWMA model:

$$\sigma_t = \sqrt{\frac{1-\lambda}{1-\lambda^N} \sum_{n=1}^{N} \lambda^{n-1} r_{it-n}^2}, \tag{3}$$

with the covariance:

$$\sigma_{ij} = \frac{1-\lambda}{1-\lambda^N} \sum_{n=1}^{N} \lambda^{n-1} r_{it-n} r_{jt-n}, \tag{4}$$

where: σ_{ij} – covariance of *i.* and *j.* rate of return, r_{ij} – rate of return of the *i.* and *j.* portfolio risk factor, N – total number of observations, λ – decline factor and α_i, β_j, ω are GARCH parameters.

Returns obtained this way will be larger or smaller than the real historical returns, depending on the current volatility. Adjusting historical returns to volatility changes between historical and current volatility gives conditional, normalized returns which meet the IID criteria. This is how historical returns become suitable for historical simulation. Further evaluation of the VaR is the same as in the case of the HS model. VaR evaluation comes down to simple determination of values $(N+1)cl$ member of the arranged set of data on portfolio returns that are volatility-weighted, which can be presented as follows:

$$VaR^{cl}_{N=1|N} \equiv r^*_w((N+1)cl), \tag{5}$$

where: $r^*_w((N+1)cl)$ taken from the arranged set of returns $\{r^*_w(1), r^*_w(2)\ldots r^*_w(N)\}$. If $(N+1)cl$ is not an integer, than the rule of interpolation is applied between two adjacent observations.

The model represents direct extension of the HS model. The only difference is that filtered information is used here instead of raw data for describing current portfolio risk. This is why, in literature, the model is often referred to as the Filtered Historical Simulation model (FHS).

In theory, the basic advantage of this model is in the fact that it can capture time-varying volatility (of conditional heteroscedasticity) without setting the assumption connected to the choice of theoretical distribution. This is why the model represents the improved version of the HS model and the parametric VaR models. Model does not make the assumptions of normality or serial independence. However, relaxing these assumptions also implies that the model does not easily accommodate translations between multiple percentiles and holding periods.

3. Empirical research of applicability of the FHS model

The data used in this paper consist of the daily logarithmic returns of the stock indexes from the Baltic equities market (OMX Vilnius in Lithuania, OMX Riga in Latvia and OMX Tallinn in Estonia). All indexes are market capitalization indexes and include all shares traded on respective stock exchanges and represent market portfolios of these countries. The returns are collected for the period between January 2013 and January 2016. This period was selected primarily from the need to work with the latest data, so that the research could be beneficial for risk managers in banks and other financial institutions that operate in this market. The length of the sampling period is chosen bearing in mind the following: 1) backtesting rules of the Basel Committee, according to Basel II Accord as well as the recent proposal of the Basel Committee (Basel Committee of Banking Supervision 2014); 2) requirements for the efficient use of the FHS model. Namely, the FHS model, and all non-parametric models in general, which are derived from the HS, are based on the assumption of a constant returns distribution (IID assumption). Hence, a more efficient application of these models means optimal balance between a long period of aging, which potentially has disruptive influence on the assumption of constant returns distribution and a short one, which leads to the minimization of statistical significance of prediction models. In other words, on one side, it is necessary to choose such a sampling period which will adequately represent the distribution of future returns of the portfolio, which is equivalent to the assumption that the returns distribution is constant. On the other side, it is necessary to provide sufficient data to obtain a statistically significant estimate of quantile distribution, which poses a risk of breach of the presumption of constant distribution. Hence, the inclusion of data before the economic crisis is not advisable due to the high economic growth, which was accompanied by high returns on the stock market and low coefficient of correlation with the developed countries. The same stands for the data during the economic crisis because of the sharp decline in activity, an increase in volatility and stock returns' correlations between Baltic OECD countries and developed countries (Kuusk *et al.* 2011; Nikkinen *et al.* 2012; Brännäs *et al.* 2012; Babalos *et al.* 2014).

Daily returns were used, because the GARCH effects at lower frequencies are not so apparent. The daily returns of selected stocks indexes are generated using the logarithmic approximation:

$$r_{i,t} = \log\left(\frac{P_{i,t}}{P_{i,t-1}}\right), \tag{6}$$

where: $P_{i,t}$ – represents the closing value of indexes i on the day (t).

The VaR estimation was made for the confidence levels of 99 and 95%. Confidence levels were chosen taking the Basel II Accord into consideration as well as the basic characteristics of the VaR calculation. Since VaR does not fulfil all the characteristics of the coherent risk measures (see Artzner *et al.* 1999), Basel Committee recently proposed fundamental changes in the regulatory treatment of financial institutions′ trading book positions (Kellner, Rösch 2016). Among others, a replacement of 99% VaR by 97.5% Expected Shortfall (ES) for the quantification of market risk is recommended. This is why besides the VaR the ES estimate is also conducted in the paper. The ES estimations were calculated, also, for a one-day ahead horizon for the confidence level of 97.5%. According to the Basel Committee (2014), this confidence level provides a broadly similar level of risk capture as the existing 99% VaR threshold, while providing a number of benefits, including a generally more stable model output and often less sensitivity to extreme outlier observations.

To secure the same out-of-the-sample VaR backtesting period for all the selected indexes the out-of-the-sample data sets were formed by taking out 244 of the latest observations for each indexes.

In the beginning, the basic characteristics of the distribution of daily logarithmic returns of selected indexes were analyzed, in order to determine the degree of the compatibility between the characteristics of these markets and the assumptions of nonparametric models. Table 1 summarizes the basic statistical characteristics of the return series.

The analysis of return series shows that all index return distributions have fat tails and that they are skewed. These characteristics are suitable for the HS model. In other words, various models of HS are effective in capturing these characteristics, but they are not successful in capturing heteroscedasticity. They react slowly to changes in the market and they are subject to predictable jumps in their forecasts of volatility. The results of the Lagrange Multiplier test confirm that there is the ARCH effect in all returns, except in series of Riga stock index returns. From the aspect of the risk management process this points the necessity of using techniques for modeling conditional volatility. The indexes have a relatively high difference between the minimum and maximum values of returns. This information suggests caution when using the HS model, since it suffers from "ghost effects", the situation in which extreme loss causes VaR estimations

Table 1. Descriptive statistics of selected markets

	OMX Vilnius	OMX Riga	OMX Tallinn
Mean	0.0003	0.0005	0.0004
Standard dev.	0.0062	0.0101	0.0051
Kurtosis	10.6815	33.0665	13.3449
Skewness	–0.8029	2.3783	–0.7008
Range	0.0767	0.1748	0.0675
Min. values	–0.0516	–0.0588	–0.0384
Max. values	0.0251	0.1160	0.0291
No. Obs.	753	740	746
Jarque-Bera test (p-value)	1932.2 (0.0000)	28570.7 (0.0000)	3387.5 (0.0000)
Lagrange multiplier (p-value)	54.55662 (1.51E-13)	1.2296 (0.2674)	30.918 (2.6E-08)

Source: authors' calculations.

to be extremely high and thus the model conducts satisfactory unconditional coverage, without taking into account the actual level of risk. Whether its model produces conservative VaR estimations is equally important to the bank as it is whether it underestimates the actual level of risk, because in both cases is required higher capital load and this has a negative effect on the bank's profitability. The standard deviations are also relatively high, which indicates a relatively high degree of fluctuation in the value of the daily returns. The kurtosis estimations are ranging from 10.6815, in the case of the OMXV to 33.0665, in the case of the OMXR and show that daily returns are not normally distributed, but indexes have a significant leptokurtosis. Excess kurtosis is especially high in Latvia, indicating highly peaked return distribution and very fat tails. Coefficients of skewness of all indexes are significantly different from zero. That means that the indexes have asymmetric returns distribution. Riga stock index have a positive skewness, while the rest exhibit negative skewness. Positive skewness in combination with high kurtosis is a clear signal to investors that in this market there is a greater probability of achieving extreme positive returns. The opposite was the case in the two other markets. Previously presented data clearly indicate that returns are not normally distributed, which confirms the value of the Jarque-Bera test.

Since many studies show that it is enough to use one day lag in order to adequately capture conditional volatility using the GARCH model, the VaR estimates were obtained by using the FHS-GARCH (1,1) with Normal, FHS-GARCH (1,1)-Student-t distributions and FHS-nonlinear GARCH (1,1) with Student's t return distributions (further marked as nFHS, TFHS and NFHS, respectively). The GARCH model with Normal, Student-t distributions and nonlinear GARCH model with Student's t return distributions were chosen bearing in mind above-mentioned Rossignolo's (2012) study as well as the fact that the GARCH model is the simplest model able to capture the volatility of clusters and leptokurtosis in the data (see Franses, van Dijk 1996; Johnson 2001). The parameters of the GARCH (1,1) model were rated by maximum likelihood estimation and Quasi-maximum likelihood estimation. The estimated parameters of volatility models for each of the selected stock indexes are given in Table 2. All the estimated parameters are statistically significant. In order to test whether our models were correctly specified, we employed χ2 goodness- to-fit test. p – value for χ2 goodness- to-fit test are reported in parentheses.

Table 2. Estimates of the parameters of GARCH (1,1) models

	OMXV	OMXR	OMXT
Parameters *of GARCH*(1,1)			
α	0.0499	0.0743	0.0499
β	0.9025	0.8857	0.9025
ω	0.000002	0.000005	0.000001
χ2(99) goodness to fit test	105 (0.3208)	117 (0.1046)	111 (0.1928)
Parameters of *t*(*d*)–*GARCH*(1,1)			
α	0.0371	0.0499	0.0568
β	0.9627	0.9299	0.6934
ω	0.000	0.000	0.000
υ	4.2541	4.004	3.1733
χ2(99) goodness to fit test	93 (0.6509)	101 (0.4252)	81 (0.9062)
Parameters of *t*(*d*)–*NGARCH*(1,1)			
α	0.0103	0.4795	0.0568
β	0.9872	0.5098	0.6989
ω	3.90E-06	1.6E-06	4.8E-06
υ	4.2500	4.000	3.1860
θ	0.0953	0.0990	0.9500
χ2(99) goodness to fit test	62 (0.9987)	117.1 (0.1035)	57 (0.9998)

Notes: All estimated parameters are statistically significant. In order to test whether our models were correctly specified, we employed χ2 goodness to fit test. In parentheses the p – value for χ2 goodness to fit test are reported. θ is positive and it reflects the leverage effect, signifying that negative returns increase future volatility by a larger amount than positive returns of the same magnitude.
Source: authors' calculations.

4. Backtesting results

In this section the backtesting results for tested models are presented, analyzed and discussed. Kresta and Tichy (2016) point out that those in the banks who decide on which model is the best one need to look at numerous criteria. However, regardless of that, suitability of any risk model can be assessed by the so-called backtesting procedure (Berkowitz *et al.* 2011). According to Basel II Accord, banks that meet the requirements to use the VaR model are, according to the backtesting rules of the Basel Committee, obliged to prove that their models are correct. Since banks are, above all, interested in meeting regulatory requirements (see Goorbergh, Vaar 1999), it is accepted in literature that the applicability of a model is tested in accordance with the backtesting rules of the Basel Committee, but instead of the one-side unconditional test (the so-called traffic light approach), a two-side unconditional test is used, the Kupiec's unconditional coverage test. The reason for using this test lies in the fact that unlike the traffic light approach it takes into account a larger and a smaller number of exceedings compared to the expected. It is very important for the bank whether the model underestimates the actual market risk but it is also very important whether it overestimates it, because in that case the bank makes unnecessary allocations of additional capital, which has a negative effect on its profitability. This is why the Kupiec's test is more appropriate for the bank. The Kupiec's backtesting results are shown in Table 3.

Table 3. Results of Kupiec's unconditional coverage test

	nFHS VaR 99%			nFHS VaR 95%			TFHS VaR 99%		
Stock index	No. of breaks	Critical value of LR_{uc} test	p-value	No. of breaks	Critical value of LR_{uc} test	p-value	No. of breaks	Critical value of LR_{uc} test	p-value
OMXV	4	0.733	0.392	10	0.628	0.428	4	0.733	0.392
OMXR	3	0.140	0.708	4	7.489	0.006	2	0.071	0.789
OMXT	2	0.093	0.761	14	0.237	0.626	3	0.112	0.738
	TFHS VaR 95%			NFHS VaR 99%			NFHS VaR 95%		
Stock index	No. of breaks	Critical value of LR_{uc} test	p-value	No. of breaks	Critical value of LR_{uc} test	p-value	No. of breaks	Critical value of LR_{uc} test	p-value
OMXV	10	0.628	0.428	3	0.083	0.773	8	2.058	0.151
OMXR	7	2.563	0.109	3	0.083	0.773	7	2.563	0.109
OMXT	10	0.425	0.514	1	1.128	0.288	11	0.150	0.699

Notes: The test was used at 5% significance level. The number of VaR breaks over backtesting period are given in the frst column.
Source: authors' calculations.

As can be seen from Table 3, the worst performer according to Kupiec's test is the FHS model. The model did not satisfy this test in the case of the OMXR index, at a confidence level of 95%. Other two models satisfied this test in all of the markets for both confidence levels. Theoretically speaking, the extreme losses, that occurre prior to and during the backtesting period, can cause high VaR estimates and this way unconditional risk coverage is automatically achieved without taking into consideration the actual market risk (Radivojevic *et al.* 2016a). This is why, for those markets in which the models passed the Kupiec's test, it is necessary to employ the Christofferson's test of conditional coverage, noting that because of the imperfections of this test, it is possible that the model which did not satisfy the unconditional coverage test, passed the test of conditional coverage, which does not mean that the VaR estimate is valid. The Kupiec's test does not take into account the independence property of VaR breaks, but only the number of exceedings. The consequence of the exposure to the series of consecutive VaR breaks (clusters) can be just as problematic as the systematic incomplete reporting on exposure to market risks (Radivojevic *et al.* 2016b). The risk of bankruptcy is considerably greater (higher?) than in the situation in which VaR breaks are evenly distributed over time. Therefore, the good VaR model needs to satisfy both properties. This is why, in literature, when the applicability of a VaR model is being tested, the Christofferson's test is used besides the Kupiec's test.

The results of the Christoffersen's test results are given in Table 4.

Table 4. Christoffersen's conditional coverage test backtesting results

	nFHS VaR 99%		nFHS VaR 95%		TFHS VaR 99%	
Stock index	Critical value of LR_{uc} test	p-value	Critical value of LR_{uc} test	p-value	Critical value of LR_{uc} test	p-value
OMXV	4.871*	0.088	4.484**	0.106	4.871*	0.088
OMXR	0.140	0.932	7.489	0.024	0.071	0.965
OMXT	0.093	0.955	0.237	0.888	0.112	0.946
	TFHS VaR 95%		NFHS VaR 99%		NFHS VaR 95%	
Stock index	Critical value of LR_{uc} test	p-value	Critical value of LR_{uc} test	p-value	Critical value of LR_{uc} test	p-value
OMXV	4.484	0.106	0.083	0.959	2.058	0.357
OMXR	2.563	0.278	0.083	0.959	2.563	0.278
OMXT	0.425	0.809	1.130	0.568	0.150	0.928

Notes: In the cases where there are no cluster VaR breaks, an alternative formula was used in the paper to calculate the first-order Markov likelihood (see Brandolini, Colucci 2013). *One cluster VaR breaks. ** Two clusters VaR breaks. The test was used at 5% significance level.
Source: authors' calculations.

As can be seen form Table 4, the model has passed Christoffersen's test in all the markets. The exception is in the case of the OMXR index at a confidence level of 95%. However, valid conclusion cannot be made based only on these tests, since the main drawback of these tests is that they have a questionable statistical power when applied to finite samples.

Both of these tests are developed using asymptotic arguments, which can create difficulties when applied to the sample size defined by the Basle Accord. The studies, conducted by Christoffersen and Pelletier (2004) and Hurlin *et al.* (2008) have shown that when the number of VaR breaks is small, there are substantial differences between asymptotic probability distributions of the considered tests and their finite sample analogues. Therefore in case of a small sample size it is better to rely on Monte Carlo simulated p-values rather than on those from the $\chi 2$ distribution.

The differences between the finite sample critical values and the asymptotic critical values for the Kupiec's and the Christoffersen's test are given in Table A1[1] Dufour (2006) Monte Carlo testing technique is used for that purpose, which allowed us to obtain the null distribution of tests statistics in finite sample setting.

Following Dufour (2006) Monte Carlo test procedure: first, 9999 samples of random IID Bernoulli (p) variables were generated, where the sample size equals the actual sample. Based on these artificial samples, 9999 simulated Kupiec's tests (LR_{uc}) were calculated and named $\left\{L\tilde{R}_{uc}(i)\right\}_{i=1}^{9999}$. Finally, the simulated p-values were calculated as the share of simulated LR_{uc} values which are larger than the actually obtained LR_{uc} test value:

$$p-value=\frac{1}{10000}\left\{1+\sum_{i=1}^{9999} I\left(L\tilde{R}_{uc}(i)>LR_{uc}\right)\right\}, \tag{7}$$

where I(.) takes on the value one if the argument is true and zero otherwise.

The same procedure was repeated in the case of the Christoffersen's test, noting that the cases for which the tests were not feasible were rejected in the simulation. Average feasible rates of tests, in case of the Kupiec's tests, are 0.826 for 99% VaR and 0.881 for 95% VaR. In the case of the Christoffersen's test, average feasible rates of tests are 0.801 for 99% VaR and 0.837 for 95% VaR. The backtesting results based on the Monte Carlo p-values for both of these tests are given in Table 5.

Table 5. Backtesting results based on the Monte Carlo p-values

	Kupiec's unconditional coverage tests are					
	99% VaR			95% VaR		
	nFHS	TFHS	NFHS	nFHS	TFHS	NFHS
OMXV	0.105	0.401	0.226	0.056	0.049	0.097
OMXR	0.086	0.202	0.127	0.040	0.079	0.265
OMXT	0.127	0.153	0.110	0.008	0.201	0.140
	Christoffersen's conditional coverage test					
	99% VaR			95% VaR		
	nFHS	TFHS	NFHS	nFHS	TFHS	NFHS
OMXV	0.032	0.349	0.106	0.042	0.237	0.025
OMXR	0.130	0.286	0.056	0.088	0.301	0.067
OMXT	0.079	0.019	0.077	0.094	0.184	0.070

Note: Significante level of 5%.
Source: authors' calculations.

The Monte Carlo testing results suggest caution, especially in case of the nFHS model. According to the Monte Carlo testing results the best performer is the NFHS. This was expected bearing in mind the presence of the leverage effect at these markets. Relatively high coefficient of kurtosis and skewness, as well as degree of freedom estimates for Student's t distribution are a clear indication that the TFHS model is more appropriate for these models than nFHS, which is confirmed by the Monte Carlo testing results. Since more than one VaR model can be accepted, the problem of ranking the models

[1] The differences between the finite sample critical values and the asymptotic critical values for the Kupiec's and the Christoffersen's test are given in Table A1.

arises. Acceptable models can be ranked using their ability to forecast. For that purpose the modifed Blanco-Ihle loss function was used in the paper, proposed by Zikovic and Filer (2013), which compares the tail loss to ES while taking into account the relative size of the tail loss compared to the difference between the two. In our two-stage backtesting procedure, the best model has to primarily satisfy both the Kupiec and Christoffersen tests (for 99% VaR) and then provide superior tail loss forecasts, in the sense of minimizing error statistics. Rankings of models according to the modified Blanco-Ihle error statistics are presented in Table 6.

Table 6. Ranking of models according to modified Blanco-Ihle error statistic

	OMXV		OMXR		OMXT		
Model	Score	Rank	Score	Rank	Score	Rank	Total
nFHS	3.3854	N/A	4.4706	1	4.3910	2	2
TFHS	2.7457	1	8.6684	3	4.8245	N/A	3
NFHS	4.2770	2	5.5939	2	3.3003	1	1

Note: N / A model is not ranked because it did not pass both tests. Scores are based on 10000 Monte Carlo simulations of actual samples size for each of these markets.
Source: authors' calculations.

Results shown in Table 6 basically confirm the Monte Carlo test procedure results. The NFHS model is the best ranked model. However, it is surprising that the nFHS model is better ranked in total score than TFHS. First of all, it is a surprise that the TFHS model did not pass the Monte Carlo test procedure for the capital market of Estonia, considering the coefficients of kurtosis and skewness, and that the nFHS model did pass. The only explanation for this phenomenon can be looked for in the degree of freedom estimates. The nFHS model is also better ranked than the TFHS model on the capital market of Latvia. The explanation can be found in the fact that the ARCH effect was not recorded on this market, so that the conditional volatility model is not of such great importance in volatility prediction. Since the model of conditional volatility with normal returns distribution is simpler than the model with Student's t return distributions, in terms of parameters assessment, its assessments are more precise, and thereby its predictions of risks are better.

Based on the previously presented research results a general conclusion can be drawn that the FHS model can be applied on the Baltic equities market with great caution and considering the fact that the specification of the model must be compatible with characteristics of the specific market as well as the reasons for its use, because its specifications are ranked differently according to different criteria.

Conclusions

The contribution of this paper lies in the fact that there have been no significant research on the applicability of VaR models, especially the FHS model, on Baltic equities market in terms of meeting the backtesting rules of the Basel II Accord. Our findings suggest caution when using the model at the Baltic equities market in terms of meeting the backtesting rules of the Basel II Accord. Namely, the backtesting results (the results of the Kupiec's and the Christofferson's tests), which are verified by the Dufour Monte Carlo testing technique, suggest that the model can be reliably used at the Baltic equities market. More precisely, the results suggest a few findings: 1) due to leverage effects presence in historical stock index returns, the best performer is the variant of the model with nonlinear GARCH model with Student's t return distribution; 2) volatility models with Student's t distribution fit index return data better than those with normal return distribution; 3) the worst performer is the model with normal GARCH volatility model; 4) all variants of the model perform better at high than low confidence levels.

However, in the context of Berkowitz *et al.* (2011) statement that the choice of an adequate model for risk assessment involves testing of its applicability from the perspective of a large number of criteria, including additional criteria, the Blanco-Ihle loss function, the conclusions of researching are different. The results of the modified Blanco-Ihle error statistic suggest that the variant of the normal GARCH volatility model is more favourable for these markets than the one with Student's t return distribution.

These findings have clear indications for risk managers in banks and other financial institutions. This clearly indicates that there cannot be only one model that would be the best for all markets, for all periods and for all levels of trust. Although the use of a large number of different models increases the costs of risk management, on the other hand it reduces the risk of bank's bankruptcy due to the application of inadequate models for the assessment of market risk. Research findings also indicate that effective risk management involves the use of different criteria for assessing the validity of the model, especially evaluating the validity of the model in the context of the latest recommendations of the Basel Committee for the use of ES.

Certainly, when it is considered the acceptance of the findings of this research, one should bear in mind its limits, which primarily relates to the choice and length of the time series of data, the technique for testing the validity of the model, as well as a well-known problem in literature related to the assessment of parameters of the model of volatility used in this paper. The choice and length of the time series of data represent a compromise between the recent request and the backtesting rules of the Basel Committee, the requirements for the efficient use of the FHS model, and the need to work with the latest data, so that the research could be beneficial for risk managers in banks and other financial institutions that operate on this market, but also the fact that the inclusion of data before and during the economic crisis is not recommended due to the extreme volatility changes and the correlation coefficient of the return of shares in these markets with the markets of the developed countries of the OECD. Hence, the task for future researchers is to test the validity of the model for different time periods and lengths. Also, due to the imperfection of test techniques and the model of verification of their results, future researchers have to use a number of rigorous tests to verify the validity of this model in the markets of the Baltic countries.

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APPENDIX

Table A1. Differences between the finite sample critical value and the asymptotic critical value fot the LR_{uc} and the LR_{cc} test

	Significance levels		
	1%	5%	10%
		LR_{uc} Statistic	
Asymptotic χ2(1)	6.6348	3.8414	2.7055
Finite-sample	5.497	5.025	3.555
	(0.49%)	(9.49%)	(12.19%)
		LR_{cc} Statistic	
Asymptotic χ2(2)	9.21	5.9915	4.605
Finite-sample	6.007	5.015	5.005
	(0.20%)	(1.10%)	(11.79%)

Note: The finite sample critical values for the both test statistics for the lower 1 percent are based on 10000 Monte Carlo simulations of sample size T = 244. The percentages shown in the brackets represent quantiles that correspond to the asymptotic critical values under the finite sample distribution.

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