TRANSVERSE MODE COUPLING INSTABILITY IN THE SPS: *HEADTAIL* SIMULATIONS AND *MOSES* CALCULATIONS

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Abstract

Since 2003, single bunches of protons with high intensity ($\sim 1.2 \ 10^{11}$ protons) and low longitudinal emittance (~ 0.2 eVs) have been observed to suffer from heavy losses in less than one synchrotron period after injection at 26 GeV/c in the CERN Super Proton Synchrotron (SPS) when the vertical chromaticity is corrected ($\xi_y \sim 0$). Understanding the mechanisms underlying this instability is crucial to assess the feasibility of an anticipated upgrade of the SPS, which requires bunches of $4 \ 10^{11}$ protons. Analytical calculations and particle tracking simulations had already agreed in predicting the intensity threshold of a fast instability. The aim of the present paper is to present a sensitive frequency analysis of the HEADTAIL simulations output using SUSSIX, which brought to light the fine structure of the mode spectrum of the bunch coherent motion. Coupling between the azimuthal modes "-2" and "-3" was clearly observed to be the reason for this fast instability.

INTRODUCTION

A campaign for the reduction of the SPS impedance took place between 1999 and 2001 to allow high-intensity LHC-type beams to be accelerated in the SPS without suffering from longitudinal microwave instability [1]. Subsequent measurements in 2003 [2] and 2006 [3] showed that the SPS intensity is now limited by a fast vertical single bunch instability at injection energy (p = 26 GeV/c) if the bunch longitudinal emittance is low ($\varepsilon_L \sim 0.2$ eVs), and the vertical chromaticity is corrected ($\xi_y \sim 0$).

This vertical instability presented the signature of a Transverse Mode Coupling Instability (TMCI): (i) The resulting heavy losses appeared within less than a synchrotron period; (ii) they could be avoided if the vertical chromaticity was increased ($\xi_y = 0.8$); and (iii) a travelling-wave pattern propagating from the head to the tail of the bunch could be observed on the data recorded on the SPS "HeadTail" monitor [4].

Calculating the coherent bunched-beam modes with the MOSES code [5] and simulating the coherent behaviour of a single bunch with the HEADTAIL code [6] agree in predicting the intensity threshold of a single bunch interacting with a broadband (BB) transverse impedance [4].

In the following, further frequency analysis of the bunch spectrum of the HEADTAIL simulation output is performed and compared with the bunch mode spectrum predicted by MOSES for a round chamber. The more realistic case of a flat chamber is then addressed, along with studies of the effect of linear coupling on the instability threshold.

ANALYTICAL CALCULATIONS WITH MOSES FOR A ROUND CHAMBER

MOSES v3.3 is used to generate the bunch mode spectrum as a function of bunch intensity, for a bunch interacting with a transverse broadband impedance of a round structure. The parameters are listed in Table 1. Horizontal and vertical planes are equivalent in this section.

Table 1: MOSES calculation parameters

Parameter Name	Value	Unit
Betatron tune spread	0	
Synchrotron tune (Q_s)	$3.24 \ 10^{-3}$	
Beam energy	26	GeV
Rms bunch length	21	cm
Beta function	40	m
Revolution frequency	$4.33 \ 10^{-2}$	MHz
Momentum compaction factor	$1.92 \ 10^{-3}$	
Linear chromaticity ($\xi_x = \xi_y$)	0	
Impedance resonant frequency	1	GHz
Impedance at resonance frequency	10	MΩ/m
Quality factor	1	

Tune shift of the bunch coherent modes

The tune shift $Re(Q - Q_x)$ with respect to the 0-current-tune Q_x is normalized to the synchrotron tune Q_s to identify each of the bunch azimuthal modes, and is plotted as a function of bunch intensity (I_b) in Fig. 1.

The azimuthal modes of the bunch are observed to separate into several radial modes, which shift with their own pace as the bunch intensity is increased. Some azimuthal modes are observed to couple, in particular modes "0" and "-1" at $I_b = 0.3$ mA, which also decouple if the bunch current is increased further. Modes "-1" and "-2", as well as modes "-2" and "-3" also couple between $I_b = 0.45$ mA and $I_b = 0.5$ mA.
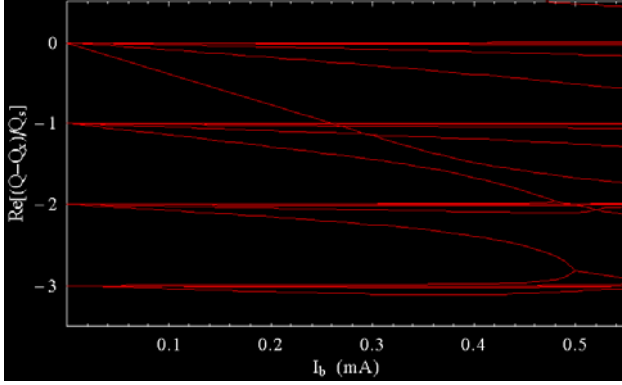
Figure 1: Zoom of the normalised real part of the mode spectrum of the bunch (from azimuthal mode "0" to azimuthal mode "-3") as a function of the bunch intensity (I_b) calculated with MOSES (see Table 1).

Growth rate

The instability growth rate τ is derived from the imaginary part of the normalized mode spectrum, and displayed in Fig. 2.
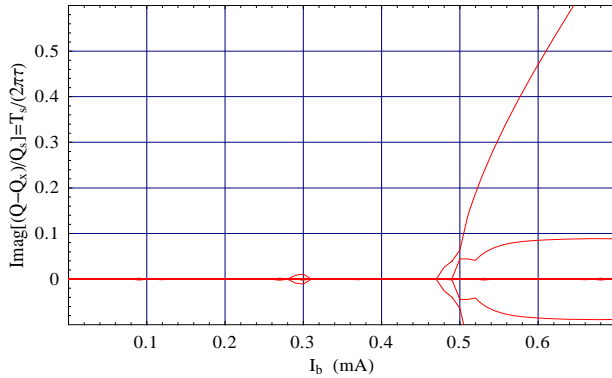

Figure 2: Imaginary part of the normalized mode spectrum of the bunch as a function of the bunch intensity (I_b) calculated with MOSES (see Table 1).

The growth rate is observed to be particularly significant for large beam intensities ($I_b > 0.47$ mA). Taking into account the observations on the real part of the mode spectrum, it can be concluded that this instability growth rate is due to coupling between azimuthal modes "-1" and "-2" from $I_b = 0.47$ mA, followed by an even stronger coupling between azimuthal modes "-2" and "-3" from $I_b = 0.5$ mA.

The coupling observed between modes "0" and "-1" at $I_b = 0.3$ mA on Fig. 1 leads to a smaller growth rate on Fig. 2. This growth rate vanishes as soon as the two modes decouple.

From these observations, it can be concluded that the instability modelled by MOSES in these conditions is the result of the coupling of transverse modes, and therefore can be referred to as a TMCI.

SIMULATIONS WITH HEADTAIL FOR A ROUND CHAMBER

The HEADTAIL code was used to simulate the interaction of a low longitudinal emittance bunch of macroparticles with a transverse impedance modelled as a broadband resonator. The main simulation parameters are given in Table 2, and are chosen to reproduce the situation modelled by MOSES in the previous section. The transverse tunes (Q_x, Q_y) are set to the working point used in the past. Q_x and Q_y have since been exchanged to enhance the lifetime. Also, assumptions include no space charge, no amplitude detuning, a linearized RF bucket, and a "frozen" wake field – i.e. the wake field is only calculated for the first turn, and remains unchanged for all remaining turns. This approximation holds because (i) the bunch is assumed to be well matched to the bucket and (ii) no longitudinal impedance is included in the simulation. In this section, the chamber is round, so horizontal and vertical planes are again equivalent.

Table 2: Main HEADTAIL simulation parameters

Parameter	Symbol	Value	Unit
Circumference	L	6911	m
Number of bunches		1	
Relativistic Gamma		27.7286	
Initial Rms beam sizes	σ_x, σ_y	1.8	mm
Horizontal Tune	Q_x	26.185	
Vertical Tune	Q_y	26.13	
Linear chromaticities	$\xi_{x,y}$	0 / 0	
Initial rms Bunch length	σ_z	0.21	m
Initial Longitudinal Momentum spread (rms)	σ_p/p_0	$9.3 \ 10^{-4}$	
Synchrotron Tune	Q_s	$3.24 \ 10^{-3}$	
Cavity Harmonic Number		4620	
Momentum Compaction Factor		$1.92 \ 10^{-3}$	
BB shunt impedance		10	MΩ/m
BB resonant frequency		1	GHz
BB quality factor		1	
Kick amplitude (in x and y)		0.9	mm
Average Beta function	β_x, β_y	40	m

Growth rate

The instability growth rate is calculated from the exponential growth of the amplitude of the bunch centroid oscillations as a function of time. The growth rate as a function of bunch intensity calculated from the output of the HEADTAIL simulations is compared with MOSES results in Fig. 3.
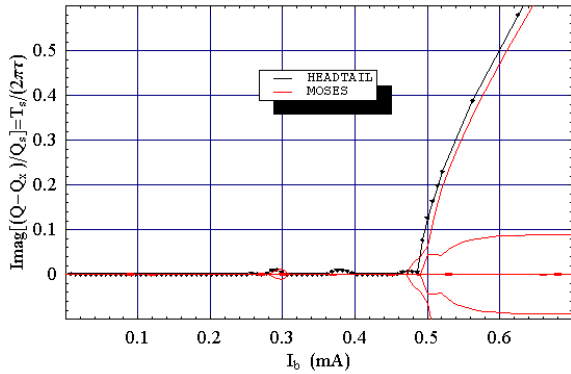
Figure 3: Comparing HEADTAIL (black full line with dots) and MOSES (red full line) growth rates as a function of bunch intensity.

Apart from a small non-zero growth rate at $I_b = 0.38$ mA, and a slightly lower growth rate in the range $I_b \in [0.45; 0.5]$ mA, HEADTAIL simulations clearly reproduce the instability growth rates predicted by MOSES calculations for the explored range of bunch intensities. However, this observation is necessary but not sufficient to prove that the transverse instability predicted by HEADTAIL is of the same nature as the one predicted by MOSES, i.e. a TMCI. To learn more about the nature of the fast transverse instability predicted by HEADTAIL, the behaviour of the transverse modes is analyzed in the frequency domain in the next section.

Tune shift of the bunch coherent modes

For each of the bunch intensities, the mode spectrum is obtained by applying a frequency analysis to the bunch transverse coherent oscillations as a function of time, which is an output of the HEADTAIL code. Two frequency analysis techniques were used to process the raw simulation data into normalized mode spectra: the Fast Fourier Transform (FFT) algorithm or the SUSSIX program [7]. The theory behind SUSSIX can be found in [8, 9]. A comparison between these two techniques for $I_b = 0.02$ mA is displayed in Fig. 4. The SUSSIX program is applied to the complex phase space normalized coordinate x-$j.p_x$ in the phase space whereas simple FFT is only applied to the coherent transverse position x of the bunch centroid, the transverse momentum p_x being left unused. It can be observed in the example in Fig. 4 that the coherent motion analyzed with SUSSIX enables to recognize azimuthal modes "-2" (2 separate radial modes), "-1", "0" (2 separate radial modes), "1", and "2" (2 separate radial modes). The same coherent motion analyzed with a classical *Mathematica* FFT algorithm only enables to observe 2 separate radial modes of azimuthal mode "0". More generally, the SUSSIX algorithm is found to be more powerful to analyze the behaviour of simulated transverse modes than a classical FFT.
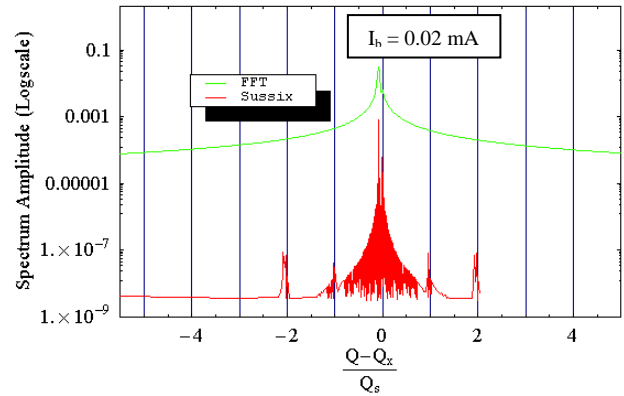

Figure 4: Comparing the performance of two frequency analysis algorithms applied to the coherent transverse oscillations simulated with HEADTAIL: FFT (green line) and SUSSIX (red line).
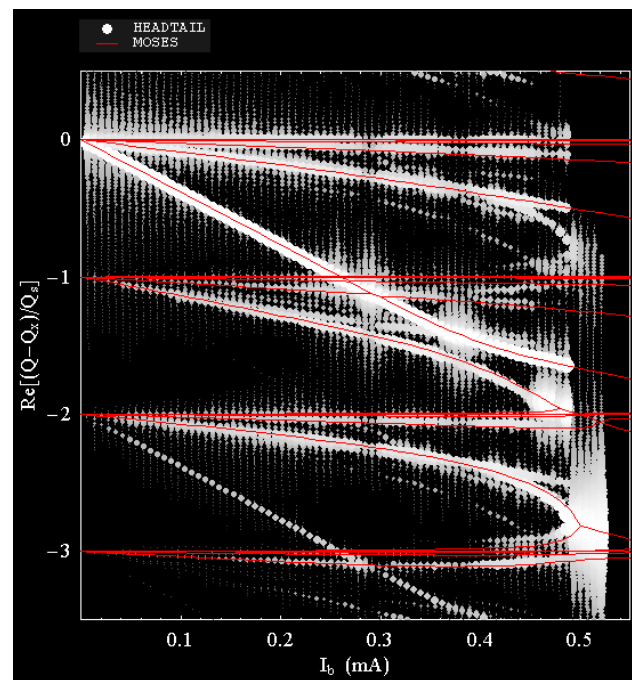

Figure 5: Comparing HEADTAIL (white dots) and MOSES (red lines). The transverse modes "0", "-1", "-2" and "-3" behaviour is plotted as a function of bunch intensity. The coherent motion as simulated with HEADTAIL was post-processed with SUSSIX and displayed using white dots, whose size and brightness are both non-linear functions of their spectral amplitude (bigger brighter dots have a higher amplitude than smaller darker dots).

The mode spectra obtained from a large number of simulations with bunch intensities ranging from $I_b = 0.01$ mA to $I_b = 0.55$ mA are displayed as a flattened 3-D plot in Fig. 5, and compared with MOSES mode spectra. From this comparison, it can be concluded that MOSES and HEADTAIL quantitatively agree in predicting most of the transverse modes shifting with increasing intensity, and transverse mode coupling at bunch intensities $I_b \sim 0.3$ mA (modes "0" and "-1"),

$I_b \sim 0.47$ mA (modes "-1" and "-2") and $I_b \sim 0.5$ mA (modes "-2" and "-3"). This latter coupled mode between modes "-2" and "-3" is clearly the main contribution to the spectrum amplitude for $I_b > 0.5$ mA, whereas the azimuthal mode "0" – also referred to as the transverse tune - carries most of the spectral power for $I_b < 0.5$ mA. This swift power swap between these two spectral lines, along with the large instability growth rate observed in the time domain (see Fig. 3), which both occur at $I_b = 0.5$ mA, proves that the resulting instability observed in HEADTAIL is indeed a TMCI.
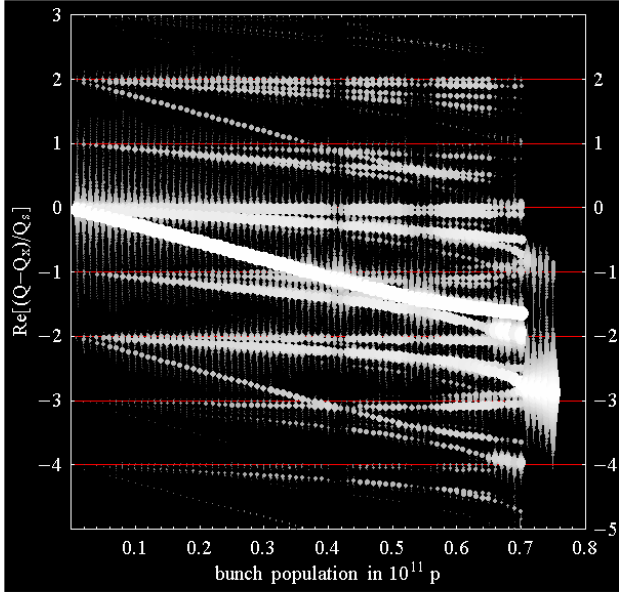

Figure 6: HEADTAIL (white dots) transverse modes "2", "1", "0", "-1", "-2", "-3", "-4" behaviour as a function of bunch population (N_b). Bunch intensity (I_b) and bunch population (N_b) are related by $I_b = N_b \, e \, f_0$, with e the proton charge and f_0 the revolution frequency. In the case of the SPS, I_b [mA] $\sim 0.7 \, N_b$ [10^{11} p] .

However, the agreement between the two codes is not perfect as it can be seen in Fig. 5 that some simulated transverse modes from HEADTAIL are not predicted by MOSES. In particular, a "-2" spectral line undergoes a shift with intensity that is comparable to the shift of the main tune. Along with other features of the HEADTAIL simulated mode spectrum - see Fig. 6 -, this tends to indicate that the mode spectrum contains echoes of the main lines translated by $+/- \, 2Q_s$, which do not seem to couple with other modes. Besides, as opposed to MOSES, the main tune couples twice with two different radial modes "-1" at $I_b \sim 0.38$ mA in HEADTAIL, which explains the non-zero growth rate at this current in Fig. 3. Work on understanding the reasons behind these discrepancies is still ongoing.

SIMULATIONS WITH HEADTAIL FOR A FLAT CHAMBER

Now that the simulations with HEADTAIL have been benchmarked with MOSES calculations for the round chamber case, we feel more confident to simulate the case

of the flat chamber, i.e. two infinite horizontal parallel plates. This flat chamber case is closer to the real elliptic chamber of the CERN SPS, but it is not yet possible to solve it with MOSES. The simulation parameters in Table 2 were left unchanged. The comparison between the simulated growth rates for both horizontal and vertical planes of the flat chamber, and the growth rate for the round chamber simulated in the previous section, is shown in Fig. 7.
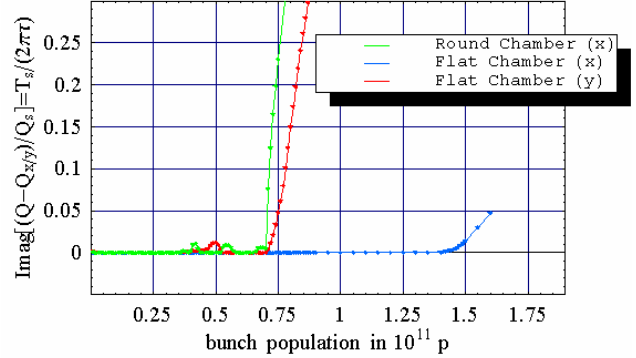

Figure 7: Comparison of HEADTAIL simulated growth rates for round chamber (green), horizontal plane of a flat chamber (blue) and vertical plane of a flat chamber (red) as functions of bunch population, for a shunt impedance $Z_s = 10$ MΩ/m.

These results are consistent with other HEADTAIL simulations presented in [10], in which the shunt impedance was set to $Z_s = 20$ MΩ/m. The instability threshold for the vertical plane of the flat chamber is slightly higher than the threshold for the round chamber case, and an instability threshold is found for the horizontal plane of the flat chamber a factor 2 higher than that of the vertical plane. As already mentioned in [10], the thresholds for the vertical and horizontal planes of the flat chamber are scaled from the round chamber threshold by the respective vertical ($\pi^2/12$) and horizontal ($\pi^2/24$) dipolar factors obtained by K. Yokoya [11].

Besides, simulated mode spectra as a function of bunch population (N_b) for both horizontal and vertical planes are presented in Fig. 8. A coupling between modes "-2" and "-3" in the vertical plane is observed. In the horizontal plane, the origin of the instability can not be proven, but a coupling between azimuthal modes "-1" and "-2" can be guessed. Moreover, the slope of the tune shift with intensity (main radial mode "0") for the vertical plane of the flat chamber case is observed to be higher by a factor $\pi^2/8$ than for the round chamber case. The slope of the tune shift with intensity for the horizontal plane of the flat chamber case is observed to be zero. These observations can be understood if we assume that both dipolar and quadrupolar parts of the flat chamber impedance have an impact on the tune shift [12]. Actually, for the vertical plane the two contributions add up resulting in a factor $\pi^2/12+\pi^2/24 = \pi^2/8$ with respect to the round chamber, whereas for the horizontal plane, the two contributions are subtracted and, in this specific case, they cancel out ($\pi^2/24-\pi^2/24=0$).
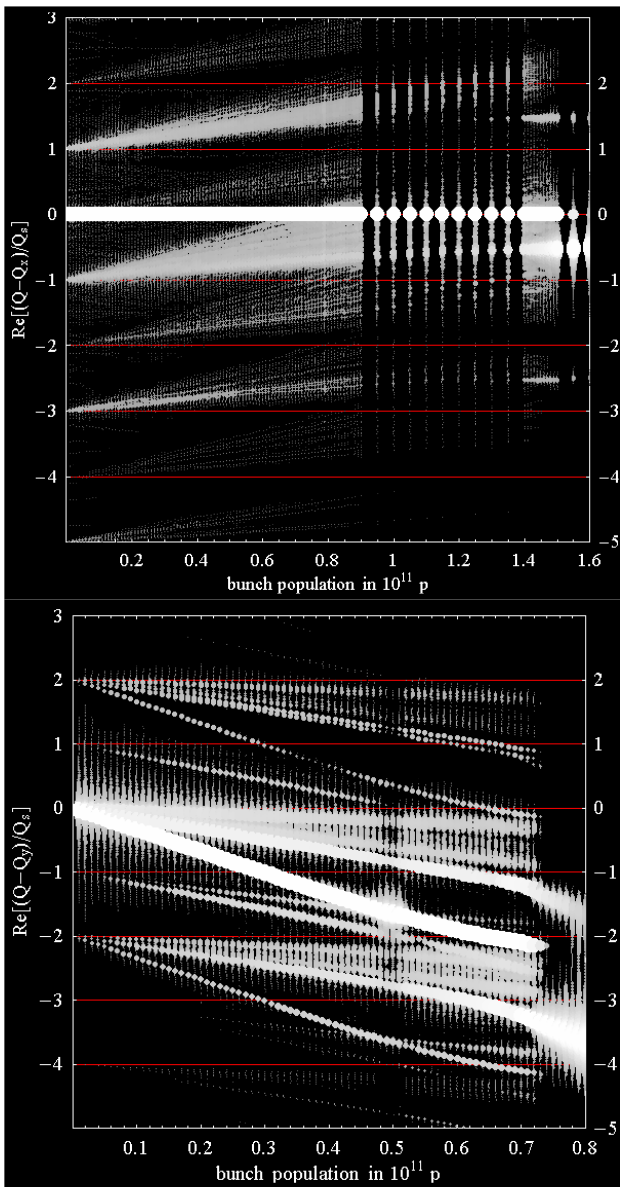
Figure 8: HEADTAIL horizontal (top) and vertical (bottom) modes behaviour as a function of bunch population for a flat chamber and a shunt impedance $Z_s = 10$ MΩ/m. It is important to notice that the horizontal scale range is larger, and that the density of simulations covering the population range $N_b \in [0.9;1.6]\ 10^{11}$ protons is reduced.

SIMULATIONS WITH HEADTAIL FOR A FLAT CHAMBER WITH LINEAR COUPLING

In the HEADTAIL simulations performed in [10], linear coupling between the transverse planes was observed to increase the TMCI threshold in the case of a flat chamber, when the transverse tunes are set to $Q_x = 26.18$ and $Q_y = 26.185$. A threshold increase is indeed obtained with a linear coupling coefficient set to K=0.005 m^{-1} (see Fig. 9).
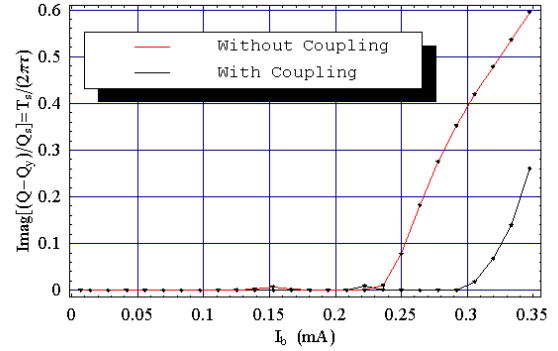

Figure 9: Comparison between HEADTAIL simulated growth rates for the vertical plane of a flat chamber without linear coupling between the two transverse planes (red), and with linear coupling (black) as a function of bunch intensity, for a shunt impedance $Z_s = 20$ MΩ/m.

The mode spectrum obtained in Fig. 10 is not normalized, so that the coupled tunes can be observed. Mode coupling is again observed to take place between mode "-2" and mode "-3" when linear coupling is present.
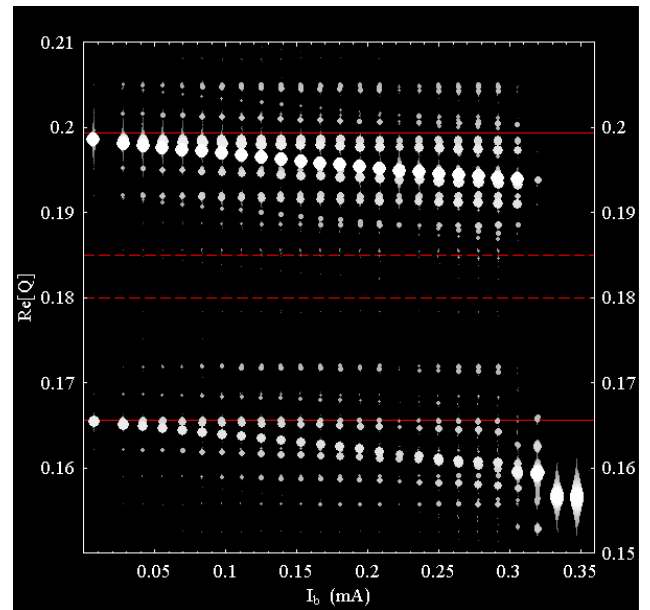

Figure 10: HEADTAIL mode behaviour as a function of bunch intensity for a flat chamber in the vertical plane, for a shunt impedance $Z_s = 20$ MΩ/m, and in the presence of linear coupling between the transverse planes. Dashed red lines are the uncoupled fractional parts of the vertical (0.185) and horizontal tunes (0.18). Full red lines are the coupled tunes predicted in [13].

CONCLUSION

Coupling between azimuthal modes "-2" and "-3" is observed to be the cause of the instabilities simulated with HEADTAIL, both in the case of the round chamber and of the vertical plane of the flat chamber (with and without linear coupling between transverse planes).

Besides, HEADTAIL simulations were benchmarked with MOSES in the case of the round chamber, and both

codes agree for the behaviour of most of the spectral lines with increasing bunch intensity. Minor discrepancies remain and work is ongoing to understand them.

The next step is to compare these simulations with measurements acquired in the SPS machine during the 2007 run.

ACKNOWLEDGMENTS

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