# THE ROLE OF CELLULAR CONVECTION WITHIN AN EXTRATROPICAL CYCLONE

by

MARTIN STEVEN TRACTON
B.S., University of Massachusetts
(1966)

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREE OF
MASTER OF SCIENCE
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
November, 1968, i.e. Jan. 1969

Signature of Author .. ..................
Department of Meteorology, 25 November 1968

Certified by ..................................................
Thesis Supervisor

Accepted by ..................................................
Chairman, Departmental Committee on Graduate Students

THE ROLE OF CELLULAR CONVECTION
WITHIN
AN EXTRATROPICAL CYCLONE

by
MARTIN STEVEN TRACTON

Submitted to the Department of Meteorology
on 25 November 1968
in partial fulfillment of the requirements
for the degree of
Master of Science

ABSTRACT

Recording rain-gauge data, supplemented by radar observations, are utilized to determine the relative contributions of convective and stratiform lifting to the total precipitation deposited by an intense extratropical cyclone. A simple cell model is then used to deduce the vertical transport of mass necessary to produce the observed convective rainfall. The vertical transport of mass accompanying production of the stratiform precipitation is obtained by assuming this precipitation was generated by uniform moist adiabatic ascent. On the basis of the computed mass transports, the vertical transports of momentum and sensible heat accomplished by each mode of lifting are determined.

It was found that at least 30 per cent of the total precipitation produced and, therefore, the same fraction of the total latent heat released by the storm was attributable to cellular convection. Approximately the same amount of air was funneled upward through the 700 mb level by convective cells as was lifted by larger-scale motions. Finally, although the convective transport of sensible heat through 700 mb was small by comparison with that transported by stratiform lifting, the cellular mode of motion was dominant at this level in the vertical transport of momentum.

The results suggest that certain physical processes usually attributed solely to the larger-scale motions of cyclonic systems may, in fact, be highly concentrated into convective cells.

Thesis Supervisor: Dr. Pauline M. Austin
Title: Research Associate

## ACKNOWLEDGMENTS

I would like to thank Dr. Pauline M. Austin and Professor James M. Austin for their encouragement and critical review of this paper.

Appreciation is also expressed to my wife, Bette, for typing of the manuscript.

Special thanks go to Steven A. Ricci for preparation of many of the figures.

# TABLE OF CONTENTS

# LIST OF TABLES

# LIST OF FIGURES

# I. INTRODUCTION

Cellular convection is recognized as an integral and indispensable factor in the development and maintenance of tropical cyclones. In the present view, hurricanes are forced circulations driven by the latent heat released in organized convection (Charney and Eliassen, 1964). In addition, the intense vertical currents of deep cumuli significantly influence the cyclonic-scale circulation by acting as diffusion agents through the vertical transports of heat, momentum, and moisture (Kuo, 1965). Riehl and Malkus (1961) have demonstrated, in fact, that the important dynamic and thermodynamic processes of a hurricane are highly concentrated into narrow convective zones and have related a storm's intensity to the mass of air channeled up through "hot towers" as opposed to that lifted by "mass circulations".

Radar observations have shown that cellular convection also frequently occurs imbedded within the broad precipitation regions of intense extratropical cyclones (e.g. Boucher, 1959; Austin, 1968). In temperate latitudes, unlike the tropics, however, cyclones originate in response to a baroclinic instability and derive their energy principally from the temperature contrast between air masses. Consequently, meteorologists generally consider that synoptic-scale motions and processes dominate within mid-latitude cyclones, and therefore they have tended to regard cellular convection as a relatively unimportant dynamic entity to the storm system.

There exists indirect evidence, however, which suggests that imbedded convection may play an important complimentary role to that of the large-scale circulation in the development and maintenance of extratropical cyclones. In examining the effects of latent heat release on storm development, Danard (1964) suggested that it is the rate of condensation which is crucial. Collectively, the rapid and localized release of latent heat in the sharp vertical currents of convective cells would enhance the larger-scale vertical velocity. The induced low-level convergence would be enough to upset the usual balance between the production of vorticity by convergence and its destruction by surface friction. According to Danard, explosive cyclogenesis might follow. Peixoto (1966) has demonstrated that on the global scale latent heat release serves as an important differential heat source to the atmosphere and thus generates eddy potential energy. It follows, then, that the rapid and localized release of latent heat by convective motions may alter the baroclinicity of the atmosphere and thereby influence the energetics of the storm system. Finally, Newell (1960) suggested that cumulus convection could accomplish substantial vertical transports of heat and momentum within extratropical storms and, in fact, may be the actual physical mechanism for the transport usually attributed solely to the cyclonic scale. The convective mode of motion may therefore play a significant part in storm evolution as well as in the overall balance of heat and momentum between the earth and its atmosphere. As Newell points out, the role of convective motions as a transport mechanism involves a basic question concerning the workings of the atmosphere.

Thus, it may well be that the structure of a storm as observed on the synoptic scale is intimately related to the dynamics and thermodynamics of the convective scale. Should this be the case, it is obvious that the nature of the interaction between convective and larger-scale motions must be more closely scrutinized before the complex phenomenon of cyclogenesis is adequately understood. In addition, numerical forecast and diagnostic models will have to simulate, either specifically or statistically, the role of small-scale convection within extratropical cyclones.

The purpose of this investigation is to assess quantitatively in a selected storm the importance of imbedded convection in relation to larger-scale motions in such processes as the production of precipitation, release of latent heat, and the vertical transports of mass, momentum, and sensible heat. The results should serve as an indication of the extent to which important physical processes are concentrated into convective cells rather than being uniformly distributed over larger-scale regions.

A direct and comprehensive analysis, either descriptive or dynamic, of convective phenomena would require a very dense observational network. Existing mesoscale networks are oriented towards investigating severe local storms such as thunderstorms and squall-lines rather than the arrays of small convective cells found imbedded within the broad precipitation areas of intense extratropical cyclones. For this study, therefore, an indirect approach is adopted which utilizes standard meteorological data. The contributions of convective and stratiform lifting to the total precipitation produced by the storm are deduced from recording rain-gauge data supplemented by radar observations. A simple cell model is then used to estimate the vertical transport of mass necessary to produce the observed convective rainfall. The vertical transport of mass accompanying the production of stratiform precipitation is obtained by assuming that this precipitation was generated by uniform moist adiabatic ascent. Although the vertical air motions cannot be determined unequivocably from the precipitation they produce, Austin (1968) has shown that the rainfall patterns do prescribe limits for the motions which are sufficiently definite for the purposes of this investigation. Once the vertical mass transports are established, knowledge of the characteristics of the atmosphere and their variation with height permits computation of the vertical transports of momentum and sensible heat accomplished individually by stratiform and convective lifting.

## II. SYNOPTIC SITUATION

The major cyclonic development that occurred over the eastern United States, 29-30 November 1963, was selected for analysis. The storm was characterized by an intense mid-tropospheric circulation upon which was superimposed widespread convective activity. An overland storm track and availability of the data described in the following section made this an ideal situation to investigate the contribution of imbedded convection to such processes as the production of precipitation, release of latent heat, and vertical transports of mass, momentum, and heat.

Synoptic analysis of the storm, which has been discussed in detail by Danielsen (1966) and Bosart (1964), showed that a sharp upper level trough and vorticity maximum phased with a weak surface cyclone over Louisiana at approximately 00Z 29 November. With the accompanying strong thermal contrast in the lower troposphere, all the ingredients necessary for classic cyclogenesis were present. As the storm developed, it moved rapidly northeastward along the Appalachians (Fig. 1), dominating the circulation both at the surface and aloft over the eastern third of North America. The sustained and rapid deepening is evidenced by the tightening pressure gradient and drop in central pressure from 1004 mb at 00Z 29 November over

Louisiana to 972 mb at 12Z 30 November over Vermont. Surface and upper level charts indicated that although the vortex intensified dramatically, the fronts did not occlude before the storm passed on into Canada.

Copious rainfall accompanied the storm, reflecting the abundant supply of moisture fed into the system by the strong southerly flow. The heaviest precipitation occurred north of the low pressure center just east of the surface inverted trough that lay parallel to and ahead of the warm front. Surface synoptic reports and radar data indicated the presence of extensive convective shower activity in this region. Rainfall rates generally decreased in average intensity from heavy to very light, sometimes just drizzle, the farther west one proceeded from the warm front. Southwest of the low center, precipitation terminated abruptly along a curved line corresponding to the northern edge of a tongue of descending dry air.

## III. DATA

The investigation covered an area of $7.4 \times 10^5 km^2$ extending southward from Maine to North Carolina and westward to eastern Tennessee and Ohio (Fig. 2). The track of the low-pressure center was such that observation points obtained a relatively symmetric sample of the developing storm system both in space and time as it moved through the region on into Canada. The basic data utilized in this study are described below.

### (a) Radar Observations

Radar data were available from the Massachusetts Institute of Technology (M.I.T.) and the U. S. Weather Bureau installations at Nantucket (ACK), Atlantic City (ACY), Washington (DCA), Buffalo (BUF), and Cincinnati (CVG). M.I.T. data were in the form of 35 mm photographs of plan position displays (PPI) showing the averaged, range-normalized signal, quantized into a series of intensity levels. Each level covered an interval of 5 db, corresponding to a factor of two in equivalent rainfall rate. PPI data were available from two radars: the AN/CPS-9 with a wave length of 3 cm, and the SCR-615-B having a wave length of 10 cm. Range-height displays were taken on the AN/CPS-9 at approximately hourly intervals.

The Washington data were from a 10-cm WSR-57 radar and were in the form of 35-mm photographs of the PPI display. Although the radar was not calibrated, a stepped gain receiver was utilized so that weak echoes were periodically eliminated to reveal the details of the intense showers. The Cincinnati, Buffalo, and Atlantic City data were also from WSR-57 radar units. The former two were recorded on 16-mm film while the last was on 35-mm film. None of these radars were provided with gain steps and the signal was uncalibrated. The Nantucket data, PPI displays recorded on 35-mm film, were taken with a 10-cm SPIM radar. Again, no gain steps were utilized and the signal was uncalibrated.

RHI photographs were not available for the Weather Bureau radars; however, the data sheets accompanying the PPI films often contained the observers' comments on the vertical structure of the precipitation.

(b) Rain-gauge Networks

Hourly precipitation amounts were obtained from pamphlets published by the U. S. Weather Bureau (1963) for a network of stations whose density varies from state to state (Fig. 2). In addition, daily amounts from a more dense network were listed in the U. S. Weather Bureau Climatological Data (1963).

(c) Detailed Rain-gauge Records

Detailed tipping-bucket gauge records showing the continuous temporal variation of precipitation rate were available from twenty-eight stations distributed rather uniformly within the area under consideration. The gauge at the M.I.T. field station in West Concord, Massachusetts, records rainfall rates with a time resolution of a few seconds. The remaining gauges, operated by the Weather Bureau, have a resolution of about one minute. This is sufficient, however, to differentiate beween rather steady stratiform rain and rainfall fluctuating rapidly in space and time as is produced in convective cells (Fig. 3).

(d) Large-scale Meteorological Data

Data from the standard synoptic network provided information on the large-scale features of the atmosphere. Advantage was taken of the availability of detailed synoptic maps and analyses performed in previous investigations of the storm by Bosart (1964) and Danielsen (1966). This material included surface, 850, 700, and 500 milibar charts as well as several cross sections through the atmosphere. In addition, computations of large-scale vertical velocity performed by Danielsen (1966) were utilized.

# IV. METHODS OF ANALYSIS

## A. Computation of the Total Precipitation and the Relative Amounts Produced by Stratiform and Convective Lifting

From totals listed in the Climatological Data, a map of the distribution of total rainfall for the storm within the $7.4 \times 10^5 km^2$ area under consideration was prepared. The region was then subdivided into squares $1^o$ latitude by $1^o$ longitude and an average precipitation amount determined for each (Fig. 4). Although the distribution presented in this manner is fairly uniform, within individual squares gradients as large as 1.5 inches over 100 miles were observed. The mean of the values assigned to the squares was taken as the average areal depth for the entire region. The product of this depth and the area yields the total volume of precipitation deposited by the storm.

The relative contributions of stratiform and convective lifting to the total precipitation were deduced from detailed analysis of the rainfall rates recorded by the tipping-bucket gauges. Both rain-gauge and radar data displayed three identifiable scales in the structure and intensity of the precipitation, similar to the patterns observed by Austin (1968) within intense extratropical cyclones. First, there were long periods of relatively steady light rain (Fig. 3) that PPI films show reflected large-scale areas of precipitation extending hundreds of kilometers and requiring on the order of 5-10 hours to pass over a point. These broad and quite uniform regions of precipitation were clearly produced by stratiform lifting associated with the synoptic-scale rising motion of the cyclonic system.

Superimposed upon the continuous periods of rain were intense convective showers whose duration over a gauge was about a minute. Precipitation rates in the showers usually exceeded 0.5 in/hr and often reached peaks greater than 2.0 in/hr. Fairly substantial vertical motions were therefore required. Radar observations indicated that individual cells were between 2 and 3 km in diameter and could be tracked as a distinguishable echo for a period averaging around eight minutes. RHI photographs from the M.I.T. radar showed that cells extended to a height of 4-5 km. Although RHI films were not available from the Weather Bureau installations, the observers' comments on the radar data sheets indicated the vertical structure of the cells was similar to those which occurred around M.I.T.

From the spacing of showers on rain-gauge traces, it is evident that cells were not imbedded randomly within the large-scale areas. Rather, as radar observations demonstrated (Fig. 5), they were organized for the most part into small bands or areas whose dimensions were roughly 30 to 40 km in length and 15 to

20 km in width. These cell arrays are often termed mesoscale areas and constitute the middle step in the three scale hierarchy observed. Within the mesoscale areas spacing between the cells was about 6 km so that one small area might contain between 15 and 20 convective elements. Figure 3 exemplifies the fact that precipitation rates between the cells of mesoscale areas generally exceeded those of the larger-scale environment by about a factor of two or three. It is not clear, a priori, whether the mesoscale component of precipitation was produced by stratiform or convective lifting. It may represent condensate diverged from convective updrafts that subsequently falls as rain in the broader region between the cells. Alternatively, the cell arrays as a whole may be areas of enhanced upward motion, which can be considered stratiform since rainfall rates between convective elements were rather uniform over horizontal dimensions large compared to the depth of the layer containing the precipitation. It is not unlikely that both mechanisms contributed to some extent. For this investigation, however, the enhanced precipitation of mesoscale areas was assumed entirely stratiform so that the rainfall attributed to convective lifting represents a lower limit to that actually produced within the cell updrafts.

Estimates of the relative amounts of precipitation produced by stratiform and convective lifting at each tipping-bucket station were obtained as follows: for times when precipitation was occurring, the fairly steady rates, or plateaus, out of which the shower peaks rise were determined. For example, the value so assigned at LGA between 1240 and 1620 was .10 in/hr (Fig. 3); between 1620 and 1820 the steady rate within the mesoscale area passing overhead was .30 in/hr. The difference between the rainfall these rates account for and that actually observed during the specified time intervals was attributed to cellular convection. For LGA, .68 in. of rain fell between 1240 and 1620. From 1620 to 1820, 1.30 in. were recorded. The amounts attributable to convective lifting were then .31 in. and .70 in., respectively. Thus, 51% of the total precipitation at LGA was formed within the sharp and transient currents of convective cells, while the balance was generated by the relatively uniform upglide motion of stratiform lifting. To estimate the percentage of the total precipitation deposited by the storm that can be ascribed to convective lifting, the sum of the convective rainfall produced at the twenty-eight rain-gauge stations was divided by the sum of the individual station totals.

The adequacy of this method for determining the relative amounts of precipitation produced by stratiform and convective lifting was tested in the following manner: a detailed and painstaking analysis was made of the PPI films taken by the M.I.T. radar, which operated continuously during periods of convective activity providing excellent data on the distribution, extent, and motion of the cells. From the analysis an estimate was made of the number of cells detected by radar within a $4 \times 10^4 km^2$ area (Fig. 6) around Boston. The total precipitation deposited by an individual cell was then estimated on the basis of the observed average cell cross section, duration and intensity (Table 1). When this was multiplied by the total number of cells occurring within the area, the amount of convectively produced rainfall ascertained agreed to within a few percent of that determined from rain-gauge information alone.

B. Cell Model for Computation of the Convective-Scale Vertical Transport of Mass

The vertical transport of mass accomplished by small-scale convection was deduced from a simple cell model which relates the total mass flux within the cell updrafts to the accumulated convective precipitation. The model is a slightly modified version of that proposed by Austin (1968) and incorporates only the essential features of the convective mechanism. It makes use of the premise that, although the precipitation rate in a non-steady state process may not be a very good measure of the rate of condensation, the total amount of lifting must be closely related to the total amount of rainfall.

It is assumed that a cell updraft starts from an initial disturbance near the bottom of the unstable layer at height $Z_o$. As the initial parcels rise, their vertical speed increases until reaching a maximum at level $Z_1$ and subsequently decreases to zero at the cell top at level $Z_2$. The cell cross section is assumed constant. Therefore, to maintain mass continuity air is entrained from the sides at all levels up to $Z_1$ and diverged from the updraft between $Z_1$ and $Z_2$. By the time the initial parcels reach the top of the layer containing the cells, the updraft column has been established. As Austin (1968) points out, however, this column may but does not necessarily extend all the way from $Z_o$ to $Z_2$; that is, convergence and lifting may cease in the lower portions of the cell before the initial disturbance actually reaches the top.

The amount of entrainment in the convergent region below the level of maximum vertical velocity and loss through divergence above that level is determined primarily by the shape of the vertical velocity profile. Austin (1968) adopts a linear updraft profile (Fig. 7a) and also neglects the variation of density with height. Under these conditions, for every kilogram of air passing upwards through $Z_1$ equal parts are drawn from all levels between $Z_1$ and $Z_o$. The fraction taken from a layer of thickness dz is then $dz/(Z_1-Z_o)$. The air entrained at any level possesses the environmental value of specific humidity, $q(z)$, when it enters the updraft and $q'(Z_1)$ when it reaches level $Z_1$. Primed quantities here refer to the conditions inside the updraft where thorough mixing is assumed to occur. For each kilogram of air rising through $Z_1$, then, the moisture which has already been condensed out is:

$$\frac{1}{(Z_1-Z_o)}\int_{Z_o}^{Z_1}[q(z)-q'(Z_1)]dz \qquad (1)$$

Above $Z_1$, moisture condenses out at the moist adiabatic rate, and simultaneously air is lost to the environment through divergence. For every kilogram which goes through $Z_1$, the fraction $dz/(Z_2-Z_1)$ is diverged into a layer of thickness dz at any level z. Hence, the moisture condensed out above $Z_1$ is:

$$\frac{1}{(Z_2-Z_1)}\int_{Z_1}^{Z_2}[q'(Z_1)-q'(z)]dz \qquad (2)$$

The sum of expressions (1) and (2) gives the total condensate for each kilogram of air lifted through $Z_1$. The total mass of air which must rise through $Z_1$ to condense the observed convective precipitation is then easily obtained.

If, instead of a linear profile, a constant updraft with no entrainment is assumed, all of the air is transported from $Z_o$ to $Z_2$. It can be shown that in such a case the total condensate for each kilogram passing through $Z_1$ is approximately doubled. For this investigation the condensate was set somewhere between that of the linear and uniform updraft profiles by assuming that the entrainment and divergence are greater near the base and top of the cell, respectively, than near level $Z_1$. To incorporate this into the model the fractions $dz/(Z_1-Z_o)$ and $dz/(Z_2-Z_1)$ were weighted with linear functions of z, $K_1(z)$ and $K_2(z)$, respectively. This in effect reflects the situation when the vertical velocity varies parabolically above and below the level of maximum updraft speed (Fig. 7b). Choice in the magnitude of the weighting functions is not especially critical, since this approach merely insures that assumptions regarding entrainment and divergence at various levels introduce an uncertainty of less than a factor of two into the computed condensate per kilogram of air rising through $Z_1$. For computation purposes $K_1(z)$ was chosen such that five times as much air is entrained at $z=Z_o$ than just below $Z_1$. That is, it was assumed that the slope of the vertical velocity profile at the cell base was five times that just below the level of maximum vertical velocity. On a similar basis $K_2(z)$ was selected so that five times as much air was diverged at $z=Z_2$ than just above $Z_1$.

Inclusion of the K's yields the following as the complete expression for the total condensate produced for each kilogram of air lifted through $Z_1$:

$$\frac{1}{(Z_1-Z_c)}\int_{Z_c}^{Z_1} K_1(z)[q(z)-q'(z_1)]dz + \frac{1}{(Z_2-Z_1)}\int_{Z_1}^{Z_2} K_2(z)[q'(z_1)-q'(z)]dz \qquad (3)$$

The cells of this storm were imbedded within a precipitating saturated environment. Since the difference in temperature between cell updrafts and their environs is not large, on the order of 0.2°C (James, 1953; Austin, 1951), the specific humidity at a particular height in the convective elements (primed q's) was assumed equal to that of the environment. The environmental values of humidity utilized for computation were those of a mean sounding (Table 2) that displays the average conditions, as depicted by standard radiosonde data, in the vicinity of convective showers. RHI films from M.I.T. and the Weather Bureau radar data sheets placed the height of the cell tops generally between 4 and 5 km. $Z_2$ was therefore assigned a value of 4.5 km. The base of the cells was assumed located just above the inversion at 0.5 km, while $Z_1$ was taken as the top of the unstable layer, 3 km (around 700 mb), above which parcels would tend to decelerate. If $K_1(z)$ and $K_2(z)$ have a value of 1 midway between the lower and upper portions of the cell, respectively, this choice of $Z_o$, $Z_1$, and $Z_2$ yields the following expressions for the weighting functions:

$$K_1(z) = -.53z + 1.94 \qquad K_2(z) = .88z - 2.20$$

In reality, the moisture condensed within an updraft exceeds the precipitation deposited by the shower. The air diverged above $Z_1$ doubtless contains condensate which may remain suspended aloft as cloud material or may contribute to the enhanced precipitation in the mesoscale areas. Therefore, in order to use equation (3) for computing total vertical transport of mass, an assumption must be made regarding the relationship between the total condensate and the observed volume of convective rainfall. Information on which to base such an assumption, however, is limited. Braham (1952) estimated that for an average air-mass thunderstorm the condensate left as cloud or evaporated from the sides is nearly twice that deposited as precipitation. In kinematic models of cells imbedded within a saturated atmosphere, a situation which more closely resembles the case under consideration, Kessler (1967) found the amount left as cloud to be about one third of that precipitated. For this investigation it was assumed that, of the condensate produced within the updrafts, two-thirds fell beneath the cell as precipitation while the balance remained aloft as cloud or contributed to the mesoscale component of precipitation.

At any level, the downward transport of mass outside convective elements must equal the amount transported upwards within the updrafts. Strong downward currents, however, could not have occurred in the region between the cells of mesoscale areas since the precipitation rate there would not have been enhanced, but rather, substantially reduced through evaporation. For computation of the convective-scale transports of momentum and sensible heat, it was assumed that the compensating downward mass flux occurred as a uniform downward shift of mass over an unspecified area large compared to that of the cells.

## C. Computation of the Stratiform Vertical Transport of Mass

An estimate of the vertical transport of mass accompanying production of stratiform precipitation may be obtained by assuming that the precipitation is generated by moist adiabatic ascent at some constant vertical velocity. For a given vertical distribution of temperature the precipitation rate is related to this value of vertical velocity in a manner illustrated by the diagram of Fulks (1935). The rate, in turn, prescribes the time necessary to deposit within an area a given volume of rainfall. The total mass flux is then the product of the mean air density, area, vertical velocity, and the value ascertained for the time interval. Since it is the product of the lifting rate and time that is involved, it is apparent that in the final analysis the actual magnitude of the vertical velocity is not critical. Changing its value and that of the associated precipitation rate merely results in a corresponding change in the required time interval but has no effect on a computed transport of mass.

For computational purposes the vertical velocity was arbitrarily set at 10 cm/sec. The mean vertical distribution of temperature obtained by averaging the 12Z Nov 29 soundings at Nantucket, Massachusetts, and Huntington, West Virginia, was assumed representative of conditions in which stratiform precipitation was formed (Table 3). The mean precipitation rate obtained from Fulks diagram was then used to determine the vertical transport of mass necessary to accumulate the total amount of stratiform rainfall deposited in the $7.4 \times 10^5 km^2$ region. The computed flux was assumed constant with height up to 6 km, the level which radar data indicated to be the top of stratiform precipitation. The compensating downward transport presumably occurred in the region of large-scale subsidence that followed immediately behind the area of rising motion and precipitation (Fig. 8).

D. Computation of the Vertical Transports of Momentum

In order to compute the vertical transports of momentum accomplished individually by stratiform and convective lifting, one must assume that each mode transports downward through a given level precisely as much mass as it has transported upward. A non-zero value for the momentum transport requires a correlation between the wind velocity and the mass flux. For this study, only the momentum associated with the zonal wind component, U, was considered. If the value of U for the air transported upward through some level is smaller than for that brought down, the correlation is negative. That is, momentum has been carried downward from higher to lower levels. Quantitatively, the net transport through a given level may be written as $M_r U'_r + M_s U'_s$, where $U'_r$ and $U'_s$ are the departures in regions of rising and sinking motion, respectively, of the zonal wind components from some space and time mean, $\bar{U}$; the M's represent the computed mass transports ($M_s = -M_r$).

The stratiform transports of momentum through the 700 and 500 mb levels, the latter lying well above the cell tops, were estimated as follows: the computations of large-scale vertical velocity performed by Danielsen (1966) were used to identify regions of rising and sinking motion at 18Z Nov 29 (Fig. 8). For each radiosonde station in the eastern United States, the average zonal wind component between 12Z Nov 29 and 00Z Nov 30 was then obtained. The mean of these values was taken as $\bar{U}$, with $U'_r$ and $U'_s$ the average departures therefrom in areas of ascent and descent, respectively.

To evaluate the momentum transport by small-scale convection, it was assumed that the vertical variation of the wind observed from radiosonde data applied to the air immediately outside the updrafts. Within an updraft column the air entrained would tend to conserve its horizontal momentum as it rose rapidly in the narrow convective channel. Therefore, the horizontal momentum at a given level in the cells is the average of the levels below from which the air was drawn. It was clear from radiosonde data and radar echo velocities that the lower portion of cells was imbedded in a region of intense vertical shear, $\partial U/\partial z > 0$. Consequently, the momentum of air at 700 mb ($Z_1$) immediately surrounding the cells was greater than that of the air within the updrafts. For computational purposes, the mean value of this difference, $U'_r$, was taken as the average of $(U_{950} - U_{700})/2$ evaluated at all radiosonde stations in regions where convective activity was occurring.

It was assumed that the compensating downward mass flux for the convective-scale motions occurred as a uniform downward shift of mass over an unspecified area large compared to that of the updrafts. In addition, there was assumed to be no mixing of air from different levels so that the descending air had the same value of momentum as the environment at a given level; hence, $U'_s = 0$.

## E. Computation of the Vertical Transports of Sensible Heat

For a non-zero transport of sensible heat there must be a correlation between the mass flux and temperature. If the air transported upward through some level, for example, is warmer than that brought down, there is a transport of heat upward from lower to higher levels. This transport may be expressed as $C_p(M_rT'_r+M_sT'_s)$ where $C_p$ is the specific heat of air at constant pressure, and $T'_r$ and $T'_s$ are the departures of the temperature from some space and time mean, $\overline{T}$. The transports of sensible heat were evaluated only for the 700 mb level.

For the case of stratiform lifting, $T'_r$, $T'_s$ and $\overline{T}$ were obtained in a manner exactly analogous for that used to ascertain $U'_r$, $U'_s$, and $\overline{U}$.

It was not possible to establish the convective-scale transport of heat directly, since the difference between environmental and updraft temperatures was not known. Empirical and dynamic considerations indicate that this difference need not be large to provide the necessary bouyancy (Austin, 1953). James (1953) presented values of the temperature measured just below the base of convective clouds which suggest the excess temperature of convective clouds is approximately $0.2^{\circ}$C. Measurements of horizontal temperature profiles through clouds mady by Cunningham (1956) indicate a similar value. Therefore, to obtain an estimate of the sensible heat transported by convective motions, the mean difference between cell and environment temperature, $T'_r$, was taken as $0.2^{\circ}C$. On arguments paralleling those for computations of the momentum transports, $T'_s$ was assumed to be zero.

F. Accuracy of the Computations

It is recognized that none of the quantities, either directly observed or derived through application of the cell model or Fulk's diagram, were known accurately, that is, within a few percent. Austin (1968) estimates there are uncertainties of a factor of 2-3 in computations of the convective mass transport from the cell model. The uncertainties arise primarily from lack of knowledge concerning the details of entrainment and the loss of condensate to the environment through divergence. It was not possible to determine the overall reliability in the accuracy of other calculations. At each step in the computations, however, the uncertainties appeared to be about a factor of two, and it is believed that the final results are valid to within at least an order of magnitude. The accuracy of the various assumptions and deductions should be assessed in future investigations of this sort.

## V. RESULTS AND DISCUSSION

### A. Contributions of Stratiform and Convective Lifting to the Total Precipitation and Latent Heat Release

The average areal depth of rainfall deposited by the storm in the $7.4 \times 10^5 km^2$ region was 35 mm (1.4 in). The total volume of water precipitated was therefore $2.7 \times 10^{10} m^3$, corresponding to a latent heat release of $1.6 \times 10^{19}$ calories. When averaged over the approximate thirty-hour storm duration, this yields for the rate of latent heat release $1.5 \times 10^{14}$ cal/sec, which is similar to a value ($2.0 \times 10^{14}$cal/sec) found by Palmen and Halopainen (1962) near the central part of an intense extratropical cyclone.

Table 4 presents the contributions of stratiform and convective lifting to the total precipitation observed at each of the recording rain-gauge stations. It should be recalled that in computation of these quantities the enhanced precipitation of the mesoscale areas was considered entirely stratiform. Since some or all of this may in fact have been diverged from cell updrafts, the values listed in Table 4 probably represent lower and upper limits to the actual amounts of convective and stratiform precipitation, respectively.

The relative contribution of cellular convection to the precipitation ranged from 56% at Nantucket to zero at gauges in western portions of the region under discussion. Figure 9 displays the distribution of intermediate values. Readily apparent is the uniformity in the distribution of the convective contribution along bands parallel to the coast and the mean position of the warm front. Thus, although the total precipitation may vary quite markedly from station to station, the relative amounts of precipitation produced by stratiform and convective lifting were rather uniformly distributed with respect to an easily identifiable synoptic entity. This result may be of some value in attempts at quantitative precipitation forecasting. It should be possible to compile statistics concerning the excess of observed precipitation over that produced by stratiform lifting for various storm situations. Such statistics, together with more precise information on the mesoscale contribution, could then be used to modify precipitation amounts predicted by large-scale dynamic models.

The overall percentage of convective precipitation was estimated by dividing the sum of column 4 in Table 4 by that of column 2. The result so obtained was 30%. That is, at least 30% of the total rainfall deposited in the $7.4 \times 10^5 km^2$ region and, therefore, the same fraction of the total latent heat released can be attributed to cellular convection.

Although the release of latent heat by convective motions takes place throughout the depth of the updrafts, the intense vertical currents carry it to the upper portion of the layer containing the cells. For stratiform lifting the vertical velocities are much smaller so that, with respect to a convective time scale, the latent heat essentially remains at the level where condensation occurs. Radar showed that in the storm under consideration cell tops were between 4 km and 5 km, while stratiform precipitation extended to a height of 6 km.

Whether or not the rapid and localized release of latent heat within convective updrafts and its subsequent transport to the top of the layer containing the cells plays a significant role in cyclone development has yet to be definitively established. The investigations of Danard (1966) and others do suggest indirectly that such effects may be of crucial importance in the process of cyclogenesis by altering the baroclinicity of the atmosphere and thereby influencing the fields of kinetic energy and vorticity. In any event, in view of the above results, the implications of concentrating a substantial fraction of the latent heat release into convective elements warrant further investigation.

## B. Vertical Transports of Mass

Application of equation (3) of the cell model indicated that for each kilogram of air passing upward through the level of maximum vertical velocity (700 mb) 4.8 grams of moisture were condensed. It was assumed that one third of this condensate was lost to the environment through divergence so that only 3.2 of the 4.8 grams were precipitated beneath the shower. Information directly available from radar and rain-gauge data concerning cell dimensions, duration, and intensity (Table 1) showed that an average cell deposited $16 \times 10^9$ grams of water. It was therefore necessary for each cell to transport through the 700 mb level $5.0 \times 10^9$ kg of air.

The results discussed above demonstrated that 30% of the total precipitation that fell within the $7.4 \times 10^5 km^2$ region, or $8.1 \times 10^{15}$ grams, was attributable to cellular convection. The convective lifting process prescribed by the cell model, therefore, required a total upward mass flux through the 700 mb level of $2.5 \times 10^{15}$ kg.

The vertical transport of mass through this level accompanying production of the stratiform precipitation was $1.5 \times 10^{15}$ kg. Within the limits of accuracy of the calculations, the difference in the convective and stratiform transports is probably not significant. They are of the same order of magnitude, however, indicating that roughly the same amount of air was funneled by the cells upward through 700 mb as was lifted by larger-scale motions of the cyclone. This result suggests that cellular convection imbedded within extratropical storms may serve as an important agent of diffusion through the vertical transport of such physical quantities as momentum and sensible heat.

C. Vertical Transports of Momentum

The stratiform transport of momentum was evaluated first at the 500 mb level, which in this storm lay well above the cell tops. From a comparison of the 500 mb analyses (Fig. 10) with the field of large-scale vertical velocity (Fig. 8), it was apparent that the descending air had a larger westerly wind component than that which was ascending; hence, a downward transport of momentum. The magnitude of this transport was $2.2 \times 10^{16}$ kg m/sec (Table 5). At 700 mb the stratiform transport of momentum was also downward, but between one and two orders of magnitude smaller. That is, there was a much smaller correlation between the mass flux and the zonal wind component.

The convective transport of momentum through 700 mb was downward with a magnitude of $1.5 \times 10^{16}$ kg m/sec (Table 6). This value is approximately equal in sign and magnitude to that obtained for the stratiform transport at 500 mb. It appears, therefore, that the convective mode of motion performed the bulk of the momentum transport at low levels, whereas at higher levels the transport was accomplished by the larger-scale motions of the cyclonic system.

An interesting result was obtained by comparing the magnitude of the computed transports of momentum with the downward transport required from considerations of the global momentum balance. Starr and his collaborators have shown that a transfer of eastward (U component) momentum takes place from low to high latitudes in the upper troposphere. Since the frictional drain of momentum occurs at the surface, there is a necessity for a downward transport in middle latitudes. Starr and White (1951) have demonstrated that the downward transport necessary in one year between $31^{\circ}$ and $65^{\circ}$ north latitude is $16.3 \times 10^{6}$ gr cm/sec $cm^{2}$. This would require $1.2 \times 10^{16}$ kg m/sec for an area of $7.4 \times 10^{5}$ $km^{2}$. It is evident, therefore, that intense mid-latitude cyclones such as the one now under discussion may play a significant role in the global balance of momentum. Furthermore, it appears that cellular convection may be the dominant physical mechanism responsible for the transport at low levels (around 700 mb).

D. Vertical Transports of Sensible Heat

The stratiform transport of sensible heat through the 700 mb level was from lower to higher levels with a magnitude of $3.9 \times 10^{18}$ cal (Table 7). This is approximately equivalent to one quarter of the total latent heat released by the storm. The convective transport of heat was computed to be $1.0 \times 10^{17}$ cal, also upward but some forty times smaller than the stratiform transport. Thus, although large quantities of sensible heat were transported vertically within convective cells, ostensibly to relieve the instability producing them, this transport was small by comparison to that accomplished by stratiform lifting.

## VI. CONCLUSION

The results of this investigation have shown that cellular convection imbedded within an intense extratropical cyclone accounted for at least 30 per cent of the total precipitation produced and, therefore, of the latent heat released. The computed value of 30 per cent represents a lower limit to the actual convective rainfall, since the enhanced precipitation of the mesoscale areas was considered entirely stratiform although it may have in fact been produced in convective updrafts. Future investigations should be designed to determine explicitly whether the mesoscale component of precipitation represents condensate diverged from cell updrafts, or whether the cell arrays are as a whole regions of enhanced upward motion.

It was also found that approximately the same amount of air was funneled upward through the 700 mb level as was lifted by the larger-scale motions. In addition, although the convective transport of sensible heat through 700 mb was small by comparison with that transported by stratiform lifting, the convective mode of motion was dominant at this level in the vertical transport of momentum.

Thus, it appears that certain physical processes usually attributed solely to the larger-scale motions of cyclonic systems may, in fact, be highly concentrated into the narrow ascent regions of convective cells. Whether or not such localization may alter the course of a storm's development has yet to be established either theoretically or observationally. Indirect evidence obtained by several investigators does suggest that the effects of imbedded convection may be quite important. One approach in further investigation of this question might be to determine whether there is a relationship between the extent of convective activity in extratropical cyclones and the departure of actual storm evolution from that predicted by large-scale dynamic models. A consistent relationship would indicate that numerical models can be improved through inclusion of the effects of small-scale convection and, moreover, demonstrate that the complex phenomenon of cyclogenesis is indeed closely related to processes occurring on the convective scale. The next step in future research would be to develop adequate methods for simulating, in dynamic models, the role of cellular convection in the evolution of extratropical cyclones. In this regard, reference to the techniques utilized in incorporating convective effects into hurricane models might be useful.

Table 1. Average Cell Characteristics

| | |
|---|---|
| Diameter | 2.8 km |
| Cross Section | 6.0 $km^2$ |
| Height | 4.5 km |
| Lifetime | 8 minutes |
| Intensity (Average over cross section) | 20 mm/hr |
| Water Deposited | $16 \times 10^9$ grams |

Table 2. Mean Environmental Conditions in the Vicinity of Convective Activity

| Height (km) | T ($^{o}$C) | q (gr/kg) |
|---|---|---|
| Sfc. | 12.0 | 8.2 |
| 0.5 | 16.3 | 11.6 |
| 1.0 | 13.8 | 10.1 |
| 1.5 | 10.2 | 8.7 |
| 2.0 | 7.5 | 7.5 |
| 2.5 | 4.8 | 6.3 |
| 3.0 | 2.0 | 5.4 |
| 3.5 | -0.2 | 4.8 |
| 4.0 | -2.5 | 4.7 |
| 4.5 | -5.0 | 3.6 |
| 5.0 | -7.4 | 3.2 |
| 5.5 | -10.0 | 2.8 |
| 6.0 | -13.0 | 2.0 |

Table 3. Mean Vertical Distribution of Temperature Obtained by Averaging the 12Z Nov 29 Soundings at Nantucket, Mass. and Huntington, W. Va.

| Height | T ( $^{o}$C) |
|---|---|
| Sfc. | 10.5 |
| .5 | 9.0 |
| 1.0 | 5.8 |
| 1.5 | 3.5 |
| 2.0 | 3.0 |
| 2.5 | 1.5 |
| 3.0 | 0.0 |
| 3.5 | -2.5 |
| 4.0 | -5.0 |
| 4.5 | -7.6 |
| 5.0 | -10.1 |
| 5.5 | -12.8 |
| 6.0 | -15.5 |

Table 4. Contributions of Stratiform and Convective Lifting to the Total Precipitation Recorded at the Tipping Bucket Gauges

| Station | Total Precip. (inches) | Stratiform Precip. (inches) | Convective Precip. (inches) | % Con-vective |
|---|---|---|---|---|
| Nantucket, Mass. (ACK) | .96 | .42 | .54 | 56% |
| Boston, Mass. (BOS) | 2.29 | 1.13 | 1.16 | 51% |
| New York, N.Y. (LGA) | 1.98 | .97 | 1.01 | 51% |
| New Haven, Conn. (HVN) | 1.26 | .63 | .63 | 50% |
| Norfolk, Va. (ORH) | 2.04 | 1.04 | 1.00 | 48% |
| Hartford, Conn. (BDL) | 1.77 | .98 | .79 | 45% |
| W. Concord, Mass. (WCON) | 1.76 | .97 | .79 | 45% |
| Concord, N. H. (CON) | 1.31 | .79 | .52 | 40% |
| Richmond, Va. (RIC) | 1.96 | 1.22 | .74 | 38% |
| Charlotte, N. C. (CLT) | 1.65 | 1.05 | .60 | 36% |
| Philadelphia, Pa. (PHIL) | .86 | .61 | .25 | 31% |
| Lynchburgh, Va. (LYH) | 1.41 | 1.01 | .40 | 28% |
| Albany, N. Y. (ALB) | .51 | .37 | .14 | 27% |
| Binghamton, N. Y. (BGM) | 1.15 | .86 | .29 | 26% |
| Reading, Pa. (RDG) | 1.08 | .82 | .28 | 26% |
| Asheville, N. C. (AVL) | 2.45 | 1.85 | .61 | 25% |
| Harrisburg, Pa. (HAR) | 2.30 | 1.72 | .58 | 25% |
| Scranton, Pa. (AVP) | .63 | .50 | .13 | 25% |
| Greensboro, N. C. (GSO) | .99 | .79 | .20 | 20% |
| Rochester, N. Y. (ROC) | 1.66 | 1.55 | .11 | 7% |
| Syracuse, N. Y. (SYR) | 1.63 | 1.52 | .11 | 7% |
| Buffalo, N. Y. (BUF) | 1.46 | 1.40 | .06 | 4% |
| Pittsburgh, Pa. (PIT) | 1.17 | 1.17 | .00 | 0% |
| Cincinnati, Ohio (CVG) | .13 | .13 | .00 | 0% |
| Cleveland, Ohio (CLE) | .18 | .18 | .00 | 0% |
| Columbus, Ohio (CMH) | .20 | .20 | .00 | 0% |
| Parkersburg, W. Va. (PKB) | .72 | .72 | .00 | 0% |

Table 5. Quantities Utilized in Computation of the Stratiform Transport of Momentum at 500 mb

$\overline{U}$ = 35 m/sec

$U'_r$ = -7.5 m/sec

$U'_s$ = 7.5 m/sec

$M_r = 1.5 \times 10^{15}$ kg

$M_s = -1.5 \times 10^{15}$ kg

Transport = $-2.2 \times 10^{16}$ kg m/sec (downward)

Table 6. Quantities Utilized in Computation of the Convective Transport of Momentum at 700 mb

$U'_r$ = -6 m/sec

$M_r = 2.5 \times 10^{15}$ kg

$M_s = -2.5 \times 10^{15}$ kg

Transport = $-1.5 \times 10^{16}$ kg m/sec (downward)

Table 7. Quantities Utilized in Computation of the Stratiform Transport of Sensible Heat at 700 mb

$\overline{T}$ = 7.2°C

$T'_s$ = -8.3°C

$T'_r$ = 8.0°C

$M_r = 1.5 \times 10^{15}$ kg

$M_s = -1.5 \times 10^{15}$ kg

Transport = $3.9 \times 10^{18}$ cal (upward)

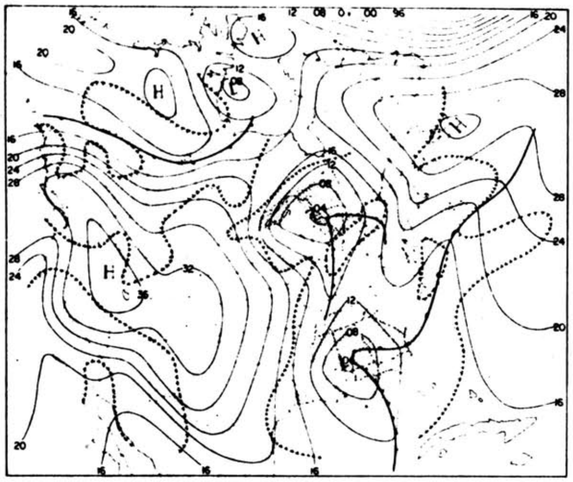


Fig. 1a. Surface Analysis 00Z 29 Nov. 1963

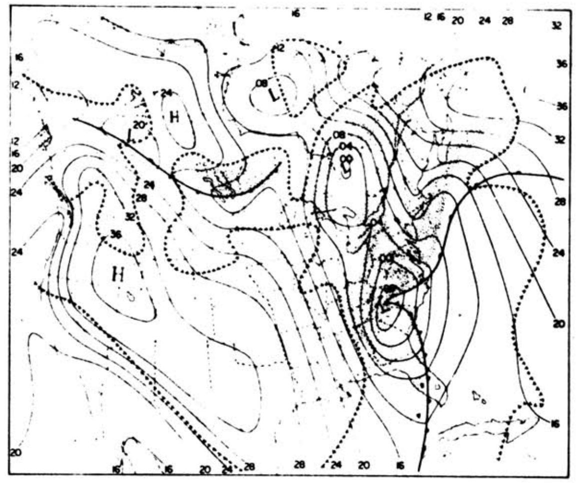


Fig. 1b. Surface Analysis 12Z 29 Nov. 1963

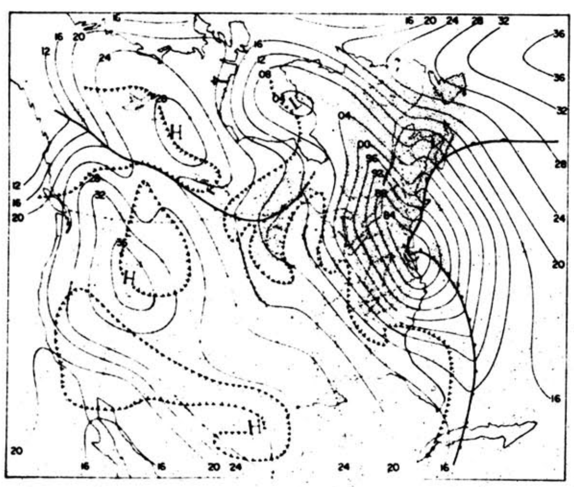


Fig. 1c. Surface Analysis 00Z 30 Nov. 1963

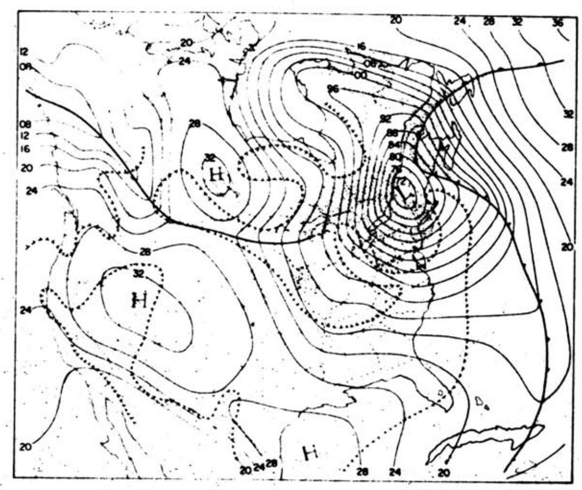


Fig. 1d. Surface Analysis 12Z 30 Nov. 1963

Figure 1. Surface analyses. Shaded areas indicate regions of precipitation(after Danielsen).

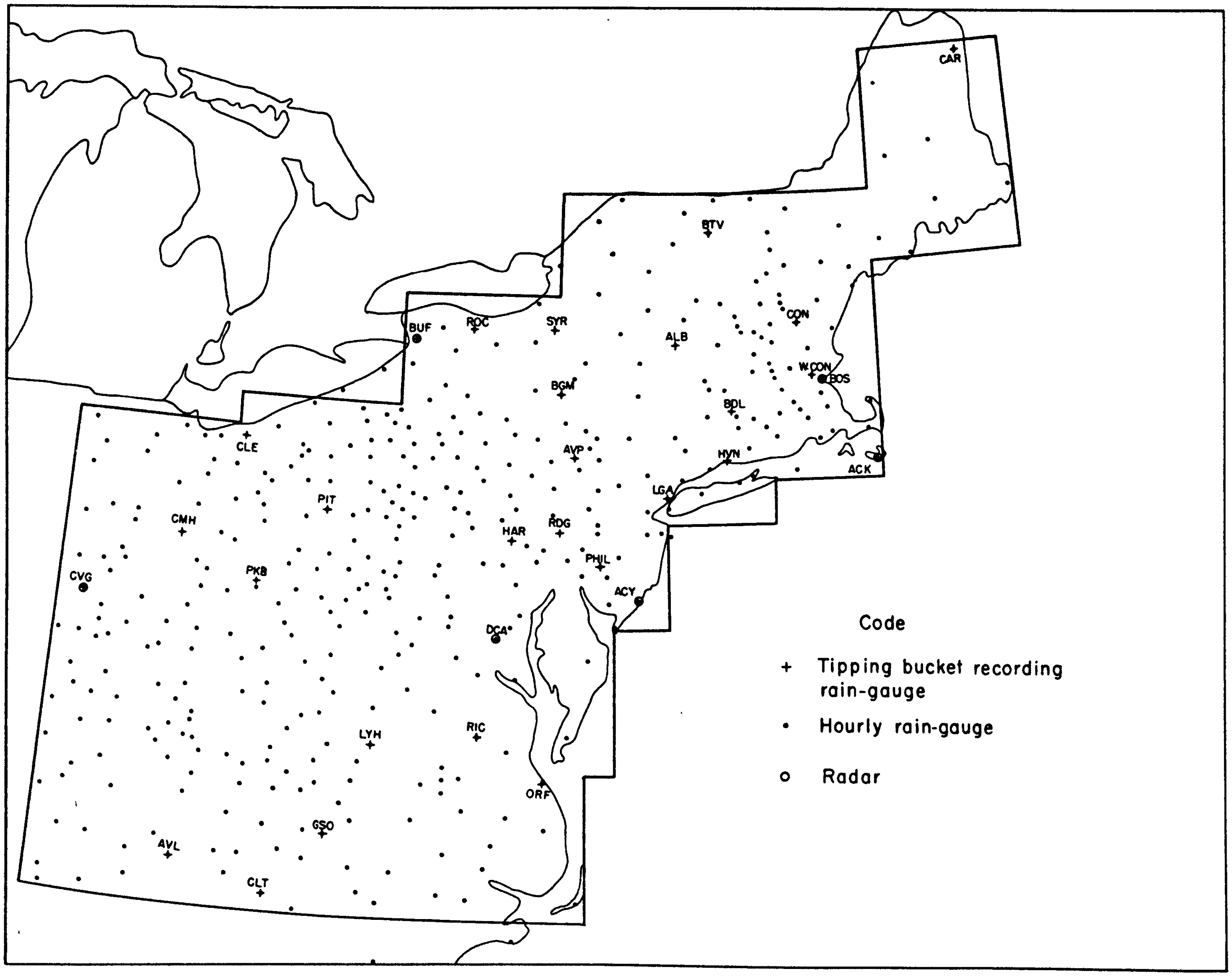



Figure 2. Location of observation points within the $7.4 \times 10^5$ $km^2$ region.

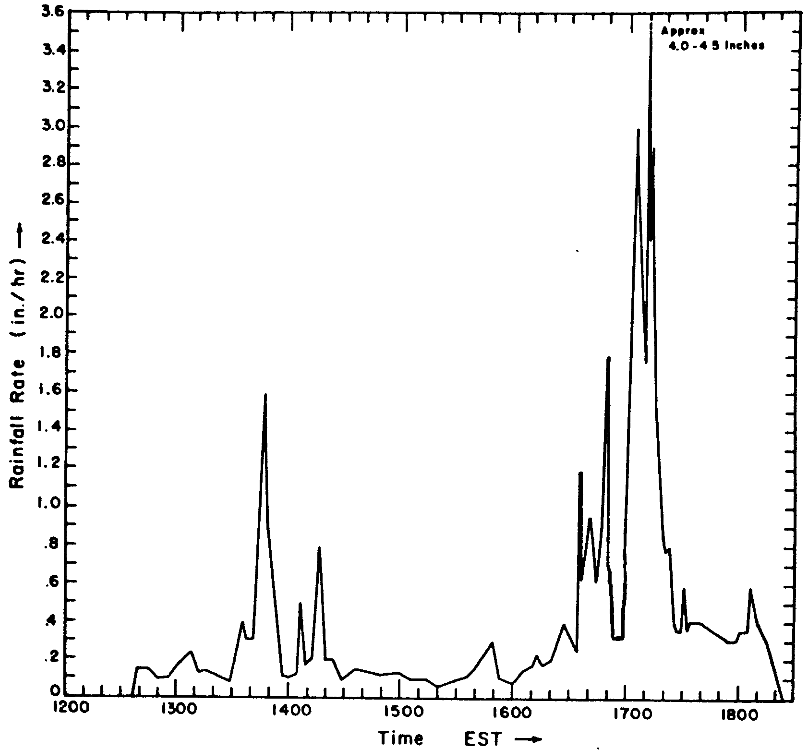



Fig. 3a LGA 29 Nov. 1963

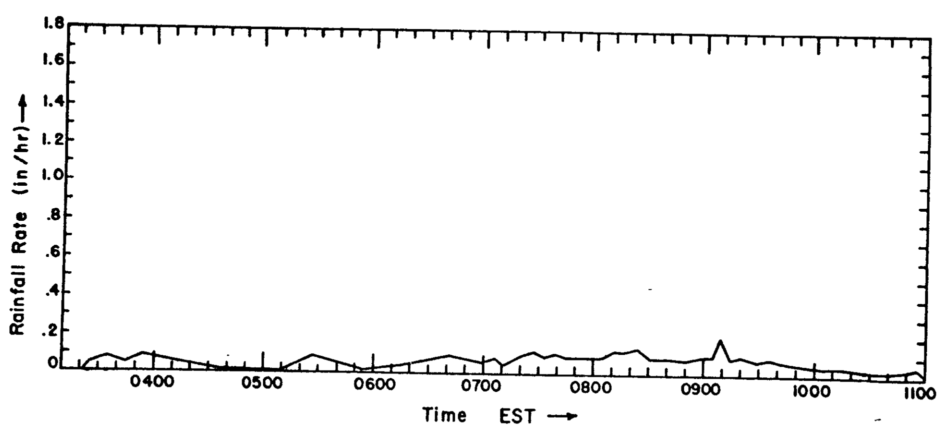



Fig. 3b PKB 29 Nov. 1963

Figure 3. Rain-gauge traces.

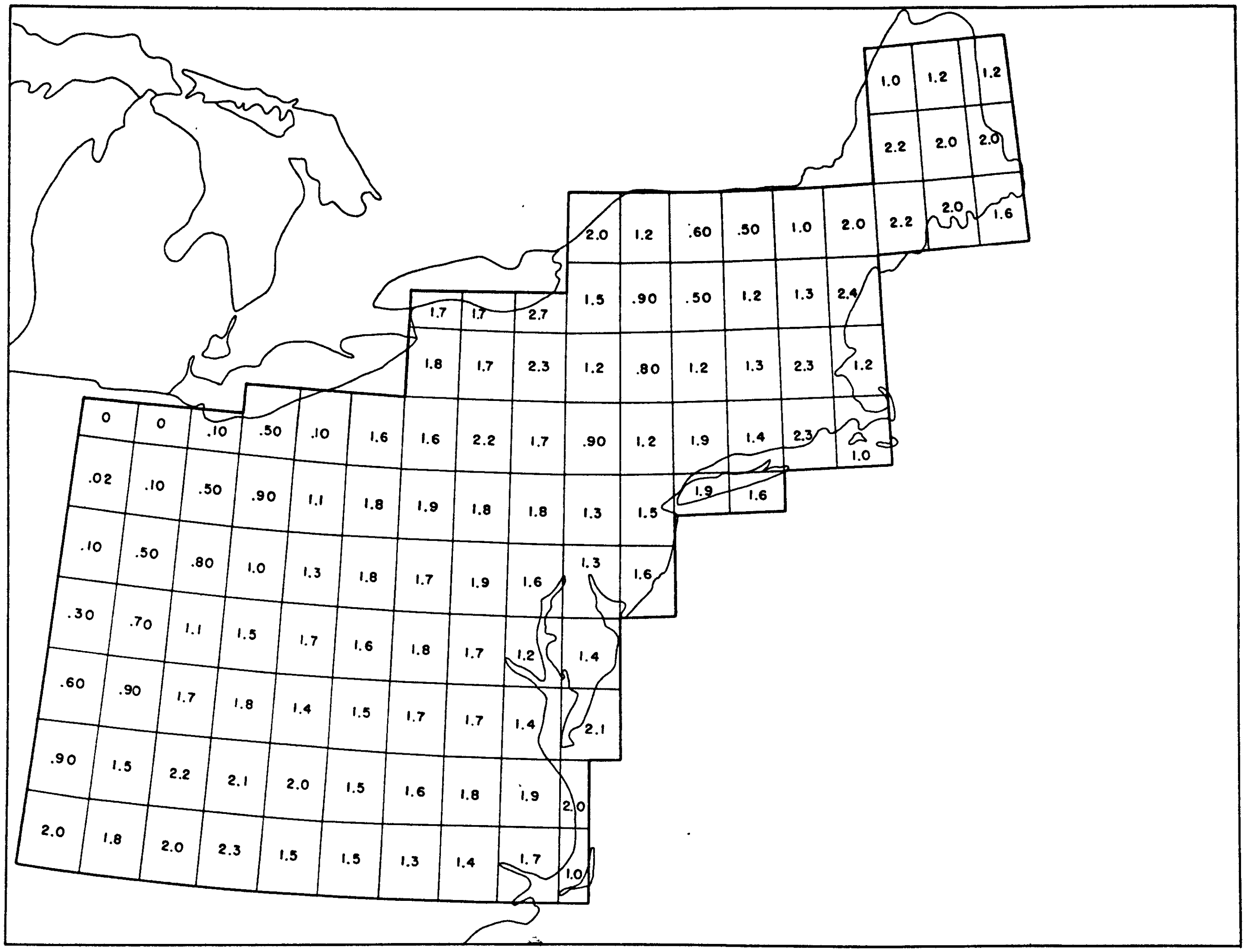
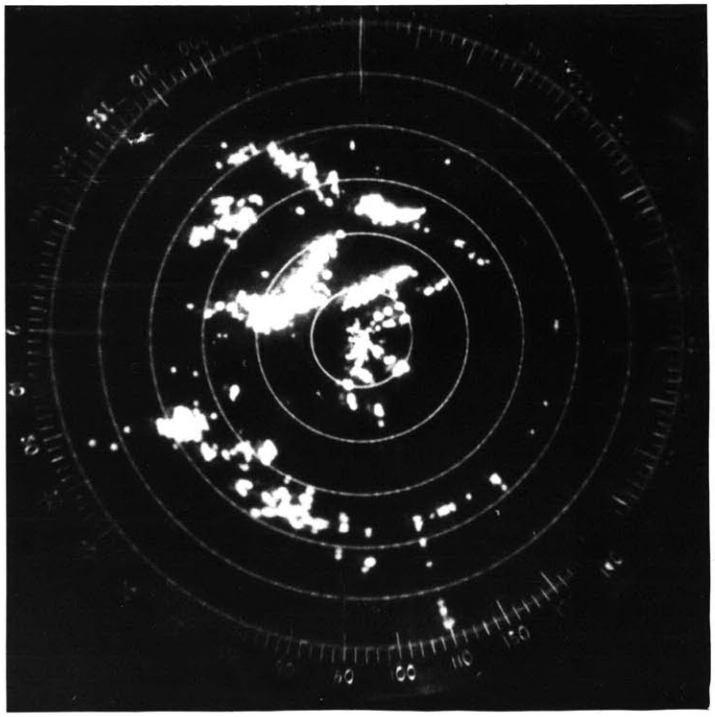
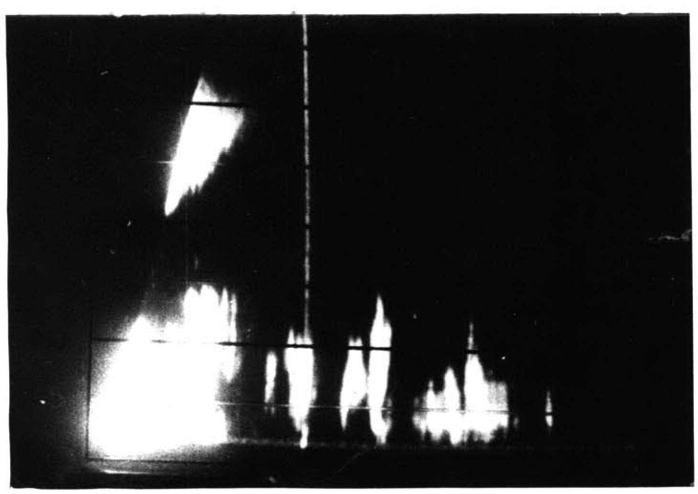



Figure 4. Distribution of the total precipitation (inches).



AN/CPS-9 120-mi range
0030 EST Nov 30



AN/CPS-9 50-mi range
0010 EST Nov 30 Azimuth 234°

Figure 5. M.I.T. radar scope photographs showing groups of intense small showers.

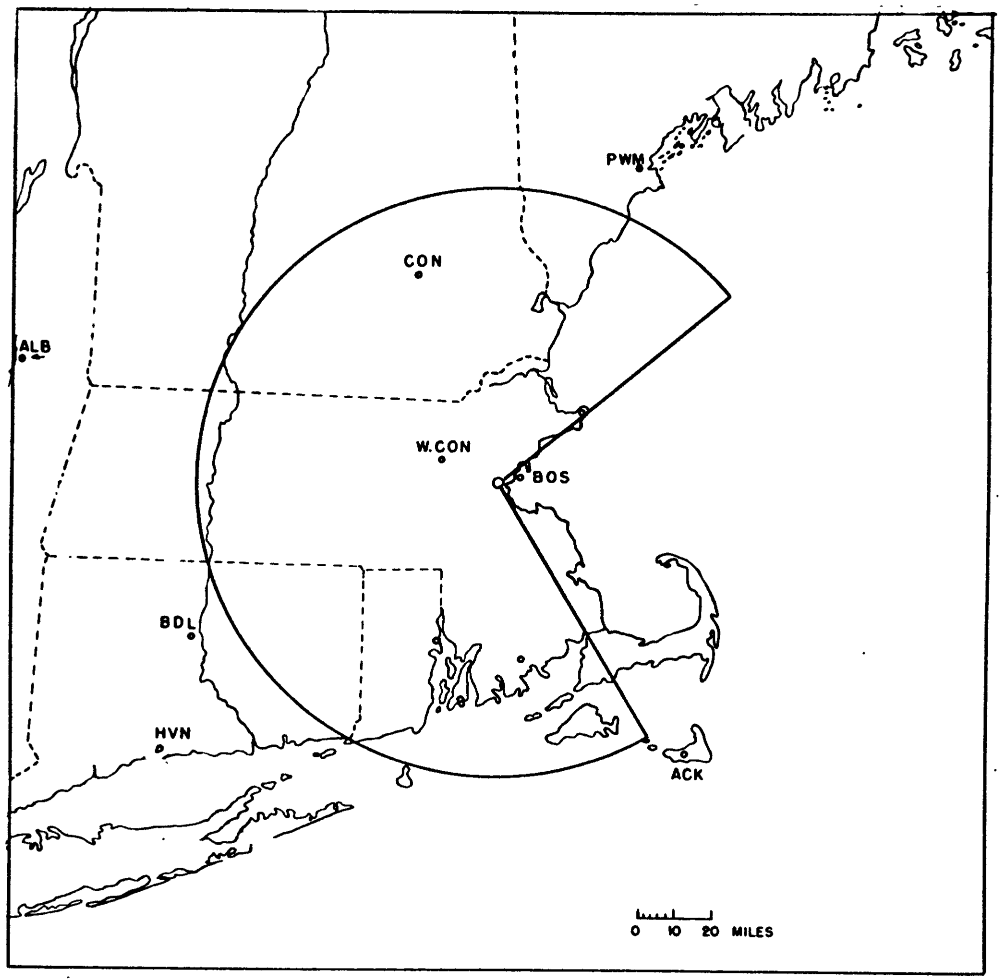



Figure 6. 4 X $10^4$ $km^2$ area considered in test of rain-gauge technique for computing the convective precipitation.

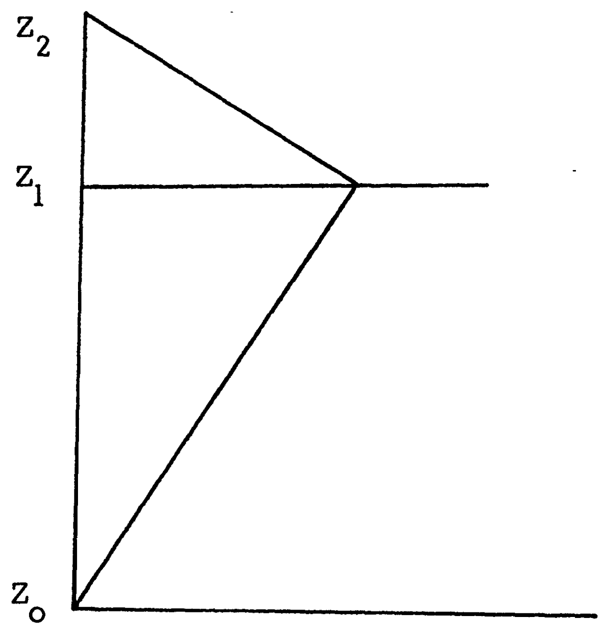



Fig. 7a Linear updraft

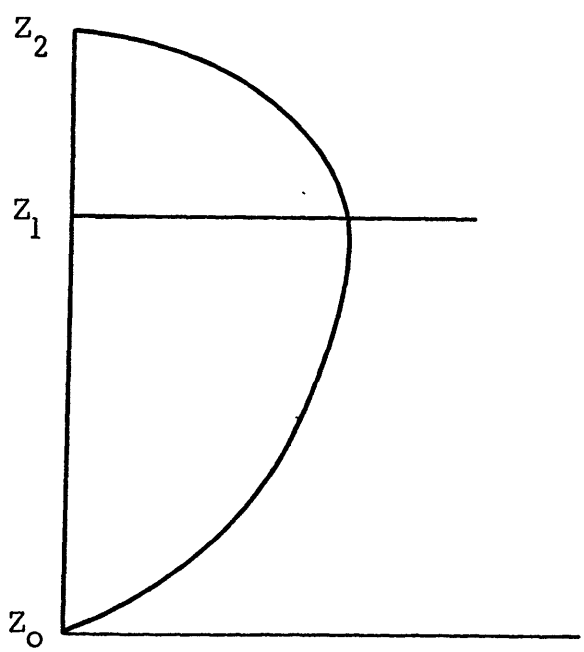



Fig. 7b Parabolic updraft above and below $Z_1$

Figure 7. Updraft velocity profiles. Maximum vertical velocity at level $Z_1$.

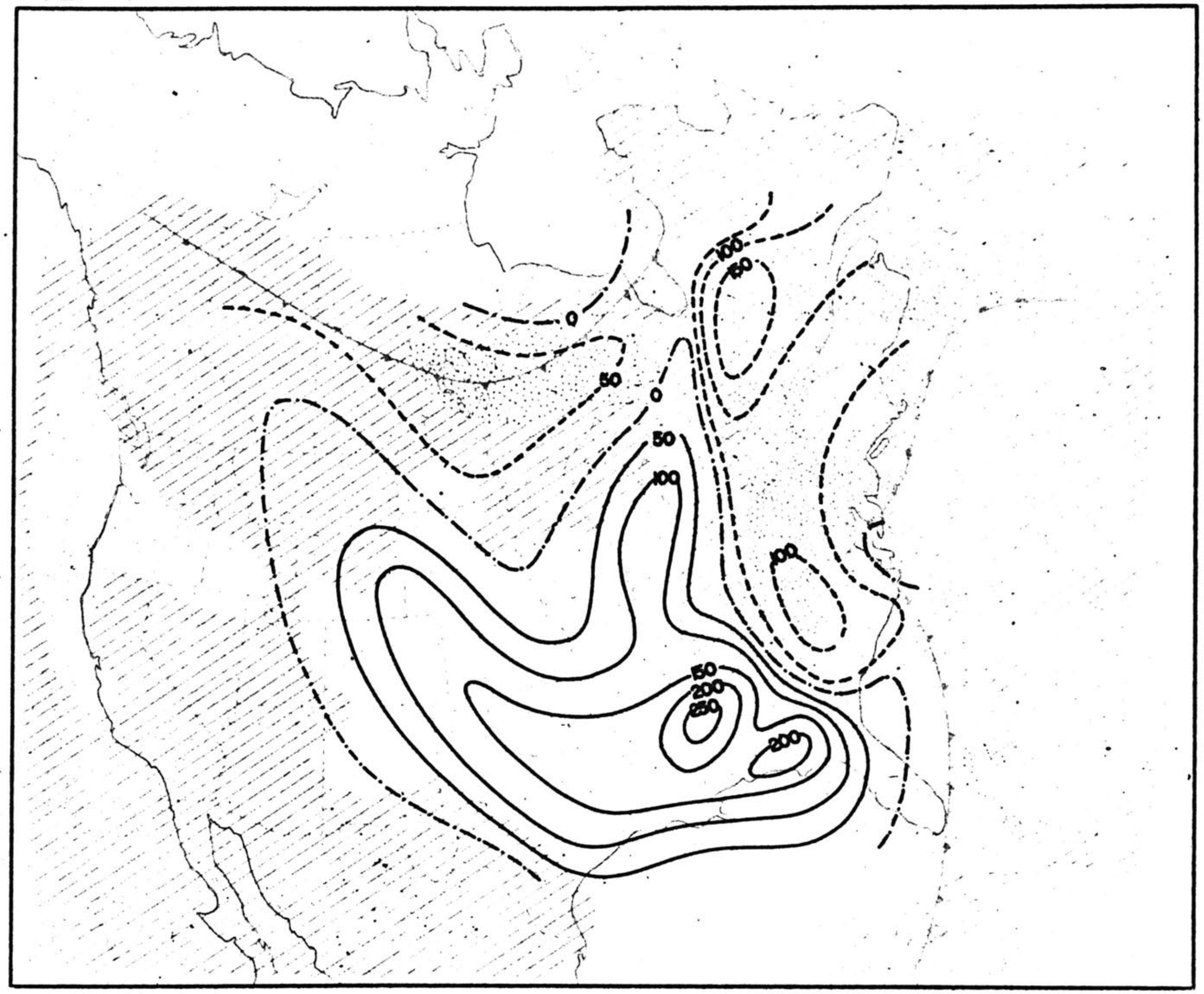



Figure 8. Vertical velocities at 500 mb 18Z 29 Nov 1963 (mb/12 hours) (after Danielsen).

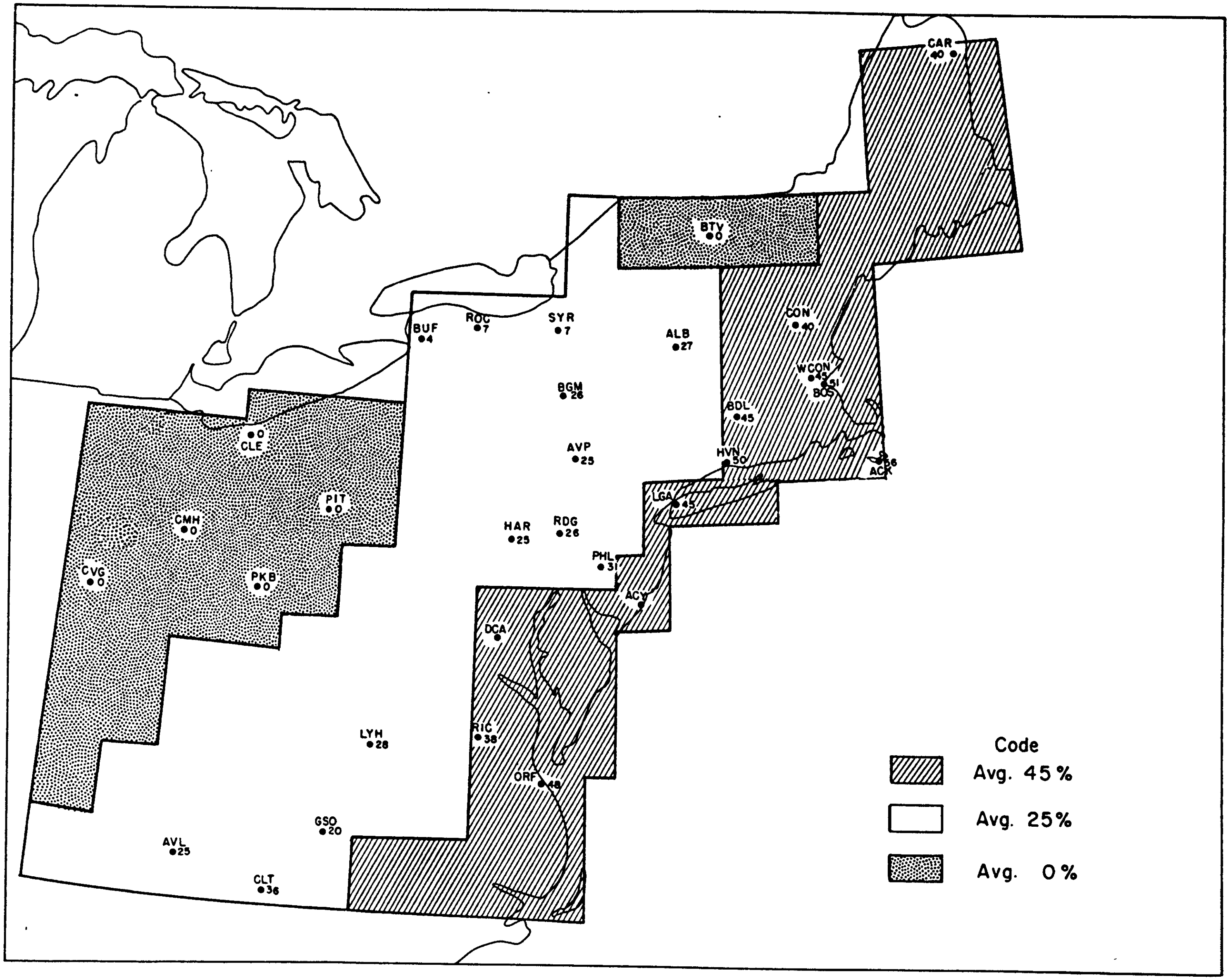



Figure 9. Distribution of the convective contribution to the total precipitation.

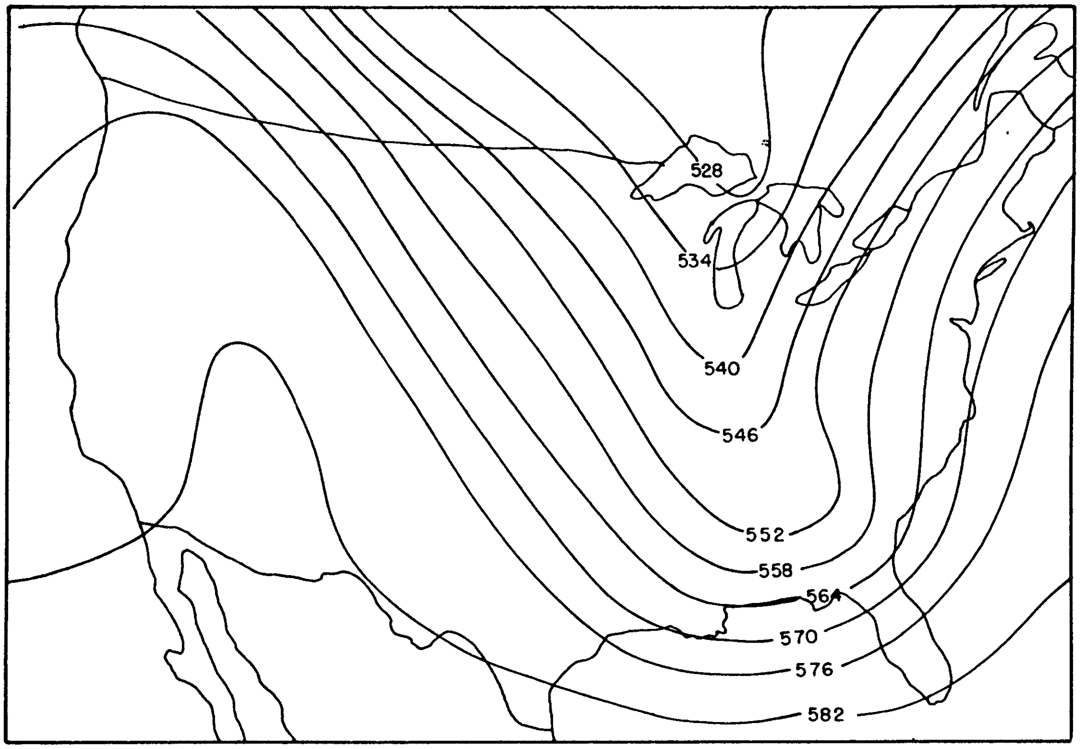



Fig. 10a 500 mb analysis, 12Z 29 Nov 1963

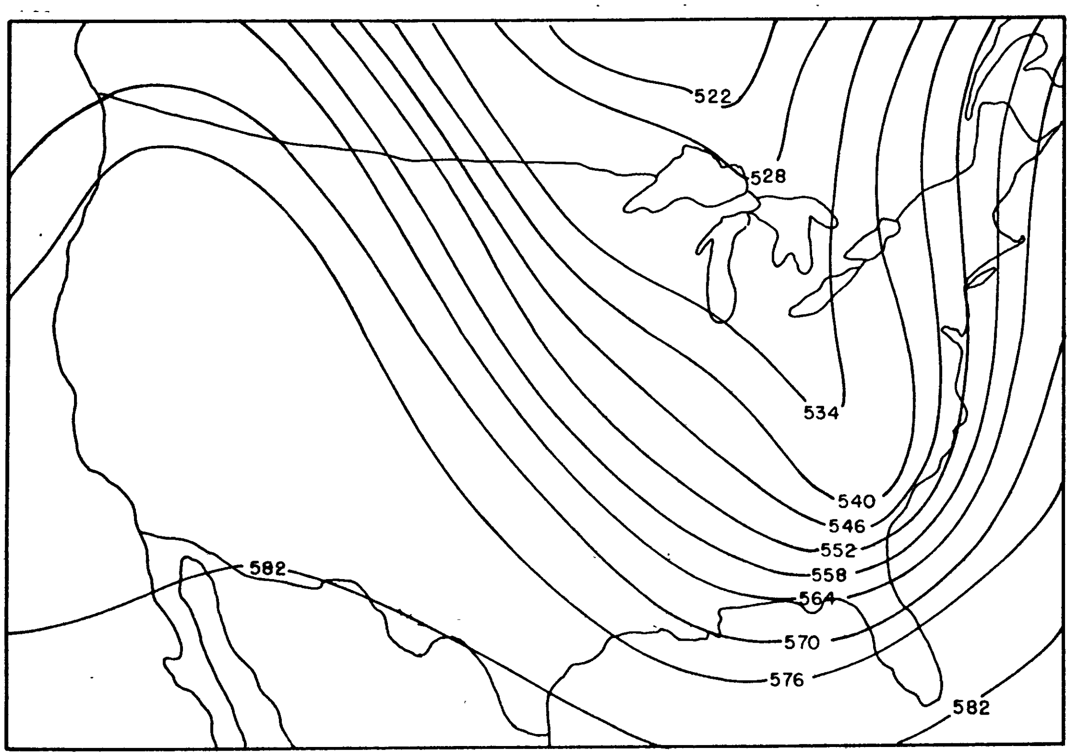



Fig. 10b 500 mb analysis, 00Z 30 Nov 1963

Figure 10. 500 mb analyses.

REFERENCES

Austin, J. M., 1951: Cumulus convection and entrainment, Compendium of Meteorology, Amer. Meteor. Soc., Boston, Mass., 694-701.

Austin, P. M., 1968: Analysis of small-scale convection in New England, Proc. of the Thirteenth Radar Meteorological Conference, American Meteorological Society, Boston, Mass., 210-215.

Bosart, L. F., 1964: Bomb of the year: 1963, Dept. of Meteor., M.I.T., Cambridge, Mass. (unpublished manuscript).

Boucher, R. J., 1959: Synoptic-physical implications of 1.25-cm vertical beam radar echoes, J. Meteor., 12, 312-326.

Braham, R. R., 1952: The water and energy budget of the thunderstorm and their relation to thunderstorm development, J. Meteor., 9, 227-242.

Charney, J. G., and A. Eliassen, 1964: On the growth of the hurricane depression, J.A.S., 21, 68-75.

Cunningham, R. M., V. G. Plank, and C. M. Campen, 1956: Cloud refraction index studies, Geophysical Research Papers, No.51, G.R.D., Air Force Cambridge Research Center, Bedford, Mass.

Danard, M. B., 1964: On the influence of released latent heat on cyclone development, J.A.M., 3, 27-37.

Danielsen, E. F., 1966: Research in four dimensional diagnosis of cyclonic storm cloud systems, Scientific Report No.1, Contract No. AF19(628)-4762, Penn. State Univ., Univ. Park, Penn. 53p.

Fulks, J. R., 1935: Rate of precipitation from adiabatically ascending air, Mon. Wea. Rev., 63, 291-294 (see also Smithsonian Meteor. Tables, 1958, Washington, D.C., Smithsonian Inst., 325-326).

James, D. G., 1959: Observations from aircraft of temperature and humidities near stratocumulus clouds, Q.J.R.M.S., 85, p.120.

Kessler, E., 1967: On the continuity of water substance, Tech. Memo. IERIM-NSSL 33, U.S. Dept. of Commerce, ESSA, 125p.

Kuo, H. L., 1965: On the formàtion and intensification of tropical cyclones through latent heat release by cumulus convection, J.A.S., 22, 40-63.

Newell, R. E., 1960: A study of tropospheric cellular convection and of its role in vertical transport from weather radar and radioactivity data, Sc.D. Thesis, Dept. of Meteor., M.I.T., Cambridge, Mass., 53p.

Palmen, E., and E. O. Halopainen, 1962: Divergence, vertical velocity, and conversion between potential and kinetic energy in an extratropical disturbance, Geophysica, 8, No.2, 1962, 87-113.

Peixoto, J. P., 1965: On the role of water vapor in the energetics of the general circulation of the atmosphere, Studies of the Gen. Circ.,V(1966), 138-168.

Riehl, H., and Malkus, J., 1961: Some aspects of hurricane Daisy, 1958, National Hurricane Research Report No.33, 63p.

Starr, V. P., and R. M. White, 1951: A hemispherical study of the atmospheric angular-momentum balance, Q.J.R.M.S., 77, 215-225.

U.S. Dept. of Commerce, Weather Bureau, 1963: Hourly Precipitation Data, New England, Ken., New Jersey, N.Y., North Carolina, Tenn., Pa., Ohio, Va., W.Va., Md. and Delaware, Gov. Printing Off., Washington, D. C.

U.S. Dept. of Commerce, Weather Bureau, 1963: Climatological Data, New England, Ken., N.Y., New Jersey, North Carolina, Tenn., Pa., Ohio, Va., W.Va., Md. and Delaware, Gov. Printing Off., Washington, D. C.