

Section 2 Quantum-Effect Devices

Chapter 1 Statistical Mechanics of Quantum Dots

Chapter 2 Single-Electron Transistors

Chapter 3 Edge Structure of a Quantum Dot in a Large
Magnetic Field

Chapter 4 Nanostructures Technology, Research, and
Applications

Chapter 5 Single-Electron Spectroscopy

Chapter 1. Statistical Mechanics of Quantum Dots

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1.1 Project Description

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Quantum dots—isolated systems of electrons whose dimensions are smaller than typical electron dephasing length—are being investigated as potential elements of novel electronic devices. In addition to their interest for such applications their fundamental physical properties are of a substantial interest. Combination of quantum interference of electronic waves with geometrical restrictions in all directions and with the interaction between electrons yields a rich variety of behavior.

To develop a statistical approach to the description of chaotic energy spectra in the presence of external fields, we have started¹ with a disordered mesoscopic ring with Aharonov-Bohm flux Φ . The energy of a given electron state E_α is a periodic function of Φ with a period $\Phi_0 = hc/e$. At the same time on the interval $\Phi < \Phi_0$ this function $E_\alpha(\Phi)$ is random. We have found that the autocorrelation function of the derivatives $dE_\alpha/d\Phi$ (i.e., of the currents carried by the level at different fluxes) determined as

$$C(\Phi) = \Delta^{-2} \left\langle \frac{dE_\alpha(\Phi_1)}{d\Phi_1} \frac{dE_\alpha(\Phi_1 + \Phi)}{d\Phi} \right\rangle \quad (1)$$

is universal, i.e., independent on the disorder and on particular characteristics of the quantum dot:

$$C(\Phi) = -(\Phi_0/\pi\Phi)^2 \quad (2)$$

in the interval $\Phi_0 \ll \Phi \ll \Phi_c = g^{1/2}\Phi_0$, where g is the conductance of the ring.

As soon as Φ becomes of the order of or bigger than Φ_0 , the universality disappears, and $C(\Phi)$ turns out to be dependent on sample geometry, degree of disorder, etc. However, for a disordered quantum dot, the $C(\Phi)$ -function at $\Phi \geq \Phi_0$, given the geometry, can be evaluated within the same perturbative approach as equation 2. At the same time, $C(\Phi)$ at $\Phi < \Phi_0$ cannot be evaluated perturbatively: equation 2 apparently diverges when $\Phi \rightarrow 0$. We have developed a nonperturbative formalism to treat the region of very small fluxes. This allowed us to discover an even higher level of universality than is demonstrated by equation 2.²

Suppose the Hamiltonian of our quantum chaotic system depends on an external perturbation through some parameter X (in our previous example $X = \Phi$), having eigenvalues given by the random functions, $E_\alpha(X)$. Such a hamiltonian can describe an atom, a molecule, or a whole variety of mesoscopic quantum systems and devices in the presence of external field. It can include disorder and interaction between electrons. With no loss of generality, we assume that $\langle \delta E_\alpha(X)/\delta X \rangle = 0$, where $\langle \dots \rangle$ denote an average over X and over a typical range of levels. We have shown that the rescaling

$$x = X[C(0)]^{1/2}, \quad \varepsilon_\alpha(X) = E_\alpha(X)/\Delta \quad (3)$$

¹ A. Szafer and B.L. Altshuler, "Universal Correlations in the Spectra of Chaotic Systems With an Aharonov-Bohm Flux," *Phys. Rev. Lett.* 70: 587 (1993).

² B.D. Simons, A. Szafer, and B.L. Altshuler, "Universality in Quantum Chaotic Spectra," *JETP Lett.* 57: 276 (1993); B.D. Simons and B.L. Altshuler, "Universal Velocity Correlations in Disordered and Chaotic Systems," *Phys. Rev. Lett.* 70: 4063 (1993).

where $C(0)$ is determined equation 1 (with the substitution of X for Φ) makes the statistics of ϵ_α universal, i.e., independent on particular quantum system and on the nature of the parameter X .

To demonstrate this remarkable universality together with Professor Daniel Kleppner's group, we compared the correlation functions $c(x) = C(X)/C(0)$ for a disordered quantum dot and for the hydrogen atom in a magnetic field and found very good agreement for the two completely different systems.³

We have calculated exactly a number of correlation functions such as the correlator of densities of states at different energies and values of the dimensionless parameter X .⁴ This allowed us to discover another remarkable fact: the universal correlations that describe the response of energy levels of, for example, a disordered metallic grain to a magnetic field or any other perturbation are equivalent to time-dependent correlations with inverse square interaction.⁵

Also, the methods that we have developed allowed us to study dielectric and transport properties of periodic systems (periodic arrays of quantum dots or antidots) when electron motion within a primitive cell is chaotic.⁶ For a large conductance g of a single primitive cell, integration over the Fermi surface provides an effective averaging that leads to the universal answers. We have determined the dynamic dielectric function and ac conductivity exactly. It is possible to also describe exactly the transition between the diffusive motion of electrons and ballistic motion of Bloch waves.

In connection with the experiments on the current switching in a mesoscopic conductor, we have studied the effect of an external magnetic field on a defect that tunnels incoherently between two locations in a mesoscopic metal.⁷ The magnetic field influences the energy asymmetry (bias) and the polaron effect due to the conduction electrons.

Both effects contribute to the magnetic field dependence of the tunneling rate. For a two-level system that has no bias without magnetic field, the modification of the polaron effect dominates at low fields and increases slightly the tunneling rate, while at large fields the bias increase can significantly suppress the average tunneling rate.

1.2 Publications

Faas, M., and B.L. Altshuler. "Magnetic Field Dependence of Defect Tunneling in a Mesoscopic Metal." *Phys. Rev. B* 48: 18043 (1993).

Simons, B.D., A. Szafer, and B.L. Altshuler. "Universality in Quantum Chaotic Spectra." *JETP Lett.* 57: 276 (1993).

Simons, B.D., and B.L. Altshuler. "Universal Velocity Correlations in Disordered and Chaotic Systems." *Phys. Rev. Lett.* 70: 4063 (1993).

Simons, B.D., A. Hashimoto, M. Courtney, D. Kleppner, and B.L. Altshuler. "New Class of Universal Correlations in the Spectra of Hydrogen in a Magnetic Field." *Phys. Rev. Lett.* 71: 2899 (1993).

Simons, B.D., P.A. Lee, and B.L. Altshuler. "Exact Results for Quantum Chaotic Systems and One-dimensional Fermions from Matrix Models." *Nucl. Phys. B* 409(FS): 487 (1993).

Szafer, A., and B.L. Altshuler. "Universal Correlations in the Spectra of Chaotic Systems With an Aharonov-Bohm Flux." *Phys. Rev. Lett.* 70: 587 (1993).

Taniguchi, N., and B.L. Altshuler. "Universal ac Conductivity and Dielectric Response of Periodic Chaotic Systems." *Phys. Rev. Lett.* 71: 4031 (1993).

³ B.D. Simons, A. Hashimoto, M. Courtney, D. Kleppner, and B.L. Altshuler, "New Class of Universal Correlations in the Spectra of Hydrogen in a Magnetic Field," *Phys. Rev. Lett.* 71: 2899 (1993).

⁴ B.D. Simons, A. Szafer, and B.L. Altshuler, "Universality in Quantum Chaotic Spectra," *JETP Lett.* 57: 276 (1993); B.D. Simons and B.L. Altshuler, "Universal Velocity Correlations in Disordered and Chaotic Systems," *Phys. Rev. Lett.* 70: 4063 (1993).

⁵ B.D. Simons, P.A. Lee, and B.L. Altshuler, "Exact Results for Quantum Chaotic Systems and One-dimensional Fermions from Matrix Models," *Nucl. Phys. B* 409(FS): 487 (1993).

⁶ N. Taniguchi and B.L. Altshuler, "Universal ac Conductivity and Dielectric Response of Periodic Chaotic Systems," *Phys. Rev. Lett.* 71: 4031 (1993).

⁷ M. Faas and B.L. Altshuler, "Magnetic Field Dependence of Defect Tunneling in a Mesoscopic Metal," *Phys. Rev. B* 48: 18043 (1993).