THE EVOLUTIONARY **STATE**

OF THE β CMa VARIABLE STARS

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List of Tables

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Abstract

General properties of, the latest observations of, and current theories attempting to explain the pulsations of the PCMa variable stars are reviewed. Stellar models are used to estimate the fractions of time spent in the S-bend region of the H-R diagram for the core hydrogen burning phase $(\sim 79\%)$, the overall contraction phase $(\sim 19\%)$, and the establishment of the shell source $(\sim 2\%)$. A homogeneous sample of 353 early B stars within 500 pc is established. β CMa stars are found to make up **~5%** of.that sample, a proportion consistent with the lifetime in the overall contraction phase. Observations of the remainder of the star sample for β CMa variability are needed since many of the remaining stars are listed as possible variables. Enough possible variables exist, if all or mosters are found to exhibit. β CMa variability, to require that the core hydrogen burning phase be responsible for those pulsations.

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I. The General Problems of Variable Stars.

Perhaps the most important fact about a pulsating star is that the pulsation must be driven **by** some process that supplies mechanical energy. This is easilr evidenced **by** seeing what happens as a star pulsates. **A** pulsating star will expand past equilibrium size until the expansion is slowed and reversed **by** gravity. Contracting, the star will again overshoot its equilibrium size. Gas pressure builds up, contraction is slowed to a stop, and the surface is pushed outward again. Dissipative forces, such as friction, should eventually cause the pulsation to stop. Thus, some form of mechanical energy must be fed to the system to maintain the pulsation.

Another important axium is that variable stars have **dif**ferent modes in which it is possible for them to vibrate. Both fundamental and higher modes may be excited. In the fundamental radial mode, the amplitude of the gas's movement decreases smoothly toward the confined and of the vibration (toward the star's interior). The vibrational periods of the higher modes are shorter and the amplitudes of the gas's movement are distributedin a different way. Nodes (stationary points) are present and the amplitude is appreciable only at the free end of the vibration (at the surface of the star). More complex non-radial modes may also be excited, and which modes are excited of course depends on how and where the driving force is applied.

The way in which vibrations are started and maintained

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are fairly well understood for some types of variables. For example, ionization zones of Hydrogen and Helium, near the surfaces of Cepheids and RR Lyrae stars, can act as values creating a heat engine to drive pulsations (Percy, **1975).**

The stars with which we are dealing, the β Cma variables, are not nearly so well understood.

II. Properties of the pCMa Variables.

The β CMa variable stars are a group of bright early class B stars. They exhibit rapid variations in radial velocity, strongly suggestive of pulsation. The Light curves are sinusoidal and lag behind the radial velocity curves **by 90** degrees $(\frac{1}{4}$ period). Some of the light curves show just one period, while others can be explained only **by** the superposition of two or more periods. the stars with the shortest periods show the smallest changes in brightness (0.03m) and in radial velocity $(90km-sec⁻¹)$. All are slightly bluer at maximum than at minimum light (Glasby, **1969).**

Observations of the narrowness of the spectral lines in the β CMa stars, until recently, seemed to indicate the absence of rotation. Until Hill's **1967** survey, all known PCMa stars had rotational velocities (v sin i) of 40 km-sec⁻¹. Nonvariables of the same spectral class have rotational velocities \sim 150 km-sec⁻¹. Projected rotational velocities (McNamara and Hansen, 1961) for eleven β CMa variables averaged 22 km-sec⁻¹. It was then thought that high rotational velocities for pulsating ρ CMa stars seemed to be about 50 km/sec^{-1} . New observations have since changed this premise.

Figure one is a color-luminosity diagram for the β CMa sequence of stars. Table one is a list of 21 definite and one likely β CMa stars.

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Figure **1:** Color-luminosity diagram for the PCMa sequence **of** stars. (Glasby, **1969)**

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Table **1:**

Definite ρ CMa Stars

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Table **1** con't:

- **@** More distant than **500** Pc.
- $#$ Not confirmed
- ***** Spectral classes from Hoffeit (1964)

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REFERENCES:

- 1 Lesh and Aizenman *(1973)*
- 2 Van Hoof **(1972)**
- **3** Dukes **(1973)**
- 4 Shobbrook **(1972)**
- **5** Shobbrook and Lamb **(1972)**

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III. New Observations Concerning g CMa Variable Stars.

Different^{ic} photometry of 18 β CMa stars (Percy, 1970) indicates that the FCMa instability strip lies about **1.85** magnitudes above the main sequence at its upper end $(B-V=-0.28)$ and about *1.35* magnitudes above the main sequence at its lower end $(B-V=-0.23)$. Those 18 known $gCMa$ stars show that the sequence ends at about $M_{\nu}=-3.0$ with no stars dimmer than that value.

Instability.strips are also known for other types of variable stars. To become a variable, the star's evolutionary track must cross such an instability region. How long the star remains a pulsating star depends on the rate of which the star's track crosses the instability region. This rate of evolution depends on three factors: first, the mass of the star's nuclear fuel; second, the energy content of the fuel per unit mass; and third, the rate of radiation of that ener**gy** (Cox et. al., 1974).

These evolutionary changes in luminosity and effective temperature occurs far too slowly to be observed directly. However, there is a way out of this observational dilemma since $\boldsymbol{\beta}$ CMa stars (like Cepheids and other variables) exhibit a period-luminosity relation. Such a relation is given **by** Strohmeier **(1972)** as:

$$
M_v = 0.4 - (18.1 \pm 2.3)P_0
$$
 days.

Thus, a luminosity change is always accompanied **by** a period

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change. This change is slow but it can be measured because the charge accumulates over the years, leaving a phase shift in the period **of** the star. **A** search of the literature (Eagleton and Percy, **1973)** has yielded values for dP/dt, for some FCMa stars, given in Table 2.

Those authors also calculated a \dot{P} (dP/dt) due to evolution for a β CMa star of 16 M_{\odot} with a theoretical evolutionary track. However, there is no way, at present, to determine the relative importance of various means of mass loss, magnetic effects, and other factors affecting \dot{P} . Observations do indicate that period changes occur, But a "simple picture of smooth evolutionary period changes is not sufficient..." to explain those observations.

The narrowness of the β CMa instability strip is one question that comes to mind since many seemingly normal stars are scattered about in the area of the strip. It is thought **by** some that smaller photometric errors would decrease the width of the strip, leaving in it a higher proportion of variables. Shobbrook (1974) estimates that as many as **75%** of the non-variables could be eliminated from the strip **by** careful observations.

New fCMa stars are constantly being discovered. In **1967,** (Hill, **1967)** photometry of a sample **of 153** early B stars in nearby galactical clusters yielded 24 new $$CMa$ candidates. Their spectra ranged from **09.5** to B3; their luminosity classes

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 $Table 2: Rate of change of period for some confirmed β CMa$ </u> stars. (Eagleton and Percy, **1973)**

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ranged from I to V, they had periods from three to ten hours, and projected rotational velocities from 0 to 300 km-sec⁻¹. Those having periods greater than **0.35** days did not obey a period-luminosity or a period-color radiation, while those with periods less than **0.3** days did appear to obey such relations.

Several other definite $\boldsymbol{\beta}$ CMa stars have been discovered since Hill's survey. Shobbrook (1973) summarized the properties of the variables he had discovered (and other previously discovered β CMa variables that he studied with the same photometric system) with the following table, Table **3.** He concludes that both fast rotating and slow rotating β CMa stars exist and exhibit widely divergent behavior in light variation from star to star. The possibility of their following different periodluminosity relations could not be ruled out. An apparent ρ CMa star has also been found in **0** VEL (Van Hoof, **1972).**

Much work has been done in the last few years in attempting to obtain better values for the luminosities, colors, and spectral types of the β CMa stars. Luminosities for six β CMa variables have been deduced from their membership in the Scorpio-Centaurus cluster (James and Shobbrook, 1974). The distance of σ VIR has been measured interferometrically, and luminosities for the remaining variables were estimated from $H \gamma$ and $H \beta$ line strengths.

Balona and Feast **(1975)** have carried out further work on the Sco-Cen association. Taking all of the stars in thatand the control of the cont $\sigma_{\rm{G}}(Q_{\rm{G}}) = \sigma_{\rm{G}}(Q_{\rm{G}}) = \sigma_{\rm{G}}(Q_{\rm{G}}) = \sigma_{\rm{G}}(Q_{\rm{G}}) = \sigma_{\rm{G}}(Q_{\rm{G}}) = \sigma_{\rm{G}}(Q_{\rm{G}}) = \sigma_{\rm{G}}(Q_{\rm{G}})$ ~ 1

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Table 3: Properties of β CMa stars obsreved by Shobbrook

(Shobbrook, **1973)**

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association, they noted a serious discrepancy between the astrometric distance modulas, **5.78** mag., and the HX distance modulas, 6.46 mag. This inconsistancy is probably due to the cut-off applied to eliminate poor proper motions.

Using the H Y luminosity calibrations of Balona and Crampton (1974), they derive a Zero-Age Main Sequence (ZAMS), apply the H **Y** calibration to the **G**CMa variables, and, plotting those stars on an H-R diagram, find that most fall in the **S**bend region for stars with $z=0.06$ (fraction of heavy elements). See Figure 2, a graph of these results.

Lesh and Aizenman **(1973)** have found simlar results. Presenting spectral types on the MR system for 17 bright g CMa variables, seven bright suspected variables, and **23** fainter suspected variables, they found that the variables occupy a very restricted region in their observational H-R diagram. In the theoretical H-R diagram this "instability strip" is shown to coincide with the region traversed three times **by** a star in the course of its post main sequence evolution: once in the core hydrogen-burning phase, once in the secondary contraction phase, and once in the shell hydrogen burning phase. Here one might hastily assume that the normal B stars in this region are in the core hydrogen burning phase and that the ρ CMa variables are in one of the two later stages of evolution. However, many theorists disagree with this assumption. Figure **3** shows some theoretical evolutionary tracks in the S-bend re-

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gion along with some observational and theoretical main sequences.

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Figure 2: $H-R$ diagram of β CMa Variables (Lesh and Aizenman, **1973)**

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Figure **3:** This diagram (Lesh and Aizenman, **1973)** shows the observational and theoretical main sequence for class B stars, emphasizing how the normal stars and theoretical S-bend regions overlap.

IV. Theories Proposed for the β CMa Stars.

With the stage of evolution of the β CMa stars still somewhat uncertain we are also left with the two problems with which we started; the mechanism and modality of the pulsations. Many possible explanations have been put forward **by** many different authors, and the only general consensus is that the radial modes cannot be excited directly. Aizenman and Cox **(1975b)** do suggest, however, that the non-radial modes that are excited might couple to the radial fundamental and harmonic modes; the modes that we are most likely observing.

Aizenman and Cox (1975a),investigated possible non-adiabatic effects and concluded that the phase shifts between the pressure and density variations brought about **by** those small non-adiabatic effects cannot be neglected in a star that is in thermal imbalance. The other larger effects are dynamical in nature and can be descibed **by** the analogy of a slowly varying spring constant in a simple mechanical system.

Adiabatic non-radial oscillations have also been investigated. Harper and Rose **(1970)** give their results of calculations using Parkers forth order linear differential equation that describes the adiabatic non-radial pulsations of a compressable self-gravitating gas sphere. They computed a sequence of 10 \mathbb{M}_{\odot} models from the main sequence to the stage of hydrogen exhaustion in the core. For several of those models they calculated the fundamental radial mode and the Kelvin

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mode to see if the period of the variability and the possible resonance between the two modes fitted the observations. They found that when their model had evolved off the main sequence about one magnitude, the pulsation period was about 4 hours and the periods of the two modes were close enough to be in resonance. They concluded that the observed double periodicity of some β CMa stars could be due to resonance between the lowest radial mode and the Kelvin mode.

As a follow up to this work, Dupree (1974a) calculated models for 8, 10, and 12 M_o stars. But periods found in these calculations agree quite well with those observed for the stars FCMa, 12 **LAC,** and KP PER. **)** ERI fits the model less well while **gSCO** and **16 LAC,** both definite binaries that show large orbital motion, do not fit the computations. This is not likely due to the effects of the binary companion.

The four stars for which the models fit well can be represented **by** models in the late core hydrogen-burning stage, but not lose to the gravitational contraction stage. The modes that seem to be involved are the radial fundamental modes and the non-radial fundamental modes.

Using nonlinear methods, Dupree **(1974b)** concludes that in slowly rotating β CMa stars two modes will be present, and that those modes will interact even if they are not precisely degenerate.

Percy (1970) has examined models of stars 10 M_{\odot} in the

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terminal stage of hydrogen burning and found that there should be a semi-convective zone present. For stars brighter than M_v=-3.0 the zone should be large enough to overcome stabilizing processes and allow pulsation. He suggests that this **hy-** pothesis be studied using a non-linear treatment of convection, in the presence of rotation.

Osaki (1974) presents a model with a rapidly spinning core in the late stages of hydrogen-burning in the core. He thinks that oscillatory large scale convective motion of the core having a frequency coinciding with the eigen-frequency of a non-radial oscillation of the whole star may excite such a non-radial oscillation of the whole star. Large amplitudes are found only in the convective core and at the surface of the star. The mode most likely to be excited is a sectoral mode, which is a wave traveling around the equator in the same direction as the rotation.

Differential rotation of the core and the surface is obviously indicated but the possible presence of angular momentum transport makes estimation of the velocity of rotation at the surface a very difficult matter. Thus this model cannot be tested observationally.

Strothers and Soimon (1969) attempt to explain β CMa stars that are members of binary systems. They set forth two possible mechanisms for the variability. In both cases the β CMa component has accreted a He rich envelope from the companion

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star and is still burning hydrogen in the core.

The first mechanism, the μ mechanism, is the "reversal of the gradient of near molecular weight". Here heavy surface material compresses the envelope while the central density remains almost unchanged. Their models indicate that this mechanism will operate in inhomogeneous stars of **6** to **95 Mo.**

If the He in the envelope mixes rapidly with the rest of the star, a higher degree of homogenity results and radiation pressure will be increased. Pulsational instability will develop in this "g mechanism" for stars of 12 to 60 Mg.

In trying to sum up the theoretical situation Iben (1974) states that the solution probably lies in the fact that the PCMa stars lie right in the overall contraction phase rather than in the core hydrogen-burning phase. There should be a huge convective shell and, taking into account time depardent convection, "it could be that fluctuations of convective motions act as the continual perturbation that drives the pulsation".

Thus, among the theorists, disagreement prevails with regard to the mechanism, modality, and evolutionary state responsible for the variability of the β CMa stars.

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V. The Evolutionary State of the β CMa Variables.

Again, observations require that the β CMa stars be in one (or perhaps more) of three evolutionary states: core **hy**drogen-burning, secondary contraction, or shell hydrogen-burning. **A** natural assumption here is that the presence of variability is directly dependent on the evolutionary state; i.e. pCMa stars should be in evidence in only one of the three states. Thus we ask, which evolutionary state is responsible for the pulsations, and can we determine this state observationally?

One such observational test would be to determine the proportion of pCMa stars in a statistically complete sample of early B stars; a number that one could compare directly to the proportion of its lifetime that an early B star should spend in each of the three evolutionary phases. To establish the lifetime expected for each phase we can select stellar models appropriate to the early B stars/ β CMa stars. To select these models we need to know the mass range of stars exhibiting β CMa variability.

We note that all confirmed β CMa variable stars are found in the range of spectral classes BO.5 to B2 and luminosity classes III to V. The most luminous star is classified BO.5III and the least luminous star, B2IV. This corresponds to a range in absolute visual magitude of -4.74 to -3.54 and a mass range of approximately 25 to 15 M_{σ} (Allen, 1973). β CMA

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candidates (Hill, **1967)** are reported with spectral classes as late as B3V. This corresponds to M_{v} = -2.30 and a mass of approximately 10 M_o. Thus the models that we choose should correspond to stars in the mass range between **10** and **25 Me.**

Three stellar models spanning this mass range were selected from the vast literature on the subject. A 20 M_o model (Barbaro et. al., 1973), a 15 M_o model and a 9 M_o model (Iben, **1972).** These models all provided figures for the time spent in each evolutionary state and, most importantly, were similar in the initial compositions assumed for the model stars. The models of both authors used **X=0.7** while Barbaro et. al. used Z=0.044 and Ihen used Z=0.06. The several models calculated **by** Barbaro et. al. showed that the value of Z had very little effect on the resulting models.

None of the models provided calculations of the time spent in the S-bend region during core hydrogen-burning. One could, however, estimate the proportion of the total core **hy**drogen-burning phase spent in the S-bend region using a **10** M0 model (Lesh and Aizenman, **1973b)** which provided several ages along the core hydrogen-burning track. The estimate thus adopted was **10%** of the total core hydrogen-burning time. Thus the time spent in the S-bend region during core hydrogen-burning is defined as **1/10** of the total core hydrogen-burning time.

The figures for time spent in overall contraction are of

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course entirely within the S-bend region; and the times required for establishment of the shell source are, in any event, far too short to account for the observed number of variables.

Roughly, the fractions of time spent in the S-bend region during each phase are as follows: core hydrogen-burning in S-bend region, approximately **79%;** overall contraction, approximately **19%;** and establishment of the shell source, approximately 2%. The results of the models and calculations of the time spent in the three evolutionary states are summarized in Table 4.

Next, one must determine the proportion of β CMa stars in a statistically complete sample of B stars. Percy (1974) has made a small attempt at this. He found that of the 42 B1-B2 stars within 200 pc. of the sun, at least 8, or 14% , are β CMa variables. However, lack of information about the variability of B1-B2 stars in the region may cause the figure to be too low, and the area having an abnormally high proportion of giants (the presence of the Sco-Cen open cluster) may cause the figure to be too high.

A more complete sample (Lesh and Aizenman, **1973b)** should extend to **500** pc. Such a sample includes a number of open clusters and associations (Allen, **1973)** including the Sco-Cen cluster, the II PER association, the I Ori association, and the Perseus cluster, plus a large number of field stars.

I found a total **of 238** stars (spectral classes BO to B3,

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Table 4: Lifetime in the evolutionary states of the S-bend

region for 20, 15, and 9 M_{\odot} models.

REFERENCES:

***** Barbaro et. al., **1973**

* Iben, **1972**

Iben, **1972**

luminosity classes **II** to V) between -20 and **-90** declination (Hoffeit, 1964). Fifty-nine of these stars did not have luminosity classes. **Of** the **179** remaining stars, **130** (using spectroscopic parallaxes (Allen, **1973)** assuming negligable interstellar absorption) were within **500** pc. of the sun. This was **73% of** the stars with luminosity classes. If the same percentage of stars without luminosity classes are within **500** pc., then we would expect 43 of these stars to be within the required distance. Thus between -20 and **-90** we find approximately **173** (BO to B3, II to V) stars within **500** pc. of the sun. North of -20 dec. we find 224 early B stars within **500** pc. **of** the sun (Lesh, **1968).** This is a total of approximately **396** stars. Bright Star Catalogue Numbers are listed in Table **5.**

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Of those approximately **396** stars, **19** (about **5%)** are confirmed β CMa variables. This proportion of variables is compatiable with the expected lifetime of the overall contraction phase of evolution. One must be wary of this result, however; since far from all of the stars in the sample have been observed for FCMa variability. **A** total of **67** (about 20%) of the stars in the sample with reasonable distance estimates were listed in the Bright Star Catalogue as being of questionable variable status. Thus many variable candidates are evident in the sample. Table **6** lists those "VAR?" stars and constitutes a list of top priority stars to be observed for g CMa variability. It is noted that several confirmed g CMa

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Table $5:$ $(con't)$

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 $\frac{1}{2} \sum_{i=1}^{n} \frac{1}{i} \sum_{j=1}^{n}$

variables (i.e.- 19CMa and σ LUP) are listed as "VAR?" stars.

The conclusion that the overall contraction phase is responsible for the variability will remain in doubt until many more of the sample stars can be classed as either normal or CMa stars. Such observations are direly needed.

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Bibliography:

Aizenman, M. L., and Cox, I.P. (1975a); AP. J,,195, **175-185.** Aizenman, M. L., and Cox, I.P. (1975b); **Ap. J ¹ 2** , 399-404. Allen, C.W. *(1973);* Astrophysical Quantities, 3rd ed.

University of London Press, **p.200** Balona, L.A., and Feast, M.W. (1975); **M.N.R.A.S., 172,191-203.** Balona, **L.A.,** and Crampton, **D.** (1974); **M.N.R.A.S.,156,181-188.** Barbaro, G., Bertelli, G., Chisi, C., Nasi, E. (1973)

Astronomy and Astrophysics, 29, 185-198.

- Cox, I.P., and Tabor, J. (1974); I.A.U. Symposium #59, Stellar Instability and Evolution, **p. 73.**
- Dukes, R.J. *(1973);* **Ap. J., 192, 81-91.**
- Dupree, R.G. (1974a); **Ap. J.,** 190, *631-636.*
- Dupree, R.G. **(1974b); Ap. J.,** 194, **393.**
- Eggleton, P.P., and Percy,J.R. **(1973); M.N.R.A.S.,161,** 421-425
- Glasby, **J.A. (1969);** Variable Stars, 94-98.
- Harper, R.R., and Rose, W.K. **(1970); Ap. J., 162,963-969.**

Hill, **G. (1967); Ap. I.** Suppl., 14,263-300.

- Hoffeit, P. (1964); Catalogue of Bright Stars, Yale **U.** Press.
- Iben, I. **(1972);** Stellar Evolution, ed. **by** Chin and Murial,

"Normal Stellar Evolution", M.I.T. Press, *58-62.* Jones, D.P.H., and Shobbrook, $R.R.$. (1974); $M.N.R.A.S.$,

166,649-661.

- Lesh, J.R. **(1968); Ap. J.** Suppl., **17,** 371-444.
- Lesh, J.R. and Aizenman, M.L. (1973a); Astr. and An., 22, **229-237.**

Bibliography: (con't)

- Lesh, J.R. and Aizenman, M.L. **(1973b);** Astr. and **Ap., 26, 1-9.**
- McNamara, **D.** H. and Hansen, K. **(1961); Ap.J.,** 134,207.

Osaki, Y. (1974); *Ap.* **J., 189, 673-684.**

- Percy, J.R. **(1970);** Ap.J., **159, 177-182.**
- Percy, J.R. (1974); <u>Astr. and Ap</u>., 30, 465-466.
- Percy, J.R. (1975); "Pulsating Stars", Scientific American *67-75.*
- Shobbrook, R.R., and Lamb, N.R. (1972); M.N.R.A.S., 156,181-188.
- Shobbrook, R.R.: (1972) ; M.R. $\underline{R}.\underline{A}.\underline{S}.\underline{A}.\underline{S}$., $\underline{156}$, $5P-9P$.
- Shobbrook, R.R. **(1972);** M.N.R.A.S., **156, 181-188.**
- Shobbrook, R.R. (1973); M.N.R.A.S., 162, 25-35.

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- Shobbrook, R.R. **(1973);** M.N.R.A.S., **161,** 257-267.
- Shobbrook, R.R. (1974); I.A.U. Symposium #59, Stellar Insta-

bility and Evolution, **69-71.** Strohmeier, M. **(1972);** Variable Stars, 134-140. Strothers, R., and Simon, N.R., (1969); <u>Ap</u>. J., 157,673-681. Van Hoof, **A. (1972);** Astr. and AR., **18,** 51-54.

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