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# Nonstandard neutrino-neutrino refractive effects in dense neutrino gases

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We investigate the effects of nonstandard four-fermion neutrino-neutrino interactions on the flavor evolution of dense neutrino gases. We find that in the regions where the neutrino-neutrino refractive index leads to collective flavor oscillations, the presence of new neutrino interactions can produce flavor equilibration in both normal and inverted neutrino mass hierarchy. In realistic supernova environments, these effects are significant if the nonstandard neutrino-neutrino interaction strength is comparable to the one expected in the standard case, dominating the ordinary matter potential. However, very small nonstandard neutrino-neutrino couplings are enough to trigger the usual collective neutrino flavor transformations in the inverted neutrino mass hierarchy, even if the mixing angle vanishes exactly.

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## I. INTRODUCTION

New neutrino interactions are predicted by several extensions of the standard electroweak theory. These new interactions can be treated in an effective (low-energy) framework by using four-fermion operators, e.g., $\mathcal{O}_{\alpha\beta} \sim [\bar{\nu}_\alpha \gamma^\mu P_L \nu_\beta][\bar{f}\gamma_\mu P_L f]$ with strength $G_{\alpha\beta}$, inducing either flavor-changing ($\alpha \neq \beta$) or diagonal but flavor nonuniversal neutrino transitions. Nonstandard interactions (NSI) of neutrinos with charged fermions in matter, and their interplay with the oscillation phenomenon, have been investigated in many different contexts. An incomplete list includes analyses related to the solar neutrino problem [1–5], to the atmospheric neutrino anomaly [6–9], to supernova neutrinos [10–16], to primordial neutrinos [17], to the production or detection of laboratory neutrinos [18,19], and to future long-baseline projects [20–22]. Neutrino-neutrino interactions are even more difficult to constrain experimentally [23–25]. In particular, four-fermion (left-handed) neutrino-neutrino interactions as large as the standard model neutral current couplings are viable without violating astrophysical or laboratory bounds [26] (see also Ref. [27] for a recent discussion on how to generate this type of interaction in a gauge invariant way).

Dense astrophysical environments like the Early Universe, core-collapse supernovae and accretion disks of coalescing neutron stars are among the few systems where neutrino-neutrino interactions play a role, via the refractive index that can affect the flavor evolution of the system. In particular, it has been shown that flavor oscillations in dense neutrino gases exhibit collective phenomena caused by neutrino-neutrino interactions [28–62]. Since physics beyond the standard model can plausibly induce at least small deviations from the standard predictions, it seems interesting to investigate the impact of new neutrino-neutrino interactions on collective flavor oscillations in dense neutrino gases.

We devote our work to this purpose. In Sec, II, we generalize the equations of motion for a neutrino ensemble in the presence of new neutrino-neutrino interactions. We present our results for the flavor evolution of a homogeneous and isotropic gas of neutrinos with decreasing density in Sec. III, where we find that NSI among neutrinos could completely equilibrate the flavor content of the system in both normal and inverted neutrino mass hierarchy. In the case of neutrinos streaming off a supernova (SN) core, these new effects are inhibited by the strong ordinary matter potential unless the strength of the NSI is of the same order of magnitude as the one predicted by the standard model. However, also in the case of small values of nonstandard neutrino-neutrino couplings, these are enough to trigger collective flavor transformations in inverted mass hierarchy for vanishing mixing, where in the standard case no evolution is expected. Finally, in Sec. IV we comment on our results and conclude.

## II. EQUATIONS OF MOTION WITH NONSTANDARD NEUTRINO-NEUTRINO INTERACTIONS

Flavor oscillations of a homogeneous ensemble of neutrinos and antineutrinos are described by an equation of motion (EOM) for each mode **p** [63]

$$i\dot{\rho}_{\mathbf{p}} = [(\Omega^0_{\mathbf{p}} + V + \Omega^S_{\mathbf{p}}), \rho_{\mathbf{p}}], \quad (1)$$

$$-i\dot{\bar{\rho}}_{\mathbf{p}} = [(\Omega^0_{\mathbf{p}} - V - \Omega^S_{\mathbf{p}}), \bar{\rho}_{\mathbf{p}}], \quad (2)$$

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where $[\cdot,\cdot]$ denotes the commutator. For ultrarelativistic neutrinos of momentum $p$, the matrix of vacuum oscillation frequencies, expressed in the mass basis, is $\Omega^0_{\mathbf{p}} = \mathrm{diag}(m_1^2, m_2^2, m_3^2)/2p$, $m_{i=1,2,3}$ being the neutrino masses. At leading order, the only nonvanishing element of the matter potential matrix in the weak-interaction basis is due to the $\nu_e$ forward scattering on background electrons, leading to $V = \sqrt{2}G_F n_e \,\mathrm{diag}(1, 0, 0)$, where $G_F$ is the Fermi constant and $n_e$ is the net electron number density. Neutrino-neutrino self-interactions introduce an additional contribution $\Omega^S_{\mathbf{p}}$ to the refractive energy shift. One finds [63]

$$\Omega^S_{\mathbf{p}} = \sqrt{2}G_F \int d\mathbf{q}(1 - \mathbf{v_q}\cdot\mathbf{v_p})\{G(\rho_{\mathbf{q}} - \bar{\rho}_{\mathbf{q}})G + G\,\mathrm{Tr}[(\rho_{\mathbf{q}} - \bar{\rho}_{\mathbf{q}})G]\}, \tag{3}$$

where $d\mathbf{q} \equiv d^3\mathbf{q}/(2\pi)^3$ and $\mathbf{v_p}$ is the velocity. The factor $(1 - \mathbf{v_q}\cdot\mathbf{v_p})$ implies "multiangle effects" for neutrinos moving on different trajectories [38]. In the standard model, the dimensionless coupling matrix $G$ is the identity matrix. If we consider the possibility of new physics beyond the standard model, the coupling matrix can assume a nonuniversal and/or nondiagonal structure.

We now focus on a two-flavor system $\{\nu_e, \nu_x\}$, where we expand all the $2\times 2$ matrices in the EOM in terms of the $2\times 2$ unit matrix 1 and the Pauli matrices $\boldsymbol{\sigma}$. Explicitly, we define [39]

$$\Omega^0_{\mathbf{p}} = \frac{1}{2}(\omega_0 1 + \omega_{\mathbf{p}}\mathbf{B}\cdot\boldsymbol{\sigma}), \qquad V = \frac{\lambda}{2}(1 + \mathbf{L}\cdot\boldsymbol{\sigma}),$$
$$\rho_{\mathbf{p}} = \frac{1}{2}(f_{\mathbf{p}} 1 + n_{\bar{\nu}}\mathbf{P_p}\cdot\boldsymbol{\sigma}), \qquad \bar{\rho}_{\mathbf{p}} = \frac{1}{2}(\bar{f}_{\mathbf{p}} 1 + n_{\bar{\nu}}\bar{\mathbf{P}}_{\mathbf{p}}\cdot\boldsymbol{\sigma}),$$
$$G = \frac{1}{2}(g_0 1 + \mathbf{g}\cdot\boldsymbol{\sigma}). \tag{4}$$

Here, the overall neutrino (antineutrino) density is given by $\int d\mathbf{p} f_{\mathbf{p}} = n_\nu$ ($\int d\mathbf{p}\bar{f}_{\mathbf{p}} = n_{\bar{\nu}}$). For simplicity and without loss of generality, we here assume that initially only $\nu_e$ and $\bar{\nu}_e$ are present with an excess neutrino density of $n_{\nu_e} = (1+\xi)n_{\bar{\nu}_e}$; in numerical examples, we shall assume that the asymmetry parameter between neutrino species is $\xi = 0.25$ [42]. The vectors $\mathbf{P_p}$ and $\bar{\mathbf{P}}_{\mathbf{p}}$ are the neutrino and antineutrino polarization vectors. We define the total polarization vectors $\mathbf{P} = \int d\mathbf{p}\mathbf{P_p}$, $\bar{\mathbf{P}} = \int d\mathbf{p}\bar{\mathbf{P}}_{\mathbf{p}}$, which are initially normalized such that $\mathbf{P}(0) = (1+\xi)\mathbf{e}_z$ and $\bar{\mathbf{P}}(0) = \mathbf{e}_z$, $\mathbf{e}_z$ being the unit vector in the positive $z$-direction. Here, we have chosen our coordinate system in such a way that a polarization vector pointing in the positive $z$-direction represents electron neutrinos, whereas an orientation in the negative $z$-direction corresponds to a combination of muon and tau neutrinos, which we denote $\nu_x$. We also have $\omega_0 = (m_1^2 + m_2^2)/2E$, the vacuum oscillation frequency is $\omega_{\mathbf{p}} = (m_2^2 - m_1^2)/2E$ with the energy $E = |\mathbf{p}|$; $\mathbf{L}$ is a unit vector pointing in the direction singled out by the neutrino potential in the charged fermion background, and $\lambda$ its normalization; for the cases considered here, $\mathbf{L} = \mathbf{e}_z$ and $\lambda = \sqrt{2}G_F n_e$. The unit vector $\mathbf{B}$ points in the mass eigenstate direction in flavor space, such that $\mathbf{B}\cdot\mathbf{L} = -\cos 2\theta$, where $\theta$ is the vacuum mixing angle. The neutrino-neutrino interaction couplings are given by $\{g_0, \mathbf{g}\}$, with the standard model case corresponding to $g_0 = 2$, $|\mathbf{g}| = 0$; it is thus the vector $\mathbf{g}$ which is characteristic of NSI, its third component setting possible nonuniversal couplings (i.e., $\nu_e - \nu_e$ different from $\nu_x - \nu_x$) while its first two components characterize flavor-violating operators. Note that we have assumed that the effective four-neutrino vertex is a good description of the new dynamics. This is done having in mind an effective field theory correction to the standard model dynamics induced by an energy scale $M \gg M_Z$, with $|\mathbf{g}| \sim (M_Z/M)^2$.

With above definitions, the neutrino EOMs assume the form

$$\dot{\mathbf{P}}_{\mathbf{p}} = (\omega_{\mathbf{p}}\mathbf{B} + \lambda\mathbf{L} + \boldsymbol{\Omega}^S_{\mathbf{p}})\times\mathbf{P_p}, \tag{5}$$

where the neutrino-neutrino interaction "Hamiltonian" $\boldsymbol{\Omega}^S_{\mathbf{p}}$ is

$$\boldsymbol{\Omega}^S_{\mathbf{p}} = \mu\int d\mathbf{q}(1 - \mathbf{v_q}\cdot\mathbf{v_p})\Big\{[g_0\xi + \mathbf{g}\cdot(\mathbf{P_q} - \bar{\mathbf{P}}_{\mathbf{q}})]\mathbf{g} + \frac{1}{4}(g_0^2 - |\mathbf{g}|^2)(\mathbf{P_q} - \bar{\mathbf{P}}_{\mathbf{q}})\Big\}, \tag{6}$$

and we have defined the parameter $\mu = \sqrt{2}G_F n_{\bar{\nu}}$ which normalizes the neutrino-neutrino interaction strength. For antineutrinos, the EOMs are the same as for neutrinos with the substitution $\omega_{\mathbf{p}} \to -\omega_{\mathbf{p}}$. It is also trivial to check that in the limit of standard model-only couplings ($g_0 = 2$, $|\mathbf{g}| = 0$), no direction is singled out in the flavor basis and one recovers the standard neutrino-neutrino Hamiltonian.

In our analysis we will assume the single-angle approximation, in which we substitute the angular structure of neutrino-neutrino interactions in Eq. (6) with an effective neutrino-neutrino interaction strength $\mu_r$ such that:

$$\mu(1 - \mathbf{v_p}\cdot\mathbf{v_q}) \to \mu_r. \tag{7}$$

This approximation has been numerically shown to be a good description of the flavor evolution in the supernova environment, where a significant asymmetry between neutrinos and antineutrinos is expected. We have explicitly checked that the single-angle approximation is a good description of the flavor evolution in presence of nonstandard interactions also for the examples shown in this work. In particular, the asymmetry between neutrinos and antineutrinos prevents possible effects of multiangle decoherence in the neutrino ensemble [42]. Inspired by this case, in our numerical examples we use an effective neutrino interaction strength [39]

$$\mu_r = 0.35\times 10^5\ \mathrm{km}^{-1}[1 - (1 - r_{10}^{-2})^{1/2}]r_{10}^{-2}, \tag{8}$$

where $r_{10} = r/(10\text{ km})$. We consider a spherically symmetric system in which neutrinos are emitted by a sphere at $R = 10$ km. In this situation, the only spatial variable that characterizes the flavor evolution is the radial coordinate $r$. Concerning the other input parameters, in our numerical examples we will consider a single mode system with $\omega = 0.3\text{ km}^{-1}$, corresponding to typical supernova neutrino energies for the atmospheric mass squared difference. We choose $\theta = 0$ which, in the standard case, would mean that no flavor evolution can occur. We always assume $g_0 = 2$, which essentially means that the overall strength of the neutrino-neutrino interactions are given by the standard model.

There is an important property which is worth noting: If $\mathbf{g} \parallel \mathbf{L}$, then the quantity $\mathbf{B} \cdot \mathbf{D} = \mathbf{B} \cdot (\mathbf{P} - \bar{\mathbf{P}})$ is conserved. This property is nothing but the conservation of lepton number found in the standard case [39,40]. In this situation, one expects to recover the standard dynamics. In the following, we shall thus concentrate on the cases where $\mathbf{g}$ is not parallel to $\mathbf{L}$, i.e., when flavor-changing currents are present. In the standard case, where $\mathbf{B} \cdot \mathbf{D} = -D_z$ (for $\theta = 0$) is preserved, to get a diagnostic of the system it is enough to follow, e.g., the evolution of $\bar{P}_z$, which describes the polarization state of antineutrinos. In the present case, it is useful to monitor also $D_z$, where the differences with respect to the standard model case manifest more clearly.

## III. FLAVOR EVOLUTION

### A. Vacuum case

To illustrate the effect of nonstandard neutrino-neutrino couplings on the flavor evolution of a neutrino gas we first consider a case in which the matter term $\lambda = 0$. Since we have chosen $\theta = 0$, we assume that only $g_1 \neq 0$ without loss of generality. We start our discussion of NSI effects referring to values of $|\mathbf{g}| \leq 10^{-1}$. In this regime, the terms $\mathcal{O}(|\mathbf{g}|^2)$ in Eq. (6) are subleading and it is clear that the new terms have the form $\mathbf{g} \times \mathbf{P}_\mathbf{p}$.

The nonstandard couplings are expected to play a significant role when the transverse nonstandard term $\xi \mu_r g_1$ in Eq. (6) dominates over the vacuum term $\omega B_z = \omega$. Thus, we expect NSI effects to be relevant for interaction strengths $g_1 \gtrsim 10^{-4}$. For smaller values, we expect to recover the standard picture. However, we note that also smaller values of nonstandard coupling can have an interesting consequence. If we assume vanishing vacuum mixing, we do not expect any flavor conversion in the standard case, since the polarization vectors are exactly aligned with $\mathbf{B}$. However, in this situation, the nonstandard term transverse to $\mathbf{B}$ is enough to trigger an instability in flavor space, producing a small offset with respect to the $\mathbf{B}$ direction. This is enough to obtain flavor transformations in inverted hierarchy analogous to the standard case [39]. In particular, we initially observe the synchronized oscillations in which the polarization vectors essentially remain fixed to their initial value until the start of the bipolar oscillations, which lead to the inversion of the polarization vectors, conserving lepton flavor number at each step. In normal hierarchy the system is initially in its stable configuration and the nonstandard effects have no impact. The behavior of $\bar{P}_z$ is represented in the top left panel of Fig. 1 for the case of $g_1 = 10^{-7}$.

For $g_1 \gtrsim 10^{-4}$ we observe that a transverse nonstandard component is enough to produce wild oscillations in the system. Essentially, the nonstandard interactions act as an external force on the standard system, violating the symmetry that keeps $D_z$ constant. Naively, it seems that the effect can be easily accounted for by going into a frame corotating along $\mathbf{g}$ [40,58]: we have just introduced a nonstandard matter potential profile. The angle between $\mathbf{B}$—now rotating—and $\mathbf{g}$ is not necessarily small if $\mathbf{g}$ has a component perpendicular to $\mathbf{B}$, even for a small vacuum mixing angle. More important, however, is that in this case the initial condition of the neutrino state might be significantly offset compared with the $\mathbf{g}$ vector, i.e., $\mathbf{P}(0)$ and $\bar{\mathbf{P}}(0)$ might have a significant component orthogonal to $\mathbf{g}$, resulting in a large amplitude of oscillation.

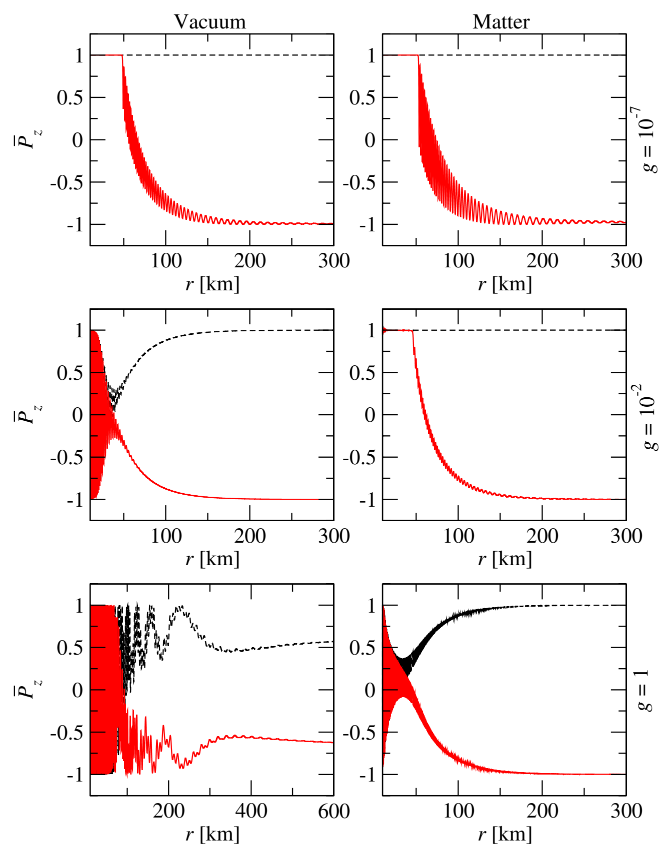


FIG. 1 (color online). The single-angle evolution of $\bar{P}_z$ in normal (dashed curves) and inverted hierarchy (solid curves) for $\theta = 0$ in vacuum (left panels) and in presence of matter (right panels). We consider values of nonstandard coupling $g_1$ equal to $10^{-7}$ (top panels), $10^{-2}$ (central panels) and 1 (bottom panels). Note the different scale for $r$ in the lower left panel.

In presence of transverse nonstandard terms, the positive electron lepton number we started with is not conserved, since

$$\dot{D}_z \simeq \mu_r \xi [g_1 D_y - g_2 D_x]. \tag{9}$$

In Fig. 2 (top left panel) we show the behavior of $D_z$ for the case of $g_1 = 10^{-2}$. At small radii $D_z$ oscillates with a full amplitude. However, reducing $\mu_r$ the amplitude of the oscillations diminishes: The decline of the upper envelope of $D_z$ follows the decrease of $g_1 \mu_r$ compared to the vacuum oscillation frequency $\omega$. Referring to the "pendulum" analogy [39,40], the external force introduced by the NSI is initially strong enough to completely dominate the dynamics. However, as the strength $g\mu_r$ of the external force decreases, it is finally so much weaker than the potential $\omega$ that it can no longer lead to significant oscillations. If the NSI term is sufficiently strong, the excess in electron neutrinos we started with will relax to complete flavor equilibration.

In Fig. 3, we show the behavior of $D_z$ at the end of the flavor conversions as a function of $g_1$. We observe that nonstandard effects start to arise for $g_1 \gtrsim 10^{-4}$, in agreement with our prediction. For $10^{-2} \lesssim g_1 \lesssim 10^{-1}$ we see that $D_z$ saturates at zero. The corresponding behavior of $\bar{P}_z$ is shown in Fig. 1 for $g_1 = 10^{-2}$ (central left panel) for both normal and inverted hierarchy. At the beginning, it oscillates with full amplitude which is then reduced following the decrease of $\mu_r$. When the NSI effects have saturated and $D_z$ has reached zero, the polarization vectors follow the standard pendulum dynamics with null lepton number [39]. As $\mu_r \to 0$, the kinetic energy of the flavor pendulum tends to zero and in both normal and inverted hierarchy the polarization vectors tend to minimize the potential energy [39]

$$E_\omega = \frac{\omega}{2}(|\mathbf{S}| + \mathbf{B}\cdot\mathbf{S}), \tag{10}$$

where $\mathbf{S} = \mathbf{P} + \bar{\mathbf{P}}$. In normal neutrino mass hierarchy, where $\omega\mathbf{B}$ points in the negative $z$-direction, $\bar{P}_z$ rises again and returns to its initial value. In inverted hierarchy, where $\mathbf{B}$ points in the positive $z$-direction and the original equilibrium position was unstable, the initial dynamics are enough to trigger significant flavor changes. The resulting

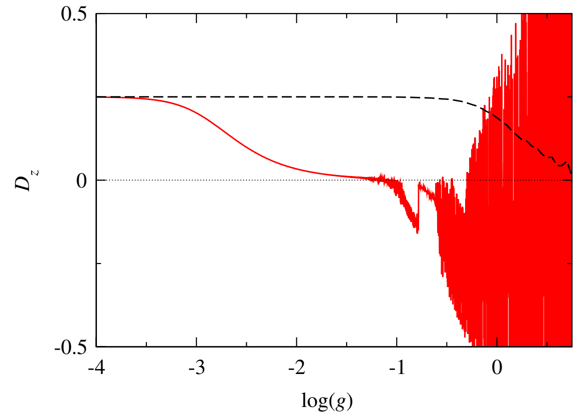


FIG. 3 (color online). The value of $D_z$ at the end of flavor evolution as a function of $g_1$ for inverted neutrino mass hierarchy in the vacuum case (solid curve) and in presence of matter (dashed curve). Similar to the normal mass hierarchy case (not shown).

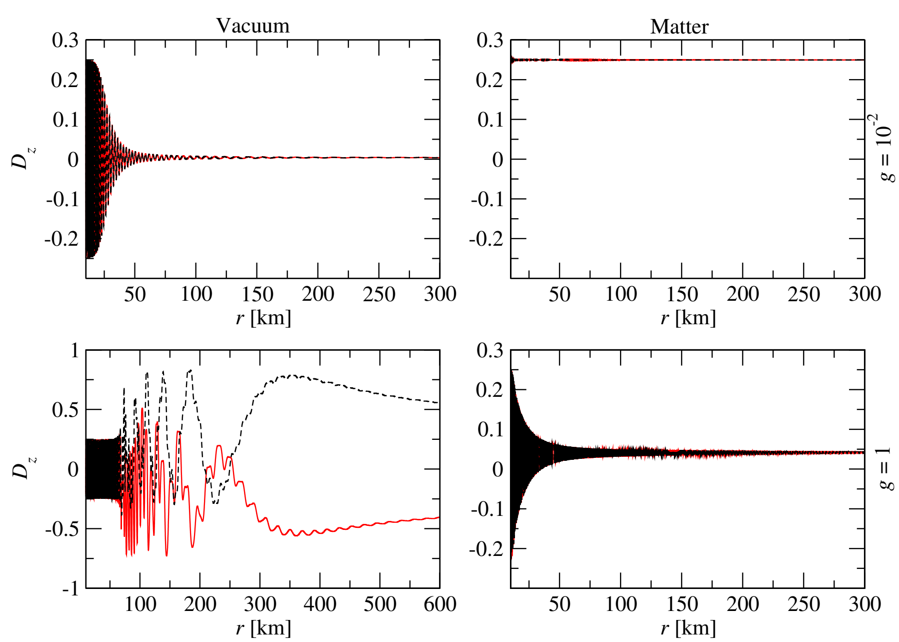


FIG. 2 (color online). The evolution of $D_z$ in normal (dashed curve) and inverted (solid curve) in vacuum (left panels) and in presence of matter (right panels). We use $g_1 = 10^{-2}$ (upper panels) and $g_1 = 1$ (bottom panels). Note the different scale for $r$ in the lower left panel.

behavior is the total inversion of $\bar{P}_z$. The main effects of a subleading nonstandard neutrino-neutrino interaction is thus to produce a flavor equilibration in the ensemble.

We now consider large values of the $\mathbf{g}$ couplings. Even if from a theoretical perspective these are quite unnatural, the loose experimental bounds on these interactions allow us to play also with these wilder scenarios. Large couplings produce a modulation of the standard neutrino-neutrino term in Eq. (6). Note that in the extreme case of $|\mathbf{g}| = 2$, the NSI term exactly compensates the standard interaction term. If it were not for the other nonstandard term on the right-hand side of Eq. (6), the system would become linear. This term produces a precession of the polarization vector $\mathbf{P}$ around the fixed $\mathbf{g}$ direction in flavor space, with a speed modulated by $\mu_r \mathbf{g} \cdot \mathbf{D}$. For generic large values of the NSI couplings, the motion of the polarization vectors in flavor space is thus a combination of two precessions, one around $\mathbf{D}$ and the other around $\mathbf{g}$, with comparable speed. Under this condition, the final value of $D_z$ at the end of the evolution is quite unpredictable as shown in Fig. 3. It is determined by the detailed combination of the two different forces that drive the system: in particular, for $g_1 =$ few $10^{-1}$, it is mostly the linear nonstandard term that matters, while for larger couplings the quadratic corrections are important. The behavior of $\bar{P}_z$ and $D_z$ as functions of $r$ is shown in Figs. 1 and 2 (lower left panels), respectively, for the case of $g_1 = 1$ in both normal and inverted hierarchy. Initially, we observe wild oscillations in these vectors that decrease their frequency following the decline of $\mu_r$. The evolution seems to maintain no track of the standard behavior. The nice pendulum analogy, driving the polarization vectors in different directions for the different neutrino mass hierarchies, remains. However, the fast oscillations of the final value of $D_z$ as a function of $g_1$ inhibit the ability of making precise predictions.

### B. Effect of matter background

In a realistic supernova environment, the propagation of a dense neutrino gas is also affected by the ordinary matter background. To characterize the matter potential $\lambda$ we refer to the time-dependent parametrization used in Refs. [46,64], inspired by supernova shock-wave simulations. In the following, we will focus on late times, fixing $t = 7$ s after the core bounce. In Fig. 4, we show the $\mu_r$ and $\lambda$ potentials for our input choices.

For typical supernova conditions, the ordinary matter term $\lambda\mathbf{L}$ dominates over subleading nonstandard interaction terms. The effect of a dominant dense matter term is to project the EOMs along the weak-interaction direction [58]. In the corotating frame around $\mathbf{L}$, the fast-rotating $\mathbf{g}$ transverse component is enough to trigger the flavor conversions in inverted hierarchy in the case of $\theta = 0$, but otherwise plays no crucial role.

In particular, for our input choices, in Fig. 3 we observe that the matter effect guarantees the conservation of $D_z$ for

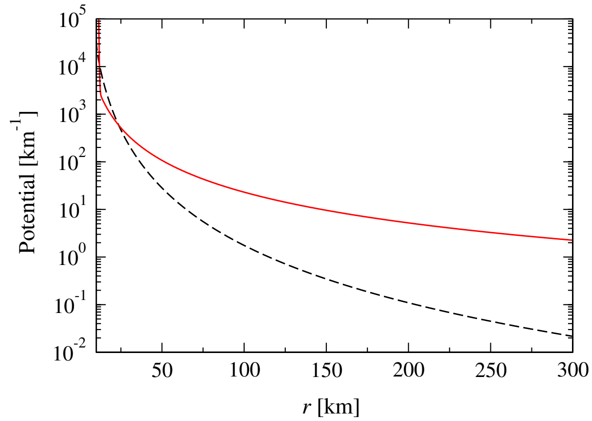


FIG. 4 (color online). Radial profile for the matter potential $\lambda$ (solid curve) and for the neutrino-neutrino potential $\mu_r$ (dashed curve).

$g_1 \lesssim 0.3$. For smaller values of $g_1$, the behavior of $\bar{P}_z$ and $D_z$ as functions of $r$ is analogous to the standard case as shown in Figs. 1 and 2 (upper right panels), respectively, for the case of $g_1 = 10^{-2}$. If we allow for $g_1 \sim \mathcal{O}(1)$, it can dominate over the $\lambda$ term close to the neutrinosphere. The behavior of $\bar{P}_z$ and $D_z$ as functions of $r$ for $g_1 = 1$ is shown in Figs. 1 and 2 (lower right panels), respectively. We observe that, for $g_1 = 1$, the vectors initially undergo very rapid oscillations. However, when $\mu_r g_1 < \lambda$, the standard matter potential dominates the dynamics and the oscillations only occur around the flavor direction, thus decreasing the flavor oscillation amplitude. Thus, at the end of flavor evolution, the value of $D_z$ saturates.

## IV. CONCLUSIONS AND OUTLOOK

In this work we have investigated the impact of nonstandard neutrino-neutrino interactions on the flavor evolution of a dense neutrino gas. First, we have generalized the equation of motion for a neutrino ensemble in presence of nonstandard neutrino-neutrino interaction terms. Then, we have shown that if ordinary matter effects are subdominant, flavor-changing neutrino-neutrino couplings lead to a dynamics very similar to the standard model analogue, with the additional feature of a complete flavor equilibration of the neutrino ensemble, both in normal and in inverted hierarchy. Including matter effects of the magnitude expected in a realistic supernova environment, subleading NSI corrections are suppressed by the dominant dense matter effect and a picture similar to the "standard" one emerges. Small NSI thus have negligible effects, except for triggering flavor conversions in inverted hierarchy even in the limiting case of $\theta = 0$. All in all, we conclude that the phenomenon of collective neutrino flavor evolution in supernovae is generally robust with respect to subleading neutrino-neutrino interactions of the four-fermion type. Potentially large effects can arise for values of the nonstandard neutrino-neutrino couplings comparable to the ordinary ones, which are phenomenologically allowed but theoretically disfavored.

Still, to make the problem tractable we made several approximations compared to a realistic physical system. Let us conclude this section by speculating on possible consequences of our findings in more realistic environments. For example, in some situations the ordinary matter effect might be weak enough for subleading NSI effects to affect the dynamics. Perhaps the most notable example is in O-Ne-Mg SN progenitors with rapidly decreasing density profiles, where, in particular, the neutronization burst might be affected [48,49,65]. Also, peculiar time-dependent signatures in the supernova neutrino burst could be produced by non-negligible NSI effects, since, at least at sufficiently late time after core bounce, the nonstandard neutrino-neutrino term could dominate over the dense matter term in the low-radii region. Another physically interesting environment is provided by accretion disks formed by coalescing neutron stars, where emitted neutrinos propagate in a region of relatively low matter density along the disk axis [57]. Nonstandard interactions could play some role in this context, perhaps with an impact on the gamma-ray burst production by neutrino-neutrino annihilations in such engines. Also note that a more realistic treatment of the problem might require a detailed three-flavor study, which goes beyond our present purpose.

Further directions of investigation may be related to the presence of nonstandard neutrino-charged fermion interactions. Their impact on matter transitions in supernovae has been explored in different works. In particular, in Ref. [12] it has been shown that these could induce new effects just above the neutrino sphere. Their impact on the collective neutrino flavor evolution still remains to be characterized.

Finally, we would like to remark that this study assumes that the nonstandard effects do not affect the formation of the neutrino spectra and the dynamics in the supernova core. It has been argued in [15,16] that sufficiently large NSI of neutrinos with SN matter may lead to drastic changes in the core electron fraction and neutrino spectra. Since the NSI considered here do not enter directly into the weak equilibrium, nor enhance neutrino-nucleus collisions, one would naively guess that at least for sufficiently small $g$ their effects should be subleading. Of course, a definitive proof of this working hypothesis would require detailed supernova simulations, including nonstandard interaction effects in the dynamics of the core-collapse.

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