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Axionic $D3$ - $D7$ inflation

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ABSTRACT: We study the motion of a $D3$ brane moving within a Type IIB string vacuum compactified to 4D on $K3 \times T_2/Z_2$ in the presence of $D7$ and $O7$ planes. We work within the effective 4D supergravity describing how the mobile $D3$ interacts with the lightest bulk moduli of the compactification, including the effects of modulus-stabilizing fluxes. We seek inflationary solutions to the resulting equations, performing our search numerically in order to avoid resorting to approximate parameterizations of the low-energy potential. We consider uplifting from D -terms and from the supersymmetry-breaking effects of anti- $D3$ branes. We find examples of slow-roll inflation (with anti-brane uplifting) with the mobile $D3$ moving along the toroidal directions, falling towards a $D7$ - $O7$ stack starting from the antipodal point. The inflaton turns out to be a linear combination of the brane position and the axionic partner of the $K3$ volume modulus, and the similarity of the potential along the inflaton direction with that of racetrack inflation leads to the prediction $n_s \leq 0.95$ for the spectral index. The slow roll is insensitive to most of the features of the effective superpotential, and requires a one-in- 10^4 tuning to ensure that the torus is close to square in shape. We also consider D -term inflation with the $D3$ close to the attractive $D7$, but find that for a broad (but not exhaustive) class of parameters the conditions for slow roll tend to destabilize the bulk moduli. In contrast to the axionic case, the best inflationary example of this kind requires the delicate adjustment of potential parameters (much more than the part-per-mille level), and gives inflation only at an inflection point of the potential (and so suffers from additional fine-tuning of initial conditions to avoid an overshoot problem).

KEYWORDS: Flux compactifications, Cosmology of Theories beyond the SM, Supergravity Models

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1 Introduction

The advent of tools for fixing moduli in string theory has opened up the possibility for surveying where slow-roll inflation occurs among string vacua, with the result (so far) that it appears to be relatively rare, but not impossible. This survey has revealed a variety of potential inflationary mechanisms, with the inflaton residing either among open or closed string modes [1].

Among the most interesting of these mechanisms is that of $D3$ - $D7$ inflation [2], for which the inflaton is the separation between mobile $D3$ branes as they approach static stacks of $D7$ branes. Besides sharing many of the attractive features of brane-antibrane models [3–6], this scenario potentially has the additional advantage that the final $D3$ - $D7$ collision may be better understood, with the possibility of the $D3$ dissolving into the $D7$ to leave a supersymmetric state. Furthermore, stacks of coincident $D7$ and $O7$ planes can source flat transverse geometries and constant dilaton configurations, among which are the well-studied compactifications on $K3 \times T_2/Z_2$. One might expect the prospects for finding slow roll for $D3$ motion for such geometries to be better than for a generic Calabi-Yau.

Additional progress became possible with the application of Type IIB modulus-stabilization techniques [7, 9] to $K3 \times T_2/Z_2$ geometries [10–12]. This opened up the possibility of understanding the low energy dynamics within the framework of the effective 4D supergravity, with all the additional control over the calculation that this brings. Until recently one ingredient remained missing for performing a more systematic 4D study of $D3$ motion in these systems, and this was the 4D supergravity formulation of the forces acting on a mobile $D3$ brane once supersymmetry becomes broken (such as by the addition of magnetic fluxes in the 7-brane world volume). This missing step was removed with the analysis [13–15] of how the $D3$ back-reacts on the $D7$ geometry, and thereby introduces a dependence on the $D3$ position into the energetics of $D7$ physics (like gaugino condensation or magnetic fluxes).

A first study of $D3$ motion in $K3 \times T_2/Z_2$ was recently performed in ref. [16], who also made a preliminary search for inflationary solutions using a semi-phenomenological potential. This potential was meant to parameterize the important features of the low-energy supergravity in the limit when the $D3$ and $D7$ are in close proximity. In particular, it includes a combination of a logarithmic ‘Coleman-Weinberg’ (CW) potential describing the attraction of a $D3$ towards a $D7$ on which supersymmetry has been broken by fluxes, the D -term energy generated by this flux [17], plus the nonperturbative superpotential generated by gaugino condensation on a $D7$ stack located at a different fixed point. Their search identified a putative slow-roll inflationary regime when the mobile $D3$ approaches very closely one of the $D7/O7$ stacks.

In this note we extend their analysis in several ways.

- First we follow the evolution of more of the twenty-odd bulk moduli of $K3 \times T_2/Z_2$. After describing the low-energy supergravity in some generality, we follow the dynamics of two of these complex moduli in addition to that of the $D3$ -brane position. We do so because it is only when at least two of the bulk moduli are kept that the full no-scale form of the leading low-energy potential is manifest.
- Second, we search numerically for inflationary solutions, allowing the use of the actual F - and D -term potentials of the low-energy supergravity, rather than an approximate semi-phenomenological potential. Since we need not rely on expansions in the $D3$ - $D7$ distance, we can both test the domain of validity of the approximate forms used by earlier workers, and can search for inflation when the $D3$ is far from the $D7$.
- Third, we consider two types of ‘uplifting’ physics, required to assure the potential is minimized at a Minkowski vacuum after inflation ends. Following [16] we examine D -term uplifting as generated by $D7$ fluxes. But due to present difficulties in obtaining these from explicit string vacua on $K3 \times T_2/Z_2$ we also explore uplifting due to anti- $D3$ branes *à la* KKLT [9].

Our search reveals several examples of slow-roll inflation, in all cases requiring some degree of tuning of the parameters of the potential. We focus on inflationary trajectories with the $D3$ moving along the torus. This is because the Kähler potential has a shift symmetry in the torus coordinate which may protect that direction from getting large

corrections from the non-perturbative F -term potential. Our best example occurs when the $D3$ falls between two stacks of $D7$'s, due to forces ultimately driven by nonperturbative physics (like gaugino condensation or Euclidean D -branes) occurring on yet a third such stack. Inflation occurs when the $D3$ starts at the antipodal point, within the torus, of the $D7$'s on which the nonperturbative physics occurs. In this case the tuning required is quite mild, with the inflationary roll largely insensitive to other parameters once the torus is adjusted to be close to square. The inflaton direction turns out to be a combination of the $D3$ position and the axionic partner of the $K3$ volume modulus, leading to a situation similar to the racetrack inflation model [18]. Since the starting position is at a local maximum of the inflaton direction, eternal topological inflation can remove the need for explaining the initial conditions. Uplifting is provided in this example by the presence of an anti- $D3$ brane.

We also search for inflation in the regime of ref. [16], where the $D3$ is close to a stack of $D7$'s on which supersymmetry-breaking fluxes provide the inflationary energy density. In this case we find inflation much more difficult to achieve, largely because we are unable to realize the parameter choices required for their slow roll within our 4D supergravity. We *are* able to obtain slow-roll inflation in this regime, however, although only by using a delicately tuned (to within a part per million) choice of potential parameters. What is troublesome, however, is that the inflationary regime that results arises near an inflection point of the potential, rather than a local maximum. This has the disadvantage of requiring a several percent tuning of the initial conditions to avoid having an overshoot problem.

Our discussion is organized as follows. In section 2 we review the underlying theoretical ingredients leading to the low-energy effective action for the inflaton. In section 3 we develop the Lagrangian explicitly, in terms of the F-term and D-term contributions and possible uplifting by anti- $D3$ branes in both warped and unwarped backgrounds, but restricted to the fields whose dynamics we wish to follow. Section 4 describes the two examples of inflation described above: the racetrack-like model starting from the antipodal point of the attracting $D7$ brane (using $\overline{D3}$ uplifting), and the inflection point model (with D -term uplifting) where the $D3$ is near the $D7$. We present our conclusions in section 5. The appendix contains results concerning the no-scale property of the Kähler potential, our conventions for Jacobi theta functions, and scaling properties of the potential under certain re-scalings of the Lagrangian parameters.

2 Low energy dynamics on $K3 \times T_2/Z_2$

In this section we develop the general properties of the 4D supergravity describing the low-energy behaviour of $K3 \times T_2/Z_2$, before specializing in the next section to the moduli playing a direct role in the inflationary scenario.

2.1 The field content

Our starting point is a Type IIB string vacuum compactified on $K3 \times T_2/Z_2$, in the presence of moduli-stabilizing 3-form fluxes [7, 9], such as studied by [10, 11]. The orientifold Z_2 acts on the torus by reflecting its (complex) coordinates, $z \rightarrow -z$, leading to $O7$ -planes

located at four fixed points. Taking the torus to be defined by the parallelogram $z \simeq z + 1$ and $z \simeq z + \tau$, with τ the complex modulus satisfying $\text{Im } \tau > 0$, these fixed points are situated at $z = 0, \frac{1}{2}, \frac{1}{2}\tau$ and $\frac{1}{2}(1 + \tau)$. The $D7$ tadpole conditions are satisfied when each $O7$ plane is accompanied by 4 $D7$'s, all wrapping the $K3$. If the 4 $D7$'s are coincident with the corresponding $O7$, they do not source the dilaton field, which can therefore remain constant along the toroidal directions. The $D3$ tadpole condition requires the number of $D3$ branes plus a flux integral to sum to 24 [10].

$K3$ is a Ricci flat space having two complex dimensions which naturally arises in supersymmetric compactifications of string theory to 4D [19], being in many ways the lower-dimensional analog of the three (complex) dimensional Calabi Yau spaces. It has a very rich topology [20], with Hodge numbers $h_{10} = h_{01} = 0, h_{00} = h_{20} = h_{02} = h_{22} = 1$ and $h_{11} = 20$, leading to an Euler number $\chi = 24$. These are the same as for the orbifold T_4/Z_2 , say, whose 16 fixed points can be regarded as the degenerate limit of 16 of the 22 nontrivial 2-cycles on $K3$.

For Type IIB string compactifications this topology leads to low-energy moduli, $T^\alpha = \xi^\alpha + i\beta^\alpha$. Some of these moduli are stabilized (at leading order in the α' and string loop expansions) once the 3-form fluxes are turned on, and these fluxes can preserve zero, one or two low-energy 4D supersymmetries [10]. The rest of the moduli can be stabilized in principle by nonperturbative effects [11]. The dynamics of this stabilization can be described by a low-energy 4D supergravity provided that the supersymmetry breaking scales are kept parametrically small compared with the Kaluza-Klein (KK) scale, as we assume to be the case in what follows.

To this geometry we imagine adding one or more of the following optional features.

- For inflationary purposes, we imagine adjusting the fluxes to allow the presence of a mobile $D3$ brane situated at a point in the extra dimensions. We argue below that the physics that stabilizes the various Kähler moduli on $K3$ tends also to stabilize the motion of this brane in the $K3$ directions, although it can be relatively free to move along the toroidal directions, with complex coordinate z .
- It is often useful to entertain the presence of an anti- $D3$ brane, in order to uplift the minimum of the potential to zero. Ultimately, the necessity for doing so reflects our poor understanding of the cosmological constant problem, and we regard such an anti-brane to represent a parametrization of whatever mechanism properly solves this problem in the string vacuum of interest. When doing so it is often useful to sequester the antibrane into a warped throat on $K3 \times T_2/Z_2$, such as was studied in ref. [8]. This has several advantages. Besides helping to localize the $\overline{D3}$, which reduces its energy by sitting in the throat, it also reduces its impact on the dynamics of the mobile $D3$ brane, by suppressing their direct ‘Coulomb’ attraction.
- It is also possible to add background magnetic 2-form fluxes, \mathcal{F} , for gauge fields residing on the $D7$ branes, in order to uplift the potential at its minimum [17]. If such fluxes are present they typically gauge some of the axion symmetries under

which the imaginary parts of the moduli shift, $\beta^\alpha \rightarrow \beta^\alpha + \eta^\alpha$. In particular,¹ if \mathcal{F} is turned on in the world volume of a brane wrapping a 4-cycle Σ^d , and if its expansion in terms of basis harmonic 2-forms is $\mathcal{F} = f_\alpha \omega^\alpha$, then $\eta^\alpha = k^{\alpha\beta d} f_\beta$, where $k^{\alpha\beta\gamma}$ denotes the intersection number for a triplet of 2-cycles [21].

The significance of the gauged shift symmetry is that it implies that the positive magnetic energy (which is proportional to the integral of $\mathcal{F}_{mn}\mathcal{F}^{mn}$ over the $D7$ volume) is captured by a supersymmetry-breaking D -term in the low-energy 4D supergravity. When nonzero this energy breaks supersymmetry in the 4D theory, just as does the magnetic flux in the underlying brane picture. The situation becomes more complicated should other multiplets, Q^x , also exist that are charged under this symmetry. Such scalars complicate the picture because they must also appear in the corresponding D term potential, and typically prefer to adjust their expectation values to try to cancel out the magnetic energy and thereby restore the supersymmetry broken by the flux. Furthermore, such scalars are often required to exist, either by anomaly-cancellation arguments or by gauge invariance if the axion fields should appear in the low-energy superpotential [17, 21, 22].

2.2 The low-energy supergravity

The interactions of these complex moduli with one another and with gravity are described at low energies by an effective 4D theory, that is close to an $\mathcal{N} = 1$ supergravity provided that the supersymmetry-breaking effects of the compactification to 4D are sufficiently weak. As such it is characterized by specifying its Kähler potential, K , its holomorphic superpotential, W , and gauge kinetic function, f_{ab} .

Kähler potential. The Kähler potential for the leading order 4D supergravity has the general Type IIB form,

$$K = -2 \ln \mathcal{V}, \tag{2.1}$$

where \mathcal{V} is the Calabi-Yau volume in units of the string length, $l_s = 2\pi\sqrt{\alpha'}$. When expressed in terms of the decompositions, t_α , of the Kähler form in terms of a basis of 2-cycles, $J = t_\alpha \omega^\alpha$, the volume becomes $\mathcal{V} = \frac{1}{6} k^{\alpha\beta\gamma} t_\alpha t_\beta t_\gamma$, where $k^{\alpha\beta\gamma}$ denotes the appropriate intersection number for the basis 2-cycles.

For use in the supergravity action the above expression for \mathcal{V} must be expressed in terms of the complex coordinates, $T^\alpha = \xi^\alpha + i\beta^\alpha$, and the complex position, z , of the $D3$ brane, corresponding to the chiral scalars of the effective theory. The expression for \mathcal{V} in terms of T^α and z can be obtained explicitly in the case of $K3 \times T_2/Z_2$. Because this is a product geometry its volume factorizes,

$$\mathcal{V} = \frac{1}{2} k^{ij} t_s t_i t_j, \tag{2.2}$$

and is linear in the volume, t_s , of the torus. Here $\{t_\alpha\} = \{t_s, t_i\}$, and $k^{sij} = k^{ij}$ is a known matrix that describes the intersection numbers of 2-cycles within $K3$ [20].² In principle,

¹We thank Michael Haack and Marco Zagermann for helpful conversations on this point.

²We adopt the convention that Greek indices α, β, \dots run over moduli of $K3 \times T_2/Z_2$; mid-alphabet Latin indices, i, j, \dots , label only the moduli of $K3$ (and not t_s or ξ^s), while early-alphabet Latin indices, a, b, \dots , collectively denote ξ^s and z . Capitalized indices, A, B, \dots generically denote all moduli together.

the sum on i, j is over all of the independent 2-cycles on $K3$ and so runs from 1 to 22. However we can imagine some of the corresponding moduli to have been stabilized (by fluxes or nonperturbative effects) at energies that are hierarchically large compared with those of later interest for inflationary dynamics, and in this case i, j range only over the number of remaining moduli that are lighter than these.

When restricted to a single $D3$ moving only in the toroidal directions, the relation between the ξ^α and the t_α becomes [23]

$$\xi^i = \frac{\partial \mathcal{V}}{\partial t_i} = k^{ij} t_s t_j \quad \text{and} \quad \xi^s = \frac{\partial \mathcal{V}}{\partial t_s} + \omega(z, \bar{z}) = \frac{1}{2} k^{ij} t_i t_j + \omega(z, \bar{z}), \quad (2.3)$$

where $\omega(z, \bar{z})$ is the Kähler form on the 2-torus (whose explicit form is given below). Inverting these expressions for the torus volume, t_s , and the $K3$ 2-cycle volumes, t_i ,

$$t_s = \left(\frac{k_{ij} \xi^i \xi^j}{X} \right)^{1/2} \quad \text{and} \quad t_i = \frac{k_{ij} \xi^j}{t_s}, \quad (2.4)$$

with $k^{ij} t_i t_j = X = 2[\xi^s - \omega(z, \bar{z})]$ and $k^{ij} k_{jk} = \delta^i_k$. Using these in eqs. (2.1) and (2.2), and dropping additive constants in K , gives

$$K = -\ln X - \ln Y \quad (2.5)$$

where $Y = \frac{1}{2} k_{ij} \xi^i \xi^j$. In terms of the complex fields, $T^i = \xi^i + i\beta^i$ and $S = T^s = \xi^s + i\beta^s$, we have

$$X = S + \bar{S} - 2\omega(z, \bar{z}) \quad \text{and} \quad Y = \frac{1}{8} k_{ij} (T^i + \bar{T}^i)(T^j + \bar{T}^j). \quad (2.6)$$

The first Kähler derivatives then are $K_A = \partial_A K$ (with $A = z, S, T^i$):

$$K_S = -\frac{1}{X}, \quad K_z = \frac{2\omega_z}{X} \quad \text{and} \quad K_i = -\frac{k_{ij} \xi^j}{2Y}, \quad (2.7)$$

where $\omega_z = \partial_z \omega$. The Kähler metric becomes

$$K_{A\bar{B}} = \begin{pmatrix} K_{a\bar{b}} & 0 \\ 0 & K_{i\bar{j}} \end{pmatrix} \quad (2.8)$$

with $(a, b = S, z)$

$$K_{a\bar{b}} = \begin{pmatrix} 1/X^2 & -2\omega_{\bar{z}}/X^2 \\ -2\omega_z/X^2 & (2\omega_{z\bar{z}}X + 4\omega_z\omega_{\bar{z}})/X^2 \end{pmatrix} \quad (2.9)$$

and

$$K_{i\bar{j}} = \frac{-k_{ij}Y + k_{ik}k_{jn}\xi^k\xi^n}{4Y^2}. \quad (2.10)$$

These have inverses

$$K^{\bar{B}A} = \begin{pmatrix} K^{\bar{b}a} & 0 \\ 0 & K^{\bar{j}i} \end{pmatrix} \quad (2.11)$$

with

$$K^{\bar{b}a} = \begin{pmatrix} X(2\omega^{z\bar{z}}\omega_z\omega_{\bar{z}} + X) & X\omega^{z\bar{z}}\omega_{\bar{z}} \\ X\omega^{z\bar{z}}\omega_z & \frac{1}{2}X\omega^{z\bar{z}} \end{pmatrix} \quad (2.12)$$

and

$$K^{\bar{i}} = 4(-k^{ij}Y + \xi^i \xi^j). \quad (2.13)$$

In terms of the real and imaginary parts of the torus coordinate, $z = z_1 + iz_2$, and complex structure modulus, $\tau = \tau_1 + i\tau_2$, the Kähler form on the torus is

$$\omega = -\frac{ic(z - \bar{z})^2}{2(\tau - \bar{\tau})} = -\frac{c(z - \bar{z})^2}{4\tau_2} = \frac{cz_2^2}{\tau_2}, \quad (2.14)$$

where c is a constant to be determined below. Its derivatives become $\omega_z = -\omega_{\bar{z}} = -ic(z - \bar{z})/(\tau - \bar{\tau})$, $\omega_{z\bar{z}} = ic/(\tau - \bar{\tau})$ and so $\omega^{z\bar{z}} = (\tau - \bar{\tau})/(ic)$, $\omega^{z\bar{z}}\omega_{\bar{z}} = z - \bar{z}$ and $\omega^{z\bar{z}}\omega_z\omega_{\bar{z}} = -ic(z - \bar{z})^2/(\tau - \bar{\tau}) = 2\omega$.

Finally, there are two further properties of K worth special mention. First, as is shown in appendix A.1, this Kähler potential satisfies the no-scale identity

$$K^{\alpha\bar{\beta}}K_{\alpha}K_{\bar{\beta}} = 3. \quad (2.15)$$

Second, K displays the periodicity of the underlying torus, although in a subtle way [15]. In particular, since $X = 2[\xi^s - \omega] = S + \bar{S} - 2\omega$ is explicitly periodic under the shifts $z \rightarrow z + 1$ of the torus, K also shares this property. Similarly, eq. (2.14), shows that K and X are also invariant under $z \rightarrow z + \tau$ provided at the same time we shift

$$S \rightarrow S - ic(2z + \tau). \quad (2.16)$$

Holomorphic functions. Full specification of the low-energy 4D supergravity also requires its holomorphic superpotential, W , and gauge kinetic functions, f_{ab} .

- *Gauge kinetic function.* The gauge kinetic function, $f_{ab}(S, z)$, may be computed as a threshold effect when computing open-string loops [13], or as the classical back-reaction of the $D3$ on the $D7$ geometry in the dual closed string picture [14]. For the $D7$'s located at fixed point r in $K3 \times T_2/Z_2$ either approach gives $f_{ab,r} = f_r \delta_{ab}$, where (up to z - and S -independent quantities)

$$f_r = S - \frac{1}{a} \left\{ \ln \vartheta_1[\pi(z_r - z)|\tau] + \ln \vartheta_1[\pi(z_r + z)|\tau] \right\} + \mathfrak{f}_r. \quad (2.17)$$

where the four fixed points on T_2/Z_2 are located at $z_r = 0, \frac{1}{2}, \frac{1}{2}\tau$ and $\frac{1}{2}(1 + \tau)$, and \mathfrak{f}_r denotes a potential contribution that is independent of the fields S, T^i and z [21]. Typically $a = 2\pi$, and ϑ_1 denotes a Jacobi theta function, for which our conventions are specified in appendix A.2.

- *Superpotential.* The appearance of a superpotential is the hallmark of an underlying stabilization mechanism, and in the present instance we envision the stabilization to give

$$W = W_0 + w(S, z) + \sum_i B_i e^{-b_i T^i}. \quad (2.18)$$

Here W_0 is contributed by the flux compactification and so is completely independent of the Kähler moduli. $|W_0|$ must be chosen as small as the various nonperturbative

terms in W in order to trust the shape of the potential while neglecting corrections to K .

The T^i -dependent terms are imagined to be generated by euclidean $D3$ branes ($ED3$ s) wrapped about the torus together with one of the various 2-cycles of $K3$ [11], in which case $b_i = 2\pi$. The coefficients B_i could depend on the position of the $D3$ in the $K3$ directions, and we imagine this dependence to have provided the forces which prevent $D3$ motion in these directions (allowing the neglect of this motion in what follows).

Wrapping such $ED3$ branes about the $K3$ similarly can stabilize its volume, as can gaugino condensation on the $D7$'s localized at the fixed points of the torus. (Which of these obtains depends on the details of the fluxes that are applied [12].) This is what generates the S -dependent term, $w(S, z)$, of eq. (2.18), which is predicted to take the following form³

$$w(S, z) = \sum_r w_r(S, z) = \sum_r \left\{ A_r e^{-a f_r(S, z)} \right\}^{1/N_r} = \sum_r \left\{ e^{-aS} F_r(z, \tau) \right\}^{1/N_r} \quad (2.19)$$

where for future notational convenience we introduce the function

$$F_r(z, \tau) \equiv A_r \vartheta_1[\pi(z_r - z)|\tau] \vartheta_1[\pi(z_r + z)|\tau]. \quad (2.20)$$

Here $N_r = 1$ if $w_r(S, z)$ arises due to an $ED3$, but N_r depends on the gauge group involved if $w_r(S, z)$ is generated by gaugino condensation. For instance, $N_r = N$ if gaugino condensation arises for an $SU(N)$ or $SO(N + 2)$ gauge group.

The quantity w is invariant under the shifts $z \rightarrow z + 1$ and $z \rightarrow z + \tau$ provided that S also shifts appropriately. The transformation properties of appendix (A.2) show in particular that invariance under the transformation $z \rightarrow z + \tau$ requires $S \rightarrow S - 2\pi i(2z + \tau)/a$. Comparing this with condition (2.16), required for invariance of K , shows that invariance of the complete scalar potential requires the constants c and a must be related by [15]

$$c = \frac{2\pi}{a}, \quad (2.21)$$

and so $c = 1$ if $a = 2\pi$.

Whether gaugino condensation occurs on the stack of branes at a given fixed point depends on the low energy gauge group and field content. We assume there is enough freedom to turn on condensation at one or more fixed points.

2.3 Low-energy scalar interactions

The low-energy scalar interactions are generically governed by $\mathcal{L} = \mathcal{L}_{SG} + \delta\mathcal{L}_{sb}$, where \mathcal{L}_{SG} denotes the relevant part of the 4D supergravity lagrangian,

$$\mathcal{L}_{SG} = -\sqrt{-g} \left[V_F(T, \bar{T}) + V_D(T, \bar{T}) + K_{A\bar{B}} \partial_\mu T^A \partial^\mu \bar{T}^{\bar{B}} + \dots \right], \quad (2.22)$$

³Notice that our definition of a does not contain the factor of $1/N_r$ in the case of gaugino condensation on the $D7$ stack at fixed point r ; our notation differs from that of ref. [16] in this respect.

and the SUSY-breaking term

$$\delta\mathcal{L}_{sb} = -\sqrt{-g} \left[V_{\text{up}}(T, \bar{T}) + \dots \right] \quad (2.23)$$

denotes the derivative expansion of any terms which cannot be put into the 4D $\mathcal{N} = 1$ supergravity form, such as low-energy terms due to the presence of a supersymmetry-breaking anti- $D3$ brane. Any such terms must be perturbatively small in order for the 4D supergravity form to be a good approximation, such as might occur if the $\overline{D3}$ were localized in a strongly warped throat.

The F - and D -term potentials are given as usual by

$$V_F = e^K \left[K^{A\bar{B}} D_A W \overline{D_{\bar{B}} W} - 3|W|^2 \right], \quad (2.24)$$

and

$$V_D = \frac{1}{2} \sum_r \mathcal{F}_r^{ab} D_{a,r} D_{b,r}, \quad (2.25)$$

where the sum is over the 4 fixed points of T_2/Z_2 and \mathcal{F}_r^{ab} denotes the inverse matrix for $\text{Re } f_{ab,r}$. The auxiliary fields, $D_{a,r}$, are given by

$$D_{a,r} = \delta_a K = \frac{\partial K}{\partial T^\alpha} \eta_a^\alpha + \frac{\partial K}{\partial Q^x} (t_a Q)^x + (\overline{Q} t_a)_x \frac{\partial K}{\partial \overline{Q}_x}, \quad (2.26)$$

where t_a denotes the appropriate gauge generator acting on any low-energy charged chiral fields, Q^x , that happen to be present (often arising as low-energy open string states). The quantity η^α denotes the shift of the moduli fields, whose imaginary parts transform as $\delta_a \beta^\alpha = \eta_a^\alpha$. Such shifts arise when the corresponding axionic shift symmetry is gauged by background 2-form fluxes localized on $D7$ brane, as described in more detail in section 2.1 above. Notice in particular that for fluxes localized on the $D7$'s wrapping the $K3$, η^α never points in the direction of the $K3$ volume modulus, S : $\eta^S = 0$. This follows from the vanishing for $K3 \times T_2/Z_2$ of all intersection numbers of the form $k^{\alpha s s} = 0$.

The z -dependence of the D -term potential, eq. (2.25), has a simple physical implication. If $D_{a,r} = 0$ after the Q^x are minimized, then V_D is z -independent. If $D_{a,r} \neq 0$, on the other hand, the z -dependence of V_D arises from the gauge coupling function, f_r . For small $D3$ - $D7$ separation this varies logarithmically, with $\text{Re}(f_r - S) \propto -\ln|z - z_r|$. This describes a force acting on the $D3$ due to the $D7$'s that vanishes (by supersymmetry) in the absence of the magnetic flux, but is otherwise nonzero. Furthermore, this force arises due to tree-level closed-string exchange (since the z -dependence of f_r arises due to the classical back-reaction on the bulk geometry by the $D3$ brane [14]). Equivalently, because of open-closed string duality, this force can be regarded as being due to open string loops, and as such can be regarded as the 4D supergravity description of the ‘Coleman-Weinberg’ part of the $D3$ -brane potential used for inflationary purposes in ref. [16].

The difficulty with using V_D is that the energetics of the charged fields, Q^x , if present, usually prefers them to adjust to ensure $D_r = 0$ ref. [17, 21, 22], thereby turning off the flux-induced $D3$ - $D7$ force. This is the low-energy 4D supergravity’s way of expressing how the $D7$'s can prefer to adjust internally to preserve supersymmetry, and thereby eliminate

the $D3$ - $D7$ Coleman-Weinberg interaction. Furthermore, such charged field very often *must* exist. They are typically required, for instance, to understand the gauge invariance of W once W depends — such as in eq. (2.18) — on fields like T^i if these shift under a gauge symmetry.

SUSY-breaking terms. Following KKL T [9] we take the contribution of any anti- $D3$ branes (should these be present) to be perturbatively small and contained in V_{up} , whose detailed form depends on whether or not the antibrane is located in a strongly warped region. Warped type IIB compactifications of $K3 \times T_2/Z_2$ were examined in ref. [8].

The 4D potential due to the tension of an anti- $D3$ brane is (in the 4D Einstein frame)

$$V_{\text{up}} = \frac{\hat{E} e^{4\mathcal{A}}}{\mathcal{V}^2}, \tag{2.27}$$

where the constant \hat{E} is proportional to the $D3$ tension, T_3 . The warp factor, \mathcal{A} , is defined by the form of the string frame metric,

$$ds_{10}^2 = e^{2\mathcal{A}} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2\mathcal{A}} g_{mn} dy^m dy^n, \tag{2.28}$$

where g_{mn} is the metric of $K3 \times T/Z_2$ such that $\mathcal{V} = \int \sqrt{g_6} d^6y$. To leading order the warp factor \mathcal{A} depends only on the $K3$ coordinates, although this changes once one includes corrections in α' and the string coupling, g_s [8].

When evaluated in a strongly warped throat it happens that $e^{4\mathcal{A}} \propto e^{-\zeta} \mathcal{V}^{2/3}$, where $\zeta = 8\pi n_1 / (3g_s n_2)$ is a combination of certain integer flux quantum numbers, n_i , and so the total volume-dependence of an antibrane uplifting potential is

$$V_{\text{up}} = \frac{E}{\mathcal{V}^p}, \tag{2.29}$$

with $E = \hat{E} e^{-\zeta}$ and $p = 4/3$ (or $E = \hat{E}$ and $p = 2$) if the anti- $D3$ is (is not) located in a warped throat. In the absence of a better understanding of the cosmological constant problem we imagine E to be tuned to ensure that the scalar potential vanishes at its minimum.

Locating the antibrane within a warped throat has several well-known advantages.

- Warping suppresses the scale of the supersymmetry-breaking physics relative to other scales, and this helps to sequester its effects from the SUSY-breaking sector [24]. This is required to justify regarding the SUSY-breaking terms of $\delta\mathcal{L}_{sb}$ as small perturbations to the 4D supergravity action.
- Warping allows the scale of the uplifting to be tuned in small steps, potentially allowing a closer approach to a vanishing potential at the minimum.
- Warping decreases the Coulomb potential between the $D3$ and $\overline{D3}$, largely because it suppresses the $\overline{D3}$ charge. This is important for inflationary applications because without the warp factor the Coulomb force tends to ruin slow roll for $D3$ motion in the z directions. Asymmetric compactifications with the $K3$ radius much larger than the torus radius do not improve this situation [4].

- Finally, warping tends to localize the $\overline{D3}$ by making it settle into the bottom of the throat. This keeps it from migrating to one of the branes and perhaps annihilating. Furthermore, although the $D3$ is mobile, we imagine that the stabilization of the $K3$ moduli in V_F fixes its position within $K3$ (see the discussion below eq. (2.18)), and does so far from the throat.⁴ This keeps the $D3$ from migrating to the $\overline{D3}$ and annihilating, leaving it free to play an inflationary role as it moves along the torus.

3 The inflationary model

To make the search for inflationary solutions manageable we imagine all but one of the moduli T^i to be stabilized with masses larger than those relevant for the inflationary motion, allowing us to specialize the previous setup to only three complex fields: the $K3$ volume, S , the $D3$ position on the torus, z , plus the one remaining modulus T . Our motivation for keeping one of the T 's is to maintain the no-scale form of the low-energy supergravity, whose Kähler potential (up to an irrelevant additive constant) then is

$$K = -\ln\left[S + \overline{S} - 2\omega(z, \bar{z})\right] - 2\ln(T + \overline{T}). \quad (3.1)$$

Although we follow T numerically when searching for inflationary dynamics, it turns out to play a negligible role in the actual inflationary slow rolls we eventually find.

To simplify the notation in this section we denote the real and imaginary components of the fields S and T by $\xi^s = s$ and $\xi^t = t$, so

$$S = s + i\alpha, \quad T = t + i\beta \quad (3.2)$$

while as before, $z = z_1 + iz_2$.

The superpotential, eq. (2.18), for this reduced theory becomes

$$W = W_0 + \sum_r w_r(S, z) + Be^{-bT}, \quad (3.3)$$

where $w_r = [F_r(z)e^{-aS}]^{1/N_r}$. For simplicity we restrict in what follows to the case where $N_r = N$ is independent of r . Then the S -dependent part of W becomes

$$w(S, z) = A(z)e^{-aS/N} \quad (3.4)$$

with $A(z) = \sum_r F_r(z)^{1/N}$. Notice that the scalar potential derived from this superpotential is periodic under $\alpha \rightarrow \alpha + 2\pi N/a$ and $\beta \rightarrow \beta + 2\pi/b$.

3.1 F-term potential

The F -term potential is found by specializing the earlier results to the three fields of interest. The derivatives of the superpotential are

$$W_z = \sum_r \frac{w_r}{N_r} (\partial_z \ln F_r) = w \partial_z \ln A, \quad W_S = -a \sum_r \frac{w_r}{N_r} = -\frac{aw}{N}, \quad W_T = -bBe^{-bT}, \quad (3.5)$$

⁴An explicit construction with the $D3$ stabilized away from the throat's tip can be found in ref. [25].

and (keeping in mind the no-scale form of the Kähler potential) the F -term potential becomes

$$V_F = \frac{1}{X(T + \bar{T})^2} \left\{ \sum_{A=S,T,z} K^{A\bar{A}} \left[W_A \bar{W}_A + (K_A W \bar{W}_A + c.c.) \right] + \left[K^{S\bar{z}} (W_S \bar{W}_z + K_S W \bar{W}_z + K_{\bar{z}} W_S \bar{W}) + c.c. \right] \right\}, \quad (3.6)$$

where $X = 2[s - \omega(z, \bar{z})]$. Notice that for large $K3$ volume, s , we have $X \approx 2s$ and so when all else is equal it is the $K^{S\bar{S}} \propto s^2$ term that dominates.

Axion minimization. The axion fields, α and β , can now be minimized explicitly. The only terms involving these fields come from V_F and are given by

$$V_{ax} = \frac{1}{X(T + \bar{T})^2} \left\{ K^{T\bar{T}} K_T \bar{W}_T (W_0 + w) + K^{b\bar{a}} K_b \bar{W}_a (W_0 + B e^{-bT}) + c.c. \right\},$$

where, as before, the indices $a, b = S, z$. This contains terms proportional to $\cos(b\beta)$, $\cos(a\alpha/N)$ and $\cos(b\beta - a\alpha/N)$. It is convenient to use an overall phase rotation to choose W_0 to be real and negative, since for the parameter range of later interest this ensures that $\text{Im} T = \beta = 0$ at its minimum. Minimizing α similarly amounts to replacing $A \rightarrow |A|$ in the remaining equations.

The supersymmetric AdS minimum. V_F as described above has a supersymmetric AdS minimum, corresponding to the solutions to $D_A W = 0$. The condition $D_S W = D_T W = 0$ to be solved for $s = s_0(z)$ and $t = t_0(z)$ may be written as

$$|B| = \frac{a|A(z)|X_0}{Nb t_0} e^{bt_0 - as_0/N}, \quad (3.7)$$

$$W_0 = -|A(z)| e^{-as_0/N} \left[1 + \frac{aX_0}{N} \left(1 + \frac{1}{b t_0} \right) \right], \quad (3.8)$$

where $X_0 = 2[s_0 - \omega(z, \bar{z})]$. Using these conditions in $D_z W = 0$ then implies z must satisfy

$$|A(z)| e^{-as_0/N} \left[\partial_z \ln |A(z)| + \frac{2\pi(z - \bar{z})}{N\tau_2} \right] = 0, \quad (3.9)$$

which uses the explicit form, eq. (2.14), of ω as well as the condition $ac = 2\pi$, eq. (2.21).

Eq. (3.9) is always solved by $A(z) = 0$, but in this case $w(S, z) = 0$ and so the S -modulus is not stabilized. If S -stabilization occurs at a single fixed point, r_0 , then $A(z) = [F_{r_0}(z)]^{1/N}$ can vanish when the $D3$ approaches $z = z_r$. Similar solutions also exist for $ED3$'s located at all four fixed points, for which $A = \sum_r F_r$, since in this case $A = 0$ when $(z_1, z_2) = (1/4 + n/2, 0)$ for n an arbitrary integer. (These last solutions are most easily seen in the limit $\tau_2 \gg 1$, for which appendix A.2 shows $\sum_r F_r \propto \cos(2\pi z)$ — see eq. (A.12).)

To obtain a supersymmetric extremum without destabilizing S requires the bracket in (3.9) to be zero. When $A = F_{r_0}^{1/N}$ these extrema are at $z = (n/2, m\tau_2/2)$ (which

includes as special cases the points where $A(z) = 0$). When $A = \sum_r F_r$ the solutions instead are $(z_1, z_2) = (n/2, 0)$, and $(n/4, \tau_2/2)$, with n an integer. (Again these latter solutions are simplest to see in the large τ_2 -limit.) Which of these are maxima or minima depends on the parameters used (and in any case can change after including an uplift term, as we shall see).

The potential at this minimum becomes

$$V_F^{\text{AdS}} = -\frac{3|W_{\text{min}}|^2}{X(T + \bar{T})^2} = -\frac{3a^2|A|^2X_0}{4N^2t_0^2} e^{-2as_0/N}. \tag{3.10}$$

For $\tau_2 \gg 1$, evaluation of the potential near this minimum shows it to be flattest along the z_2 direction, while for $\tau_2 \ll 1$ it is instead flatter along z_1 .

3.2 Uplifting

We next consider lifting this solution to positive values of the potential at the minimum, using either a D -term potential or that of an anti- $D3$ brane.

D-term potential. If a D -term potential due to magnetic flux located at a brane at fixed point z_r were to exist, either due to the absence of charged matter fields, Q^x , or if their complete potential is minimized at $Q^x = 0$, it would depend on X and T in the following way: $V_{D,r} \propto (K_\alpha \eta^\alpha)^2 / \text{Re}(f_r)$. Here η^α measures the linear combination of axion fields which is gauged by the magnetic flux in question, which the discussion of previous sections shows never points in the S direction. So for the fields of present interest $K_\alpha \eta^\alpha \propto K_T \propto 1/t$, and so

$$V_D^r(S, z) = \frac{E_r}{\text{Re}(f_r)t^2}, \tag{3.11}$$

where E_r is a constant.

But there is a consistency problem with having T shifting in this way under a $U(1)$ symmetry without also having charged fields Q^x be present. The problem is that if T shifts in the way required to appear in D , then this same symmetry precludes the existence of a term in the superpotential like Be^{-bT} , as was required to stabilize T . Charged fields like Q^x can resolve this kind of paradox because their presence in W can combine with T to make the superpotential invariant. We refer the reader to refs. [21, 22] for more detailed discussions of these issues.

Because of this issue, we perform our main search for inflation using an alternative source of uplifting, such as from an anti- $D3$ brane. This is what we use in our most successful inflationary scenario, described below. However, following [16] we also seek inflation using eq. (3.11), in the spirit that it might ultimately turn out to capture the low-energy dynamics of some better motivated, but more sophisticated, string constructions. In particular, we use this form of uplifting when exploring the limit where the $D3$ and $D7$'s are in close proximity, in order to try to follow as closely as possible the analysis of ref. [16].

Anti-D3 brane. When uplifting with an anti- $D3$ brane, we assume the potential to depend only on the volume, with the form discussed above

$$V_{D3} = \frac{E}{\mathcal{V}^p} = \frac{E}{X^{p/2}(T + \bar{T})^p} \tag{3.12}$$

which uses $\mathcal{V} = \sqrt{X}(T + \bar{T})$. As before, the power is $p = 2$ for anti-branes in unwarped regions and $p = 4/3$ when the antibrane is deep within a warped throat [6], and so we use $p = 4/3$ in our main search for inflationary solutions. We have checked that similar solutions also exist when $p = 2$, however.

Since neither X nor $\text{Re}(f_r)$ depend on the axions, α, β , an uplifting potential of either D -term or antibrane type would not alter their minimization.

4 Slow-roll Inflation

We next search the potential for the fields S, T and z , seeking slow-roll regions for which the effective single-field slow-roll parameters, ϵ and η , can be made small. We find that inflation does not generically arise, but — as for many other brane-inflation models — slow roll can occur provided some of the parameters in the potential are mildly tuned (see, however, [26] for potentially less tuned alternatives). In this section we describe two such examples.

We first search for slow-roll regimes that do not rely on the existence of a D -term potential, by uplifting using an antibrane. We find that slow-roll inflation is possible to obtain near a saddle point, where we use a superpotential generated by gaugino condensation localized at a single fixed point on the torus, with the $D3$ located as far away as possible from this fixed point. In order to achieve inflation the shape of the torus must be tuned to be very close to square, to within a part in 10^4 , but once this is done the resulting slow roll is largely insensitive to the other parameters in the F -term potential.

The second example we present is a direct analog of the scenario proposed in ref. [16], with the $D3$ very close to one of the $D7$ fixed points. Two D -terms are added in this setup, one to drive inflation and the other to uplift the potential. In doing so we follow [16] and put aside the concerns given above whether the charged matter fields cause the D -term to relax to zero. We perform our search numerically, using the full expressions for V_D and V_F , rather than searching analytically using a simplified parametrization of the potential. Taking the inflaton to be primarily in the z_1 -direction, we do not find any example that resembles standard D -term driven hybrid inflation (and we identify the reasons for this difference with [16]). Instead, we find that inflation can occur at an inflection point of the potential, for which the D -term and F -term contributions to $\epsilon = 0$ and η are fine-tuned to be small.

4.1 Axionic inflation

We start with our best example of $D3$ - $D7$ inflation. We imagine gaugino condensation to occur only at a single fixed point, $z_r = 0$, leading to a superpotential term as given by (3.4),

$$W = W_0 + [A_0 e^{-aS} \vartheta_1^2(\pi z | \tau)]^{1/N} + B e^{-bT} \tag{4.1}$$

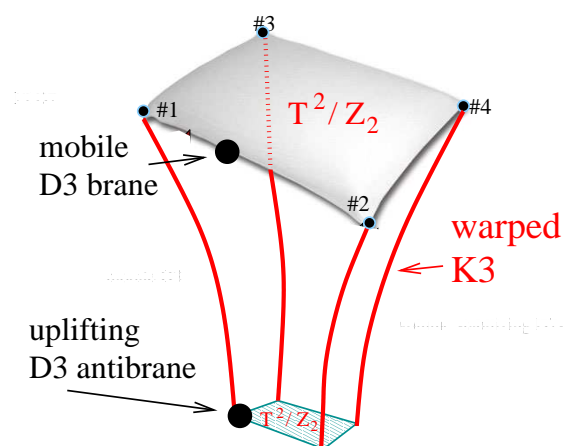


Figure 1. Illustration of warped $\overline{D3}$ uplifting with $D3$ confined to top of throat.

which absorbs a conventional sign into the constant A_0 .

We take the uplifting potential provided by a $\overline{D3}$, localized in a throat, leading to a contribution as in eq. (3.12) with $p = 4/3$. (Although we find the warped version of this picture most appealing, we have also checked that inflation can work without warping.) Finally, as discussed at length above, we imagine the $D3$ cannot move in the $K3$ directions but is mobile within the T_2 . We assume the $D3$ is not near the throat, so the uplifting potential is the only antibrane perturbation to the $D3$ motion. Notice that this uplifting depends on the $D3$ position through the factor $X = 2[\text{Re}S - \omega(z, \bar{z})]$ of eq. (3.12), and this plays an important role in the shape of the $D3$ potential. Although we believe this construction — illustrated in figure 1 — to be plausible, we leave a detailed derivation for future work.

For the warped configuration, it is straightforward to find an almost-flat saddle point in the potential, for any values of the superpotential parameters a, b, A_0, B . This is done simply by tuning the single parameter $\tau_2 \equiv \text{Im}(\tau)$, which determines the shape of the torus. Setting $\tau_1 \equiv \text{Re}(\tau) = 0$ for simplicity, we find that if τ_2 is close to 1 (so that T_2/Z_2 is nearly square) we get a flat potential close to the antipodal point, $z = \frac{1}{2}(1 + \tau)$, of the fixed point source at $z = 0$. The surprise is that the unstable direction turns out not to be in the z_1 - z_2 plane, but rather is a linear combination of z_1 and $\alpha = \text{Im} S$, the axion associated with the volume modulus.

An explicit example leading to an inflationary slow roll is given by the parameter choices

$$W_0 = -4.14 \times 10^{-7}, \quad a = b = 2\pi, \quad A_0 = 0.538, \quad B = 0.912, \quad N = 4, \quad \tau_1 = 0, \quad z_2 = \frac{\tau_2}{2}, \quad (4.2)$$

which corresponds to a minimum at $s_0 = 11.54$, $t_0 = 2.802$ and $\beta = 0$. Uplifting requires taking $E = 1.70217 \times 10^{-13}$ (when $p = 4/3$). These values were chosen to satisfy the COBE normalization of the power spectrum, $P = 4 \times 10^{-10}$, at the scale which we take to be 55 e -foldings before the end of inflation. For this purpose we approximate P as $H^4/(50\pi^2 L_{\text{kin}})$ where L_{kin} is the kinetic energy of the fields.

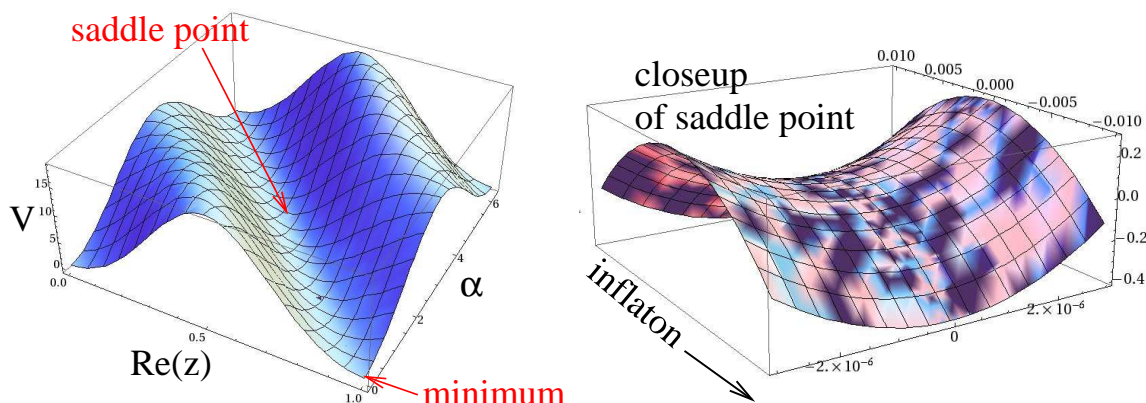


Figure 2. Left: the potential for the warped uplifting saddle point in the the z_1 - $2\pi\alpha$ plane. Right: closeup of the saddle point region at $z_1 = 1/2$, $2\pi\alpha = \pi$.

The resulting potential is displayed in figure 2 as a function of the remaining fields z_1 and α . The inflationary saddle point at $z_1 = \alpha = \frac{1}{2}$ is visible in the figure, at which the unstable direction numerically evaluates to

$$\hat{\phi} = \frac{1}{\sqrt{2}} \hat{\alpha} - \frac{1}{\sqrt{2}} \hat{z}_1, \quad (4.3)$$

where \hat{z}_1 and $\hat{\alpha}$ denote unit vectors in these two coordinate directions of field space. (The components in the directions of the other, heavy fields are smaller than this by a factor of 10^{-8} .) An initial condition near this saddle point initially moves in the direction given by eq. (4.3), which is also the direction towards the local minimum at $z_1 = \alpha = 0$. In the resulting motion the $D3$ falls from the fixed point at $z = \frac{1}{2}(1 + \tau)$ to the fixed point at $z = \frac{1}{2}\tau$, driven by the nonperturbative physics situated at $z = 0$.

We regard the values of t_0 and s_0 to be just within the domain of validity of the α' and g_s expansions, although ideally larger values would be preferable.⁵ With the parameters chosen we have $as_0/N \simeq bt_0 \simeq 18$, ensuring the suppression of the nonperturbative superpotential. Furthermore $\mathcal{V} \simeq \sqrt{s_0} t_0 \simeq 10$, so known α' corrections to K are of order [27]

$$\frac{\delta K}{K} \simeq \frac{\chi \zeta(3)}{2(2\pi)^3 g_s^{3/2} \mathcal{V} \ln \mathcal{V}} \simeq \frac{0.1}{g_s^{3/2} \mathcal{V} \ln \mathcal{V}}, \quad (4.4)$$

where $\chi = 48$ is the Euler number of $K3 \times T_2/Z_2$ and $\zeta(3) \simeq 1.2$. For $\mathcal{V} \simeq 10$ this is of order $3/\mathcal{V} \ln \mathcal{V} \simeq 0.1$ (or $0.6/\mathcal{V} \ln \mathcal{V} \simeq 0.03$) if $g_s \simeq 0.1$ (or $g_s \simeq 0.3$). Comparatively small values for t_0 and s_0 are driven by the requirement that the potential be large enough to reproduce the observed primordial scalar perturbations, and are a reflection of a common tension in brane inflation models between this condition and the control over the g_s and α'

⁵It is our use of $N = 4$ in $w(S, z)$ while $N = 1$ for the Euclidean D3-brane superpotential Be^{-bT} , together with our requirement that $A_0, B < 1$ (to avoid having a large energy scale in the nonperturbative superpotential) that leads to our obtaining the hierarchy $t_0 \cong s_0/4$. (Note that eq. (3.7) shows that exponentially large values of B would be required to make $t_0 \sim s_0$.) On the other hand, if we take $N = 1$ for $w(S, z)$ then the values $s_0 \sim t_0 \sim 10$ put the energy scale of the potential far below that needed for the COBE normalization.

expansions. We regard the present calculation as being sufficiently accurate to demonstrate the existence of a slow roll, motivating a more detailed search for inflation with larger s_0 and t_0 .

We remark in passing that our numerics follow all six of the fields s, t, α, β, z_1 and z_2 , but for the inflationary example considered here it turns out that the variation of the heavy fields s, t, z_2 and β found numerically during inflation proved to be negligible, the largest being 1 part in 10^6 for s . This can be understood analytically, and is consistent with the suppression of the perturbations of these fields by their masses, which are heavy compared with the inflaton directions. We thus find it to be a good approximation to ignore the slight motion of the heavy fields during inflation, even though our numerical code evolves all six of the fields subject only to the slow-roll approximation.⁶

Remarkably, the potential at the saddle point is acceptably flat for slow-roll inflation for a reasonable range of parameter values in the vicinity of those of eq. (4.2), *provided* we tune τ_2 to be in the range

$$\tau_2 = 1.00174 - 1.00184. \tag{4.5}$$

This range corresponds to the η parameter at the saddle point in the interval $-0.04 < \eta_{\text{saddle}} < 0$; see figure 3. The lower value of τ_2 gives the larger value of η_{saddle} , and $\tau_2 = 1.00174$ yields 230 e -foldings when starting at a displacement of 0.001 from the saddle point. Although τ_2 must be tuned at the level of 1 part in 10^4 , it is only this one parameter in the superpotential that needs such fine adjustment. It is suggestive that the required value for τ_2 is so close to $\tau_2 = 1$, which corresponds to a square torus, and although the symmetry of this geometry has been argued to lead to special cancellations amongst inter-brane forces [4], we do not have a symmetry argument for why the inflationary value of τ_2 is not precisely at 1.

If we change the parameter values in (4.2), the position of the saddle point typically shifts, as does the unstable direction. For example with $W_0 = -10^{-6}/(2\pi)^{3/2}$, $a = 3\pi$, $A = 2/(2\pi)^{3/2}$, $B = 3/(2\pi)^{3/2}$, $b = \pi$ and $N = 1$ we find the saddle point moves to $z_1 = \frac{1}{2}$ and $\alpha = \frac{1}{3}$, with the unstable direction becoming $\hat{\phi} = 0.97266 \hat{a} - 0.23221 \hat{z}_1$. However, the tuning needed to get inflation again simply requires τ_2 close to unity; with $\tau_2 = 1.00673$ giving about 300 e -foldings of inflation.

It is also possible to get inflation from unwarped $\overline{D3}$ uplifting, where the dependence of V_{up} on the volume goes like $1/\mathcal{V}^2$ instead of $1/\mathcal{V}^{4/3}$. In this case, using exactly the same superpotential parameters as (4.2), we find that the locations of the minima and saddle points get interchanged, with the minimum at $(z_1, \alpha) = (\frac{1}{2}, \frac{1}{2})$ and the saddle at $(z_1, \alpha) = (0, 0)$ (and $z_2 = \frac{1}{2}\tau_2$ as before). However the value of τ_2 needed for flatness is now farther from unity: $\tau_2 = 1.61683$. The direction of the inflaton is exactly the same as for the corresponding warped case, eq. (4.3).

Although the qualitative features of our scenario are robust to changes in the superpotential parameters, they are on the other hand rather sensitive to the detailed form of

⁶Making the slow-roll approximation in the numerical evolution greatly reduces the computational burden, while still giving the correct trajectory during inflation, and also allowing sufficiently accurate determination of when inflation ends.

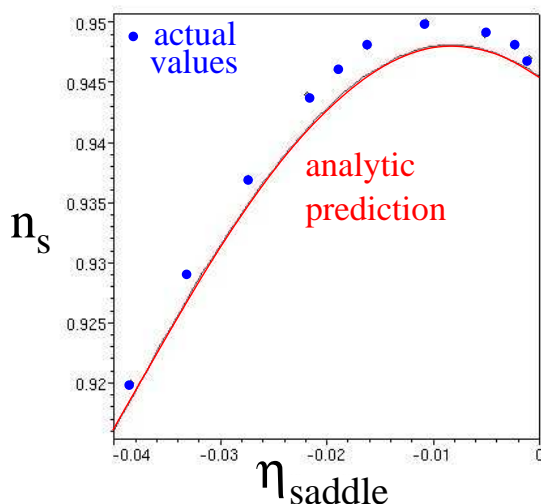


Figure 3. Spectral index versus η evaluated at the saddle point; dots are exact numerical values, while the line is the analytic prediction using the method of ref. [29].

the uplifting potential (3.12). We find that the inflationary mechanism fails if one tries to replace the $\overline{D3}$ uplifting with a D-term, for example located at the $D7$ stack at $z = \frac{1}{2}$. Either the η parameter cannot be tuned to be small, or the $D3$ is attracted to the $D7$ which sources the D-term, leading to annihilation and the removal of the uplifting. Moreover, the z_2 -dependence appearing in the uplifting term through X is also important; neglecting this dependence leads to an additional negative eigenvalue of the curvature matrix, along a linear combination of the s and z_2 directions, which spoils the slow roll at the saddle point.

Another potential source of tuning arises because the antipodal point, $z = \frac{1}{2}(1 + \tau)$, where the inflationary saddle arises is coincident with one of the T_2/Z_2 fixed points. As a result one might worry about additional $D3$ interactions with the $D7$ and $O7$ planes that reside there. Although neither the superpotential or uplifting physics is located at this point, this need not preclude there being other $D3$ - $D7$ instabilities which might compete successfully with the inflationary slow roll. (On the other hand, the same processes may be quite welcome once the $D3$ encounters the $D7$ stack at the endpoint of the roll, when $z = \frac{1}{2}\tau$.) To avoid these difficulties we therefore demand either that no such physics exist (such as if the $D3$ and relevant $D7$'s remain mutually BPS), or that the $D3$ not approach the apex of the saddle point to within closer than the string scale. This latter condition is easier to achieve the larger is the torus volume, although there can be some tension between having sufficiently large volumes and keeping the potential large enough during inflation to get acceptably large primordial scalar perturbations.

Not surprisingly, this scenario is very similar in its predictions for primordial fluctuations to those of racetrack inflation [18], which is also based on axion motion from near a saddle point in the potential. Just like in the racetrack model, we find by numerical evolution that the spectral index — which we define at the canonical 55 e -foldings before the end of inflation — cannot exceed $n_s = 0.95$ even when the potential is arbitrarily flat near the saddle point. A simple explanation of the robustness of this result is given in ref. [29],

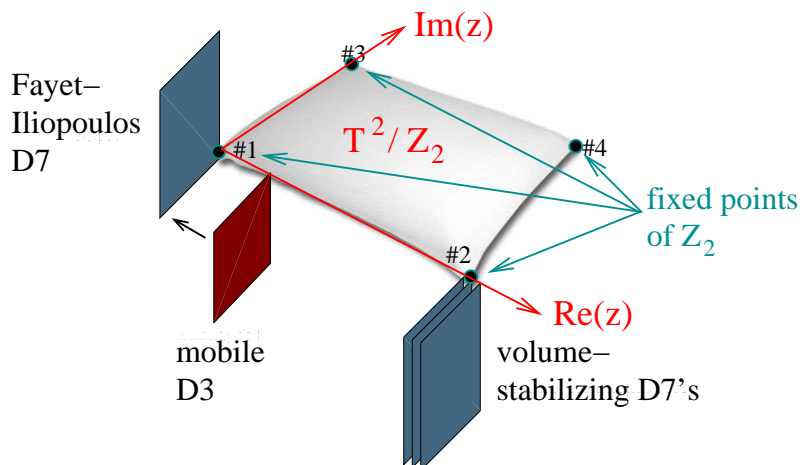


Figure 4. A figure adapted from ref. [16] which illustrates their inflationary configuration.

which shows how it can be understood from the dominance of the terms $V_0 - \frac{1}{2}m^2\phi^2$ in the inflaton potential until the end of inflation. In figure 3 we display the variation of n_s with the value of the η parameter evaluated at the saddle point (η_{saddle}), both for the exact numerical determination (dots) and the analytic approximation (line) of ref. [29].

4.2 A D -term driven example

Our second inflationary example is motivated by the inflationary solution found in ref. [16], which we first briefly describe.

The D -term inflationary setup. Ref. [16] seeks a stringy analogue of D -term inflation [30]. To do so they consider a nonperturbative superpotential W_{np} generated by a stack of $D7$ branes at the fixed point $z = \frac{1}{2}$ of T_2/Z_2 . The inflationary energy density and the uplifting potential is modelled as a Fayet-Iliopoulos D -terms, given by adding fluxes to the $D7$ branes situated at $z = 0$. Inflation occurs when the $D3$ is in close proximity to $z = 0$, to which it falls driven by the one-loop Coleman-Weinberg (CW) potential obtained using the threshold corrections obtained from integrating out massive $D3$ - $D7$ string modes

$$V_{FI} = \frac{g^2\xi^2}{2} \left[1 + \frac{g^2U(x)}{16\pi^2} \right], \tag{4.6}$$

where g is the gauge coupling for the $D7$ brane at $z = 0$, ξ is a constant and x is related to the canonically normalized inflaton, $\phi \propto z_1$, by $x = \phi/\sqrt{\xi}$. (The large mass associated with the z_2 coordinate allows it to be set safely to zero.) The potential $U(x)$ is

$$U(x) = (x^2 + 1)^2 \ln(x^2 + 1) + (x^2 - 1)^2 \ln(x^2 - 1) - 4x^4 \ln x - 4 \ln 2. \tag{4.7}$$

The slow roll occurs as z rolls down the CW-potential, and ends when the $D3$ - $D7$ waterfall fields condense to cancel the D -term. To obtain a Minkowski or dS vacuum after inflation an additional uplifting term must be added, perhaps elsewhere on the torus, although its explicit form is not considered in ref. [16]. This setup is illustrated in figure 4.

To gain an analytic understanding of the inflationary dynamics near $\phi = 0$ the authors of ref. [16] observe that $U(x)$ can be well approximated by a logarithm in this regime,⁷ and the F -term potential can be expanded as a power series about $z = 0$, leading to

$$V \simeq V_0 + D \ln \left(\frac{\phi^2}{\xi} \right) - \frac{m^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4, \quad (4.8)$$

where $V_0 = \frac{1}{2} g^2 \xi^2$ and $D = g^2 V_0 / (8\pi^2)$. The coefficients of this potential are regarded as implicit functions of S, T and τ — all of which are regarded as being stabilized. m^2 and λ are obtained by expanding V_F in powers of z .

With these approximations the potential approaches that of usual D-term hybrid inflation in the regime as $\phi \rightarrow 0$, where the first two terms dominate. A slow roll in this regime is possible provided $D/V_0 = g^2/(8\pi^2)$ can be made much smaller than $(\phi/M_p)^2$. But this is not the only possibility; inflation can also occur for larger $D3$ - $D7$ separations. Indeed, provided $m^4 > 4\lambda D$ the potential V has a local maximum at $\phi_{\max}^2 = (m^2 - \delta)/(2\lambda)$, whose curvature is given by $V''(\phi_{\max}) = -2\delta$ where $\delta = \sqrt{m^4 - 4\lambda D}$. A slow roll may therefore also be sought near this local maximum. If $m^4 \simeq 4\lambda D$ (and so $\delta \simeq 0$) this maximum coalesces with a local minimum at $\phi_{\min}^2 = (m^2 + \delta)/(2\lambda)$ to produce an inflection point. The idea is thus to identify parameters to ensure that the slow roll parameter $\eta \cong 0.015$, so that the spectral index matches the WMAP5 value of $n_s \cong 1 - 2\eta = 0.96$, and once these are found see if such parameters can be obtained from underlying brane dynamics on $K3 \times T_2/Z_2$.

Supergravity search. We seek to reproduce this scenario numerically within the low-energy supergravity. This differs from the analysis of ref. [16] in two ways. First, we describe the D -term physics using the full D -term potential, eq. (3.11), when numerically seeking a slow roll. Second, we also add an explicit uplifting term, which for definiteness we also take to be a D -term arising from a flux localized on the $D7$'s located at $z = \frac{1}{2}$ (*i.e.*, at the same location as the gaugino condensation $D7$'s). Using this potential we numerically compute the potential for z by evaluating the moduli fields at their instantaneous minima $V(z) = V(z, S_0(z), T_0(z))$. We then search for an inflationary slow roll with the $D3$ close to the $D7$'s at $z = 0$.

The best example of slow-roll inflation we found in this setup arises near an inflection point of the scalar potential (described below), corresponding to tuning the parameter $\delta \simeq 0$ in the approximate potential of eq. (4.8). We did not find examples of inflation arising at the local maximum, for which the potential was also large enough to satisfy the COBE normalization. We find that once m^2 is made sufficiently small to get a small curvature at the maximum, the quartic term in eq. (4.8) becomes important, leading to the limit $\delta \simeq 0$. This can be seen in the numerical evaluation of the potential shown in figure 5.

Although we did examine a broad class of parameters, we were not exhaustive enough to preclude the potential existence of the inflationary example obtained in ref. [16]. In particular, our choice of $f_r = 0$ in eq. (2.17) relates our effective value for g^2 to the value of s at its minimum, and as a consequence the choices leading to a slow roll tend to destabilize

⁷The log approximation is valid in the regime with $\phi \gg \xi$ during inflation.

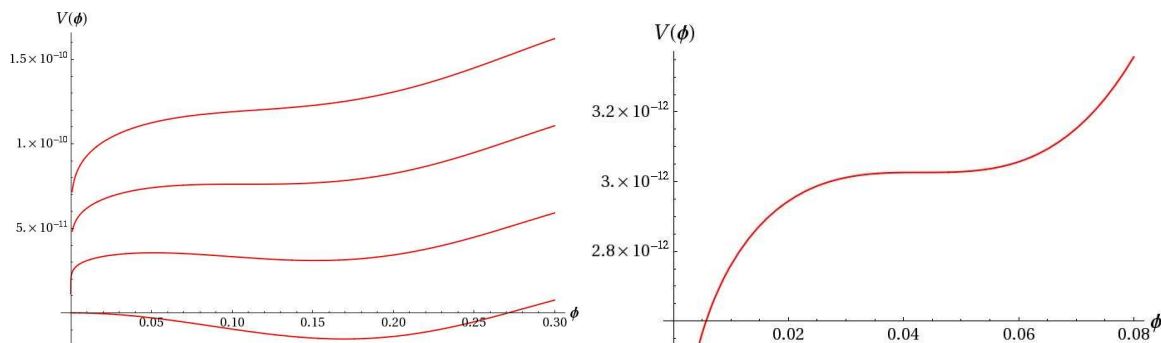


Figure 5. Left: Lowest curve is uplifted F-term potential; upper curves are $V_F + \epsilon_i V_D$, where V_D is the inflationary D-term, for an increasing series of values of $\epsilon_i \approx 10^{-6}$ of the strength which would be needed to uplift using this V_D (from $D7$'s at $z_1 = 0$) rather than the uplifting V_D coming from $D7$'s at $z_1 = 1/2$. Right: fine-tuned inflection point potential with parameters given by (4.9, 4.10).

the potential for s . We do not know if the same need be true once the freedom to adjust f_r is used.

Inflation at an inflection point can be found by tuning the inflationary D -term (while adjusting the uplifting D -term to ensure a Minkowski vacuum at the end of inflation). To see this, choose for definiteness $\tau_1 = 0$ and $\tau_2 \lesssim 2$ and turn off the inflationary D -term. Then the potential has a local minimum at small values for z_1 . As illustrated in figure 5, this local minimum becomes increasingly shallow until it eventually turns into the desired inflection point as the inflationary D -term is turned back on.

A specific example uses $\tau_1 = 0$, $\tau_2 = 0.6$, $z_2 = 0$, with F - and inflationary D -term potential parameters

$$N = 1, \quad E_2 = 1.56512 \times 10^{-8}, \quad A_0 = \frac{1}{(2\pi)^{3/2}} \quad \text{and} \quad a = b = 2\pi. \quad (4.9)$$

where E_2 is the strength of the uplifting D -term located at $z = \frac{1}{2}$. The values of B and W_0 are chosen by solving $D_T W = D_S W = 0$, using eqs. (3.8), to obtain minima at $s_0 = t_0 = 5/\pi = 1.592$. Of these parameters only τ is relatively important, since its value determines the sign of m^2 . The other parameters are randomly chosen, apart from the coefficient of the inflationary D-term, whose value must be tuned to

$$E_1 = 2.062673254 \times 10^{-9} \quad (4.10)$$

in order to obtain the desired inflection point, which occurs near $z_1 = 0.107024$.

Starting sufficiently close to this point, and tuning E_1 as above, one can obtain 5100 e -foldings of inflation. However, this number decreases rapidly with less tuning. To get 60 e -foldings, E_1 must be increased only by 1 part in 10^6 , otherwise the potential is not sufficiently flat. Other examples we tried require comparable levels of fine-tuning, which is somewhat more severe than the part-per- 10^3 tuning that is required in other brane-inflation models.

Although it is encouraging that inflation in this regime is possible at all, it is a disadvantage that inflation occurs at an inflection point rather than a maximum since this

makes it is much more sensitive to the initial conditions. For the above numbers the initial value of z_1 cannot be increased by more than 6% from the inflection point without overshooting it and so ending inflation too quickly. Furthermore, since inflation is not at a maximum we cannot appeal to general arguments of eternal inflation [28] to explain these initial conditions.

5 Conclusions

This paper reports on the results of a detailed numerical search for inflation in Type IIB vacua compactified (with modulus-stabilizing fluxes) on $K3 \times T_2/Z_2$. The search is performed using the 4D field equations of the low-energy effective theory, which is constructed using familiar ingredients: an F -term potential generated by fluxes and branes together with some sort of uplifting physics.

There are two underlying motivations for performing this search. The first starts from the observation that so much is known about Type IIB compactifications on $K3 \times T_2/Z_2$, because much is known about string behaviour on both $K3$ and the orbifolded torus. The relative simplicity of these geometries makes the study of their dynamics valuable, since most other instances of string inflation arise in much more complicated contexts where corrections can be more difficult to identify and control.

A second motivation for this study is the great appeal of the $D3$ - $D7$ inflationary mechanism [2], which has the promise of providing inflationary examples with a supersymmetric final state, inflation driven by D terms, and a potentially interesting cosmic-string signature. $K3 \times T_2/Z_2$ is a natural place to study this mechanism in detail because it naturally contains stacks of $D7$ and $O7$ planes wrapping $K3$ as well as a nice flat toroidal geometry in which to hope to find slow-roll $D3$ motion. Furthermore, new tools to describe this motion in terms of the low-energy 4D effective field theory have recently been developed [13–16], and their use allows a more systematic study of the degree to which modulus stabilization interferes with the conditions required for slow-roll inflation.

Using this 4D theory we numerically search for slow-roll inflation. To do so we follow three of the possible low-energy complex fields: the $D3$'s toroidal position, z ; the $K3$ volume modulus, S ; and a modulus, T , dual to one of the 22 nontrivial 2-cycle volumes on $K3$. We consider two kinds of uplifting, either that due to an anti- $D3$ brane [9], or by a flux-induced D term potential [17]. We follow earlier workers in using this last type despite some of the consistency problems [21, 22] it raises when realized in string vacua. We do so in the spirit that similar terms might arise from more complicated string constructions, and it may therefore be worth seeing whether they can support nontrivial inflationary dynamics.

Our search identifies two kinds of slow-roll regime. What we regard to be the most attractive has a $D3$ fall slowly between two $D7$ stacks, driven by a modulus-stabilizing superpotential (perhaps produced by gaugino condensation) located on a third stack. Uplifting is achieved by adding a $\overline{D3}$ in a warped throat. A slow roll is then possible when the brane is at the antipodal point from the modulus-stabilizing stack, provided the torus

is adjusted to be almost perfectly square (i.e. $\tau = i$)⁸. The low-energy scalar potential has a saddle point at this position, whose unstable inflaton direction turns out to be a linear combination of the $D3$ position, z_1 , and the axion, $\alpha = \text{Im } S$, associated with the $K3$ volume modulus. The resulting inflationary picture resembles earlier ‘racetrack’ models [18], and shares their generic prediction $n_s \lesssim 0.95$. Although the inflation is robust against changes to the superpotential parameters, it is sensitive to the kind of uplifting involved and requires a 1-in- 10^4 tuning in the value of τ .

The second inflationary regime found generates a superpotential (and places an uplifting flux) at one fixed point, $z = \frac{1}{2}$, and places another inflationary D -term generating flux on a second brane stack at $z = 0$. Inflation is then sought with the $D3$ very close to $z = 0$, in the hopes of obtaining standard hybrid D -term inflation as the $D3$ dissolves into the $D7$ ’s there. Our search here led to inflation at an inflection point, provided the inflationary D term is tuned to a part in 10^6 . But because it arises at an inflection point, this inflationary scenario is sensitive to the inflaton’s initial conditions due to a potential overshoot problem. In this case use of the full effective 4D potential makes finding inflation more difficult than might be thought based on simpler approximate potentials.

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A Useful relations

In this appendix we collect various useful relations and identities that are used in the text.

A.1 No-scale condition

We briefly show here that any Kähler function, $K(T, \bar{T}) = K(T + \bar{T})$, which satisfies the scaling identity

$$K(\lambda T, \lambda \bar{T}) \equiv K(T, \bar{T}) - 3 \ln \lambda, \tag{A.1}$$

for arbitrary moduli T^α and constant λ , must also satisfy the no-scale condition

$$K^{\alpha\bar{\beta}} K_\alpha K_{\bar{\beta}} \equiv 3. \tag{A.2}$$

⁸Antipodal inflation with unwarped $\overline{D3}$ lifting is also possible; in this case the tuning of the torus is not so close to being square.

This establishes the no-scale property of the Kähler function of interest in the main text, $K = K(S + \bar{S}, T_i + \bar{T}_i, i(z - \bar{z}), i(\tau - \bar{\tau}))$, which satisfies these assumptions.

To establish the result we first recognize that because K is a real function only of the combination $(T + \bar{T})^\alpha$, we may ignore the distinction between derivatives with respect to T^α and \bar{T}^α : $K_\alpha = K_{\bar{\alpha}} = \partial K / \partial X^\alpha$, where $X^\alpha = (T + \bar{T})^\alpha$. Next we differentiate eq. (A.1) once with respect to λ , and then a second time with respect to T^α , giving

$$X^\beta K_\beta(\lambda X) \equiv -\frac{3}{\lambda} \quad (\partial/\partial\lambda) \quad (\text{A.3})$$

$$K_\alpha(\lambda X) + \lambda X^\beta K_{\alpha\beta}(\lambda X) \equiv 0 \quad (\partial^2/\partial T^\alpha \partial \lambda). \quad (\text{A.4})$$

Contracting two copies of eq. (A.4) together using the inverse matrix $K^{\alpha\gamma}$ then gives

$$\begin{aligned} K^{\alpha\gamma}(\lambda X) K_\alpha(\lambda X) K_\gamma(\lambda X) &= \lambda^2 K^{\alpha\gamma}(\lambda X) K_{\alpha\xi}(\lambda X) K_{\gamma\rho}(\lambda X) X^\xi X^\rho \\ &= \lambda^2 K_{\alpha\beta}(\lambda X) X^\alpha X^\beta, \end{aligned} \quad (\text{A.5})$$

which may be further simplified by contracting eq. (A.4) with X^α and using eq. (A.3), to get

$$K^{\alpha\gamma}(\lambda X) K_\alpha(\lambda X) K_\gamma(\lambda X) = \lambda^2 K_{\alpha\beta}(\lambda X) X^\alpha X^\beta = 3. \quad (\text{A.6})$$

The desired result is now obtained by evaluating at $\lambda = 1$.

A.2 Theta functions and periodicity

We adopt the following definition for the Jacobi theta function

$$\begin{aligned} \vartheta_1(u|\tau) = \vartheta_1(u; q) &\equiv -i \sum_{n=-\infty}^{\infty} (-)^n e^{(2n+1)iu} q^{(n+1/2)^2} \\ &= 2q^{1/4} \sum_{n=0}^{\infty} (-)^n q^{n(n+1)} \sin[(2n+1)u], \end{aligned} \quad (\text{A.7})$$

with $q = e^{i\pi\tau}$. This satisfies

$$\begin{aligned} \vartheta_1(u \pm \pi|\tau) &= -\vartheta_1(u|\tau) \\ \vartheta_1(u \pm \pi\tau|\tau) &= -q^{-1} e^{\mp 2iu} \vartheta_1(u|\tau), \end{aligned} \quad (\text{A.8})$$

under the displacements that define the periods of the torus T_2 .

The combination $F_r(z, \tau) = \vartheta_1[\pi(z_r - z)|\tau] \vartheta_1[\pi(z_r + z)|\tau]$ appearing in the superpotential therefore transforms as

$$\begin{aligned} F_r(z + 1, \tau) &= F_r(z, \tau) \\ F_r(z + \tau, \tau) &= e^{-4i\pi z - 2i\pi\tau} F_r(z, \tau), \end{aligned} \quad (\text{A.9})$$

for any z_r . Clearly the combination $e^{-aS} F_r(z, \tau)$ is therefore invariant under the combined transformations $(z, S) \rightarrow (z + 1, S)$ and

$$z \rightarrow z + \tau, \quad S \rightarrow -\frac{2\pi i}{a}(2z + \tau). \quad (\text{A.10})$$

When $\tau_2 \gtrsim 1$ we have $|q| \ll 1$ and so the above series for ϑ_1 is well approximated by its first terms, $\vartheta_1(u|\tau) \simeq 2q^{1/4} \sin u$, and so

$$F_r \simeq 4q^{1/2} \sin[\pi(z_r - z)] \sin[\pi(z_r + z)] = 2q^{1/2} \left[\cos(2\pi z) - \cos(2\pi z_r) \right]. \quad (\text{A.11})$$

Using $z_0 = 0$, $z_1 = \frac{1}{2}$, $z_2 = \frac{1}{2}\tau$ and $z_3 = \frac{1}{2}(1 + \tau)$, we have respectively $\cos(2\pi z_0) = 1$, $\cos(2\pi z_1) = -1$, $\cos(2\pi z_2) = \cos(\pi\tau)$ and $\cos(2\pi z_3) = -\cos(\pi\tau)$. Notice in particular that in this limit

$$\sum_{r=0,1} F_r \simeq \sum_{r=2,3} F_r \simeq \frac{1}{2} \sum_{r=0}^3 F_r \simeq 4q^{1/2} \cos(2\pi z). \quad (\text{A.12})$$

A.3 Scaling behavior

This appendix displays a useful scaling property that allows one to relate numerical results for different choices of parameters.

The scalar potential arises as the sum of an F -term, D -term and an up-lifting contribution, $V = V_F + V_D + V_{\text{up}}$, with the D -term and uplifting contributions having the form

$$V_{\text{up}} = \frac{E}{X^m t^n}, \quad (\text{A.13})$$

and

$$V_D = \frac{E_r}{\text{Re} f_r t^2}, \quad (\text{A.14})$$

where $t = \text{Re} T$, E and E_r are constants, $X = 2[s - cg(z, \bar{z})]$, $s = \text{Re} S$ and $f_r = S - (1/a)h(z)$. Here g and h are functions whose form is not important in what follows. The constant c is related to a by $ac = 2\pi$, due to the requirement that the potential be periodic under the toroidal shift $z \rightarrow z + \tau$. The F -term potential is similarly computed using the Kähler potential

$$K = -\ln X - 2\ln(T + \bar{T}), \quad (\text{A.15})$$

and superpotential

$$W = W_0 + A(z)e^{-aS} + Be^{-bT}. \quad (\text{A.16})$$

Here W_0 , B , b and a are constants — with a the same constant as appears in f_r — and $A(z)$ is a function whose detailed form is not important for the argument now to be made.

These contributions to the scalar potential have the property that they scale simply under the following redefinitions

$$s \rightarrow \lambda_1 s, \quad t \rightarrow \lambda_2 t, \quad a \rightarrow a/\lambda_1, \quad b \rightarrow b/\lambda_2, \quad \text{and} \quad E \rightarrow \lambda_1^{m-1} \lambda_2^{n-2}, \quad (\text{A.17})$$

with z , E_r , A , B and W_0 held fixed. With these choices we have $X \rightarrow \lambda_1 X$, $\text{Re} f_r \rightarrow \lambda_1 \text{Re} f_r$ and so $W \rightarrow W$, $e^K \rightarrow e^K / (\lambda_1 \lambda_2^2)$. This makes V_F , V_D and V_{up} all scale very simply:

$$V \rightarrow \frac{V}{\lambda_1 \lambda_2^2}. \quad (\text{A.18})$$

Because V_F is quadratic in W and its derivatives, the F -term potential also rescales as $V_F \rightarrow \lambda^2 V_F$ under the scalings

$$x \rightarrow \lambda x, \quad \text{where} \quad x = \{A, B, W_0\}. \quad (\text{A.19})$$

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