PHYSICAL REVIEW D **80,** 083508 (2009)

Zipping and unzipping of cosmic string loops in collision

H. Firouzjahi*
School of Physics, Institute for Research in Fundamental Sciences (IPM), P.O. Box 19395-5531, Tehran, Iran

J. Karouby†
Physics Department, McGill University, Montreal, H3A 2T8, Canada

S. Khosravi‡
Physics Department, Faculty of Science, Tarbiat Mo'alem University, Tehran, Iran
School of Astronomy, Institute for Research in Fundamental Sciences (IPM), Tehran, Iran

R. Brandenberger§
Physics Department, McGill University, Montreal, H3A 2T8, Canada
Theory Division, CERN, CH-1211 Genève, Switzerland
(Received 7 August 2009; published 7 October 2009)

In this paper the collision of two cosmic string loops is studied. After collision junctions are formed and the loops are entangled. We show that after their formation the junctions start to unzip and the loops disentangle. This analysis provides a theoretical understanding of the unzipping effect observed in numerical simulations of a network of cosmic strings with more than one type of cosmic strings. The unzipping phenomena have important effects in the evolution of cosmic string networks when junctions are formed upon collision, such as in a network of cosmic superstrings.

DOI: 10.1103/PhysRevD.80.083508 PACS numbers: 98.80.−k, 98.80.Cq

I. INTRODUCTION

Cosmic strings are copiously produced at the end of brane inflation [1,2] (for reviews see e.g. [3–6]). These strings are in the form of fundamental strings (F strings), D1-branes (D strings) or their bound states. F and D strings can combine to form bound states—(p, q) strings—which are constructed from p F strings and q D strings on top of each other. Because of charge conservation, when two (p, q) cosmic strings intersect generally a junction is formed. This is in contrast to what happens in the case of U(1) gauge cosmic strings: When two U(1) gauge cosmic strings intersect, they usually exchange partners and intercommute with probability close to unity. In this view, the formation of junctions may be considered as a novel feature of the network of cosmic superstrings. Networks of strings with junctions have interesting physical properties, such as the formation of multiple images [7,8] and nontrivial gravity wave emission [9,10]. Different theoretical aspects of (p, q) string construction were studied in [11–16] while the cosmological evolution of a string network with junctions has been investigated in [17].

The evolution of a network containing two types of cosmic strings was studied by Urrestilla and Vilenkin [18]. In their model, the cosmic strings are two types of U(1) gauge strings with interactions between them. Let us label these strings as A and B strings. Because of the interaction, the strings cannot exchange partners and a bound state, string AB, will form if the strings are not moving too fast. It was shown that the length and the distribution of the string network are dominated by the original A and B strings and there is a negligible contribution to the string network length and population from the bound states strings AB. This can be understood based on the following two reasons. First, the junctions may not form if the colliding strings are moving very fast so they can simply pass through each other [19–24]. Second and more curiously, if the junctions are formed, they start to unzip during the evolution. The process of zipping and unzipping of cosmic strings in collision is a nontrivial dynamical property. Our aim here is to provide some theoretical understanding of how this process happens in the collision of cosmic strings loops.

II. THE SETUP

Here we provide our setup. We consider two cosmic string loops moving in opposite directions. At the time of the collision, junctions are formed. This can be viewed as a generalization of straight strings collision [20–22]. However, due to topological constraints, there are new nontrivial effects which can lead to the unzipping of junctions. This is unlike what happens in the case of straight strings, where for two colliding straight strings, once the junctions are formed, they will always grow with time and do not unzip [20–22].

In order to simplify the analysis, we assume the colliding loops have equal tensions and radii and that the planes

*firouz@ipm.ir
†karoubyj@physics.mcgill.ca
‡khosravi@ipm.ir
§rhb@physics.mcgill.ca

they span are parallel. Choosing the center of mass frame, we take the loops to be in the $x - y$ plane and assume they are moving along the z axis with speed $\pm v$. A schematic view of the collision is shown in Fig. 1. The collision happens at $t = 0$, $z = 0$. There are two collision points. The angle of collision 2α is defined as the angle between the tangential lines to the loops at the points of collision.

We choose the incoming strings to be of the form of simple loops in their rest frames:

$$R(t) = R_0 \cos\left(\frac{t - t_0}{R_0}\right). \tag{1}$$

This may not be a realistic configuration, but due to the complexity of the collision analysis, this ansatz is illustrative enough to capture the unzipping effect. Here t_0 is the phase at the time of collision and the radius at collision time is $R_0 \cos(t_0/R_0)$.

After the collision, four junctions are formed (in Fig. 1 they are denoted by A, B, C and D). Because of symmetry, one expects that junctions A and B and junctions C and D evolve similarly but in opposite directions. On each junction, there are three string legs; two of them are the incoming strings and the third is the newly formed string with tension μ_3. As explained above, we assume that the incoming loops have equal tensions: $\mu_1 = \mu_2$. Later we will see that for a junction to form one requires that $2\mu_1 > \gamma\mu_3$.

Because of the symmetry, the third string is stationary and is oriented either along the y axis (y link) or the x axis (x link). The orientation of the third string is controlled by the angle α. For small α (roughly $0 < \alpha < \pi/4$) we expect a y-link junction and for a larger value of α an x-link junction. For the discussion below and in Fig. 1 we consider a y-link junction.

Guided by the causality and the symmetry of the problem, one expects that, after collision, the entangled loops are divided into two secondary loops, the external loop and the internal loop. The external and the internal loops are connected by the newly formed strings with tension μ_3. Given the symmetry of the setup, a nice feature of the internal and external loops is that half of each is from the first string and the other half is from the second string. There are four kinks on each secondary loop separating the newly formed arcs from the parts of the old loops which do not yet feel the presence of the junctions (by causality). As we shall see, these nontrivial topological constraints between the internal and external loops play an essential role in the unzipping process.

At the time of collision the system has a nonzero angular momentum around the axis of collision. However, we do not expect the angular momentum to play an important role in the unzipping process. As we shall see below, the unzipping process is determined by forces in the place of the strings, whereas angular momentum induces forces in the orthogonal direction. In addition, these forces will vanish at the local of the junctions.

The world sheet of each string is described by a temporal coordinate τ and a string length parameter σ_i. We take each string to have its own σ_i parameter. Our convention for the orientation of σ_i is that on a given loop, whether an original colliding loop or a secondary loop, the σ_i coordinate increases counterclockwise from 0 to 2π. For example, at the time of collision, the point M in Fig. 1 has $\sigma_1 = \gamma R_0 \alpha$ on string 1 and $\sigma_2 = \gamma R_0(\pi - \alpha)$ on string 2. Similarly, the point N has $\sigma_1 = \gamma R_0(2\pi - \alpha)$ on string 1 and $\sigma_2 = \gamma R_0(\pi + \alpha)$ on string 2. Here γ is the Lorentz factor, $\gamma^{-2} = 1 - v^2$, which shows up due to the boost from the string rest frame to the center of mass frame.

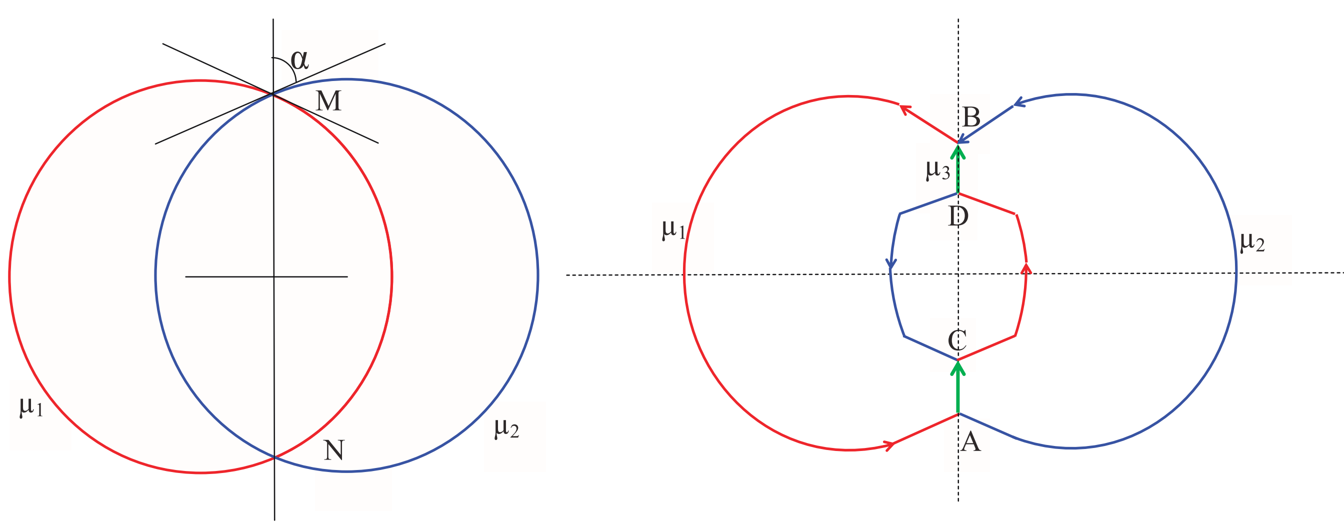

FIG. 1 (color online). A schematic view of the loops at the time of collision (left) and after collision (right). The arrows in the right figure indicate the directions in which the σ_i coordinate increases. We use the convention that on a loop σ_i runs counterclockwise. There are four junctions and eight kinks in total.

Finally on the third string, σ_3 increases from south to north.

One complexity of dealing with loops in collision is the orientation of the σ_i coordinate at junctions. We follow the prescription of [25] and use the sign parametrization for δ_i according to which δ_i can take values ± 1. If the value of σ_i of a particular string increases (decreases) towards the junction, we assign $\delta_i = +1$ ($\delta_i = -1$). With this prescription, the two ends of a piece of string ending in two neighboring junctions have opposite δ parameters. The arrows in Fig. 1 indicate this prescription. Since it is important for the later analysis, we now give the values of δ_i at each junction:

$$A\colon \left| \begin{array}{l} \delta_1 = +1 \\ \delta_2 = -1, \\ \delta_3 = -1 \end{array} \right. \qquad B\colon \left| \begin{array}{l} \delta_1 = -1 \\ \delta_2 = +1, \\ \delta_3 = +1 \end{array} \right.$$
$$C\colon \left| \begin{array}{l} \delta_1 = -1 \\ \delta_2 = +1, \\ \delta_3 = +1 \end{array} \right. \qquad D\colon \left| \begin{array}{l} \delta_1 = +1 \\ \delta_2 = -1. \\ \delta_3 = -1 \end{array} \right. \tag{2}$$

We consider a flat space-time background. The induced metric γ_{iab} on each string is given by

$$\gamma_{iab} = \eta_{\mu\nu}\partial_a x_i^\mu \partial_b x_i^\nu. \tag{3}$$

Here and in the following, we reserve $\{a, b\} = \{\tau, \sigma_i\}$ for the string world-sheet indices while Greek indices represent the four-dimensional space-time coordinates. Furthermore, x_i^μ stands for the position of the ith string in the target space-time.

After collision, the junction points correspond to the intersection of three segments of strings: two from the colliding loops and one from the bound state string that appears after collision. Including the δ parametrization on each segment of strings, the Nambo-Goto action describing the dynamics of the strings positions x_i^μ and the evolution of junction points is [25]

$$S = -\sum_i \mu_i \delta_i \int d\tau \int d\sigma_i \sqrt{-x_i'^2 \dot{x}_i^2}\,\theta(s_i^{B_i}(\tau) - \sigma_i) \times \theta(-s_i^{A_i}(\tau) + \sigma_i) + \sum_i \sum_J \int d\tau f_{i\mu}^J. \times [x_i^\mu(\tau, s_i^J(\tau)) - X_J^\mu(\tau)]. \tag{4}$$

Here an overdot and a prime denote derivatives with respect to τ and σ, respectively. The function $s_i^J(\tau)$ indicates the value of the σ_i coordinate for the ith string at the junction J. The theta functions indicate the fact that each piece of string exists only between junctions B_i and A_i. In this notation, the σ_i coordinate for the piece of the string which stretches from junctions B_i to A_i is increasing from B_i to A_i and

$$s_i^{B_i}(\tau) \leq \sigma_i \leq s_i^{A_i}(\tau). \tag{5}$$

It should be noted that in our case $\{A_i, B_i\}$ collectively stand for the junction points J with

$$J \in \{A, B, C, D\} \tag{6}$$

in Fig. 1. Finally, the functions $f_{i\mu}^J$ are the Lagrange multipliers which enforce the constraints that at the junction J, the three strings meet and

$$x_i(s_i^J(\tau), \tau) = X^J(\tau), \tag{7}$$

where $X^J(\tau)$ is the junction position in the target space-time.

As explained above, the value of the σ_i coordinate for the ith string at junction J is given by the function $s_i^J(\tau)$. It is a dynamical variable which controls the evolution of the junction. For example, at junction B in Fig. 1 the process of zipping for the string μ_3 happens when $\dot{s}_3^B(\tau) > 0$ whereas its unzipping happens when $\dot{s}_3^B(\tau)$ vanishes at some time during evolution and $\dot{s}_3^B(\tau) < 0$ afterwards. Our goal in the next section is to find the dynamical equations for $\dot{s}_i{}^J$ to understand the process of zipping and unzipping of strings in junctions.

The derivation of the equations of motion coming from action (4) is given in [25]. Here we summarize the equations which are necessary for our colliding loop analysis.

We impose the conformal temporal gauge on the string world sheet, namely, $X_i^0 = t = \tau$ and $\gamma_{i0\sigma} = 0$. This is equivalent to

$$\dot{\mathbf{x}}_i.\mathbf{x}_i' = 0, \qquad \dot{\mathbf{x}}_i^2 + \mathbf{x}_i'^2 = 1, \tag{8}$$

where the $\mathbf{x}_i$ represent the spatial components of the ith string. The solution of string equation of motion $\ddot{\mathbf{x}}_i - \mathbf{x}_i'' = 0$, as usual, is given in terms of left- and right-mover waves:

$$\mathbf{x}_i(t, \sigma) = \frac{1}{2}\mathbf{a}_i\left(\frac{\sigma + t}{2}\right) + \frac{1}{2}\mathbf{b}_i\left(\frac{\sigma - t}{2}\right) \tag{9}$$

with $\mathbf{a}_i'^2 = \mathbf{b}_i'^2 = 1$. Imposing the junction conditions obtained from varying the action (4), one can find expressions for $\mathbf{a}_i'$ and $\mathbf{b}_i'$ at the position of the junction J. Imposing the conditions $\mathbf{a}_i'^2 = \mathbf{b}_i'^2 = 1$ one finds the following equations for s_i^J:

$$\dot{s}_i{}^J = \delta_i\left(1 - \frac{\mu M_i(1 - c_i^J)}{\mu_i \sum_k M_k(1 - c_k)}\right), \tag{10}$$

where $\mu \equiv \sum_i \mu_i$, and

$$M_i = \mu_1^2 - (\mu_j - \mu_k)^2, \qquad c_i^J(t) = Y_i.Y_k \tag{11}$$

with $i \neq j \neq k$ and

$$Y_j = \begin{cases} \mathbf{b}_j' & \text{if } \delta_i = +1, \\ -\mathbf{a}_j' & \text{if } \delta_i = -1. \end{cases} \tag{12}$$

It should be noted that the Y_i are constructed at the point of each junction J where $\sigma_i = s_i^J(\tau)$.

Finally, energy conservation at each junction $J = \{A, B, C, .D\}$ requires that

$$\delta_1^J \mu_1 \dot{s}_1{}^J + \delta_2^J \mu_2 \dot{s}_2{}^J + \delta_3^J \mu_3 \dot{s}_3{}^J = 0. \tag{13}$$

One can check that this also follows from Eq. (10).

III. ZIPPING AND UNZIPPING

Here we study in detail the equations of motion for $s_i(t)$. Let us start with junction B. At the time of collision $s_3^B(0) = 0$. For the junction to form, $s_3^B(t)$ should be increasing initially. For unzipping to happen, $s_3^B(t)$ should come to a stop (i.e. $\dot{s}_3^B = 0$) at some time $t = t_u^B$ corresponding to the time of unzipping at junction B. Then $s_3^B(t)$ decreases. Similarly, $s_3^D(0) = 0$ initially and after collision $s_3^D(t)$ decreases, reaching a minimum negative value before turning back. The unzipping at junction D happens at $t = t_u^D$ (when $\dot{s}_3^D = 0$). Interestingly, we find that $t_u^D \neq t_u^B$. The loops disentangle and separate from each other at the time $t = t_f$ when the junctions B and D meet, corresponding to $s_3^D(t_f) = s_3^B(t_f)$. As we shall see, the loop disentanglement does not happen when $s_3^D(t) = s_3^B(t) = 0$. It turns out that $s_3^D(t_f) = s_3^B(t_f) < 0$. Because of our symmetric construction, the same arguments go through for junctions A and C and we can restrict the analysis to the pair of junctions B and D.

Going to the center of mass frame, it follows from Eq. (1) that

$$\mathbf{x}_{1,2} = \begin{pmatrix} \mp b + R_0 \cos(\frac{t-t_0}{\gamma R_0}) \cos(\frac{\sigma_{1,2}}{\gamma R_0}) \\ R_0 \cos(\frac{t-t_0}{\gamma R_0}) \sin(\frac{\sigma_{1,2}}{\gamma R_0}) \\ \pm v t \end{pmatrix}, \tag{14}$$

where the impact parameter $2b$ is the separation between the centers of the loops (see Fig. 1). Decomposing $\mathbf{x}_i$ into left movers as in Eq. (9) yields

$$\mathbf{a}'_{1,2} = \begin{pmatrix} -\gamma^{-1} \sin(\frac{\sigma_{1,2}+t-t_0}{\gamma R_0}) \\ \gamma^{-1} \cos(\frac{\sigma_{1,2}+t-t_0}{\gamma R_0}) \\ \pm v \end{pmatrix},$$
$$\mathbf{b}'_{1,2} = \begin{pmatrix} -\gamma^{-1} \sin(\frac{\sigma_{1,2}-t+t_0}{\gamma R_0}) \\ \gamma^{-1} \cos(\frac{\sigma_{1,2}-t+t_0}{\gamma R_0}) \\ \mp v \end{pmatrix}. \tag{15}$$

For the third string which stretches between the D and B junctions one has (following the arrows in Fig. 1 where σ_3 increases from south to north)

$$\mathbf{x}_3 = (0, \sigma_3, 0), \qquad \mathbf{a}'_3 = \mathbf{b}'_3 = (0, 1, 0). \tag{16}$$

Let us start again with the junction B. Based on symmetry considerations (both loops have equal tensions and radii) one expects that $\dot{s}_1^B(t) = -\dot{s}_2^B(t)$. From Eq. (10) one can check that $\dot{s}_1^B(t) = -\dot{s}_2^B(t)$ is a consistent solution. This in turn leads to $s_1^B(t) + s_2^B(t) = s_1^B(0) + s_2^B(0)$. However, at the time of collision, $s_1^B(0) = \gamma R_0 \alpha$ and $s_2^B(0) = \gamma R_0(\pi - \alpha)$ so

$$s_1^B(t) = -s_2^B(t) + \gamma R_0 \pi. \tag{17}$$

Using the energy conservation equation (13) one obtains

$$s_3^B(t) = -\frac{2\mu_1}{\mu_3}[s_2^B(t) - \gamma R_0(\pi - \alpha)]. \tag{18}$$

The dynamical process of zipping, unzipping and loop disentanglement is controlled by the functions $s_3^B(t)$ and $s_3^D(t)$. To obtain the differential equation for $\dot{s}_3^B$, we first need to calculate the quantities $c_i(t)$ at junction B. One has

$$c_1 = \mathbf{b}'_2.\mathbf{b}'_3 = \gamma^{-1} \cos\left(\frac{s_2^B(t) - t + t_0}{\gamma R_0}\right) \tag{19}$$

and

$$c_2 = -\mathbf{a}'_1.\mathbf{b}'_3 = \gamma^{-1} \cos\left(\frac{s_1^B(t) + t - t_0}{\gamma R_0}\right) = c_1, \tag{20}$$

where to obtain the final relation, use was made of Eq. (17). Similarly, one obtains

$$c_3^B = -\mathbf{a}'_1.\mathbf{b}'_2 = -1 + 2\gamma^{-2}\cos^2\left(\frac{s_2^B(t) - t + t_0}{\gamma R_0}\right). \tag{21}$$

With these values of $c_i(t)$ and using Eq. (10), one obtains

$$\dot{s}_3^B = \frac{2\mu_1\gamma^{-1}\cos(\frac{\mu_3 s_3^B(t)}{2\mu_1\gamma R_0} + \alpha + \frac{t-t_0}{\gamma R_0}) - \mu_3}{2\mu_1 - \mu_3\gamma^{-1}\cos(\frac{\mu_3 s_3^B(t)}{2\mu_1\gamma R_0} + \alpha + \frac{t-t_0}{\gamma R_0})}, \tag{22}$$

where to get the final answer, the relation (18) has been used to eliminate $s_2^B(t)$ in favor of $s_3^B(t)$.

To check the validity of the above expression, one can show that in the limit where $R_0 \to \infty$, it reduces to the result of [20] for collision of two infinite straight strings.

Following the same steps, for the junction D one finds

$$\dot{s}_3^D = -\frac{2\mu_1\gamma^{-1}\cos(\frac{\mu_3 s_3^D(t)}{2\mu_1\gamma R_0} + \alpha - \frac{t-t_0}{\gamma R_0}) - \mu_3}{2\mu_1 - \mu_3\gamma^{-1}\cos(\frac{\mu_3 s_3^D(t)}{2\mu_1\gamma R_0} + \alpha - \frac{t-t_0}{\gamma R_0})}. \tag{23}$$

Comparing the equations for $\dot{s}_3^B$ and $\dot{s}_3^D$, we observe that $s_3^B \to -s_3^D$ under time reversal $t - t_0 \to -(t - t_0)$.

One can check that for the junctions A and B the evolution of $\dot{s}_3^A$ and $\dot{s}_3^C$ is identical to that of $\dot{s}_3^B$ and $\dot{s}_3^D$, with a sign difference as expected due to our symmetric construction.

With some effort, one can solve Eqs. (22) and (23) with the answer

$$\frac{s_3^B}{R_0} - \sin\left(\frac{\mu_3 s_3^B}{2\mu_1\gamma R_0} + \alpha + \frac{t - t_0}{\gamma R_0}\right) = -\sin\left(\alpha - \frac{t_0}{\gamma R_0}\right) - \frac{\mu_3}{2\mu_1 R_0} t, \tag{24}$$

which expresses $s_3^B(t)$ implicitly as a function of t. A similar equation holds for s_3^D with $(t, t_0) \to -(t, t_0)$.

The above implicit equations for s_3^B and s_3^D cannot be solved explicitly to obtain the variables as functions of t. However, some insight can be obtained by looking at the form of Eq. (22). For the junction B to materialize at $t = 0$, we need that $\dot{s}_3^B(0) > 0$. This requires that $\gamma\mu_3 < 2\mu_1$ and

$$\left(\alpha - \frac{t_0}{\gamma R_0}\right) < \alpha_c \equiv \cos^{-1}\left(\frac{\mu_3 \gamma}{2\mu_1}\right). \tag{25}$$

Interestingly, this is the same junction formation condition as for the collision of straight strings [20] where $R_0 \to \infty$. This is understandable, since the collision and junction formation is a local effect and at the points of collision large loops may be approximated as straight strings. On the other hand, for the junction D to materialize after collision, one expects that $\dot{s}_3^D(0) < 0$ which yields

$$\left(\alpha + \frac{t_0}{\gamma R_0}\right) < \alpha_c. \tag{26}$$

Interestingly, when $t_0 \neq 0$, Eq. (26) is stronger a condition than Eq. (25).

Once the junction B is formed, it grows until the time t_u^B of unzipping, when the argument inside the cos function in Eq. (22) becomes equal to α_c and $\dot{s}_3^B = 0$. As time goes by, the argument inside the cos function increases, $\dot{s}_3^B$ becomes negative and the junction B turns back. A similar argument applies to junction D except that the unzipping happens at the time $t = t_u^D$, and due to the time asymmetry in Eqs. (22) and (23), $t_u^D \neq t_u^B$. Below we will demonstrate that $t_u^D > t_u^B$. After $t > t_u^D$, the junctions B and D move towards each other. The loops disentangle at the time $t = t_f$ when the junctions meet, corresponding to $s_3^B(t_f) = s_3^D(t_f)$. In Fig. 2 we have plotted the shapes of s_3^B and s_3^D for some given parameter values of α, γ, μ_1, μ_2 and R_0. The left figure indicates that $s_3^B(s_3^D)$ increases (decreases) initially and then come to a halt, indicating the time of unzipping.

Here we would like to find the time of unzipping and loop disentanglement. Consider junction B. At the time $t = t_u^B$ of unzipping one obtains from $\dot{s}_3^B = 0$ that

$$\frac{s_3^B(t_u^B)}{R_0} = \frac{2\mu_1\gamma}{\mu_3}\left(\alpha_c - \alpha - \frac{t_u^B - t_0}{\gamma R_0}\right). \tag{27}$$

Plugging this into Eq. (24) gives the unzipping time

$$\frac{t_u^B}{R_0} = \left(1 - \frac{\mu_3^2}{4\mu_1^2}\right)^{-1}\left\{\gamma(\alpha_c - \alpha) + \frac{t_0}{R_0} + \frac{\mu_3}{2\mu_1}\left[\sin\left(\alpha - \frac{t_0}{\gamma R_0}\right) - \sqrt{1 - \frac{\gamma^2\mu_3^2}{4\mu_1^2}}\right]\right\}. \tag{28}$$

To find the unzipping time for junction D, we note that after junction formation the argument inside the cos function in Eq. (23) decreases with time. It becomes negative and the unzipping for junction D happens when the expression inside the cos function becomes equal to $-\alpha_c$. With this consideration and following the steps as above yields

$$\frac{t_u^D}{R_0} = \left(1 - \frac{\mu_3^2}{4\mu_1^2}\right)^{-1}\left\{\gamma(\alpha_c + \alpha) + \frac{t_0}{R_0} - \frac{\mu_3}{2\mu_1}\left[\sin\left(\alpha + \frac{t_0}{\gamma R_0}\right) + \sqrt{1 - \frac{\gamma^2\mu_3^2}{4\mu_1^2}}\right]\right\}. \tag{29}$$

Equations (28) and (29) are implicit equations which relate t_u^B and t_u^D to the tensions μ_i, the incoming angle of collision α, the velocity γ and the initial loop phase t_0. It is not easy to see how t_u^B and t_u^D vary as one varies these parameters simultaneously. As a simple treatment, let us take μ_i and γ as fixed properties of a network of cosmic strings and consider the unzipping times as functions of α and t_0 (which may be considered as random parameters for the network evolution). If one increases $t_0 > 0$ while keeping α fixed, then both t_u^B and t_u^D increase. There is a limit on how large t_0 can be. This is determined by Eq. (26). The dependence of the unzipping on α is more nontrivial. From Eqs. (28) and (29) we note that the dependence of these

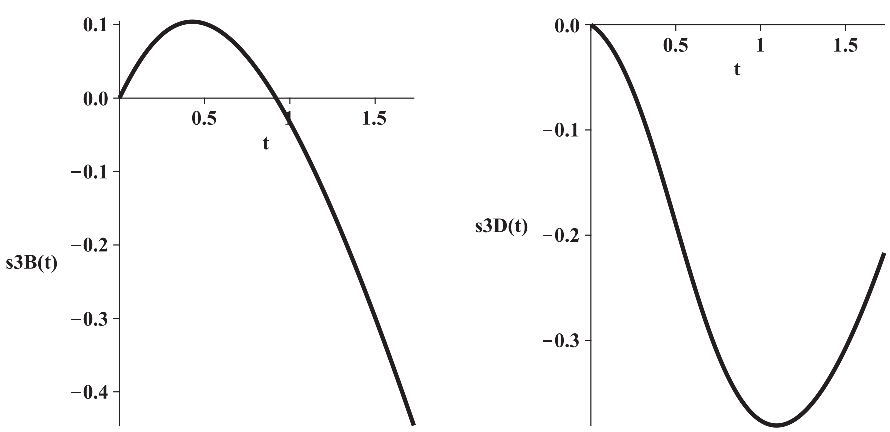

FIG. 2. Here the numerical solutions for $s_3^B(t)$ (left) and s_3^D (right) are plotted for $\gamma = 1.1$, $\mu_1/\mu_3 = 0.7$, $\alpha = \pi/8$ and $t_0 = 0.1$ in units where $R_0 = 1$. The unzipping for junction B (D) happens when s_3^B (s_3^D) reaches a maximum (minimum) value. In this example, the loop disentanglement happens at $t_f \simeq 1.4$ before the loops shrink at $t_{\rm shrink} \simeq 1.7$.

times on α is not symmetric. For a fixed value of t_0, then as α increases, t_u^D increases while t_u^B decreases almost linearly with α. Again, there is a limit on how big α can be, which is determined by Eq. (26).

It is instructive to see which of the junctions B or D starts to unzip sooner. From the above two equations, the difference in the unzipping times is calculated to be

$$\frac{t_u^D - t_u^B}{R_0} = 2\gamma\alpha\left(1 - \frac{\mu_3^2}{4\mu_1^2}\right)^{-1} \times \left[1 - \frac{\mu_3}{2\mu_1\gamma}\cos\left(\frac{t_0}{\gamma R_0}\right)\frac{\sin\alpha}{\alpha}\right]. \quad (30)$$

Since $\sin\alpha/\alpha$ is always less than unity we see that $t_u^D > t_u^B$. This means that the junction B which holds the external large arcs unzips sooner than the junction D which holds the internal small arcs. Keeping all other parameters fixed, by increasing the angle of collision α, the difference in unzipping times increases almost linearly with α.

The time t_f of loop disentanglement is given by $s_3^B(t_f) = s_3^D(t_f)$. Using Eq. (24) and the similar equation for s_3^D gives

$$\frac{2\gamma\mu_1}{\mu_3}\cos^{-1}\Gamma - \cos\left(\frac{t - t_0}{\gamma R_0}\right)\sqrt{1 - \Gamma^2} - \frac{2\gamma\mu_1\alpha}{\mu_3} + \sin\alpha\cos\left(\frac{t_0}{\gamma R_0}\right) = 0, \quad (31)$$

where

$$\Gamma \equiv \left[\frac{\mu_3 t/(2\mu_1 R_0) - \cos\alpha\sin(\frac{t_0}{\gamma R_0})}{\sin(\frac{t - t_0}{\gamma R_0})}\right]. \quad (32)$$

This is an implicit equation for t_f which should be solved in terms of μ_i, γ, α, t_0 and R_0. For this to make sense, we demand that $t_f - t_0 < \pi R_0/2$ before the loops shrink to zero.

IV. DISCUSSION

In this paper we have provided a theoretical understanding of the zipping-unzipping phenomena in cosmic string loop collisions. The process of unzipping and string disentanglement has important effects on the evolution of networks containing different types of strings. Initially, one may fear that the overabundance of junctions and the string bound states may lead to a frustrated network of cosmic strings, preventing the network to reach a scaling regime. In an interesting simulation run by Urrestilla and Vilenkin [18] it was shown that the presence of junctions and bound states is not catastrophic. Indeed, it was shown that for a network containing two different types of strings, the contribution of the bound states to the population and length is negligible compared to that of the original strings. There may be two reasons for why the contribution of the junctions and bound states to the network's string length and number density is subdominant. First, cosmic strings move with very high velocities and can simply pass through each other, and no junctions form in the first place [20–22,26]. Second, and more curiously, junctions may materialize occasionally but they soon become unstable to unzipping. This was the subject of our current study.

To simplify the analysis, here we considered the simple case when the colliding loops have equal tensions and radii. In principle one can consider more general cases when loops have different tensions and configurations.

In examples of straight strings in collision [20–22], the junctions do not stop growing in time once they are formed. In contrast, we have demonstrated that for colliding loops unzipping phenomena take place. It is energetically costly for junctions to grow indefinitely. The junctions holding the external loops and those holding the internal loops behave differently. The junctions holding the external large loops start to unzip sooner than the junctions holding the internal small ones. The onset of unzipping and eventual loops disentanglement is determined by the parameters of the colliding loops such as their tensions, the angle of collision and their velocity.

ACKNOWLEDGMENTS

We would like to thank T. Kibble, R. Myers, S. Sheikh-Jabbari, T. Vachaspati and A. Vilenkin for useful discussions and comments. H. F., J. K. and R. B. would like to acknowledge the hospitality of the KITPC, where this work was started, and the Perimeter Institute, where this work was finished. At McGill, this research has been supported by NSERC under the Discovery Grant program. R. B. is also supported by the Canada Research Chairs program.

[1] S. Sarangi and S. H. H. Tye, Phys. Lett. B **536**, 185 (2002).
[2] M. Majumdar and A. Christine-Davis, J. High Energy Phys. 03 (2002) 056.
[3] S. H. Henry Tye, Lect. Notes Phys. **737**, 949 (2008).
[4] T. W. B. Kibble, arXiv:astro-ph/0410073.
[5] A. C. Davis and T. W. B. Kibble, Contemp. Phys. **46**, 313 (2005).
[6] M. Sakellariadou, Phil. Trans. R. Soc. A **366**, 2881 (2008).
[7] B. Shlaer and M. Wyman, Phys. Rev. D **72**, 123504 (2005).
[8] R. Brandenberger, H. Firouzjahi, and J. Karouby, Phys. Rev. D **77**, 083502 (2008); T. Suyama, Phys. Rev. D **78**, 043532 (2008).
[9] R. Brandenberger, H. Firouzjahi, J. Karouby, and S.

Khosravi, J. Cosmol. Astropart. Phys. 01 (**2009**) 008; M. G. Jackson and X. Siemens, J. High Energy Phys. 06 (2009) 089; P. Binetruy, A. Bohe, T. Hertog, and D. A. Steer, arXiv:0907.4522.

[10] L. Leblond, B. Shlaer, and X. Siemens, Phys. Rev. D **79**, 123519 (2009).

[11] E. J. Copeland, R. C. Myers, and J. Polchinski, J. High Energy Phys. 06 (2004) 013; G. Dvali and A. Vilenkin, J. Cosmol. Astropart. Phys. 03 (2004) 010; L. Leblond and S.-H. H. Tye, J. High Energy Phys. 03 (2004) 055; N. Barnaby, A. Berndsen, J. M. Cline, and H. Stoica, J. High Energy Phys. 06 (2005) 075.

[12] H. Firouzjahi, L. Leblond, and S. H. Henry Tye, J. High Energy Phys. 05 (2006) 047; H. Firouzjahi, J. High Energy Phys. 12 (2006) 031; K. Dasgupta, H. Firouzjahi, and R. Gwyn, J. High Energy Phys. 04 (2007) 093; M. Lake, S. Thomas, and J. Ward, arXiv:0906.3695.

[13] M. G. Jackson, N. T. Jones, and J. Polchinski, J. High Energy Phys. 10 (2005) 013; A. Hanany and K. Hashimoto, J. High Energy Phys. 06 (2005) 021; K. Hashimoto and D. Tong, J. Cosmol. Astropart. Phys. 09 (2005) 004; M. Eto, K. Hashimoto, G. Marmorini, M. Nitta, K. Ohashi, and W. Vinci, Phys. Rev. Lett. **98**, 091602 (2007); M. G. Jackson, J. High Energy Phys. 09 (2007) 035.

[14] H. Firouzjahi, Phys. Rev. D **77**, 023532 (2008).

[15] Y. Cui, S. P. Martin, D. E. Morrissey, and J. D. Wells, Phys. Rev. D **77**, 043528 (2008).

[16] A. C. Davis, W. Nelson, S. Rajamanoharan, and M. Sakellariadou, J. Cosmol. Astropart. Phys. 11 (2008) 022.

[17] S. H. Tye, I. Wasserman, and M. Wyman, Phys. Rev. D **71**, 103508 (2005); **71**, 129906(E) (2005); L. Leblond and M. Wyman, Phys. Rev. D **75**, 123522 (2007); A. Avgoustidis and E. P. S. Shellard, Phys. Rev. D **78**, 103510 (2008); A. Rajantie, M. Sakellariadou, and H. Stoica, J. Cosmol. Astropart. Phys. 11 (2007) 021; M. Sakellariadou and H. Stoica, J. Cosmol. Astropart. Phys. 08 (2008) 038.

[18] J. Urrestilla and A. Vilenkin, J. High Energy Phys. 02 (2008) 037.

[19] L. M. A. Bettencourt and T. W. B. Kibble, Phys. Lett. B **332**, 297 (1994).

[20] E. J. Copeland, T. W. B. Kibble, and D. A. Steer, Phys. Rev. Lett. **97**, 021602 (2006).

[21] E. J. Copeland, T. W. B. Kibble, and D. A. Steer, Phys. Rev. D **75**, 065024 (2007).

[22] E. J. Copeland, H. Firouzjahi, T. W. B. Kibble, and D. A. Steer, Phys. Rev. D **77**, 063521 (2008).

[23] P. Salmi, A. Achucarro, E. J. Copeland, T. W. B. Kibble, R. de Putter, and D. A. Steer, Phys. Rev. D **77**, 041701 (2008).

[24] N. Bevis and P. M. Saffin, Phys. Rev. D **78**, 023503 (2008).

[25] N. Bevis, E. J. Copeland, P. Y. Martin, G. Niz, A. Pourtsidou, P. M. Saffin, and D. A. Steer, arXiv:0904.2127.

[26] A. Achucarro and R. de Putter, Phys. Rev. D **74**, 121701 (2006).