

Simple Models of Magnetism

Ralph Skomski

*Department of Physics and Astronomy
and
Nebraska Center for Materials and Nanoscience
University of Nebraska*

OXFORD
UNIVERSITY PRESS

Contents

List of abbreviations	xii
List of panels and tables	xiv
Preface	xv
1 Introduction: The simplest models of magnetism	1
1.1 Field and magnetization	2
1.2 The circular-current model	4
1.3 Paramagnetic spins	6
1.4 Ising model and exchange	8
1.5 The viscoelastic model of magnetization dynamics	10
Exercises	13
2 Models of exchange	15
2.1 Atomic origin of exchange	17
2.1.1 One-electron wave functions	18
2.1.2 Two-electron wave functions	21
2.1.3 Hamiltonian and spin structure	22
2.1.4 Heisenberg model	25
2.1.5 Independent-electron approximation	27
2.1.6 Correlations	29
2.1.7 *Hubbard model	32
2.1.8 *Kondo model	34
2.2 Magnetic ions	36
2.2.1 Atomic orbitals	36
2.2.2 Angular-momentum algebra	39
2.2.3 Vector model and Hund's rules	41
2.2.4 Spin and orbital moment	41
2.3 Exchange between local moments	44
2.3.1 Exchange in oxides	44
2.3.2 Ruderman-Kittel exchange	46
2.3.3 Zero-temperature spin structure	48
2.4 Itinerant magnetism	51
2.4.1 Free electrons, Pauli susceptibility, and the Bloch model	54
2.4.2 Band structure	58

2.4.3	Stoner model and beyond	63
2.4.4	*Itinerant antiferromagnets	66
	Exercises	69
3	Models of magnetic anisotropy	73
3.1	Phenomenological models	74
3.1.1	Uniaxial anisotropy	75
3.1.2	Second-order anisotropy of general symmetry	76
3.1.3	Higher-order anisotropies of nonuniaxial symmetry	78
3.1.4	Cubic anisotropy	78
3.1.5	Anisotropy coefficients	79
3.1.6	Anisotropy fields	80
3.2	Models of pair anisotropy	80
3.2.1	Dipolar interactions and shape anisotropy	81
3.2.2	Demagnetizing factors	82
3.2.3	Applicability of the shape-anisotropy model	83
3.2.4	The Néel model	83
3.3	Spin-orbit coupling and crystal-field interaction	84
3.3.1	Relativistic origin of magnetism	85
3.3.2	Hydrogen-like atomic wave functions	87
3.3.3	Crystal-field interaction	87
3.3.4	Quenching	89
3.3.5	Spin-orbit coupling	90
3.4	The single-ion model of magnetic anisotropy	91
3.4.1	Rare-earth anisotropy	91
3.4.2	Point-charge model	95
3.4.3	The superposition model	97
3.4.4	Transition-metal anisotropy	98
3.5	Other anisotropies	100
3.5.1	Magnetoelasticity	100
3.5.2	Anisotropic exchange	101
3.5.3	Models of surface anisotropy	102
	Exercises	104
4	Micromagnetic models	107
4.1	Stoner-Wohlfarth model	110
4.1.1	Aligned Stoner-Wohlfarth particles	111
4.1.2	Angular dependence	112
4.1.3	Spin reorientations and other first-order transitions	113
4.1.4	Limitations of the Stoner-Wohlfarth model	115
4.2	Hysteresis	116
4.2.1	Micromagnetic free energy	117
4.2.2	*Magnetostatic self-interaction	118
4.2.3	*Exchange stiffness	119
4.2.4	Linearized micromagnetic equations	120
4.2.5	Micromagnetic scaling	122
4.2.6	Domains and domain walls	123

4.3	Coercivity	128
4.3.1	Nucleation	130
4.3.2	Pinning	135
4.3.3	Phenomenological coercivity modeling	139
4.4	Grain-boundary models	141
4.4.1	Boundary conditions	141
4.4.2	Spin structure at grain boundaries	143
4.4.3	Models with atomic resolution	144
4.4.4	Nanojunctions	145
	Exercises	146
5	Finite-temperature magnetism	149
5.1	Basic statistical mechanics	150
5.1.1	Probability and partition function	152
5.1.2	*Fluctuations and response	153
5.1.3	Phase transitions	155
5.1.4	Landau theory	156
5.2	Spin-Space modeling	159
5.2.1	Heisenberg models	160
5.2.2	Ising, XY, and other n -vector models	161
5.2.3	*Other discrete and continuum spin models	162
5.2.4	Ionic excitations	163
5.2.5	Spin fluctuations in itinerant magnets	164
5.3	Mean-field models	167
5.3.1	Mean-field Hamiltonians	168
5.3.2	Basic mean-field predictions	169
5.3.3	*Ornstein-Zernike correlations	171
5.3.4	Magnetization and Curie temperature	172
5.3.5	*Mean-field Curie temperature of n -vector models	173
5.3.6	Two-sublattice magnetism	174
5.3.7	Merits and limitations of mean-field models	178
5.4	Critical behavior	179
5.4.1	One-dimensional models	180
5.4.2	Superparamagnetic clusters	181
5.4.3	*Ginzburg criterion	183
5.4.4	Fluctuations and criticality	184
5.4.5	Renormalization group	187
5.5	Temperature dependence of anisotropy	190
5.5.1	Callen and Callen model	191
5.5.2	Rare-earth anisotropy	193
5.5.3	Sublattice modeling	195
	Exercises	196
6	Magnetization dynamics	199
6.1	Quantum dynamics and resonance	199
6.1.1	Spin precession	201
6.1.2	Uniform magnetic resonance	202

6.1.3	Spin waves	203
6.1.4	Spin dynamics in inhomogeneous magnets*	206
6.2	Relaxation	208
6.2.1	Damped precession	209
6.2.2	*Physical origin of relaxation	210
6.2.3	*A mechanical model	211
6.3	Coarse-grained models	213
6.3.1	Master equation	214
6.3.2	Fokker-Planck equations	216
6.3.3	Langevin models	218
6.4	Slow magnetization dynamics	220
6.4.1	Magnetic viscosity and sweep-rate dependence	223
6.4.2	Superposition model of magnetic viscosity	223
6.4.3	Asymptotic behavior*	225
6.4.4	Energy-barrier models	226
6.4.5	*Linear and other laws	227
6.4.6	Superparamagnetism	228
6.4.7	*Fluctuations	229
	Exercises	233
7	Special topics and interdisciplinary models	237
7.1	Disordered magnets and spin glasses	237
7.1.1	Atomic disorder and electronic structure	238
7.1.2	*Green Functions	239
7.1.3	Ferromagnetic order in inhomogeneous magnets	242
7.1.4	Spin glasses	244
7.2	Soft matter, transport, and magnetism	247
7.2.1	Random walks, polymers, and diffusion	248
7.2.2	*The $n = 0$ vector-spin model	249
7.2.3	Polymers and critical dimensionality	250
7.2.4	Percolation	252
7.2.5	Diffusive transport	255
7.2.6	Gases in magnetic metals	256
7.2.7	Magnetoresistance	258
7.2.8	Other transport phenomena involving magnetism	261
7.3	Bruggeman model	263
7.3.1	Static and dynamic properties	263
7.3.2	*Parameterization	265
7.3.3	*Self-consistent materials equations	266
7.3.4	*The response parameter g	267
7.3.5	*Percolation in the Bruggeman model	267
7.4	Nanostructures, thin films, and surfaces	268
7.4.1	Length scales in nanomagnetism	270
7.4.2	Nanomagnetic effects of atomic origin	271
7.4.3	Random anisotropy	274

7.4.4	*Cooperative magnetization processes	277
7.4.5	Two-phase nanostructures	279
7.5	Beyond magnetism	282
7.5.1	Metallurgy	283
7.5.2	Biology and medicine	285
7.5.3	Social sciences	286
	Exercises	286
Appendix		289
A.1	Units and constants	289
A.1.1	Units systems and notation	289
A.1.2	Unit conversions	290
A.1.3	Physical constants	290
A.2	Mathematics	290
A.2.1	Linear equations	290
A.2.2	Eigenmode analysis	292
A.2.3	Real 2×2 matrices	293
A.2.4	Vector and functional calculus	295
A.2.5	Useful formulae	297
A.3	Basic quantum mechanics	297
A.3.1	Time dependence	298
A.3.2	Eigenvalues and eigenfunctions	298
A.3.3	Perturbation theory	299
A.3.4	Quantum statistics	300
A.3.5	Relativistic quantum mechanics	302
A.4	Electromagnetism	304
A.4.1	Maxwells equations	304
A.4.2	Simple magnetostatic solutions	306
A.4.3	Simple dynamic solutions	308
A.5	Magnetic materials	309
A.5.1	Transition-metal elements and alloys	310
A.5.2	Magnetic oxides	314
A.5.3	Rare-earth magnets	314
A.6	Forgotten and reinvented	315
References		319
Index		335