Geodesy and Cartography
ISSN 2029-6991 / eISSN 2029-7009
2019 Volume 45 Issue 2: 92–95
https://doi.org/10.3846/gac.2019.6692

UDK 528.482.5

STRUCTURAL METHOD FOR DETERMINING DEFORMATIONS BY GEODETIC MEASUREMENTS

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Received 21 November 2018; accepted 03 June 2019

Abstract. Industrial equipment is a dynamic system that deforms during installation (assembly) and during operation. Under the influence of variable load and mixing of the center of gravity of the equipment and foundations on which it is installed, uneven horizontal and vertical displacements occur, therefore individual equipment elements are unevenly deformed, which can lead to poor performance or stoppage of this equipment.
Timely measurement of the displacement of certain points of equipment (deformations) of precision equipment with the help of geodetic and other methods and their correct use for correcting the geometry of the equipment will contribute to improving the operational properties and increasing the period of uninterrupted operation of equipment's, for example, precision conveyor lines for assembling cars.

Keywords: displacement of point's equipment, deformations, relative deformations.

Introduction

For the process of plastic shaping of industrial equipment, the heterogeneity of deformation is characteristic, which manifests itself in different in magnitude and direction of deformation. The individual elements of the equipment have quite clear boundaries, which can be used for qualitative and quantitative assessment of the deformed state of the industrial equipment in question.

The proposed method (Kasatkin, Kudrin, & Lobanov, 1981) for estimating the deformed state is as follows. On the surface of the equipment, measuring marks of the necessary (small) dimensions are applied in accordance with the project and their measurements are carried out prior to deformation (design installation) and after it, and also at certain intervals of time, in order to track the dynamics of the deformation process. From the theory of finite deformations it is known that a material body (industrial equipment) having an initially spherical shape is transformed in the process of deformation into an ellipsoid, for a planar problem the circle is transformed into an ellipse, the parallelepiped is transformed into an "elliptical parallelepiped".

1. Measurements

On the surface of industrial equipment, measuring marks are installed on a grid of squares of 20×20 m (Figure 1) and distances ρ_o (projected) between different pairs of diametrically located grades are measured and the same distance ρ after deformation is measured (arrows point the direction of the points) (Gladilin, Chulanov, & Shudra, 2015).

The relation ρ_o/ρ (relative deformations) are different for different pairs of point (marks) and depend on the angle Θ associated with the direction of the segment relative to the coordinate system (Figure 1) is chosen so that it corresponds to the configuration of the equipment under study.

According to the foregoing, the functional dependence of $(\rho_o/\rho)^2$ on the angle Θ is subject to the law (Gladilin & Chulanov, 2002):

$$\left(\frac{\rho_0}{\rho}\right)_i = a_0 + a_1 \sin\Theta_i + a_2 \cos\Theta_i . \quad (1)$$

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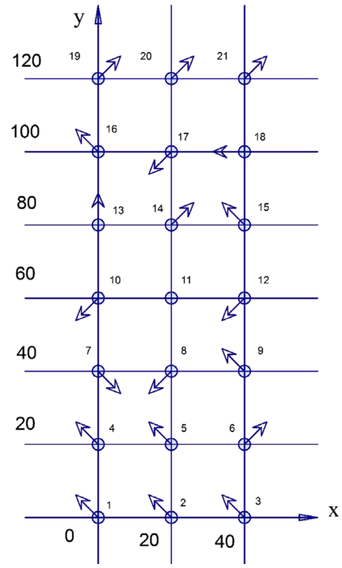

Figure 1. The scheme of placement of points on the equipment and directions of their deviation after deformation

The presence of an error in the geodetic measurements and the heterogeneity of the material structure of the elements of the industrial equipment for the reliable determination of the coefficients a_0, a_1, a_2 must be carried out more than 20 measurements (Gladilin, 2016). The subsequent mathematical processing is based on the method of least squares and leads to a system of linear algebraic equations

$$\begin{cases} a_0 n + a_1 \sum_{i=1}^{n} \sin\Theta_i + a_2 \sum_{i=1}^{n} \cos\Theta_i = \sum_{i=1}^{n} \left(\frac{\rho_0}{\rho}\right)_i, \\ a_0 \sum_{i=1}^{n} \sin\Theta_i + a_1 \sum_{i=1}^{n} \sin^2\Theta_i + \\ a_2 \sum_{i=1}^{n} \sin\Theta_i \cos\Theta_i = \sum_{i=1}^{n} \left(\frac{\rho_0}{\rho}\right)_i \sin\Theta_i, \\ a_0 \sum_{i=1}^{n} \cos\Theta_i + a_1 \sum_{i=1}^{n} \sin\Theta_i \cos\Theta_i + \\ a_2 \sum_{i=1}^{n} \cos^2\Theta_i = \sum_{i=1}^{n} \left(\frac{\rho_0}{\rho}\right)_i \cos\Theta_i. \end{cases} \quad (2)$$

Projected value of ρ_0, Θ_0 and measured value ρ, Θ are given in Table 1. The distances are measured with the mean square error $m_\rho = 5$ mm, the angles Θ are measured with an average quadratic error of $m_\theta = 2''$.

The greatest error in the position of the most distant extreme point, example, No. 1, 3, 19, 21. S = 63.2456 m, and a table, with the accuracy of these measurements will be with generally known formula:

$$m_0^2 = m_\rho^2 + S^2 \left(\frac{m_\Theta}{\rho}\right)^2. \quad (3)$$

Table 1. Relative and angular deformation of equipment points

No. points	ρ_{0i}, project value, meters	ρ_i, measured values, meters	Θ_{0i}, degree	Θ_i, degree	$\Delta\rho_i = \rho_i - \rho_{0i}$, mm	$1/l_i = \Delta\rho_i / \rho_{0i}$, relative	$\Delta\Theta_i = \Theta_i - \Theta_{0i}$, sec
11–10	20.0000	20.0040	180 00 00,0	180 00 10,3	+4.0	1:5000	+10.3
11–13	28.2843	28.2871	135 00 00,0	134 59 29,1	+2.8	1:10100	–30.9
11–16	44.7214	44.7325	116 33 54,2	116 34 04,5	+11.1	1:4000	+10.3
11–19	63.2456	63.2487	108 26 05,8	108 25 55,5	+3.1	1:20400	–10.3
11–14	20.0000	20.0090	90 00 00,0	89 59 29,1	+9.0	1:2220	–30.9
11–17	40.0000	39.9990	90 00 00,0	0 30,9	–1.0	1:40000	+30.9
11–20	60.0000	60.0070	90 00 00,0	89 59 49,7	+7.0	1:8570	–10.3
11–21	63.2456	63.2550	71 33 54,2	71 33 43,9	+9.4	1:6730	–10.3
11–18	44.7214	44.7200	63 26 05,8	6 18,2	–1.4	1:31940	+12.4
11–15	28.2843	28.2878	45 00 00,0	45 01 38,0	+3.5	1:8080	+98.0
11–12	20.0000	19.9960	0 00 00,0	359 59 18,7	–4.0	1:5000	–41.3
11–09	28.2843	28.2744	315 00 00,0	315 00 00,0	–9.9	1:2860	0.0
11–06	44.7214	44.7169	296 33 54,2	296 34 35,4	–4.5	1:9940	+41.2
11–03	63.2456	63.2304	288 26 05,8	288 26 12,0	–15.2	1:4160	+7.0
11–08	20.0000	20.0030	270 00 00,0	269 58 27,2	+3.0	1:6670	–92.8
11–05	40.0000	39.9910	270 00 00,0	269 59 29,0	–9.0	1:4440	–31.0
11–02	60.0000	59.9930	270 00 00,0	269 59 29,0	–7.0	1:8570	–31.0
11–01	63.2456	63.2443	251 33 54,2	251 33 15,0	–1.3	1:48650	–39.2
11–04	44.7214	44.7133	243 26 05,8	243 25 36,9	–8.1	1:5520	–28.9
11–07	28.2843	28.2800	225 00 00,0	225 00 41,3	–4.3	1:6580	–41.3

Determined that m_0 = ±5.4 millimeters, for the equal effect of liner and angular measurement, ρ = 206265". Substituting expression (3) value m_ρ, m_0, for points 8, 10, 12, 14 and S = 20.0000 will get m_0 = 5.0 mm.

2. Calculanion

The spatial position of point 11 is determined using GPS with the mean square error ±20 mm, the heights of all points are determined by high-precision leveling with an accuracy of ±1.5 mm (for georeference to WGS-84) (Burachek, Malik, Kryachok, Bryk, & Belenok, 2018).

Using the data from the Table 1, we find the coefficients of the normal equations (2). Substituting them into equations:

$$\begin{cases} 20 \cdot a_0 + 0.000276352 \cdot a_1 - 0.000891207 \cdot a_2 = \\ 20.000058, \\ 0.000276352 \cdot a_0 + 14.8004289644 \cdot a_1 + \\ 0.000755963 \cdot a_2 = -0.001850522, \\ -0.000891207 \cdot a_0 + 0.000755963 \cdot a_1 + \\ 5.1995710356 \cdot a_2 = -0,000241656. \end{cases} \quad (4)$$

We obtain the matrix of the normal coefficients of the equations:

$$A = \begin{bmatrix} 20 & 0.000276352 & -0.000891207 \\ 0.000276352 & 14.8004289644 & 0.000755963 \\ -0.000891207 & 0.000755963 & 5.1995710356 \end{bmatrix}. \quad (5)$$

That determination det A = 1539.117635042 not equal to zero.

Thus, the system of equations has a single solution. The matrix is a column of free numbers:

$$B = \begin{vmatrix} 20.000058 \\ -0.001850522 \\ -0.000241656 \end{vmatrix}. \quad (6)$$

Covariance matrix of coefficients of normal equations:

$$\operatorname{cov} A = \begin{vmatrix} 133.337432739 & -49.331879224 & -17.330265792 \\ -49.331879224 & 73.012473084 & -12.823378559 \\ -17.330265792 & -12.823378559 & 9.0089916896 \end{vmatrix} \quad (7)$$

The solution of the normal equations of the form $Aa = B$, by Gauss method will give the following values a_0, a_1, a_2

$$a_0 = 1.000002903468,$$

$$a_1 = -0.000143709974,$$

$$a_2 = 0.000124945351.$$

3. Results of calculations

Substituting these values into equations (1), we obtain:

$$\left(\frac{\rho_0}{\rho}\right)_i = 1.000078070 - 0.000143746 \sin\Theta_i + 0.000413993 \cos\Theta_i. \quad (8)$$

The calculated values $\left(\frac{\rho_0}{\rho}\right)_i$ are in the Table 2.

Table 2. Calculation of corrections to the position of equipment points

No. points	$\left(\frac{\rho_0}{\rho}\right)_i$	Calculated of formula (8)	Defined ρ_i, meters	Corrections, mm
11–10	0.99980004	0.999877965	20.00244099	–1.6
11–13	0.999901015	0.999812933	28.28959204	2.5
11–16	0.999751858	0.999818486	44.72951905	–3.0
11–19	0.999950987	0.999827061	63.25653954	7.8
11–14	0.999550202	0.999859212	20.00281615	–6.2
11–17	1.000025001	0.999859175	40.0056338	6.6
11–20	0.999883347	0.9998592	60.00844921	1.4
11–21	0.999851395	0.999906088	63.25154011	–3.4
11–18	1.000031306	0.999930232	44.72452034	4.5
11–15	0.999876272	0.999989545	28.28459573	–3.2
11–12	1.00020004	1.000127878	19.99744278	1.4
11–09	1.00035014	1.000192871	28.27884582	4.4
11–06	1.000100633	1.000187328	44.71302398	–3.8
11–03	1.000240391	1.000178752	63.23429674	3.8
11–08	0.999850022	1.000146557	19.99706929	–5.9
11–05	1.000225051	1.000146595	39.99413707	3.1
11–02	1.00011668	1.000146595	59.99120561	–1.8
11–01	1.000020555	1.000099696	63.23929527	–5.0
11–04	1.000181154	1.00007554	44.71802202	4.7
11–07	1.000152051	1.00001621	28.28384152	3.8

Table 2 shows the comparative values calculated by formula (8) and of Table 1. Differences in values and corrections in the indicated points of equipment are also determined. The equation (8) is approximated with the mean square m = ±4.3 mm.

Conclusions

Using the equation (8), we calculate the corrections to the position of the equipment points relative to its center, point 11 (figure) in the polar coordinate system and can be recalculated into rectangular coordinates. If point offsets exceed assembly tolerances, then individual elements of equipment should be shifted by the calculated value of corrections and after that control measurements of the position of equipment points should be made.

The structural method for determining deformations works well determining the deformations of industrial equipment of large length, precision conveyor lines and calculating corrections to the position of equipment points, so alignment of the equipment itself is performed. Corrections in the position of points are introduced for the precise operation of industrial equipment.

It is most convenient to introduce corrections to the position of equipment points in the polar coordinate system from the center of the equipment using an electronic total station.

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