COMPRESSIVE STRENGTH
OF
STIFFENED SHEETS OF ALUMINUM ALLOY

by

Harbert W. Gall

Submitted in Partial Fulfillment of the Requirements
for the Degree of
Bachelor of Science
in
Aeronautical Engineering
from the
Massachusetts Institute of Technology
1930

Signature of Author...

Certification by the Aeronautical Engineering Staff:

Professor in Charge of Research..................................

Head of Course...

May 29, 1930

Professor A. L. Merrill
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Mass.

Dear Sir:

Submitted herewith, in partial fulfillment of the requirement for the degree of Bachelor of Science in Aeronautical Engineering, is a thesis: Compressive Strength of Stiffened Sheets of Aluminum Alloy.

Sincerely yours,

ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation to Professor J. S. Newell, under whose direction, and with whose valued suggestions and assistance this work was done.

He is also indebted to Professor J. H. Frost for the use of testing machine equipment; to Professor R. G. Adams for the use of testing machines and measuring apparatus; and to Professor S. Ober for the Use of the Aeronautics Laboratory shop and equipment.

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INTRODUCTION

The purpose of this investigation was primarily to outline a method of procedure to be followed in further research for the determination of practicable design methods and allowable stresses for aluminum alloy airplane structures of the "stressed skin" type, though it was hoped to get a general idea of the magnitude of the stresses to be used.

Since only one specimen of each type was built, with the exception of some which were reloaded after being cut down from longer specimens which had failed elastically, it is obvious that actual design values could not be obtained.

The need for a large amount of unified research work became apparent when an attempt was made to correlate test data from different sources. Inasmuch as the number of variables was several times the number of tests, nothing definite could be determined, and it was primarily to outline a method of related research that the work presented here was undertaken.

SCOPE

Tests were made on forty-two built up specimens. Two types of stiffeners, three sheet thicknesses, four lengths and four radii of curvature (besides flat sheets) were used so as to introduce as many variables as practicable in order to reach as general conclusions as possible. A different rivet spacing was used on the two types of stiffeners.

Tensile tests were made to check the physical characteristics of the sheet.

An experimental determination of the EI of the various specimens was made to determine the stiffness characteristics.

Samples of the stiffeners used were tested as columns in compression.

Several sheets were weighed, and the average density of the material found.

Samples of the stiffeners were weighed, and their areas were calculated from the weights obtained.

DESCRIPTION OF MATERIAL AND SPECIMENS TESTED.

MATERIAL

The sheets and stiffeners used were aluminum alloy manufactured by the Bausch Machine Tool Company, Springfield, Mass., who give the following properties:

Yield Point,	30,000 lbs/sq.in. min.
Ult.Tensile Strength,	55,000 lbs/sq.in. min.
% Elongation in 2 inches,	18% min.
Modulus of Elasticity,	10,500,000 approx.

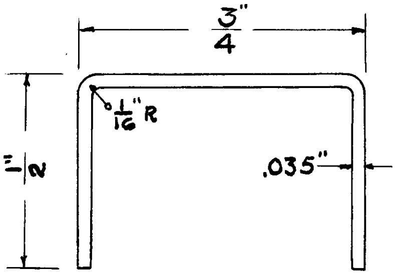
Tensile tests of the sheet made in the laboratory showed an average ultimate tensile strength of 55,300 lbs/sq.in.

RIVETING

The rivets used were #12 iron tinners rivets. The channel stiffeners were riveted single row with a 3/4" pitch. The U stiffeners were riveted double row, with a staggered pitch of 3/4"

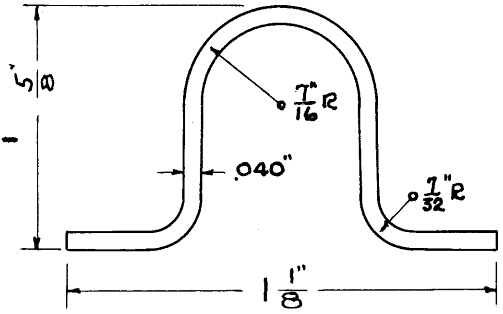
STIFFENERS

The channel stiffeners used were as sketched, and had the following characteristics:



Area	- Experimental Determination -	.0566 sq.in.
	- Calculated - Method of Army Air Corps Information Circular -	.0575 sq.in.
EI	- Experimental Determination - Calculated	11960
Weight per foot -		.067 lbs.

The U stiffeners used were as sketched, with the following characteristics:



Area	- Experimental Determination -	.0808 sq.in.
EI	- Experimental -	27400
Weight per foot -		.0959 lbs.

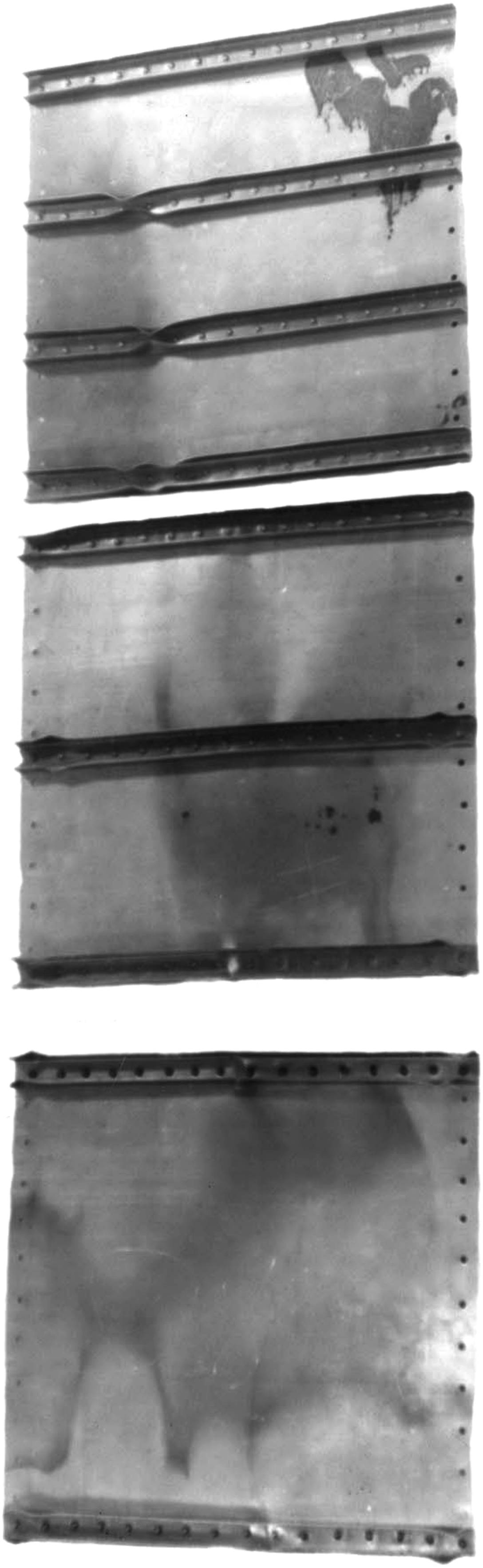
BUILT UP SPECIMENS

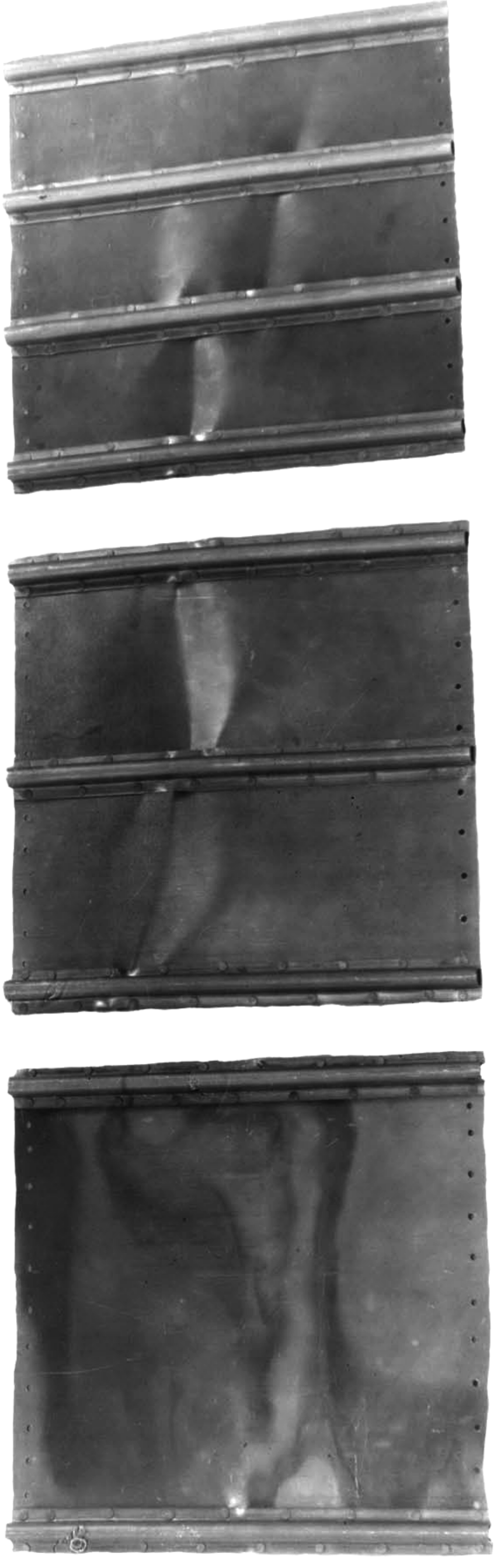
Three flat specimens were built of 12" x 12" x .032" sheet having two, three and four U stiffeners equally spaced. The other thirty-nine specimens were built in groups of three, using two, three and four channels, and having different combinations of length, plate thickness, and radius of curvature as follows:

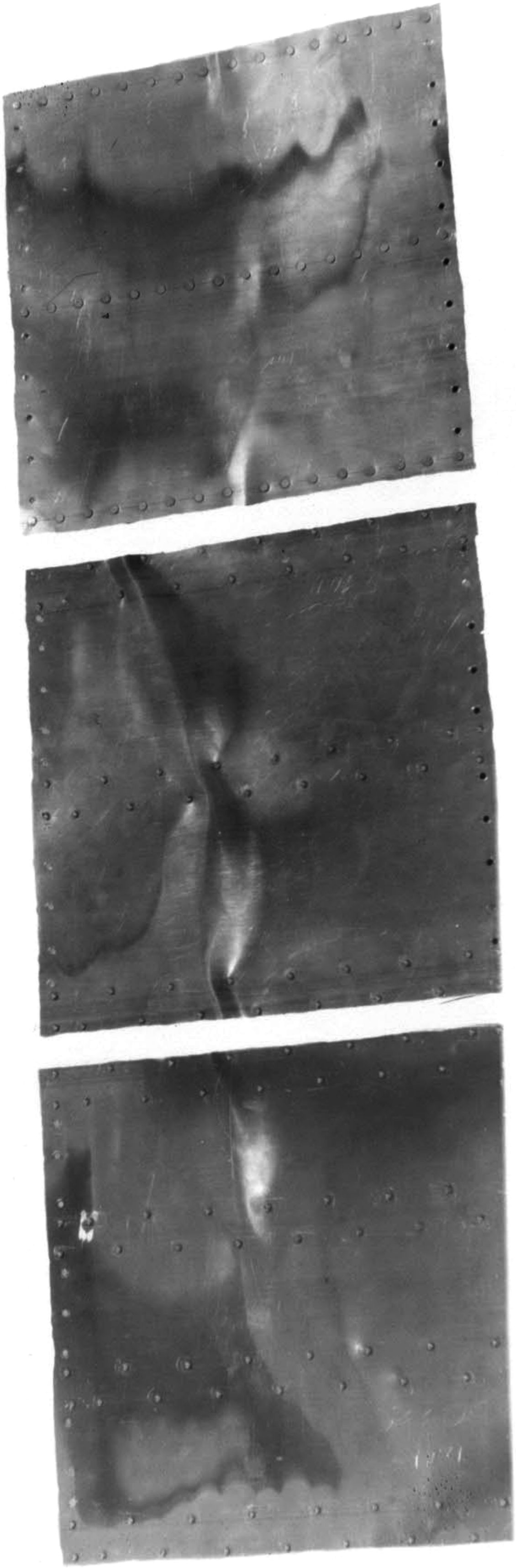
12" x 6" x .032" - Flat
12" x 12" x .032" - Flat
12" x 18" x .032" - Flat
12" x 24" x .032" - Flat
12" x 12" x .020" - Flat

12" x 12" x .031" - Flat
12" x 12" x .032" - 5" radius curvature.
12" x 12" x .032" - 10" " "
12" x 12" x .032" - 30" " "
12" x 12" x .032" - 80" " "

The specimens were as shown in the photographs on the following pages, except that a piece of 1/2" thick wood was fastened about 1/8" from each end of the specimens in order to hold them flat or to the desired radius of curvature.







METHOD OF TESTING

The built-up specimens were tested in compression as flat ended columns in a 20,000 pound Riehle Bros. machine. After riveting, the ends of all specimens bearing on the head and bed of the testing machine were milled in order to obtain as uniform load distribution as possible. On this machine, the load was read to 5 pounds.

The experimental EI of the specimens was determined by loading them as simple beams with a concentrated central load and measuring the deflections. This was accomplished by supporting the specimens on two edges placed 9" apart, and loading them by means of scale weights hung on a wire over the center of the specimen. Deflections were measured with an Ames Dial Gauge reading .001".

The stiffeners were also tested as flat ended columns. The channel stiffeners were unsupported. Two U stiffeners were tested unsupported, and two were prevented from twisting during the test by means of a short 1/4" x 1" iron bar clamped to the center of the column by means of small "C" clamps.

DATA

In the tabulation of compression test data following, the following abbriviations are used:

1. Spec. No.	Specimen Number.
2. Rad. Curve.	Radius of Curvature of Plate.
3. % Re-in.	Percent reinforcement, based on total area.
4. Exp. EI.	Experimental EI, obtained by loading specimen as a simple beam.
5. Buck. Load.	Load at which considerable wrinkling occurs.
6. Max. Load.	Maximum Load.
7. Ave. Stress.	Stress computed using total area.
8. Stiff. Type.	C - Channel stiffeners. U - U shaped stiffeners.
9. "W"	Plate width.
10. "L"	Plate length.
11. "T"	Plate thickness.

COMPRESSION TEST DATA

1 Spec. No.	2 Rad. Curve.	3 % Re-in.	4 Exp. EI	5 Buck Load	6 Max. Load	7 Ave. Stress
1	5"	22.3	43400	10100	10100	19500
2	10	22.3	28940	3500	5400	10450
3	30	22.3	28940	2330	4300	8325
4	80	22.3	31000	2000	3500	7000
5	5	30.0	37025	8080	10500	18250
6	10	31.0	28933	4180	7400	13280
7	30	31.0	25950	2650	5700	10250
8	80	31.0	28804	3500	5100	9150
9	5	36.9	35700	11360	12600	19900
10	10	37.5	33650	8000	10130	16500
11	30	37.5	29700	8000	8800	14320
12	80	36.4	27600	6000	7840	12400
13	F	22.7	--	2400	6180	12200
14	F	30.6	--	--	7040	12500
15	F	37.0	--	--	9380	15100
16	F	23.1	30350	4100	4190	8400
17	F	31.0	44000	3500	6110	10960
18	F	37.5	46000	5160	8450	13780
19	F	22.7	28100	1500	2830	5600
20	F	30.6	37900	2350	4350	7725
21	F	37.0	48000	4300	6170	9945

COMPRESSION TEST DATA

1 Spec. No.	2 Rad. Curve.	3 % Re-in.	4 Exp. EI	5 Buck Load	6 Max. Load	7 Ave. Stress
22	F	23.1	30000	1200	2000	4010
23	F	31.0	42700	3200	3430	6160
24	F	37.7	44000	3700	4240	6900
25	F	33.0	33740	1250	3070	8800
26	F	42.5	36100	3130	4710	11600
27	F	49.7	34800	3800	6550	14150
28	F	18.9	30900	--	4000	6600
29	F	26.0	43300	4500	6370	9600
30	F	31.8	43300	8000	10180	14100
31	F	23.1	30000	--	4630	9300
32	F	30.9	42700	--	6650	11900
33	F	37.5	44000	--	8800	14350
34	F	23.0	28100	1780	4030	8075
35	F	30.9	37900	4000	6025	10800
36	F	37.5	48000	4980	7570	12320
37	5"	23.1	31600	--	8150	16350
38	30"	30.9	28600	3500	6090	10900
39	F	37.5	44000	--	8315	13550
40	F	30.0	63300	2000	6500	11550
41	F	37.6	75750	--	9340	14450
42	F	44.5	104500	--	14205	19600

COMPRESSION TEST DATA

1	8	9	10	11	12	13	14
Spec.	Stiff.		Plate		Plate	Stiff.	Total
No.	Type	W."	L."	T."	Area.	Atea.	Area
1	C	11.99	12.00	.0335	.402	.115	.517
2	C	12.05	11.96	.0335	.4037	.115	.5187
3	C	11.99	12.00	.0335	.402	.115	.517
4	C	12.00	12.01	.0320	.384	.115	.499
5	C	12.02	11.98	.0335	.402	.1725	.575
6	C	12.01	11.96	.032	.384	.1725	.557
7	C	12.00	11.99	.032	.384	.1725	.557
8	C	12.02	12.00	.032	.385	.1725	.557
9	C	12.02	11.99	.0335	.403	.230	.633
10	C	12.02	12.00	.032	.385	.230	.615
11	C	12.02	12.01	.032	.385	.230	.615
12	C	12.00	12.02	.0335	.402	.230	.632
13	C	12.02	5.97	.0325	.391	.115	.506
14	C	12.02	5.95	.0325	.391	.1725	.563
15	C	12.02	5.90	.0325	.391	.230	.621
16	C	12.02	11.98	.032	.385	.115	.500
17	C	12.02	12.02	.032	.385	.1725	.557
18	C	12.02	12.01	.032	.385	.230	.615
19	C	12.02	18.03	.0325	.391	.115	.506
20	C	12.02	18.00	.0325	.391	.1725	.563
21	C	12.02	17.86	.0325	.391	.230	.621

COMPRESSION TEST DATA

1 Spec. No.	8 Stiff. Type	9	10 Plate	11	12 Plate Area	13 Stiff. Area.	14 Total Area
		W."	L."	T."			
22	C	12.00	24.02	.032	.384	.115	.499
23	C	12.02	24.03	.032	.384	.1725	.557
24	C	12.01	23.97	.032	.384	.230	.614
25	C	12.00	11.95	.0195	.234	.115	.394
26	C	12.01	11.91	.0195	.234	.1725	.407
27	C	12.00	11.94	.0195	.234	.230	.464
28	C	12.00	11.95	.091	.492	.115	.607
29	C	12.00	11.95	.041	.492	.1725	.665
30	C	12.00	11.93	.041	.492	.230	.722
31	C	12.00	6.00	.032	.384	.115	.499
32	C	12.02	5.99	.032	.386	.1725	.559
33	C	12.01	5.98	.032	.384	.230	.619
34	C	12.02	11.97	.0325	.385	.115	.500
35	C	12.02	11.97	.0325	.385	.1725	.557
36	C	12.02	11.94	.0325	.385	.230	.615
37	C	12.00	12.00	.032	.384	.115	.499
38	C	12.02	11.99	.032	.385	.1725	.557
39	C	12.00	12.01	.032	.384	.230	.614
40	U	12.00	11.92	.0335	.402	.1616	.5636
41	U	12.03	11.93	.0335	.403	.2424	.6454
42	U	12.01	11.94	.0335	.402	.3232	.7252

STIFFENER TEST DATA.

1 Type	2 Length	3 Weight (gr.)	4 Exp. EI.	5 Area Exp.	6 Load (Max)	7 Stress
C	12.035	30.3	11960	.0560	1300	23700
C	12.04	30.9	11960	.0572	1345	23500
U	11.93	43.15	27400	.0804	1610	20050
U	11.935	43.4	27400	.081	1650	20380
U	11.94	43.2	27400	.815	2300	28200
U	11.96	43.6	27400	.805	2220	27600

The above stiffeners were tested as flat ended columns. The first four listed were unsupported. The last two were restrained from twisting, but not from bending, by means of a light iron bar clamped to the middle of the section. This was done in order to get some idea of the effect of the plates on the stiffeners, as the stiffeners, when tested without support, were found to fail by twisting rather than straight bending.

DISCUSSION

Accuracy of Results.

All specimens were milled so as to have the edges bearing on the testing machine as parallel as possible, but since they were tested with a flat headed machine there are undoubtedly some errors due to loading conditions.

However, as this work is more qualitative than quantitative, this error is not as important as it may seem. In future quantitative work, it is suggested that the loads be applied through a bar free to rotate about an axis perpendicular to the plane of the plate at the middle of the upper edge. In the case of curved specimens it would rotate in the plane of the minor axis.

Loads applied were read to five pounds.

No account of the variation in length of the specimens above or below the standard used in calculations, as it was in no case more than a few hundredths of an inch.

The variation in plate thickness and plate width was considered in the calculations of area and stress.

Types of Failures.

In the 24" specimens, the failure was purely elastic, with very few humps in the plate at time of failure. The free edges of the channels bearing on the machine crumpled after the columns had failed elastically. This was common to the specimens of all lengths. Also, practically all specimens failed with the free edges in tension. This was undoubtedly due to the fact that it was easier for the channels to fail locally at the free edges than at the side to which the sheet was attached.

The failure was also elastic in the 18" specimens. In this case the sheet with three stiffeners went roughly into three buckles and that with four stiffeners into four buckles, thus checking to some degree the theory of Bryan as put forth by Timeshenko, which states that the plate will form itself as nearly as possible into rectangular sections.

In the 12" specimens, failure seemed to be a combination of elastic and local failure, the channels bending somewhat under load, and then failing locally by having the free edges bend either toward or away from each other.

In the 6" specimens failure seemed to be purely local. In the curved specimens the failure seemed to be more local than elastic.

Effect of Rivet Spacing.

Only two rivet spacings were used, as it was not thought advisable to introduce too many variables. With the channel stiffeners, #12 tinners rivets spaced 3/4" were used. This held the sheet to the stiffeners in a very satisfactory manner, even when crushed far beyond the point of maximum load. With the U stiffeners, a staggered pitch of 3/4" was used, which gave an "effective" pitch of about 1 1/16" between parallel rows of rivets. This additional distance allowed the plates to buckle considerably, as can be seen from the photographs. There were, however, no rivet failures.

From the above results, and from test data of outside sources, it is thought that 3/4" should be the maximum rivet pitch in sheets .040" and less thick.

Variation of Stress with EI.

In the flat specimens there was a marked relation between the EI determined experimentally and the stress. In the curved specimens there seemed to be no connection between EI and stress, the specimens showing in some cases an increased stress with slightly decreased EI. This tendency was also shown by the 6" flat specimens.

There are two possible reasons for this:

First, the method of determining the EI may be in error. However, as it was obtained in the same manner as in the case of the flat sheets, this is not thought to be the case.

Second, the short flat and the curved specimens fail by local buckling, in which case the failing load would not be a function of the total EI of the specimen, but would depend rather upon the EI of the plate alone, in connection with some undetermined fixity coefficient, as given by Eulers formula, where for a unit section the stress is given by $K\pi^2 E\left(\frac{\rho}{l}\right)^2$, or that of Rankine, where the stress is equal to $\frac{K_1^2}{1+K_3\left(\frac{l}{\rho}\right)^2}$. This value of K, however, is very indefinite and the flexibility of the unit section varied from zero to infinity as K varies from 1 to 4.

An example of effect of EI on long specimens, and of its ineffectiveness on short columns may be noted by referring to the curves of "Variation of Stress with EI in Flat Sheet", page 31. Here the curve for 24" specimens shows a maximum EI of 44000, while for 18" and 12" specimens of the same type the EI is 48000. It will be noted that this point lies on a theoretical curve, shown dotted, which parallels the other curves, and leads the author to believe that with an EI of 48000 it would have shown a stress of about 8000 lbs/sq.in.

The results ploted on the 6" curve are of the same specimen, which was cut down in length after elastic failure in the 24" length. In this case the stress obtained lies far above the curve, but if it were plotted against a value of 48000 EI it would be practically on the curve.

The above, in connection with the action of the curved sheets, leads the author to believe that in short flat sheets up to a foot in length (and possibly longer) no information as to strength can be determined from EI values.

Variation of Plate Thickness.

While the test values obtained on the .041 specimens with two and three channel stiffeners seem to be low, if they are raised so as to give a stiffener load# comparable with the .020 and .032 samples, it appears that the average stress for any one ratio of stiffener spacing to plate thickness is practically constant.

This is not borne out by tests made by the Navy and Charles W. Hall. In the former, the average stress increased with plate thickness, while in the latter it decreased.

If the plate had no effect on the channel, it would seem that the average stress should increase with plate thickness, as shown by the Bureau of Standards tests. However, it seems logical that the plates might possibly exert a weakening effect upon the stiffeners, since the attempt of the plate to buckle under load would place an additional side load upon the stiffener.

The figures given below indicate however, that to carry any given load, the lightest structure can be made by using the greatest percentage of stiffening, since the stiffening members, having large moments of inertia compared to that of the plate, can be much more highly stressed.

Loads Obtained by the Author.

Sheet Thickness,	.020"		.032"	
	Load	Area	Load	Area
2 Stiffeners,	3070	.349	4190	.4996
3 "	4710	.406	6110	.5571
4 "	6550	.464	13780	.6146

Loads Obtained by C. W. Hall.

Sheet Thickness,	.020"		.031"	
	Load	Area	Load	Area
2 Stiffeners,	2340	.31	3620	.422
3 "	3680	.345	5240	.477
4 "	4950	.38	5960	.512

In each of the above examples it is seen that the thin sheet with three stiffeners carried about the same load as the thicker sheet with two stiffeners, and has less area. Likewise the thin sheet with four stiffeners carried a greater load than the thicker sheet with three stiffeners.

From this it appears that without reference to cost and difficulty of manufacture, thin sheets with more stiffeners is the best construction.

Effect of Stiffener Type.

The average stress obtained using a U shaped stiffener with the free edges riveted to the sheet, was, as was to be expected, considerably higher than for channel stiffeners with free edges at the greatest distance from the neutral axis. Using the closed stiffener raised the average stress about 3000 lbs/sq.in. for all three specimens.

The results listed below show the undeniable advantage of using closed sections over open, as well as using large stiffeners over small ones of the same type.

Tests by the Author - .032 Sheets

	Channel Stiffeners Stress - lbs/sq.in.	U Stiffeners Stress - lbs/sq.in.
2 stiffeners	8400	11550
3 stiffeners	10960	14450
4 stiffeners	13780	19600

Tests by C. W. Hall - .020 Sheets

	1/4" x 9/16" x .020" Bulb Angles Stress - lbs/sq.in.	1/2" x 1" x .020" Bulb Angles Stress - lbs/sq.in.
2 stiffeners	7550	10820
3 stiffeners	10650	14940
4 stiffeners	13000	17000

These results indicate that even though the plate itself is weakened by reducing the number of stiffeners, the overall efficiency of the combination is greater with fewer but larger stiffeners. This is also advantageous in that it reduces construction costs.

Effect of Radius of Curvature.

The general effect of changing the radius of curvature was, as expected, to increase the stress as the radius of curvature was increased. Also, as is shown by the curves on page 34 , the specimens having a greater Radius/T than 937 showed a lower stress than that of similar flat plates, while those of Radius/T of less than 937 showed greater stress.

This last condition does not seem untoward, if it is considered that when the plate is bent the neutral axis moves toward the center of curvature, thus making the two edges the most stressed portion of the specimen. Then as the plate is loaded, the sheet tries to buckle, thus throwing a bending load into the stiffeners at the edges, which are already highly loaded, due to their distance from the neutral axis. It then appears that for values of Radius/T of less than 937, the additional support given the sheet more than overcomes the effect of eccentric loading, and the specimen shows a higher stress than that of a flat plate.

The author is of the opinion that for fully circular specimens, the stress at an R/T of about 4400 would approach that of a flat plate, and that it would increase from that point as the Radius of Curvature was diminished.

As shown by the curves on page 36, there are two possible effects of radius of curvature on circular specimens, one that the stress curve obtained would be moved vertically until tangent to the flat sheet curve at R/T = 4400, or that it move vertically till tangent, as in the first case, and then rotated about its point of tangency.

However, when plotted as a function of % reinforcement, the stress curves of the curved sections have the same shape as those of the flat specimens, which leads one to believe that the stresses probably developed in circular sections would be those developed in the segments tested, increased by 2000 lbs/sq.in., which is the difference between the flat specimen stresses and the probable stress for R/T = 4400, as plotted on page 36. The probable stresses developed are plotted on page 37.

As may be seen from the stress curves on page 36, the effect of decreasing R/T from 4400 to 1000 is slight, increasing steadily until an R/T of 300 is reached, after which there is a very marked increase.

In order to make a comparison between the plate stresses obtained in the flat and curved specimens, the stiffener load was assumed to be constant at 1300 lbs. (which load they held when tested as flat ended columns) and the plate stresses calculated. The ratios of the stresses are plotted on page 38. For the specimens having values of R/T of 2500, 937 and 298, the ratio of stresses in the flat and curved sheets was practically constant. For R/T of 149, the ratio was not constant.

On page 39 the ratios of average stresses in flat and curved specimens are plotted, which naturally have the same characteristics. This leads the author to believe that the test data obtained of the specimens having an R/T of 149 does not represent the true allowable stresses, and that the number of stiffeners does not affect the change in stress due to curving the specimen.

Inspection of the curves showing the effect of length on flat specimens and the effect of radius of curvature, both plotted against percent reinforcement, shows a very similar effect. For a rough comparison - a decrease in radius of curvature from 80" to 30" increases the stress about the same amount as decreasing the length from 24" to 18". Decreasing the Radius of Curvature from 30" to 10" gives about the same effect as reducing the length from 18" to 12".

No attempt was made to determine any mathematical relations for the effect of curving a flat plate other than

than the comparisons made above as it was felt that with only one type of specimen enough data was not at hand.

Since only one thickness of sheet was tested, it is not known whether the stiffening is a function of the radius of curvature divided by the plate thickness, or merely a function of the radius of curvature. If is felt, though, to be a function of R/T.

The above discussion points to the advisability of doing further work to determine whether or not the stiffening effect is a function of R/T or merely R, whether or not the percent reinforcement changes the effect, as it seems not to in this case, and the stiffening effect on various sized segments of the same radius of curvature, varying from full circles to narrow segments.

As a matter of comparison it is interesting to note the results of static tests conducted by the Naval Aircraft Factory upon a pursuit fuselage, and by the Martin Company on a bomber fuselage:

<u>N.A.F. Tests</u>

Stiffener Spacing	5"	Skin .029 in.
Radius of Curvature	13.7"	
Stress Obtained	15600 lbs/sq.in.	
Stress from Curve	13300 lbs/sq.in.	

Stiffener Spacing	10"	Skin .025"
Radius Curvature	13.7"	
Stress Obtained	10700 lbs/sq.in.	
Stress from Curve	10200 lbs/sq.in.	

Martin Tests.

Stiffener Spacing	11"	Skin .032"
Radius of Curvature	Flat	
Stress Obtained	6545 lbs/sq.in.	
Stress from Curve	8400 lbs/sq.in.	

Stiffener Spacing	6"	Skin .032"
Radius of Curvature	24" (approx.)	
Stress Obtained	7910 lbs/sq.in.	
Stress from Curve	11250 lbs/sq.in.	

While the above results by no means check with the test data, they show the same general trend in every case, which is all that can be expected, due to the difference in types of construction.

Effect of Variation of Length.

In testing specimens of different lengths, two major variables enter, namely the effect of length on the plate itself and the effect on the stiffener considered as a column. Of a secondary nature are the effect of the plate on the stiffener, for instance, putting a bending load into the stiffener by its tendency to buckle, the amount of restraint supplied the plate by the stiffener, and the effect of the restraint of the stiffeners by the head and bed of the testing machine.

The only data at hand on the strength of thin dural sheet is that obtained by the Bureau of Standards. Curves on pages 43 and 44 give this data. From the results of their tests, the Bureau derived an equation of the curve of loads obtained, where the load that a sheet will take is given by $1.11 \times 10^{6} T^{2} - 2.15 \times 10^{6} T^{3}$, where T is the plate thickness in inches. According to this, the load carried is independent of the width, being a function of thickness alone. However, the load curves, plotted on page 42, show a distinct falling off in the narrow widths. From the test results obtained in the series of built up specimens it is believed that this falling off represents more nearly the actual conditions than does the curve given above.

When the stiffener loads were calculated by taking the stress from the Faired Stress curve of Bureau of Standards tests, multiplying by the plate area to get the plate load, and subtracting this from the maximum load found in test (See Appendix),the stiffener loads were found to be progressively less as the width between stiffeners was decreased. As it seems logical that the stiffeners should carry practically the same load regardless of number, the only remaining variable is the plate stress.

The indications from the tests conducted on the built up specimens indicate that the stress with multiple stiffeners does not increase as rapidly as the Bureau tests show. This seems logical, when it is remembered that in the Bureau tests the restraint was absolutely rigid and continuous, as compared to restraint by more or less flexible columns and rivets. In the samples built, there was often a small waviness in the plates due to riveting on the stiffeners, which may help to reduce the allowable plate load. In one or two cases, the plate showed no buckles when loaded until the entire specimen collapsed at a very high load. This condition could only be obtained in laboratory practice with the use of extreme care, and should not enter considerations of commercial practice.

The above seems to indicate that before close design methods can be established a great deal of investigation on flat plates must be conducted.

The load curves were faired, and the stresses obtained therefrom are plotted on page 43 .

Timeshenko, in his work on flat sheet restrained on four sides, says that the crippling stress equals the Eu~~l~~ler stress multiplied by a constant which depends on the ratio of the lengths of the two sides of the sheet. For rectangles this constant is 4, and varies from 4 to 4.49 for values of length/width from 1 to 1.41, reducing again to 4 for a length/width ratio of 2. He also states that for length/width ratios of over 3, 4 is a good approximation for the constant. From this it would seem that there should be no great difference in allowable stresses. However, this may introduce an error in the neighborhood of 10%, which might amount to 1000 lbs/sq.in. in the size sheet commonly used in airplane construction.

This leads to the conclusion that before any precise design method can be worked out, tests will have to be made on restrained sheets with various combinations of length, width and thickness, especially in view of the fact that the ability to account for stress variations would throw a good deal more light on the results of the present tests.

The action of longitudinal stiffeners attached to thin plate must also be thoroughly investigated before a precise design method can be worked out. Roy A. Miller, in Army Air Corps Information Circular No. 598, has developed a method for calculating the strength of pin ended dural channels, but as channel shapes are very inefficient as stiffeners and the conditions as a rule far from pin ended, it is doubtful whether this will be of any great value in this connection.

It is probable that the effect of the stiffeners on the plate, and of the plate on the channels can only be found by a series of tests of plain sheets, of stiffener sections restrained in various ways, and of built up combinations of the two. The stiffeners used in these tests, when tested as unsupported columns failed by an elastic twisting. When restrained from twisting only, the ultimate load on the U stiffeners was raised from 1600 to 2200 lbs. From this it would seem that the sheet would have a very beneficial effect on the stiffener in this respect. On the other hand, the tendency of the plate to buckle undoubtedly places a bending load on the stiffener. It would seem that this is less than the restraining effect of the plate. However, this is purely a guess. Of the fact that the stiffeners support the plate there can be no doubt, but to what extent is unknown. The rivet spacing takes care of local buckling of the sheet along the stiffener, but on the stiffener alone the resistance to failure as a column seems to depend.

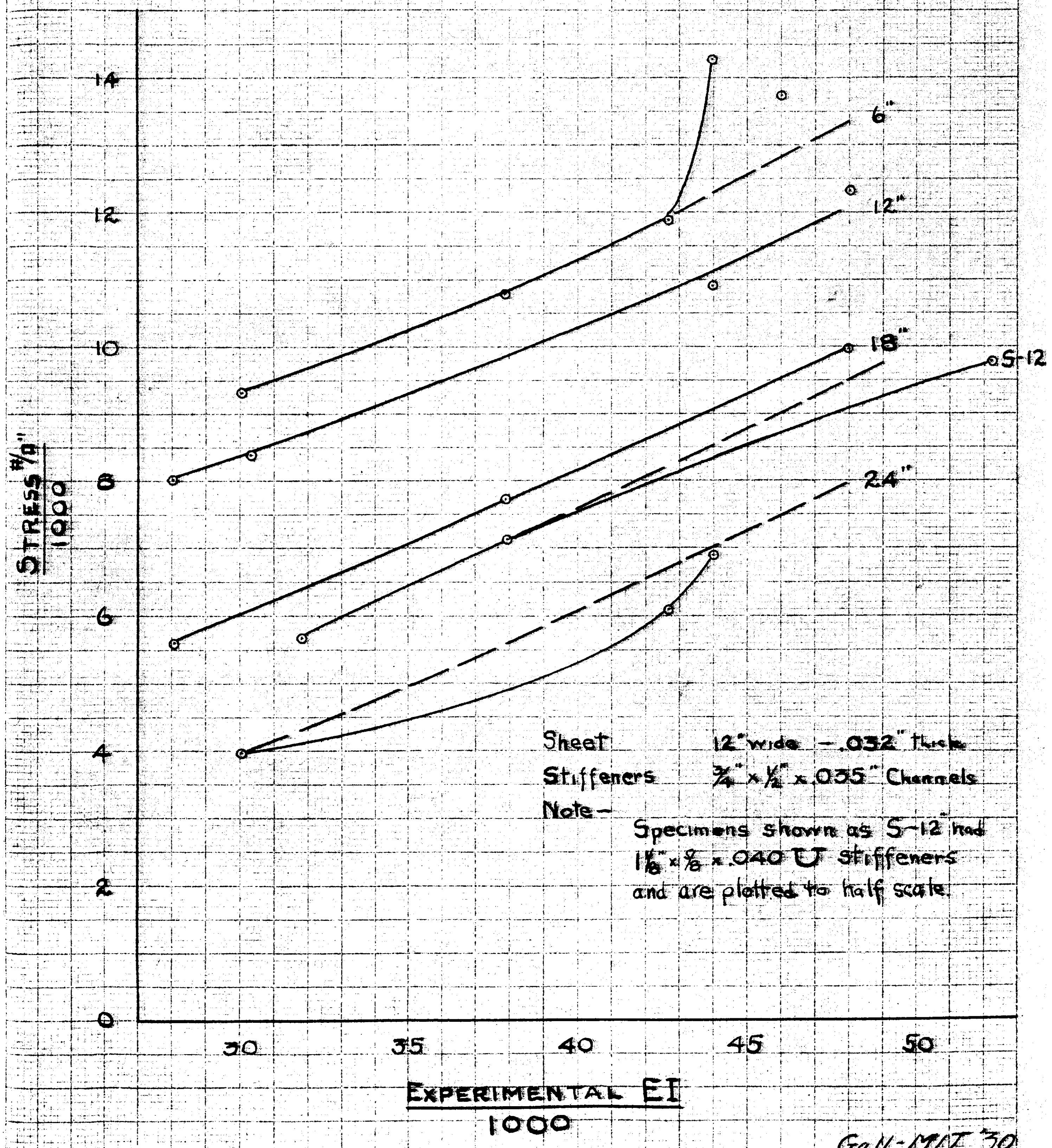
The results of the tests and the Bureau of Standards Stress Curves were juggled in various ways in an attempt to devise some logical design conditions. However, the uncertainty as to the effect of length on the plate stresses and on the columns was such as to preclude arriving at any general definite method. For conditions of a nature somewhat similar to the test, it is believed that members may be designed by calculating the plate load from the stresses computed from The Bureau of Standards derived load curve, using the distance between the centers of the stiffeners as plate width, and adding to this the Euler load carried by the stiffener, multiplied by a fixity coefficient. The following coefficients are recommended:

Length	Coefficient
6"	1.00
12"	1.35
18"	1.55
24"	1.60

These coefficients give rather good results, but for 12" stiffener spacings are about 20% on the safe side, with the exception of the 24" length, due presumably to length effects in the sheet stresses.

From the foregoing discussion it seems that the first step toward practicable design methods must be the determination of allowable stresses in sheets of different thickness, length and width combinations. The second step will be to determine the effect of various types of restraint on stiffeners of different lengths, in order to establish suitable fixity coefficients. After this must come a series of related tests of built up sheets and stiffeners similar to those separately tested. It is thought that with a sufficient amount of data at hand a method of design can be worked out, which will probably be the use of the Euler stiffener loads, varied by a fixity coefficient, in connection with plate stresses from a group of curves, similar to those developed from the Bureau of Standards tests.

VARIATION OF STRESS WITH EI
IN
FLAT STIFFENED SHEET



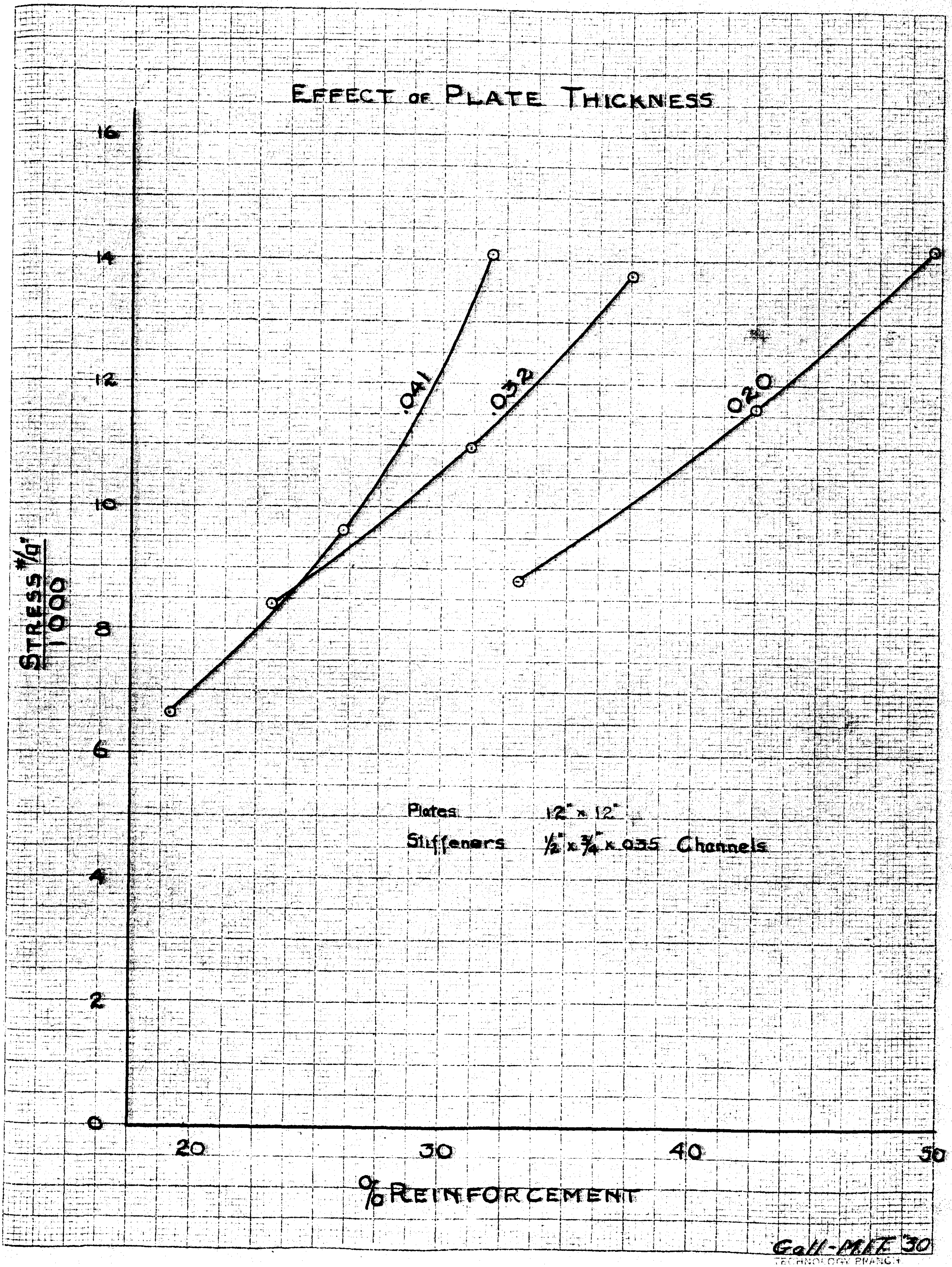
Sheet 12" wide - .032" thick

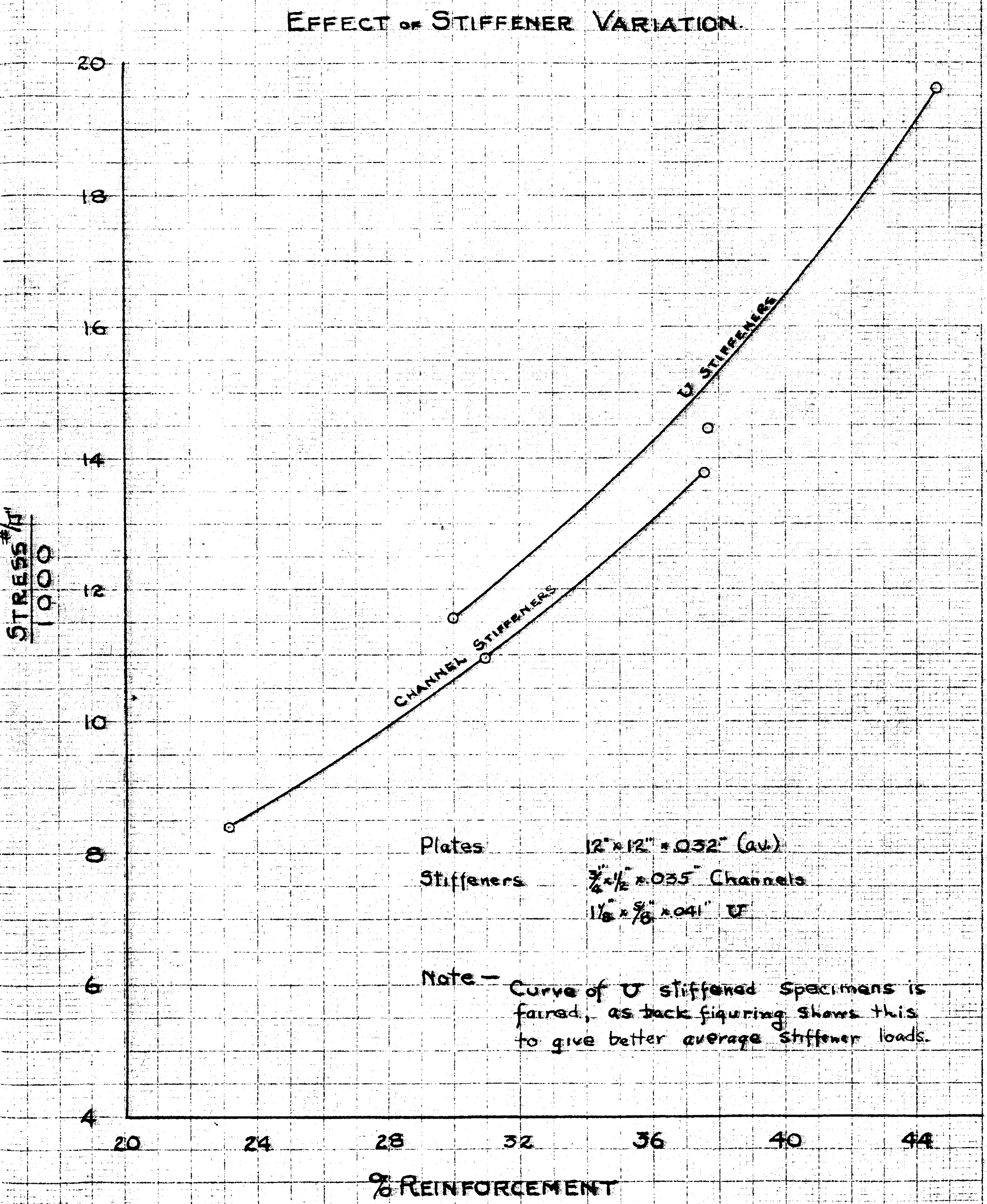
Stiffeners ¾" × ½" × .035" Channels

Note -

Specimens shown as S-12 had 1⅛" × ⅝ × .040 U stiffeners and are plotted to half scale.

Gall-M.I.T. '30

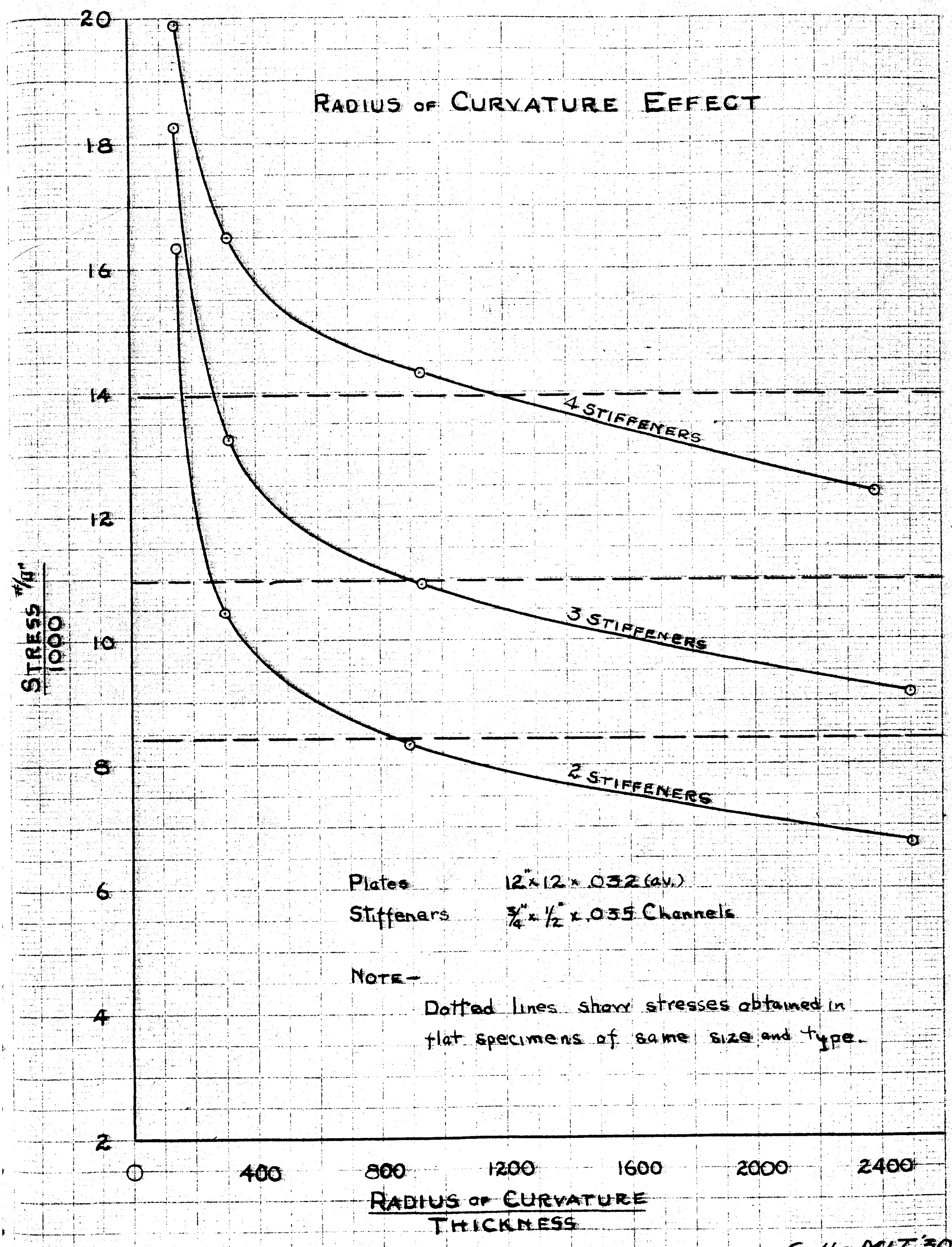

EFFECT OF PLATE THICKNESS
$\frac{\text{STRESS \#/□"}}{1000}$
0
2
4
6
8
10
12
14
16
20
30
40
50
% REINFORCEMENT
.041
.032
.020
Plates 12" x 12"
Stiffeners ½" x ¾" x .035 Channels
Gall-MIT '30
TECHNOLOGY BRANCH



Plates 12" × 12" × .032" (av.)
Stiffeners 3/4" × 1/2" × .035" Channels
1 1/8" × 5/8" × .041" U

Note — Curve of U stiffened specimens is faired, as back figuring shows this to give better average stiffener loads.

Gall-MIT '30

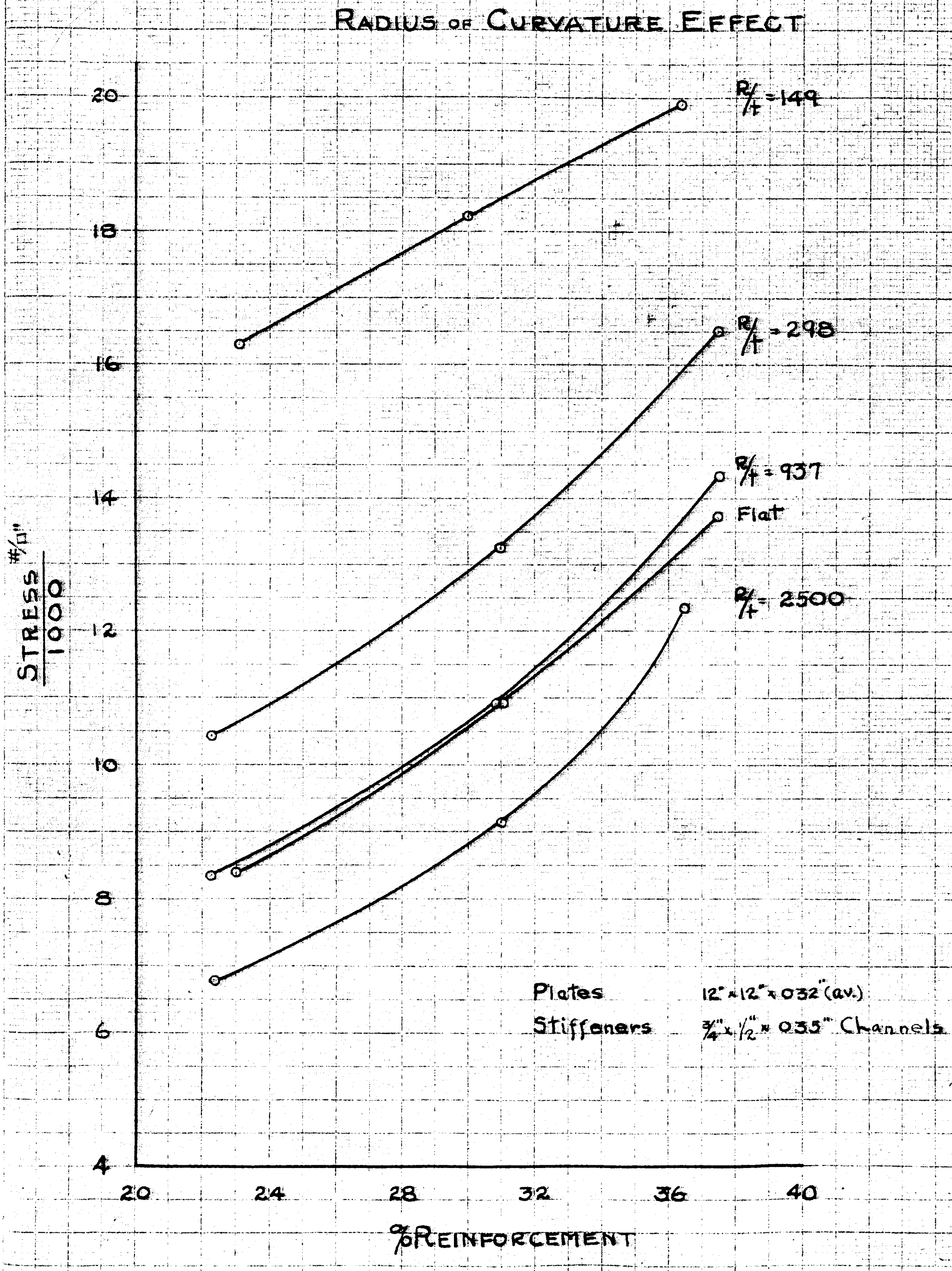


RADIUS OF CURVATURE EFFECT

Plates 12" × 12 × .032 (av.)
Stiffeners ¾" × ½" × .035 Channels

NOTE–
Dotted lines show stresses obtained in flat specimens of same size and type.

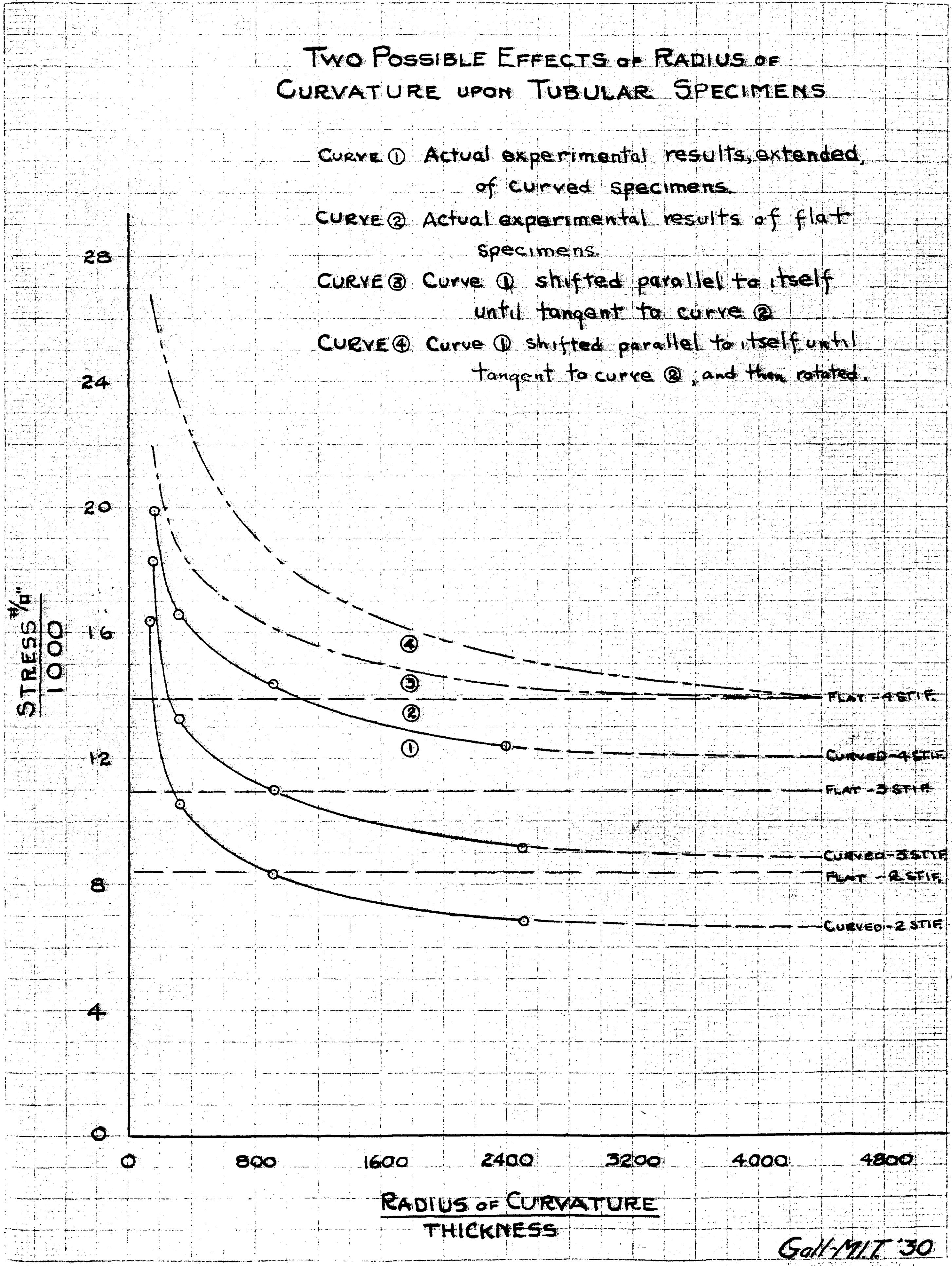
Gall - MIT '30



RADIUS OF CURVATURE EFFECT

Plates 12" × 12" × .032" (av.)
Stiffeners 3/4" × 1/2" × .035" Channels

Gall - M.I.T. '30



TWO POSSIBLE EFFECTS OF RADIUS OF CURVATURE UPON TUBULAR SPECIMENS

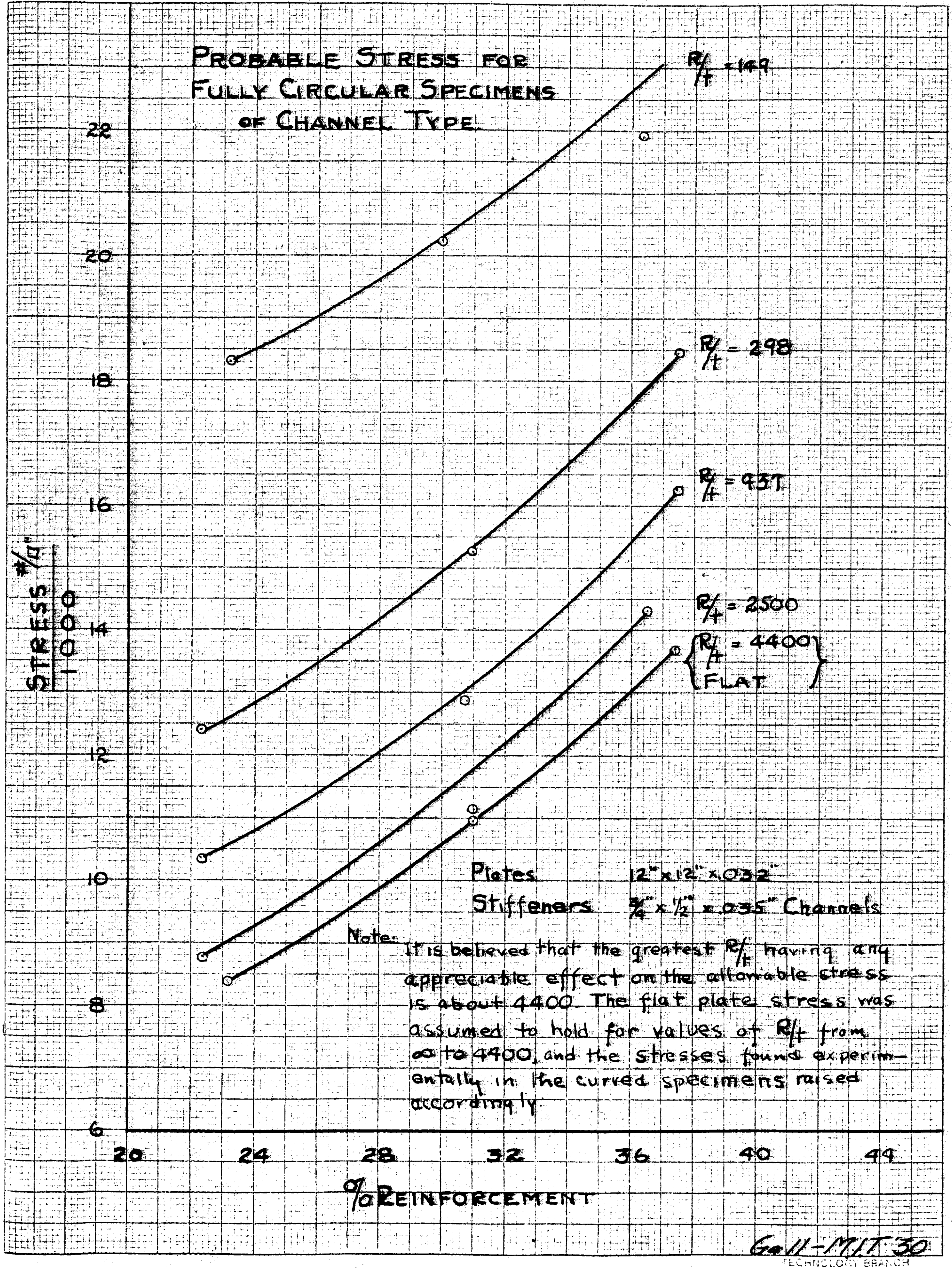
CURVE ① Actual experimental results, extended, of curved specimens.

CURVE ② Actual experimental results of flat specimens.

CURVE ③ Curve ① shifted parallel to itself until tangent to curve ②

CURVE ④ Curve ① shifted parallel to itself until tangent to curve ②; and then rotated.

Gall-M.I.T. '30



PROBABLE STRESS FOR FULLY CIRCULAR SPECIMENS OF CHANNEL TYPE

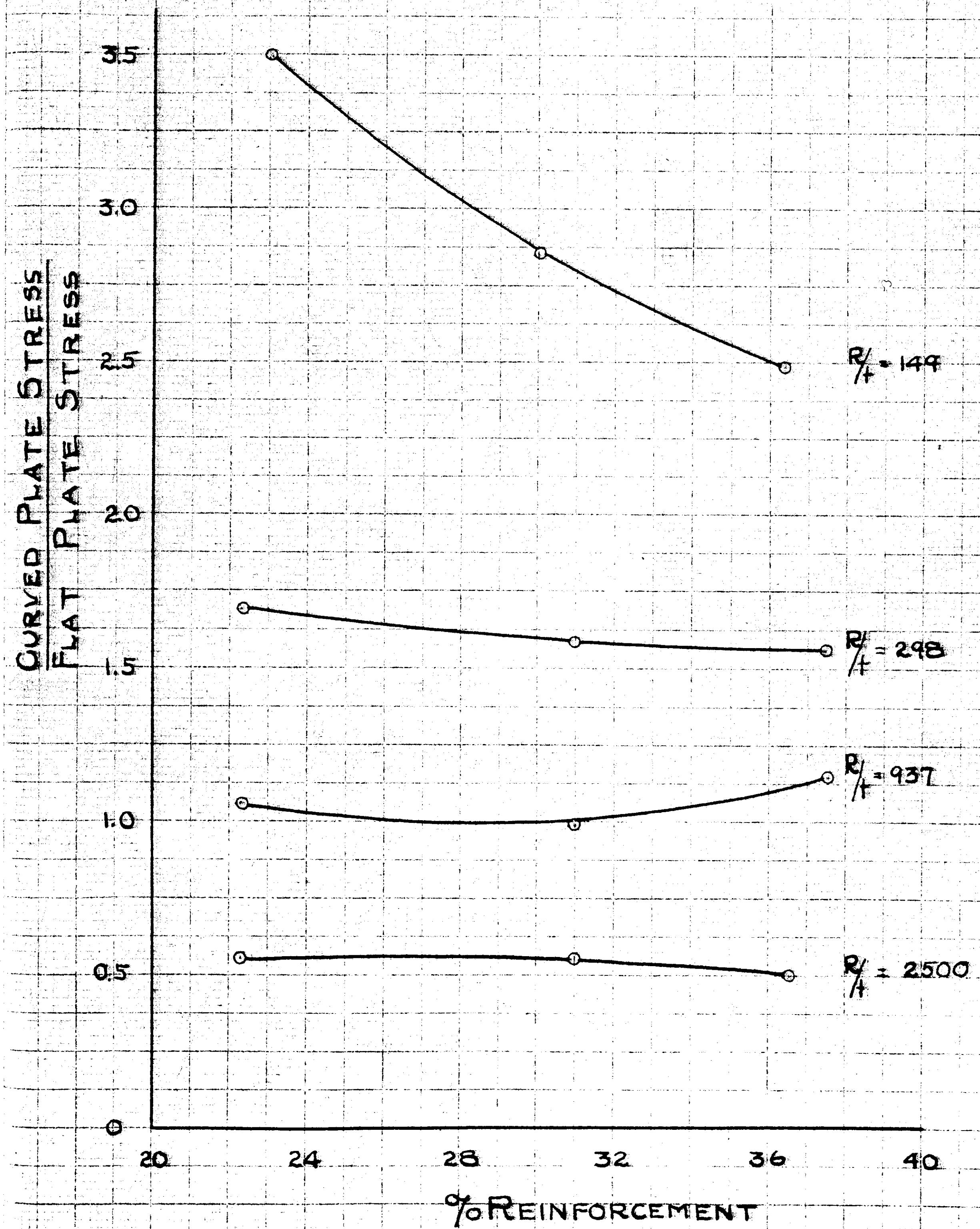
Plates 12" x 12" x .032"

Stiffeners 3/4" x 1/2" x .035" Channels

Note: It is believed that the greatest R/t having any appreciable effect on the allowable stress is about 4400. The flat plate stress was assumed to hold for values of R/t from ∞ to 4400, and the stresses found experimentally in the curved specimens raised accordingly.

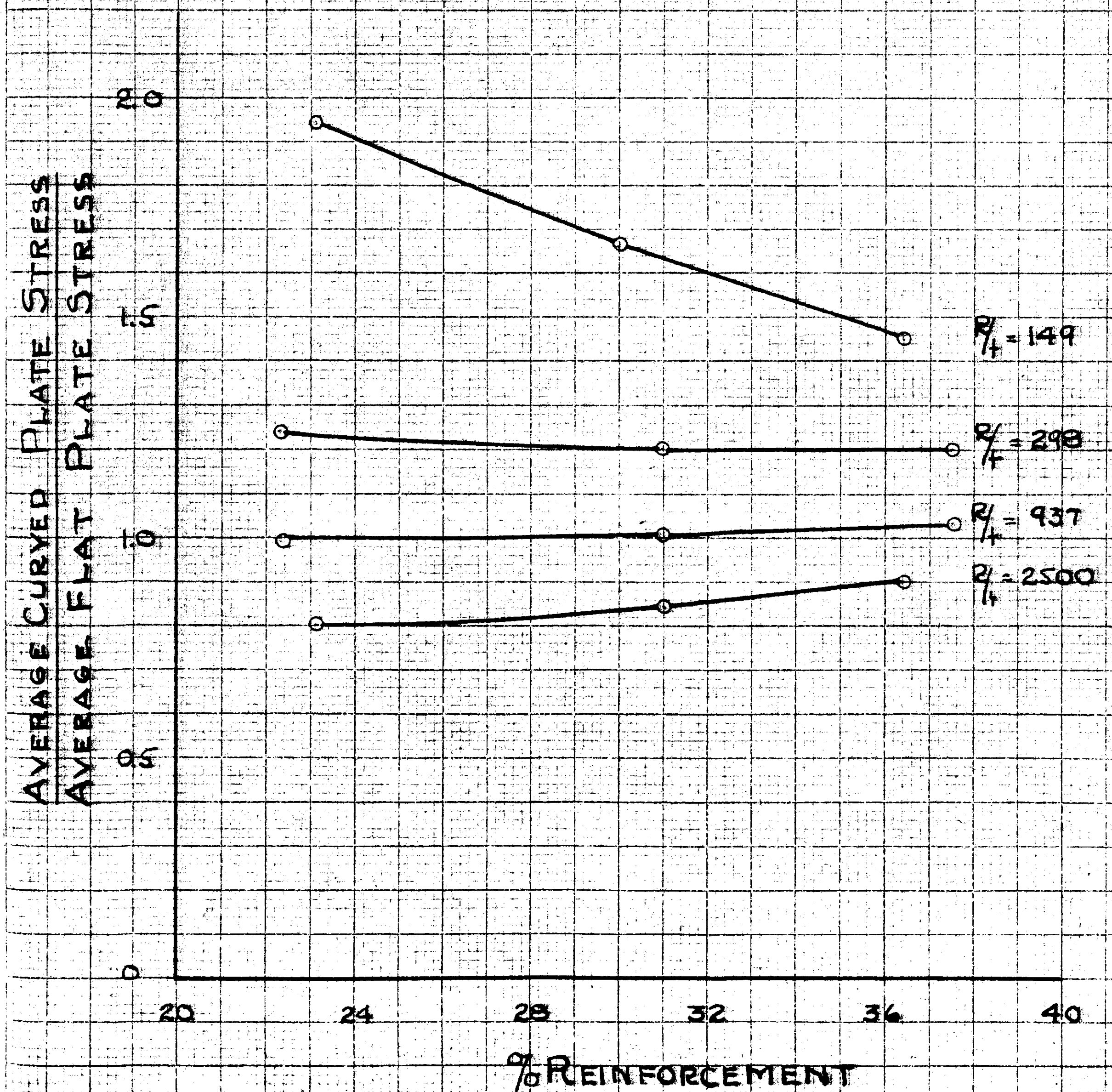
G-11-M.I.T. 30

RATIO OF CURVED PLATE STRESS
TO FLAT PLATE STRESS
STIFFENER LOAD ASSUMED CONSTANT AT 1300#

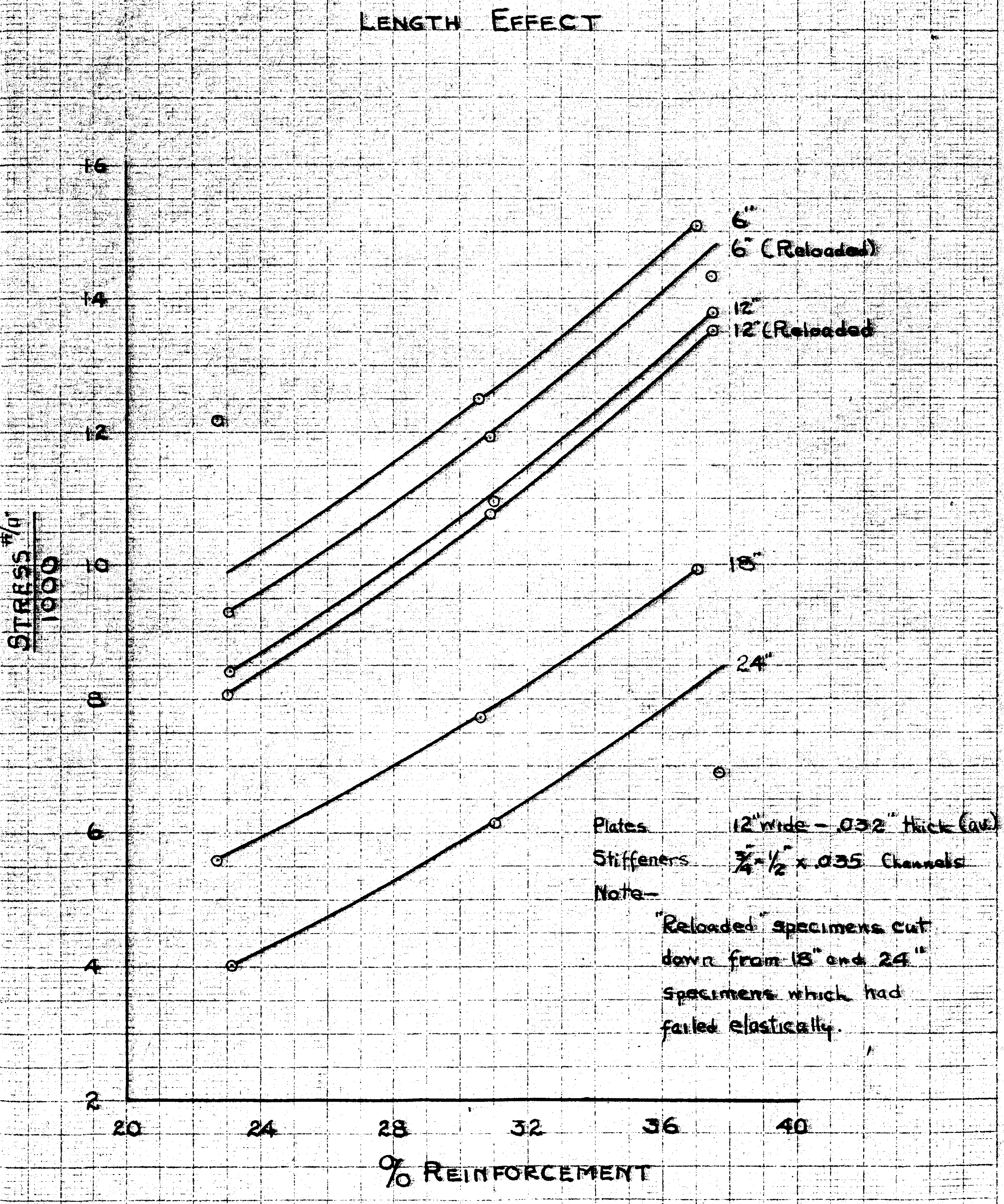


Gall-MIT '30

RATIO OF AVERAGE CURVED SPECIMEN STRESS TO AVERAGE FLAT SPECIMEN STRESS



Gall-MIT '30

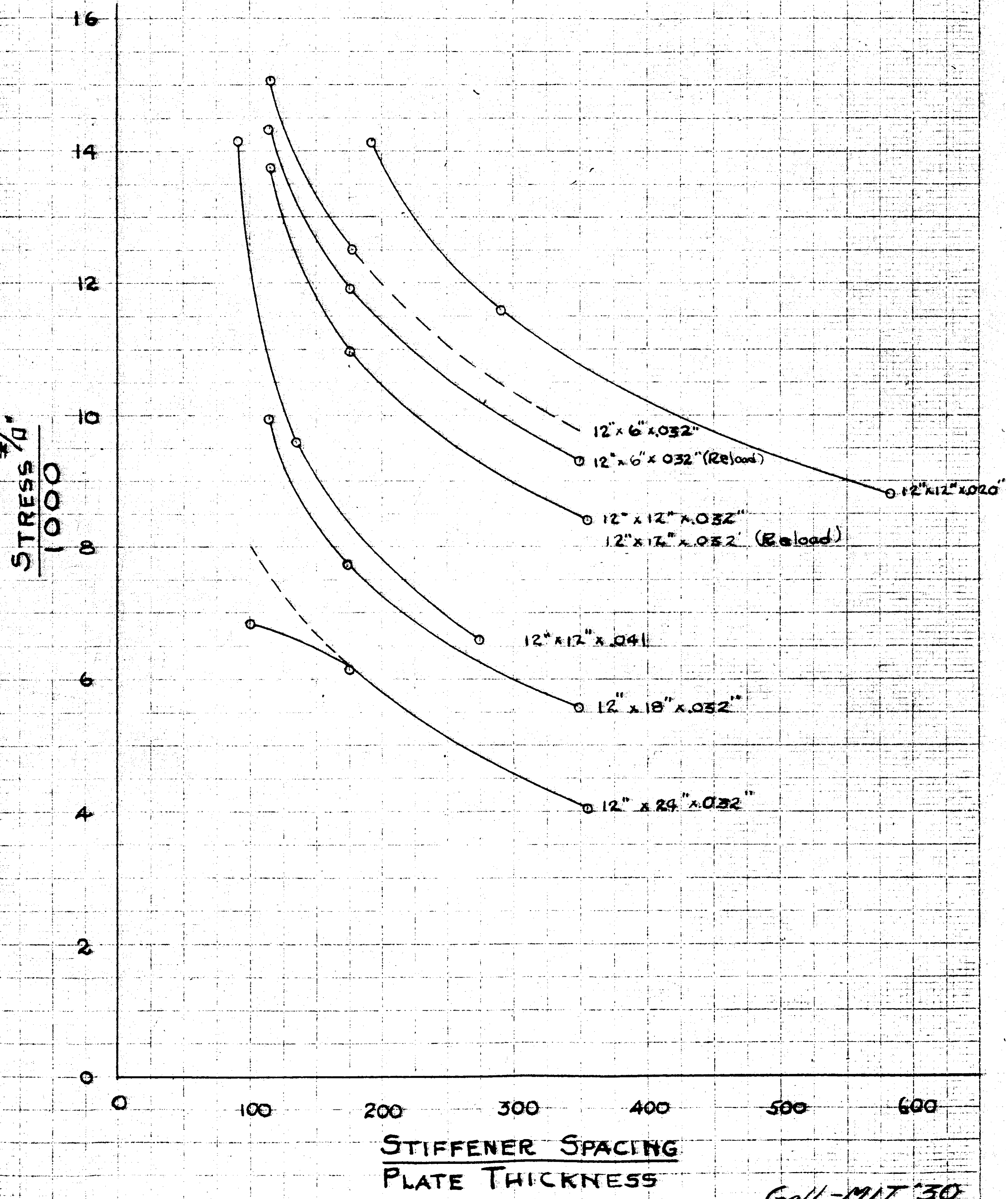


LENGTH EFFECT

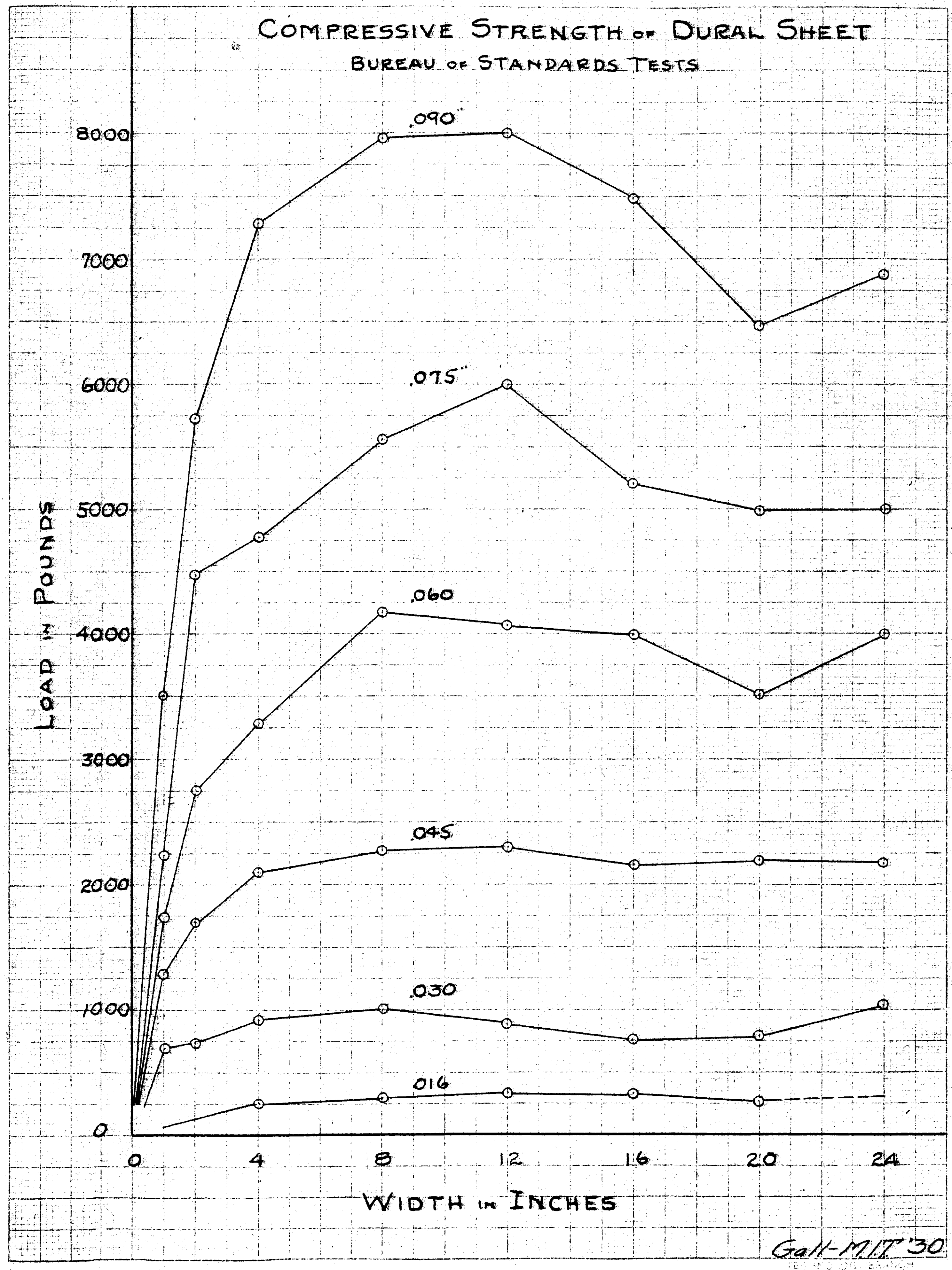
Plates 12" wide - .032" thick (av.)
Stiffeners ¾ - ½" x .035 Channels
Note -
"Reloaded" specimens cut down from 18" and 24" specimens which had failed elastically.

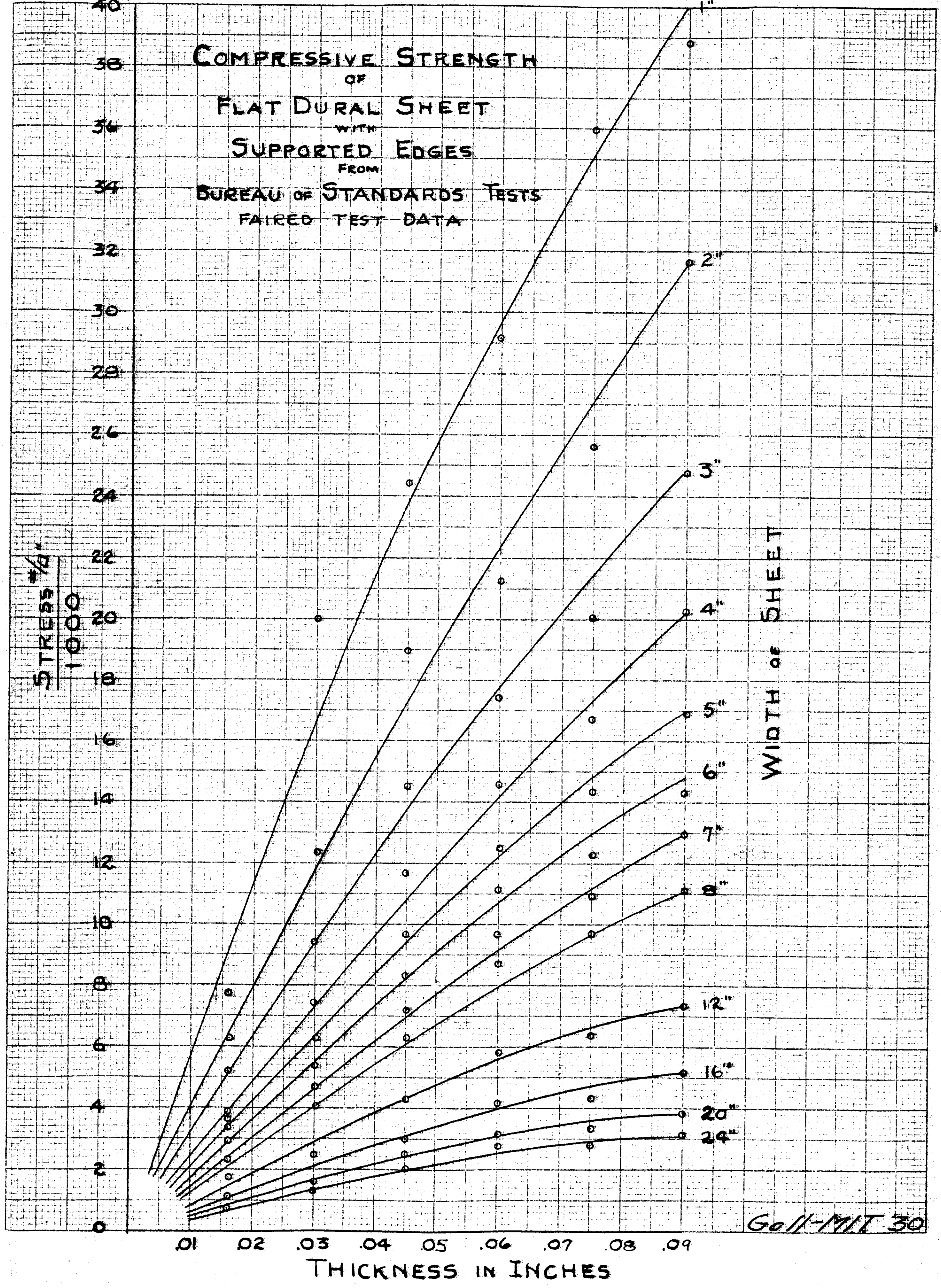
GaN-M.I.T. '30

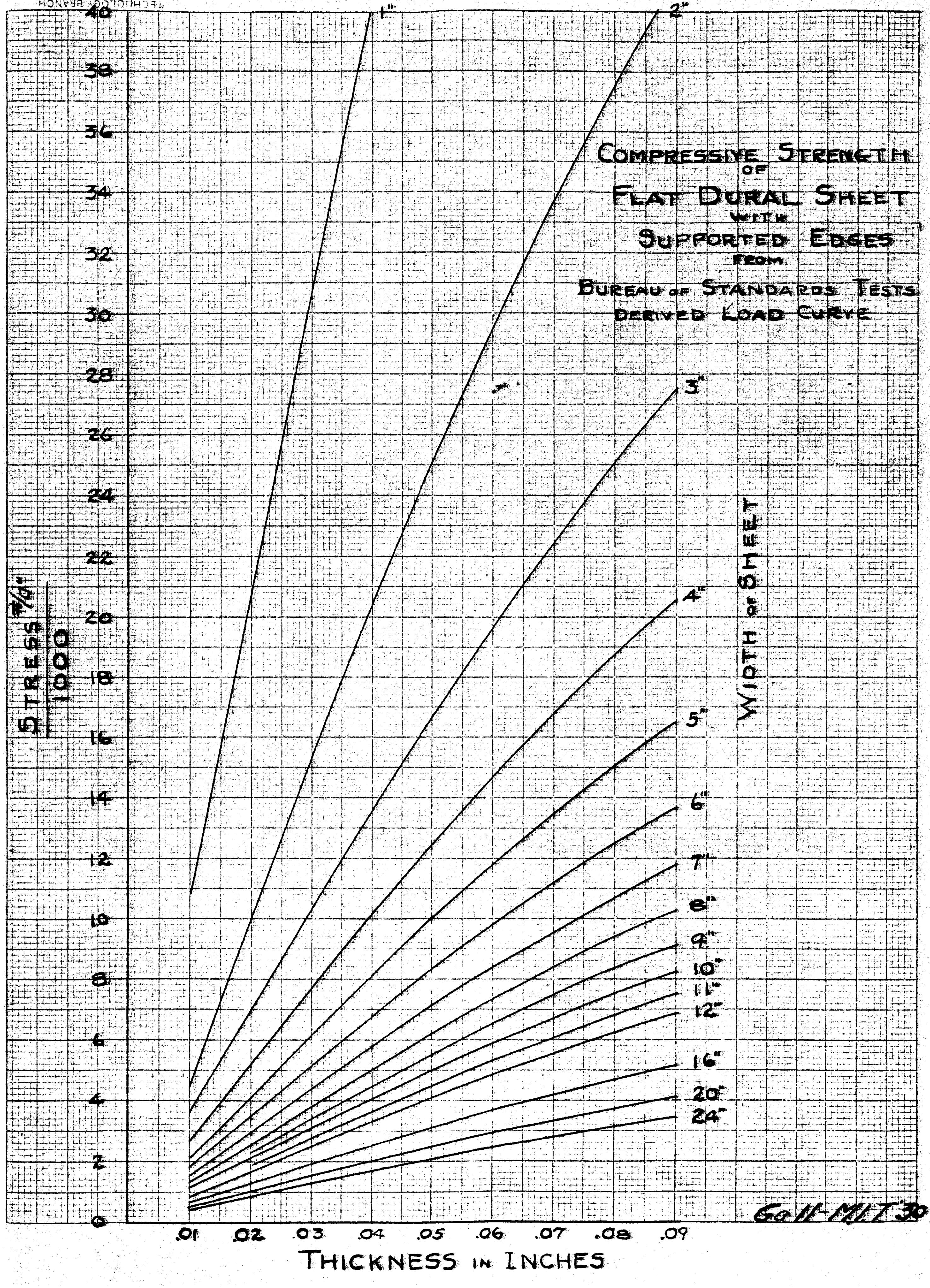
EFFECT OF VARIATION OF LENGTH AND PLATE THICKNESS



Gall - M.I.T. '30


Compressive Strength of Dural Sheet
Bureau of Standards Tests
.090"
.075"
.060
.045
.030
.016
8000
7000
6000
5000
4000
3000
2000
1000
0
Load in Pounds
0
4
8
12
16
20
24
Width in Inches
Gall-M.I.T. '30


COMPRESSIVE STRENGTH
OF
FLAT DURAL SHEET
WITH
SUPPORTED EDGES
FROM
BUREAU OF STANDARDS TESTS
FAIRED TEST DATA
STRESS #/□"
1000
40
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
THICKNESS IN INCHES
WIDTH OF SHEET
1"
2"
3"
4"
5"
6"
7"
8"
12"
16"
20"
24"
Goll-MIT 30


COMPRESSIVE STRENGTH
OF
FLAT DURAL SHEET
WITH
SUPPORTED EDGES
FROM
BUREAU OF STANDARDS TESTS
DERIVED LOAD CURVE
STRESS #/□"
1000
WIDTH OF SHEET
THICKNESS IN INCHES
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
.01
.02
.03
.04
.05
.06
.07
.08
.09
1"
2"
3"
4"
5"
6"
7"
8"
9"
10"
11"
12"
16"
20"
24"
Goll-M.I.T.30

CONCLUSIONS.

The stress curves calculated from the Bureau of Standards tests give values that are too high in the 4 and 6 inch width range for lengths shorter than 24", probably due to length and restraint effects.

Until more comprehensive tests of flat sheet, various stiffeners and built up combinations have been made, no practicable general design procedure can be set forth.

The method of design set forth in the discussion making use of the Bureau stress curves and certain fixity coefficients will serve in a general way to design for static testing.

In short specimens and in curved specimens there is no definite relation between stress and EI. In flat sections of 12" and longer, there is a marked relation between stress and EI.

Thin sheet with numerous stiffeners gives better economy of weight than thicker sheet and fewer stiffeners, though its construction cost is higher.

For any one plate thickness, two large stiffeners giving about the same area as three smaller ones are more efficient, even though the plate itself is stronger with more stiffeners.

Stress in every case increases with % reinforcement.

Closed stiffeners with the greatest I for a given area are most efficient.

For plates of .040" and under, the rivet pitch should not exceed 3/4", unless the section is lightly loaded.

The increased stress due to radius of curvature is independent of % reinforcement.

For discontinuous curved sections of a value of R/T greater than 935, the allowable stress is less than that for a flat plate.

RECOMMENDATIONS.

The desirability of establishing a practicable method of design being readily apparent, it is recommended that tests organized somewhat along the following lines be carried out:

A. For the determination of the effect of length and stiffener spacing.

1. A thorough investigation of the strength of plates of various widths, lengths and thicknesses.
2. An investigation of the strength of various types and lengths of stiffeners, preferably U shapes, and bulb angle shapes similar to those used by the Navy and by C. W. Hall.
3. Tests of built up sections, of materials similar to those suggested in 1 and 2.

B. For the determination of the effect of plate thickness.

1. Tests of specimens with varying plate thickness and stiffener arrangement, built of materials whose characteristics have been determined under A, 1 and 2.

C. For the determination of effect of varying radius of curvature.

1. Tests of specimens of various plate thickness

at several radii of curvature to determine whether the stress is a function of R/T or simply of R.

2. Tests of specimens of the same type, but of different circumferential length with the same radii of curvature to determine the effect of this on stress. The specimens should be varied from small segments to full circles in the smaller radii of curvature.

D. For the determination of the most efficient rivet spacing.

1. In the case of single riveted stiffeners, tests of variation of rivet pitch, using various sizes of rivets and varied pitch. In the case of double riveted stiffeners, different arrangements of stagger should be tried.

While the above outlined program is much too large and undertaking for a few persons in a short time, it would undoubtedly pay to decide on the specimen types and methods of the test, after which tests could be carried on at intervals and the results compared directly.

APPENDIX

CALCULATION OF STIFFENER LOADS.

Method of Calculation.

In the following computations, a theoretical stiffener load is arrived at by subtracting a "plate load", found by multiplying the plate area by the allowable stress read from the Bureau of Standards curves, from the gross load supported by the specimen in test. For comparative purposes, two sets of values are found, one using the Bureau of Standards stresses as found from their computed load curve derived from test results, and the other using the stresses found from the Bureau of Standards test data itself.

All loads are in pounds and stresses in lbs/sq.in.

The fixity coefficient, "C" is found by dividing the theoretical stiffener load by the stiffener load calculated by the method outlined by Roy A. Miller in Army Air Corps Information Circular No. 598, on the strength of dural channels.

In the case of the U stiffeners, the load as a pin ended column was determined by means of the Euler formula, using the experimental value of EI obtained. The values calculated are given below:

Channel Stiffeners

Length	6"	12"	18"	24"
Load (lbs)	1495	940	430	245

U Stiffeners

Length 12"	Load 1840 lbs.

12" x 6" x .032" Flat Specimens - Channel Stiffeners.

I. Based on "Faired Curve" Sheet Stresses.

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	5120	7040	9380
Plate Load	1160	2370	3480
Stiff. Load	3960	4720	5900
Load/Stiff.	1980	1573	1475
"C"	1.325	1.05	.988

II. Based on "Test Data" Sheet Stresses.

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross load	5120	7040	9380
Plate Load	1310	2440	3280
Stiff. Load	3810	4600	6100
Load/Stiff.	1905	1533	1525
"C"	1.275	1.037	1.020

12" x 12" x .032" Flat Specimens - Channel Stiffeners.

I. Based on "Faired Curve" Sheet Stress.

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	4190	6110	8450
Plate Load	1140	2290	3420
Stiff. Load	3050	2820	5030
Load/Stiff.	1525	1273	1257
"C"	1.66	1.36	1.335

II. Based on "Test Data" Sheet Stresses.

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	4190	6110	8450
Plate Load	1265	2380	3190
Stiff. Load	2925	3730	5260
Load/Stiff.	1463	1243	1315
"C"	1.56	1.325	1.40

III. Assuming Stiffeners to Carry 1500# each.

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	4190	6110	8450
Stiff. Load	3000	4500	6000
Plate Load	1190	1610	2450
Plate Stress	3100	4200	6400

12" x 18" x .032" Flat Specimens - Channel Stiffeners.

I. Based on "Faired Curve" Sheet Stress.

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	2830	4350	6170
Plate Load	1160	2320	3480
Stiff. Load	1670	2030	2690
Load/Stiff.	835	676	672
"C"	1.94	1.57	1.56

II. Based on "Test Data" Sheet Stress.

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	2830	4350	6170
Plate Load	1285	2420	3240
Stiff. Load	1545	1930	2930
Load/Stiff.	772	643	732
"C"	1.80	1.49	1.70

12" x 24" x .032" Flat Specimens - Channel Stiffeners.

I. Based on "Faired Curve" Sheet Stress.

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	2000	3430	5000
Plate Load	1140	2290	3420
Stiff. Load	860	1140	1580
Load/Stiff.	430	380	395
"C"	1.75	1.55	1.61

II. Based on "Test Data" Sheet Stress.

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	2000	3430	5000
Plate Load	1265	2380	3190
Stiff. Load	725	1650	1810
Load/Stiff.	362	350	452
"C"	1.475	1.425	1.84

12" x 12" x .020" Flat Specimens - Channel Stiffeners.

I Based on "Faired Curve" Sheet Stress.

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	3070	4710	6550
Plate Load	422	865	1310
Stiff. Load	2648	3845	5240
Load/Stiff.	1324	1281	1310
"C"	1.41	1.36	1.39

II Based on "Test Data" Sheet Stress

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	3070	4710	6550
Plate Load	492	725	1240
Stiff Load	2578	3985	5310
Load/Stiff.	1289	1328	1327
"C"	1.37	1.41	1.41

12" x 12" x .041 Flat Specimens - Channel Stiffeners.

I. Based on "Faired Curve" Sheet Stress.

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	4000	6470	10180
Plate Load	1770	3640	5500
Stiff. Load	2230	2730	4680
Load/Stiff.	1115	910	1170
"C"	1.185	.97	1.250

II. Based on "Test Data" Sheet Stress.

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	4000	6370	10180
Plate Load	2060	3880	5100
Stiff Load	1940	2490	5080
Load/Stiff.	970	830	1275
"C"	.98	.89	1.35

12" x 12" x .032 Flat Specimens - U Stiffeners.

I. Based on "Faired Curve" Sheet Stress.

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	6500	9340	14205
Plate Load	1365	3020	5060
Stiff. Load	5135	6320	9145
Load/Stiff.	2567	2106	2286
"C"	1.40	1.15	1.24

II. Based on "Test Data" Sheet Curves.

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	6500	9340	14205
Plate Load	1530	2975	4350
Stiff. Load	4970	6365	9855
Load/Stiff	2485	2122	2464
"C"	1.35	1.15	1.34

Flat Plate - 12" x 12" x .032 - Channels

Assuming Stiffener Load of 1300# Each.

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	4190	6110	8450
Stiff. Load	2600	3900	5200
Plate Load	1590	2210	3250
Plate Stress	4150	5750	8200

5" Radius of Curvature - R/T = 149

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	8150	10500	12600
Stiff. Load	2600	3900	5200
Plate Load	5550	6600	7400
Plate Stress	14420	16400	18350
$\frac{\text{Plate Stress}}{\text{Flat Plate Stress}}$	3.48	2.85	2.48

10" Radius of Curvature - R/T = 298

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	5400	7400	10130
Stiff Load	2600	3900	5200
Plate Load	2800	3500	4930
Plate Stress	6990	9100	12800
$\frac{\text{P.S.}}{\text{F.P.S.}}$	1.68	1.58	1.56

30" Radius of Curvature R/T = 935

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	4300	6090	8800
Stiff. Load	2600	3900	5200
Plate Load	1700	2190	3600
Plate Stress	4430	5700	9400
$\frac{\text{P.S.}}{\text{F.P.S.}}$	1.06	.99	1.15

80" Radius of Curvature R/T = 2500

	2 Stiffeners	3 Stiffeners	4 Stiffeners
Gross Load	3500	5100	7840
Stiff. Load	2600	3900	5200
Plate Load	900	1200	1640
Plate Stress	2340	3130	4090
$\frac{\text{P.S.}}{\text{F.P.S.}}$	.565	.545	.498