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Momentum and missing mass from elastic scattering on hydrogen

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Abstract

The kinematics of the elastic scattering of incoming protons and pions on protons at rest is recalled. The effect of biases and resolutions on the momentum of the recoil protons observed in the TPC, and on the missing mass of the forward-scattered beam particle, are discussed.

1 Introduction

Elastic scattering of incoming protons and pions in a liquid hydrogen target has been strongly advocated by 'official' HARP as decisive 'physics benchmark'[1].

The kinematics of elastic scattering is straightforward.

The dynamics is such that the incoming beam particle is scattered forward at a small angle (typically 5–12 degrees). Its scattering angle is measured in the forward spectrometer. The recoiling target proton is scattered at a large angle (typically 70 degrees) and its scattering angle and its momentum is measured in the TPC. The recoil particle is confirmed as proton by $\mathrm{d}E/\mathrm{d}x$ in the TPC and by time of flight in the RPCs.

There are two ways to utilize the recoiling large-angle protons as physics benchmark:

1. from the scattering angle of the beam particle measured in the forward spectrometer, the scattering angle and the momentum of the recoil proton can be calculated; this permits to check the measurement of polar angle, and to check the momentum scale and the momentum resolution in the TPC;

2. from the scattering angle of the recoil proton measured in the TPC, the proton momentum can be calculated; this permits to check the momentum scale and the momentum resolution in the TPC; alternatively, from the scattering angle and from the momentum of the recoil proton measured in the TPC, the 'missing mass' of the beam particle can be calculated which must be the mass of a proton or a pion.

Throughout this paper, we assume that the incoming beam momentum is Gaussian-distributed around its nominal value with a σ of 1%. We restrict our discussion to beam momenta of 3 GeV/c and 5 GeV/c since only at relatively large forward-scattering angles, which are favoured by a low beam momentum, the recoil proton's momentum is predicted with high precision.

Because of the rectangular aperture of the dipole magnet[2], the acceptance of the forward-scattered beam particle is unlimited only in the horizontal direction; also, because of the wire orientation of the drift chambers, only the horizontal coordinate is measured with high precision. Therefore, predictions of the momentum of recoil protons from the forward-scattering angle are limited to recoil protons that populate primarily the horizontal TPC sectors 2 and 5. In this context, we recall that sectors 2 and 5 have been excluded in our data analysis for their low quality (large crosstalk, many bad electronics channels, no cross-check of performance with cosmic-ray muons).

Throughout this paper, scattering angles are understood as polar angles with respect to the direction of the incoming beam particle.

In the following, we discuss the effect of biases and resolutions on these physics benchmarks.

[1]We wish to acknowledge U. Gastaldi who—as nearly as we can tell—came up with this suggestion.

[2]The dipole magnet aperture is 2.41 m in the horizontal and 0.88 m in the vertical direction.

2 Kinematics

Our variables are:

$p_{\rm B}$	absolute value of the 3-momentum of the incoming beam particle
$E_{\rm B}$	total energy of the incoming beam particle
$m_{\rm B}$	mass of the beam particle
$m_{\rm P}$	mass of the proton (0.93827 GeV/c^2)
m_π	mass of the pion (0.13957 GeV/c^2)
$p_{\rm R}$	absolute value of the 3-momentum of the recoil proton
θ	scattering angle of the recoil proton
$p_{\rm F}$	absolute value of the 3-momentum of the forward-scattered beam particle
α	scattering angle of the forward-scattered beam particle

From the conservation of energy and momentum we obtain the momentum of the recoil proton $p_{\rm R}$ as a function of its scattering angle θ as

$$p_{\rm R} = \frac{2m_{\rm P}p_{\rm B}\cos\theta(E_{\rm B}+m_{\rm P})}{(E_{\rm B}+m_{\rm P})^2-(p_{\rm B}\cos\theta)^2}\,; \tag{1}$$

the momentum $p_{\rm F}$ of the forward-scattered beam particle as a function of its scattering angle α as

$$p_{\rm F} = \frac{A+\sqrt{A^2-Bp_{\rm B}^2(m_{\rm B}^2-m_{\rm P}^2)}}{B}\,, \tag{2}$$

where

$$\begin{aligned} A &= p_{\rm B}\cos\alpha(E_{\rm B}m_{\rm P}+m_{\rm P}^2) \\ B &= (E_{\rm B}+m_{\rm P})^2-(p_{\rm B}\cos\alpha)^2\,; \end{aligned}$$

and the momentum of the recoil proton $p_{\rm R}$ as a function of the momentum $p_{\rm F}$ and scattering angle α of the forward-scattered beam particle as:

$$p_{\rm R} = \sqrt{(p_{\rm B}-p_{\rm F}\cos\alpha)^2+(p_{\rm F}\sin\alpha)^2}\,. \tag{3}$$

Table 1 summarizes the salient features of recoil protons from the elastic scattering of 3 GeV/c protons on protons at rest. As a function of the recoil proton's scattering angle θ are given: the proton momentum at the vertex, the proton momentum in the centre of the TPC gas (reduced by energy loss in materials), the uncertainty on the angle θ from multiple scattering in materials, and the uncertainty on the momentum arising from fluctuations of the energy loss in materials. The listed properties are obtained from a GEANT simulation study that includes a detailed description of the materials that are passed before the recoil proton enters the TPC and that give rise to energy loss and multiple scattering.

We note the strong decrease of the proton recoil momentum with increasing angle θ. In addition, there is a strong decrease of the cross-section of elastic scattering with increasing $|t|$ (larger $|t|$ corresponds to a larger forward-scattering angle of the beam particle and hence to a smaller scattering angle of the recoil proton).

The experimental uncertainty on the angle θ leads to a momentum uncertainty when comparing recoil proton momenta measured in the TPC with momenta predicted from the angle θ by the kinematics of elastic scattering. The dependence of the angle on a cross-section that falls with $|t|$ like $\exp(-7.9|t|)$, leads to a bias in the average momentum.

Table 1: Salient features of recoil protons in the elastic scattering of 3 GeV/c protons on protons at rest, as a function of their scattering angle θ.

θ [deg]	$p_{\rm vertex}$ [GeV/c]	$p_{\rm gas}$ [GeV/c]	$\sigma(\theta_{\rm msc})$ [deg]	$\sigma(p_{\rm eloss})$ [GeV/c]
61	0.802	0.790	0.356	0.0020
62	0.768	0.756	0.369	0.0022
63	0.737	0.724	0.402	0.0024
64	0.704	0.690	0.415	0.0024
65	0.673	0.658	0.467	0.0028
66	0.642	0.626	0.472	0.0028
67	0.612	0.594	0.518	0.0032
68	0.582	0.562	0.556	0.0035
69	0.553	0.531	0.607	0.0038
70	0.524	0.499	0.687	0.0045
71	0.495	0.467	0.750	0.0049
72	0.467	0.435	0.857	0.0058
73	0.440	0.402	0.934	0.0074
74	0.412	0.367	1.132	0.0093
75	0.385	0.331	1.379	0.0121
76	0.358	0.289	1.719	0.0184
77	0.331	0.242	2.137	0.0237
78	0.305	0.172	2.855	0.0276
79	0.291	0.087	3.340	0.0256

3 Reconstructing from the forward-scattering angle

For this study, the forward-scattering angles of 10° and 7° were chosen for the beam momenta of 3 GeV/c and 5 GeV/c, respectively. The angular resolutions of the forward-scattering angles were read off from the right panel of Fig. 7 in the HARP Al-paper [1]. The angular resolutions of 2 mrad and 1.5 mrad, respectively, are dominated by multiple scattering of the incoming beam particle and, primarily, of the forward-scattered beam particle[3].

Table 2 lists the salient results for the four incident beams. The results for the four beams are also shown in a set of six figures each, see Figs. 1, 2, 3, and 4.

We note that the resulting uncertainties shown in these figures stem (i) from the 1% beam momentum uncertainty and (ii) from the 2 mrad and 1.5 mrad uncertainty, respectively, of

[3]For comparison, the quoted spatial resolution along the horizontal direction of the drift chambers is 340 μm [1], the distance between the target and the first drift chamber is 2.4 m.

the measurement of the forward-scattering angle of the beam particle. At this point, no use is made of any measurement in the TPC.

Table 2: Results from reconstructing from the forward-scattering angle

	3 GeV/c p	3 GeV/c π	5 GeV/c p	5 GeV/c π
$\sigma(p_{\mathrm{B}})/p_{\mathrm{B}}$ [%]	1	1	1	1
α [°]	10	10	7	7
$\sigma(\alpha)$ [mrad]	2	2	1.5	1.5
p_{F} [GeV/c]	2.85	2.86	4.81	4.81
$\sigma(p_{\mathrm{F}})/p_{\mathrm{F}}$ [%]	0.96	0.96	0.97	0.97
θ [deg]	69.0	69.8	68.5	68.8
$\sigma(\theta)$ [deg]	0.263	0.258	0.292	0.289
p_{TR} [GeV/c]	0.496	0.497	0.586	0.586
$\sigma(p_{\mathrm{TR}})$ [GeV/c]	0.0070	0.0070	0.0087	0.0087
$\sigma(1/p_{\mathrm{TR}})$ [(GeV/c)$^{-1}$]	0.028	0.028	0.025	0.025

We note the following:

1. there is not much difference between incoming protons and pions;
2. with the chosen forward scattering angles, the p_{T} of the recoil protons is around 0.45 GeV/c, its scattering angle is around 70°;
3. the precision of the calculated momentum of the forward-scattered beam particle is at the 1% level and thus much better than the resolution of the momentum measured in the forward spectrometer;
4. the precision of the calculated scattering angle of the recoil proton is with ∼5 mrad much better than the angular resolution ∼15 mrad of a track fit in the TPC;
5. the precision of the calculated p_{T} of the recoil proton is with $\sigma(1/p_{\mathrm{T}}) \sim 0.03$ (GeV/c)$^{-1}$ much better than the resolution $\sigma(1/p_{\mathrm{T}}) \geq 0.20$ (GeV/c)$^{-1}$ of a TPC track fit;
6. the missing mass is at this level only calculated to ascertain that the proton and pion masses are correctly reproduced (no independent measurement of the recoil proton in the TPC is used).

Therefore, in principle, the comparison of the p_{T} predicted from the measured angle of the forward-scattered beam particle with the p_{T} measured in the TPC permits a direct and significant check of the TPC's p_{T} scale and resolution. The beam momentum as well as the measurement of the forward-scattering angle can reasonably be taken as unbiased, the propagation of the finite resolutions of the beam momentum and the forward-scattering angle into the predicted p_{T} of the recoil proton are almost negligible.

In practice, however, there remains the fact that only the two bad TPC sectors 2 and 5 are involved.

There is a further effect to be taken into account: the energy loss of the recoil protons in materials before entering the TPC volume, its fluctuations, and the effect of multiple scattering in these materials on the measured scattering angle.

4 Reconstructing from the scattering angle of the recoil proton

The scattering angle θ of the recoil proton is measured in the HARP TPC with a resolution of 9 mrad [2]. To this, the error of the direction of the incoming beam arising from multiple scattering must be added, however this contribution is at the level of 1 mrad and therefore almost negligible. Yet a further important contribution arises from multiple scattering in materials before entering the TPC volume, which is for the proton recoil momentum at stake about 11 mrad. We adopt for our calculations 15 mrad as effective overall θ resolution of the recoil proton, and assume again a 1% uncertainty of the absolute momentum of the incoming beam particle.

The observable that can then be predicted is the p_{T} of the recoil proton. This permits also a direct and significant check of the TPC's p_{T} scale and resolution.

Table 3 lists the salient results for the four incident beams. The results for the 3 GeV/c proton and pion beams are also shown in a set of three figures each, see Figs. 5 and 6. We note that the resulting uncertainties shown in these figures and in Table 3 stem (i) from the 1% beam momentum uncertainty, and (ii) from the 15 mrad uncertainty of the measurement in the TPC of the scattering angle of the recoil proton.

The uncertainties given so far refer to the p_{T} at the vertex, i.e., the fluctuation of the energy loss in materials and the fluctuation of the scattering angle from multiple scattering in these materials are not yet taken into account.

These fluctuations increase further the resolution of p_{T} predicted from the elastic scattering kinematics. From our GEANT simulation, we estimate the overall resolution as $\sigma(1/p_{\mathrm{T}}) \sim 0.12$ (GeV/c)$^{-1}$.

Table 3: Results from reconstructing from the scattering angle of the recoil proton

	3 GeV/c p	3 GeV/c π	5 GeV/c p	5 GeV/c π
θ [deg]	69.0	69.8	68.5	68.8
$\sigma(\theta)$ [deg]	0.861	0.861	0.862	0.862
p_{TR} [GeV/c]	0.495	0.497	0.586	0.586
$\sigma(p_{\mathrm{TR}})$ [GeV/c]	0.0195	0.0206	0.0235	0.0240
$\sigma(1/p_{\mathrm{TR}})$ [(GeV/c)$^{-1}$]	0.080	0.084	0.068	0.070

Another, albeit not independent, check is provided by calculating the missing mass of the forward-scattered beam particle and comparing it with the proton and pion mass, respectively. This calculation uses the 4-momentum of the recoil proton as measured in the TPC, and therefore permits to check the correspondence of the measured momentum of the recoil proton with its prediction from its scattering angle.

In our simulation, the forward-scattering angle is fixed to the nominal values of 10 degrees for the 3 GeV/c beam, and 7 degrees for the 5 GeV/c beam, respectively; no use is made of any measurement in the forward spectrometer.

Table 4 lists the results for the predictions of the missing mass squared for the four incident beams, for a bias in $1/p_{\mathrm{T}}$ of +0.3, for a bias of -0.30, and for a resolution of ±0.30 without bias. The results for the 3 GeV/c proton and pion beams are also shown in a set of three figures each, see Figs. 7 and 8. We note that the uncertainties in the missing mass squared shown in these figures and in Table 4 include (i) the 1% beam momentum uncertainty, and (ii) the 15 mrad uncertainty of the measurement in the TPC of the scattering angle of the recoil proton. The fluctuations of the recoil proton's energy loss in materials are not taken into account. The inclusion of results with a ±0.30 resolution in $1/p_{\mathrm{T}}$ is intended to illustrate what will be seen in the experimental measurement of the missing mass squared.

Table 4: Results for the missing mass squared from the scattering angle and the p_{T} of the recoil proton

	3 GeV/c p	3 GeV/c π	5 GeV/c p	5 GeV/c π
m_{X}^2, nominal	0.8804	0.0195	0.8804	0.0195
m_{X}^2, bias +0.3	0.9947	0.1296	1.1320	0.2676
m_{X}^2, bias −0.3	0.6832	-0.1708	0.4093	-0.4444
m_{X}^2, resolution ±0.3	0.8360	-0.0233	0.7688	-0.0776

Figures 9 and 10 show the missing mass squared for the 3 and 5 GeV/c proton and pion beams, respectively, for the following assumptions on $1/p_{\mathrm{T}}$ that we consider realistic for the analysis by 'official' HARP: (i) resolution of ±0.55, (ii) resolution of ±0.55 plus bias of +0.30, and (iii) resolution of ±0.55 plus bias of +0.20. The bias of +0.20 stems from the assumption that the momentum measured in the TPC with the use of the beam point may not have been corrected for the energy loss of the recoil protons in materials (this contributes a bias of -0.10). Another interpretation of the +0.20 bias is that the energy loss correction was made but dynamic distortions in the relevant data set are less strong than, e.g., in the data set from the interactions of +8.9 GeV/c protons in a 5% λ_{abs} target.

Figure 11 shows the expectation on the missing mass squared specifically for 3 GeV/c protons. Therein, a resolution of $1/p_{\mathrm{T}}$ of 0.55 and a bias of +0.20 for proton recoil has been assumed. The expectations are shown for three scattering angles of the recoil proton: 65°, 69°, and 73°. The contributions from data at these three angles are weighted with $\exp(-7.9|t|)$. One concludes that the sum of the contributions gives a missing mass squared distribution which has no reason to be Gaussian, and where the relation of the parameters of a Gaussian fit with the real missing mass squared is not obvious.

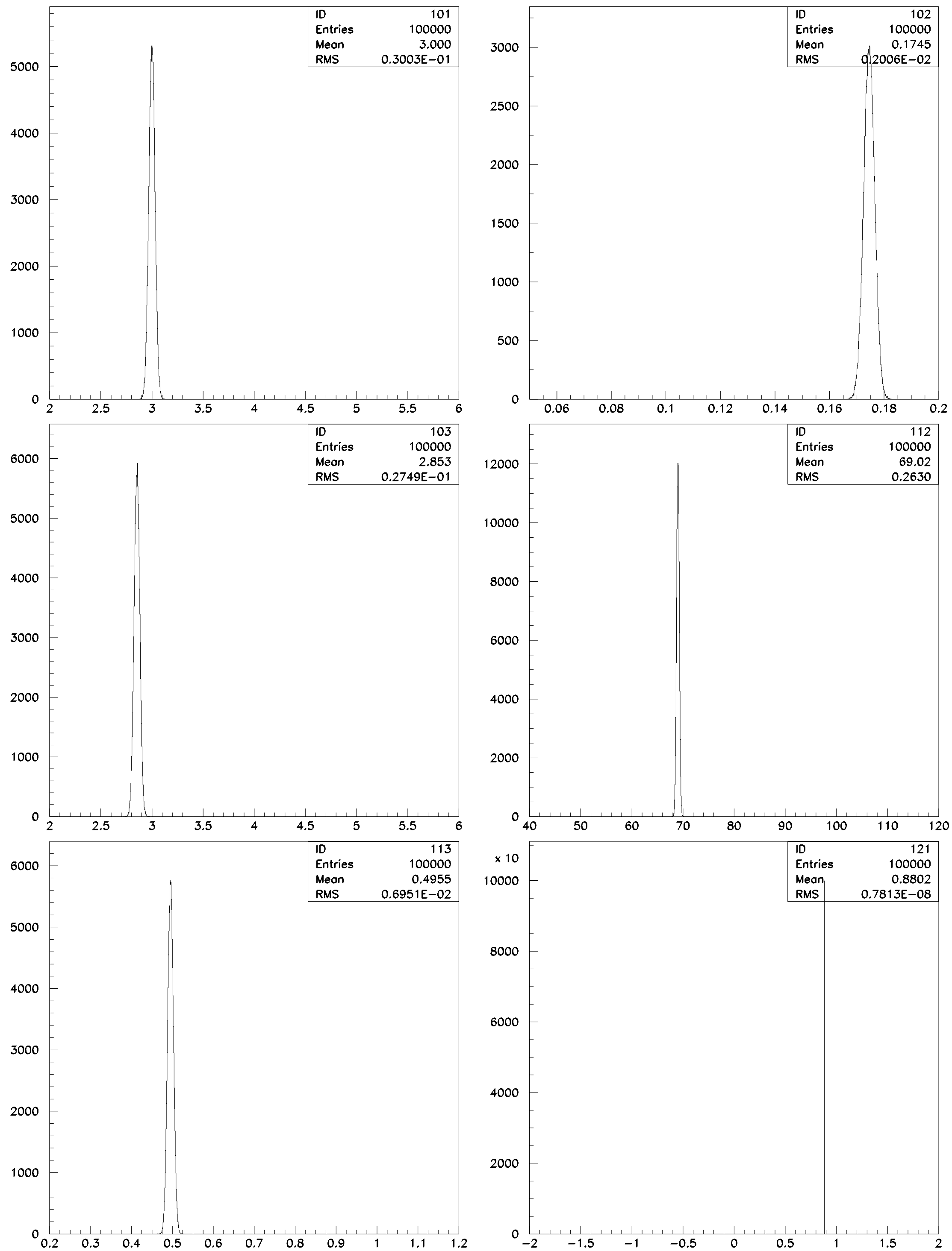

Figure 1: Elastic scattering of 3 GeV/c protons on protons at rest; upper left: beam momentum [GeV/c]; upper right: forward scattering angle [rad]; middle left: momentum of forward-scattered beam particle [GeV/c]; middle right: scattering angle of recoil proton [deg]; lower left: p_{T} of recoil proton [GeV/c]; lower right: missing mass squared [GeV/c^2].

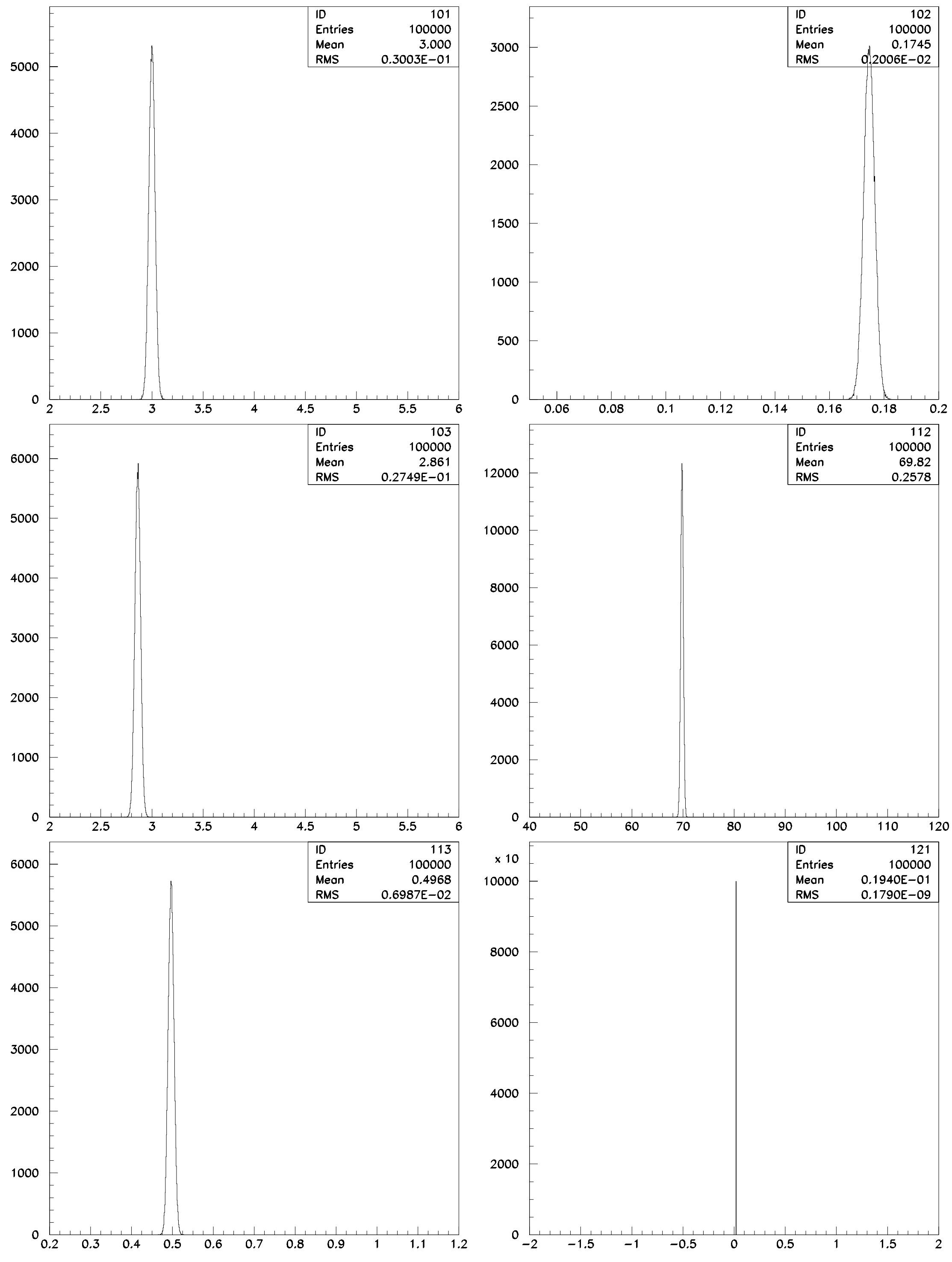

Figure 2: Elastic scattering of 3 GeV/c pions on protons at rest; upper left: beam momentum [GeV/c]; upper right: forward scattering angle [rad]; middle left: momentum of forward-scattered beam particle [GeV/c]; middle right: scattering angle of recoil proton [deg]; lower left: p_{T} of recoil proton [GeV/c]; lower right: missing mass squared [GeV/c^2].

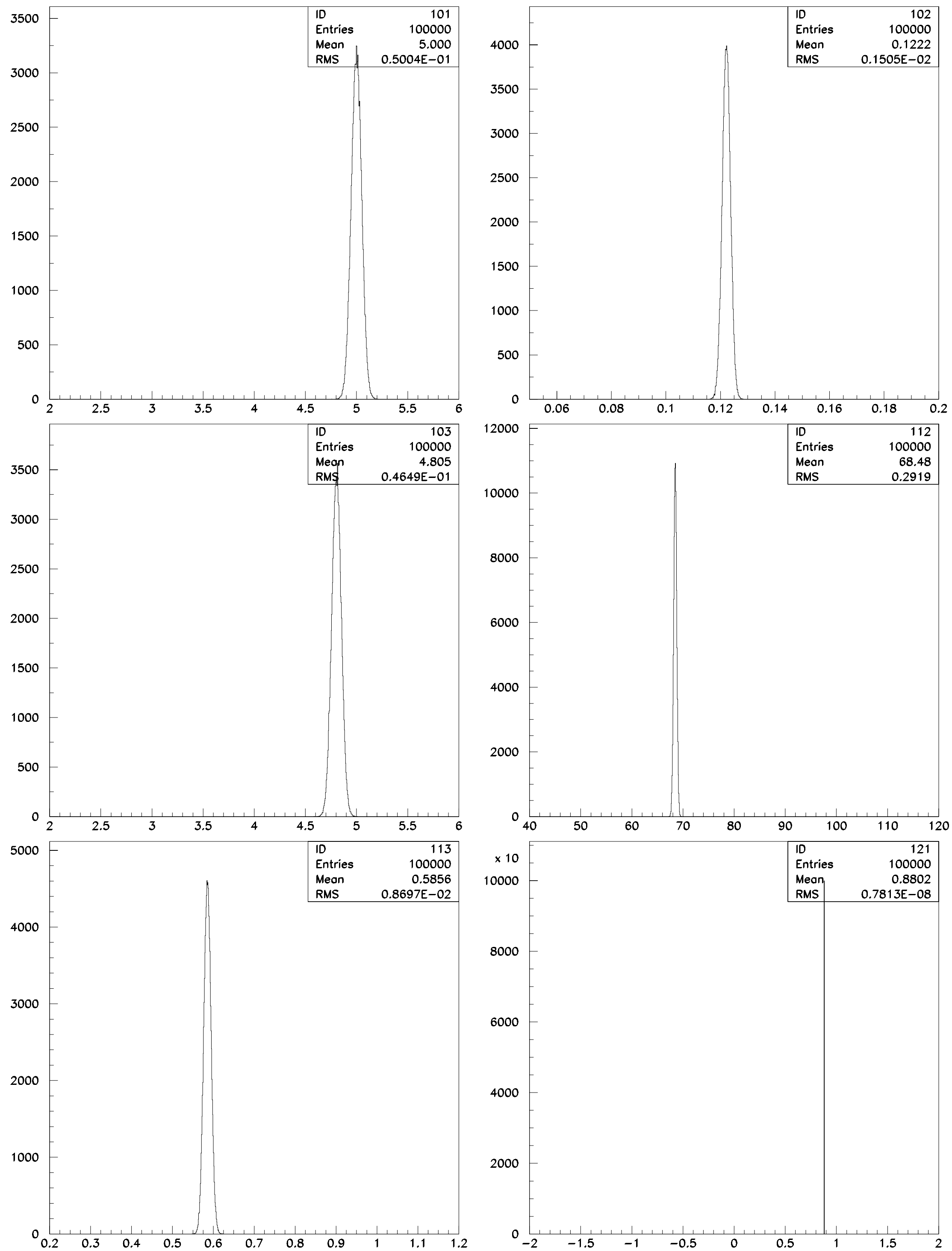

Figure 3: Elastic scattering of 5 GeV/c protons on protons at rest; upper left: beam momentum [GeV/c]; upper right: forward scattering angle [rad]; middle left: momentum of forward-scattered beam particle [GeV/c]; middle right: scattering angle of recoil proton [deg]; lower left: p_{T} of recoil proton [GeV/c]; lower right: missing mass squared [GeV/c^2].

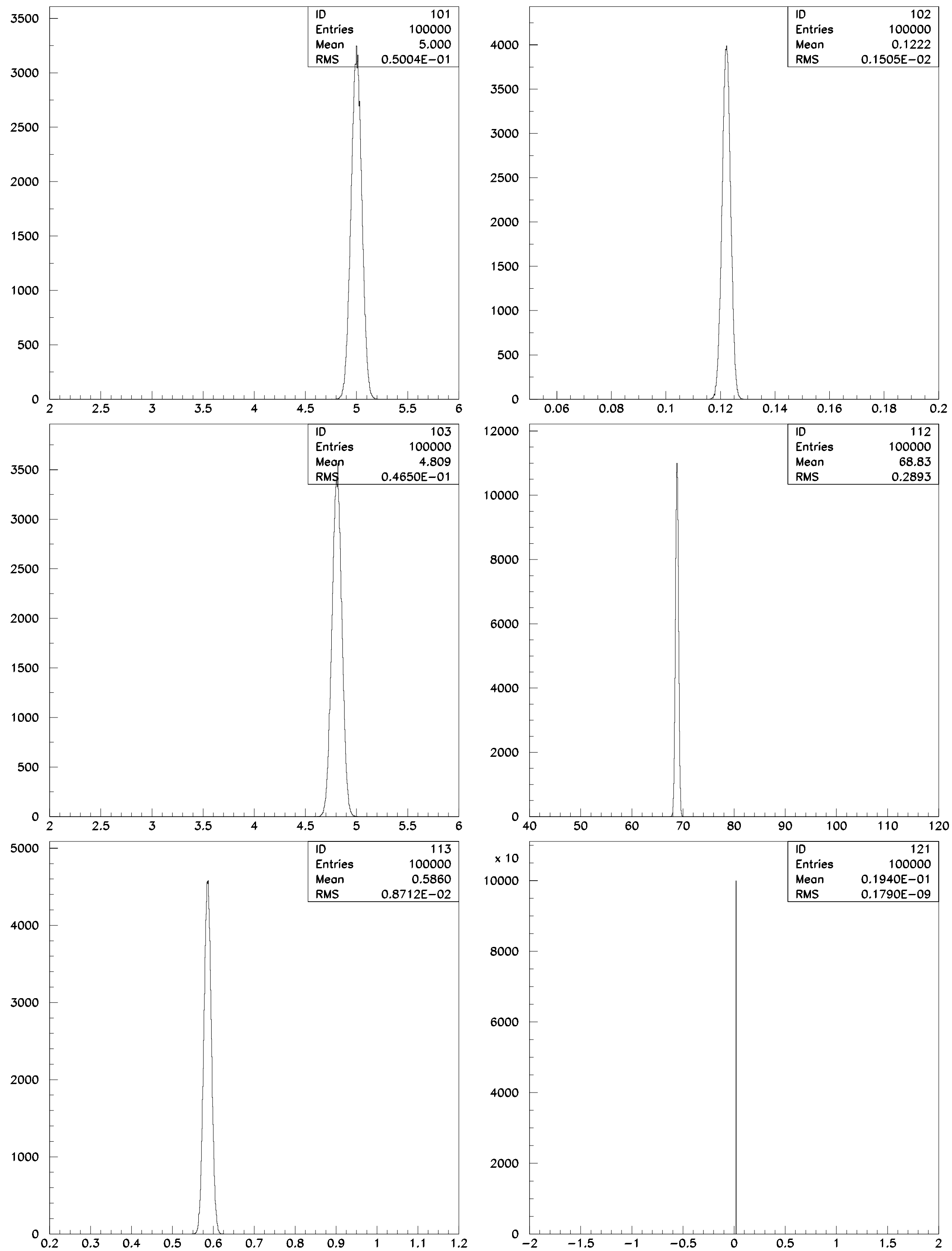

Figure 4: Elastic scattering of 5 GeV/c pions on protons at rest; upper left: beam momentum [GeV/c]; upper right: forward scattering angle [rad]; middle left: momentum of forward-scattered beam particle [GeV/c]; middle right: scattering angle of recoil proton [deg]; lower left: p_{T} of recoil proton [GeV/c]; lower right: missing mass squared [GeV/c^2].

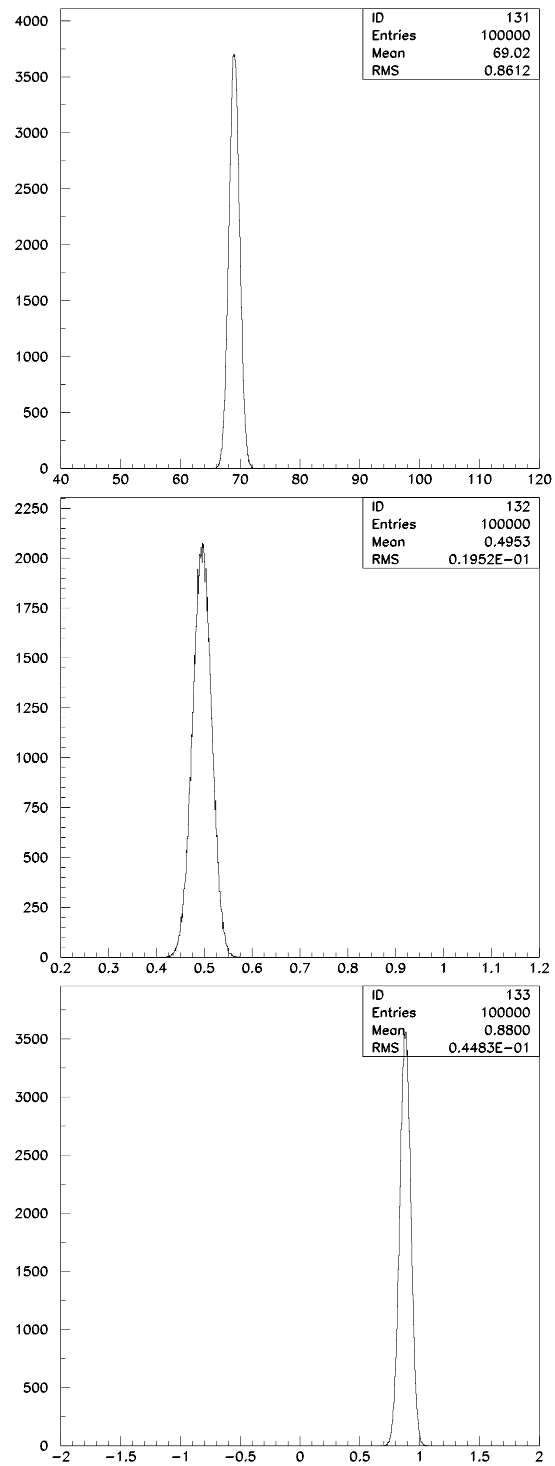

Figure 5: Elastic scattering of 3 GeV/c protons on protons at rest; upper panel: scattering angle of recoil proton [deg]; middle panel: p_{T} of recoil proton [GeV/c]; lower panel: missing mass squared with nominal proton recoil momentum but with beam uncertainty and recoil angle uncertainty included.

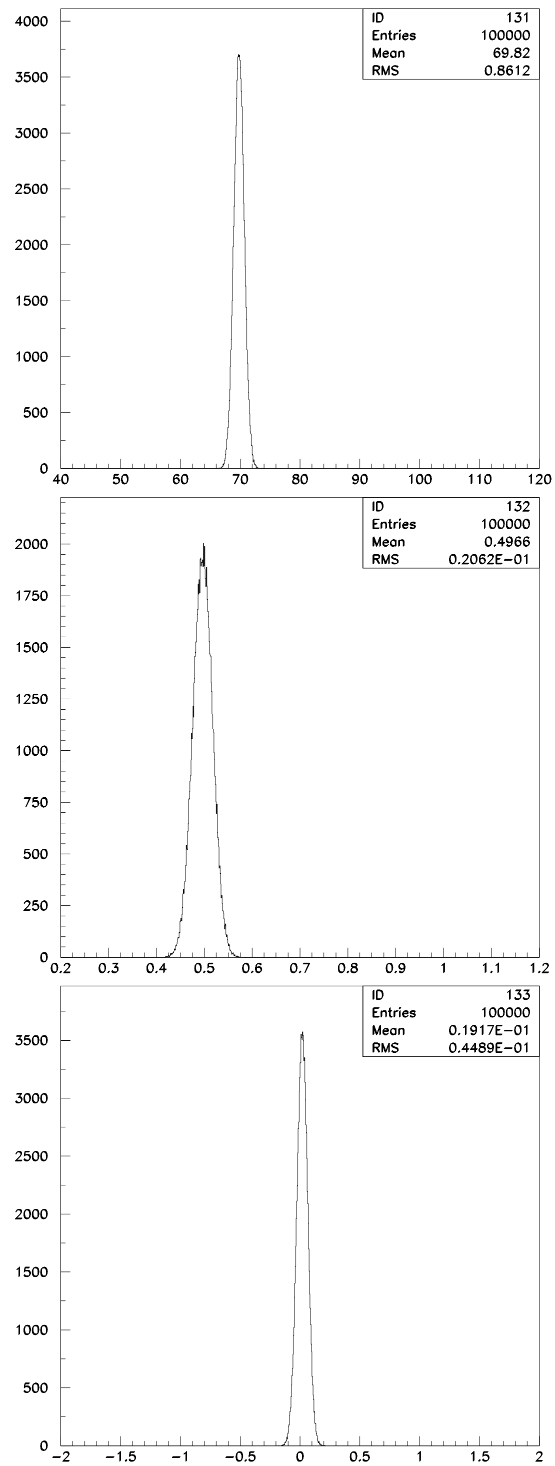

Figure 6: Elastic scattering of 3 GeV/c pions on protons at rest; upper panel: scattering angle of recoil proton [deg]; middle panel: p_{T} of recoil proton [GeV/c]; lower panel: missing mass squared with nominal proton recoil momentum but with beam uncertainty and recoil angle uncertainty included.

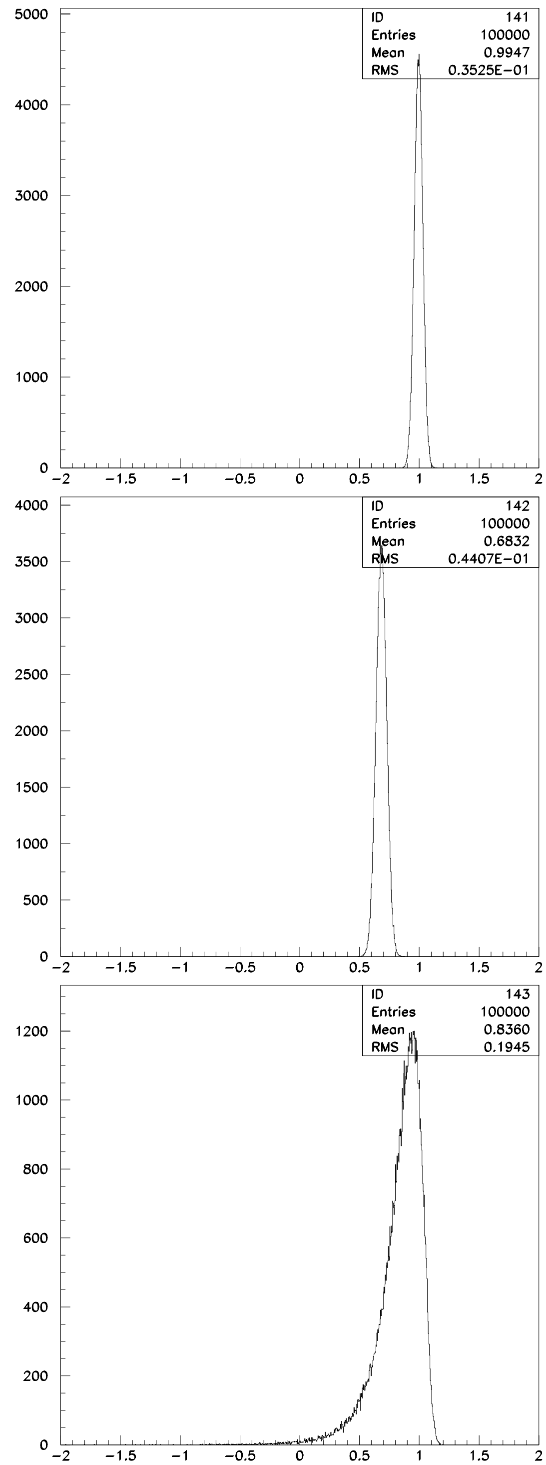

Figure 7: Effect of bias and resolution in $1/p_{\mathrm{T}}$ on the missing mass squared [GeV/c^2] in the elastic scattering of 3 GeV/c protons on protons at rest; upper panel: bias of +0.30; middle panel: bias of -0.30; lower panel: resolution of ±0.30.

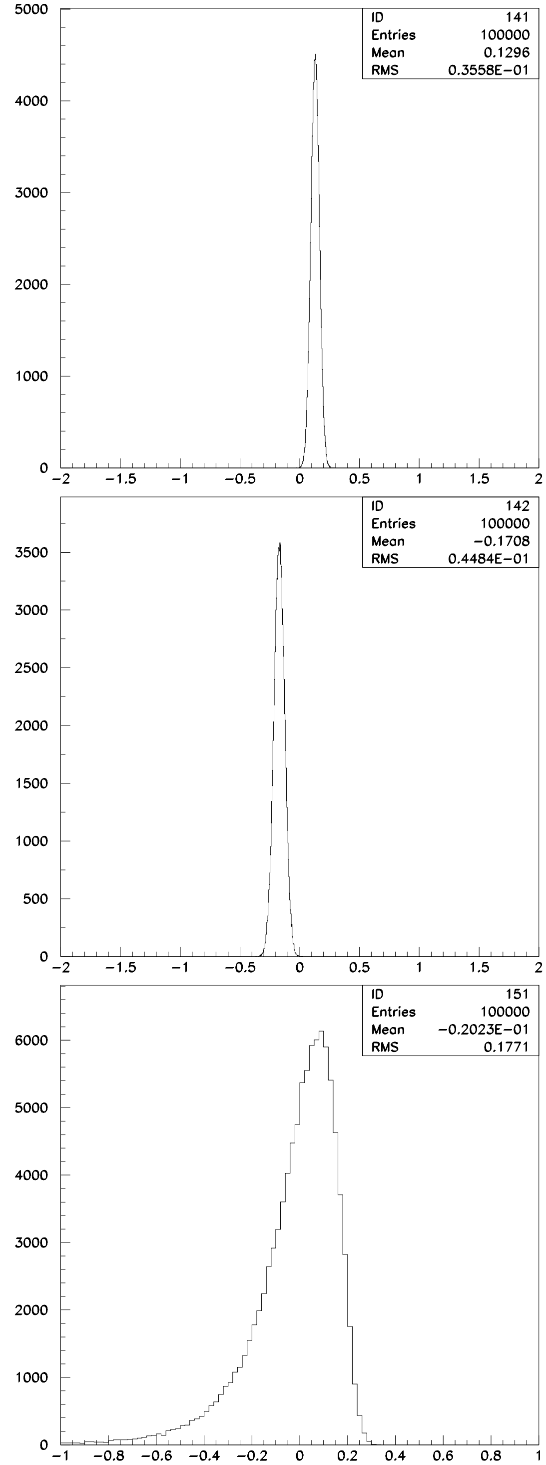

Figure 8: Effect of bias and resolution in $1/p_{\mathrm{T}}$ on the missing mass squared [GeV/c^2] in the elastic scattering of 3 GeV/c pions on protons at rest; upper panel: bias of +0.30; middle panel: bias of -0.30; lower panel: resolution of ±0.30.

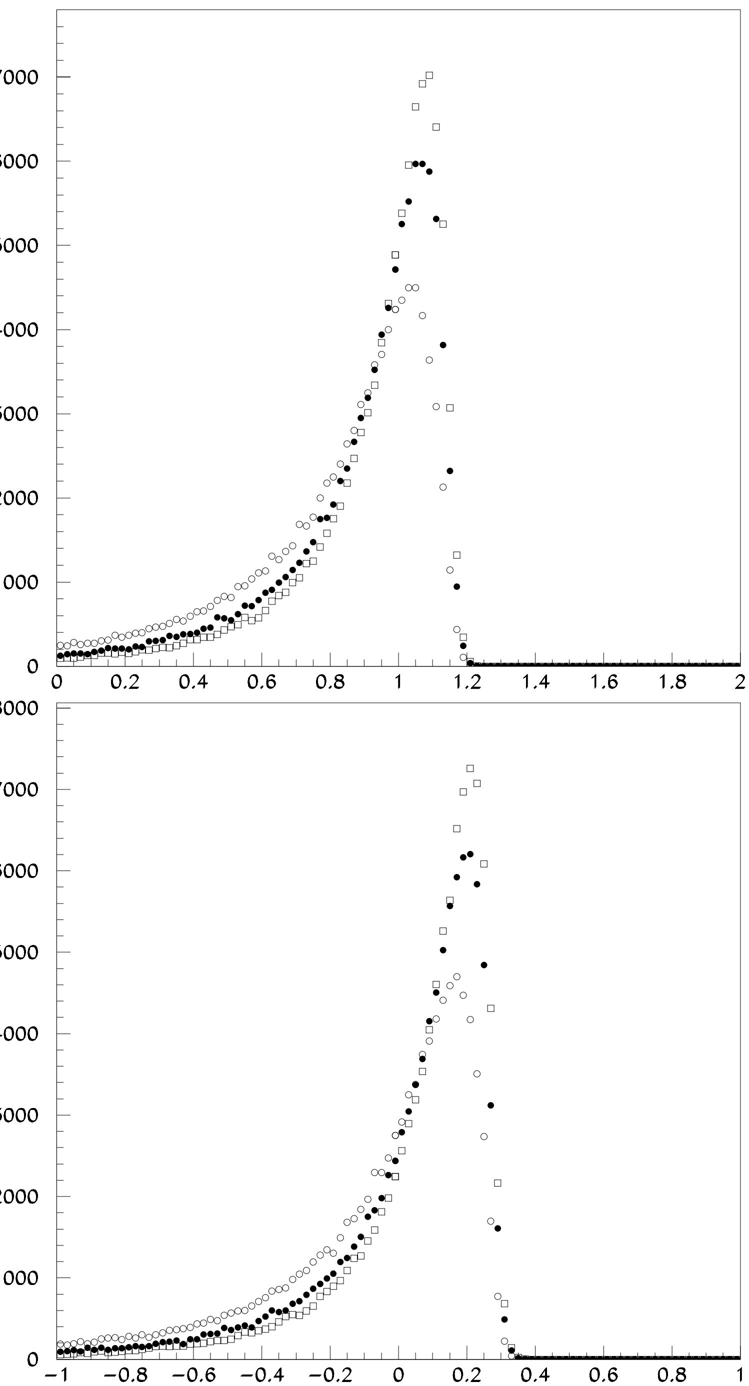

Figure 9: Effect of bias and resolution in $1/p_{\mathrm{T}}$ on the missing mass squared [GeV/c^2] in the elastic scattering of 3 GeV/c protons (upper panel) and pions (lower panel) on protons at rest; open circles: resolution of ±0.55; open squares: resolution of ±0.55 plus bias of $+0.30$; black circles: resolution of ±0.55 plus bias of $+0.20$.

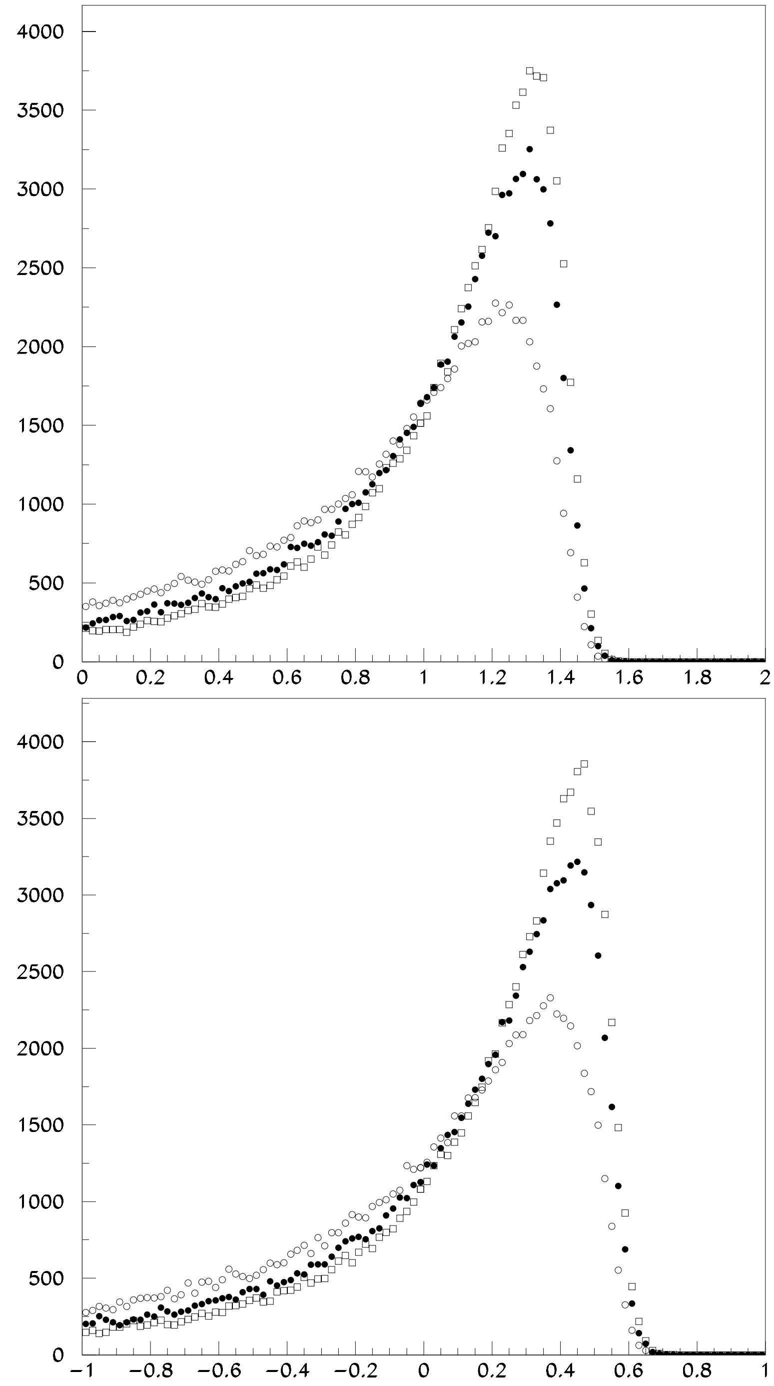

Figure 10: Effect of bias and resolution in $1/p_{\mathrm{T}}$ on the missing mass squared [GeV/c^2] in the elastic scattering of 5 GeV/c protons (upper panel) and pions (lower panel) on protons at rest; open circles: resolution of ±0.55; open squares: resolution of ±0.55 plus bias of +0.30; black circles: resolution of ±0.55 plus bias of +0.20.

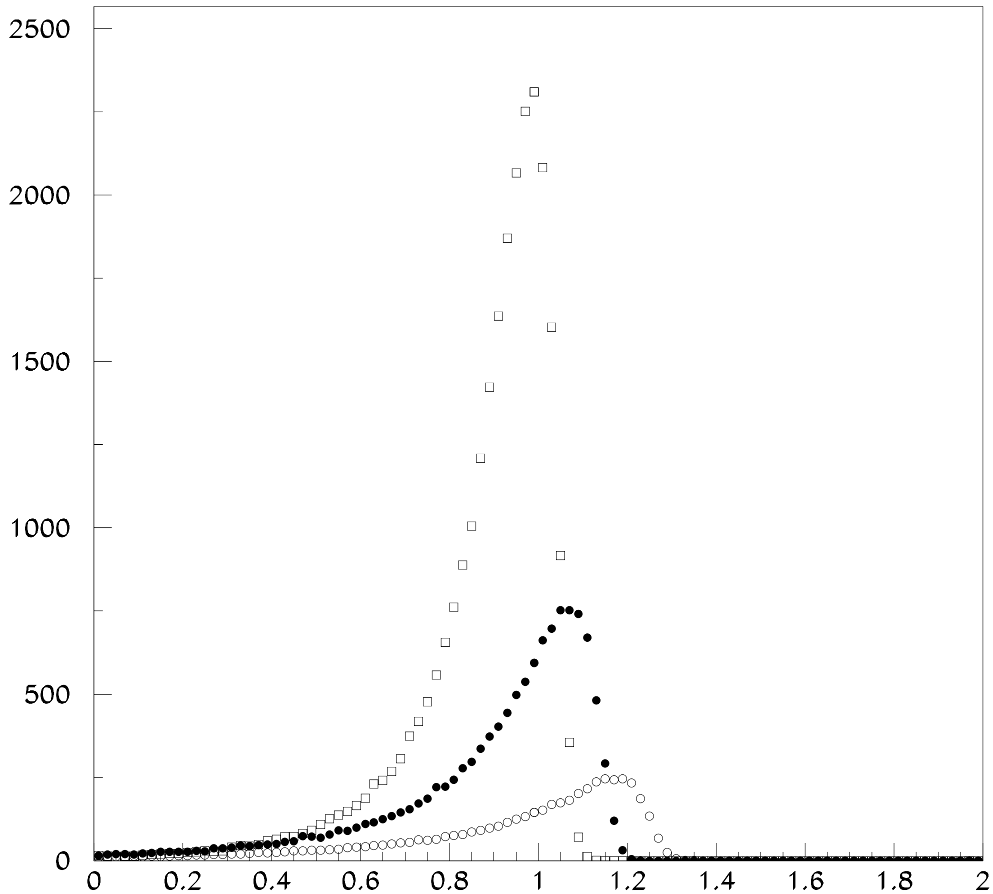

Figure 11: Simulation of the missing mass squared in the elastic scattering of 3 GeV/c protons on protons at rest; simulation with a resolution of $1/p_{\mathrm{T}}$ of 0.55 and a bias of +0.20 for proton recoil angle of 65° (open circles), 69° (full circles) and 73° (open squares).

References

[1] M.G. Catanesi *et al.*, Measurement of the production cross-section of positive pions in p–Al collisions at 12.9 GeV/c, Nucl. Phys. **B732** (2006) 1.

[2] A. Bolshakova *et al.*, The HARP Time Projection Chamber: Characteristics and Physics Performance, Preprint CERN-PH-EP-2007-030, submitted to Nucl. Instr. Meth. Phys. Res. A, http://cern.ch/dydak/CERN-PH-EP-2007-030.pdf