Section 2 Quantum-Effect Devices

Chapter 1. Statistical Mechanics of Quantum Dots

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1.1 Project Description

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In 1994, we concentrated our research on the dynamical properties of quantum dots. These closed systems which consist of a substantial but finite number of electrons usually demonstrate behavior that is called quantum chaos.[1] This behavior can be caused by disorder, the geometry of a quantum dot, or interactions between electrons. The phenomenon of quantum chaos was intensively studied by Berry for several years.[2] However, attention was concentrated for the most part on the statistics of energy spectra. It was found that these statistics are universal for systems with well developed chaos. Previously, we were interested in parametric spectra, in which exact energy levels are monitored as a function of an external tunable parameter, e.g., magnetic field. We studied the parametric spectral correlations and showed that these correlations are also universal, i.e., they depend only on the symmetries of the system and are the same for different systems and perturbations.

Our goal for 1994 was to extend the concept of universality, based on the phenomenon of quantum chaos, to the physically interesting and observable features of mesoscopic quantum objects. We found that, similar to spectral statistics, these features are determined by the relationship between the classical energy scale E_c and the quantum energy scale Δ, that is the mean level spacing.

We solved the fundamental problem of wave packet evolution within a chaotic quantum dot.[3] Consider a particle that, originally at $t = 0$, is put at a certain point $r = 0$. At short times, the density distribution $\rho(t,r)$ demonstrates the usual ballistic and later (in a disordered dot) diffusion behavior.

At longer times, the density evolution depends on the symmetry of the system with respect to time inversion. If T-invariance is violated by a strong enough magnetic field, for example, the density distribution becomes nearly homogenous, at times $t_c \approx h/E_c$ (the time it takes a particle to get from one side of the dot to another side). However, the long time evolution turned out to be unexpectedly interesting: we found a remarkable evolution towards a state with increasing correlation with the original state. In other words, an echo appears. In 1991, Berry mentioned that the exact wave functions of a particle in a chaotic system $\Psi_v(r)$ are strongly inhomogenous and the correlation function $<|\Psi_v(0)|^2\Psi_v(r)|^2>$ has a universal form.[2] For example, in two dimensions:

$$<|\Psi_v(0)|^2\Psi_v(r)|^2> = CJ_0(k|r|) \quad (1)$$

where <...> means ensemble and/or spectral averaging and $J_0(|r|)$ is a Bessel function that is known to decay very quickly for distances $|r|$ much bigger than the wavelength 1/k. In terms of the wave packet evolution, this means that at $t = \infty$, density

1 M.C. Gutzwiller, *Chaos in Classical and Quantum Mechanics* (New York: Springer Verlag, 1990).

2 M.V. Berry, in *Chaos in Quantum Physics*, eds. M.-J. Gianonni, A. Voros, and J. Zinn-Justin (North Holland, Amsterdam, 1991), p. 251.

3 V.N. Prigodin, B.L. Altshuler, K.B. Efetev, and S. Iida, *Phys. Rev. Lett.* 72: 546 (1994).

distribution has a peak at $r = 0$: $\rho(\infty, r) = \langle |\Psi_v(0)|^2 \Psi_v(r)|^2 \rangle$.

We have not only computed the constant C, but also determined the dynamics of the density evolution. We found that at $t \gg t_c$, the density distribution has a form

$$\rho(t, r) = C(t) J_0(k|r|), \tag{2}$$

and the time dependance C(t) is determined by the Fourier transformed two-point spectral correlator:

$$C(t) = K(t) = \int \langle v(\varepsilon) v(\varepsilon + \omega) \rangle \, d\omega \tag{3}$$

Therefore, the evolution of the wave packet is determined by the spectral correlation, and the dynamical echo is a consequence of the spectral rigidity.

For T-invariant systems, the phenomenon is even more complex: there is a maximum in $\rho(t,r)$ at $r = 0$ even for $t < t_c$. This additional return probability is well known in the theory of weak localization. However, the echo takes place anyway and, if the spin-orbit interaction is substantially strong, the echo becomes even much more pronounced. Later, we analyzed the time dependence of the density distribution in several physically interesting cases: (1) for T-invariant systems, (2) in the crossover regime, and (3) for a quantum dot with low conductance. We found that even in the latter case, when we do not yet know K(t) explicitly, we still can prove the validity of equation (3). We believe that the echo can manifest itself in a number of possible experimental situations.

We have also continued our work on quantum chaos in the band structure of complex crystals and superlattices. We have analyzed[4] the *ab initio* band structure of silicon and the spectrum of the alloy AlGaAs. The Bloch quasimomenta act as in the Hamiltonian of a particle in a unit cell in close analogy to Aharonov-Bohm fluxes threading a torus. We found that the statistical properties of the band structure obey the universal predictions derived from the theory of parametric quantum chaos. We have also analyzed the connection between the statistical properties and certain physical properties of a crystal with basic symmetries of the lattice.

We have started to study current fluctuations (noise) in a mesoscopic conductor using the non-equilibrium Keldysh diagram technique.[5] We have derived a general expression for the fluctuations in the presence of a time dependent voltage V(t), valid for an arbitrary relation between voltage and temperature. Two limits are then treated: a pulse of voltage and a DC voltage. A pulse of voltage causes phase sensitive current fluctuations for which we derive microscopically an expression periodic in $\int V(t)dt$ with the period h/e. Applied to current fluctuations in Josephson circuits caused by phase slips, it gives an anomalous contribution to the noise with a logarithmic singularity near the critical current. In the DC case, we get a quantum to classical shot noise reduction factor of 1/3.

1.2 Publications

Altshuler, B.L., L.S. Levitov, and A.Y. Yakovets. "Non-equilibrium Noise in a Mesoscopic Conductor: Microscopic Analysis." *JETP Letters* 59: 857 (1994).

Mucciolo, E.R., R.B. Capaz, B.L. Altshuler, and J.D. Joannopoulos. "Manifestation of Quantum Chaos in Electronic Band Structures." *Phys. Rev. B* 50: 8245 (1994).

Prigodin, V.N., B.L. Altshuler, K.B. Efetev, and S. Iida. "Mesoscopic Dynamical Echo in Quantum Dots." *Phys. Rev. Lett.* 72: 546 (1994).

Thesis

Mucciolo, E.R. *Universal Correlations in the Quantum Spectra of Chaotic Systems in Exactly Solvable Many - Body Problems.* Ph.D. diss. Dept. of Physics, MIT, 1994.

[4] E.R. Mucciolo, R.B. Capaz, B.L. Altshuler, and J.D. Joannopoulos, *Phys. Rev. B* 50: 8245 (1994).

[5] B.L. Altshuler, L.S. Levitov, and A.Y. Yakovets, *JETP Letters* 59: 857 (1994).