Analysis of resonances induced by the SIS–18 electron cooler*

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Abstract

Besides beam cooling, an electron cooler also acts as a non-linear optical element. This may lead to the excitation of resonances possibly resulting in an increase of the beam emittance. The aim of this work is the calculation of resonances driven by the electron space charge field in the cooler installed in the SIS heavy ion synchrotron at GSI Darmstadt. For our calculations, we used a numerical model consisting of a rotation matrix representing the ideal lattice together with a non-linear transverse kick element representing the electron cooler. Within this model, we studied the dominant resonance lines resulting from the interaction with the cooler.

INTRODUCTION

The space charge field in an electron cooler acts as a non-linear optical element in the lattice of a storage ring. This may lead to the excitation of additional ring resonances. Depending on the machine working point these resonances cause emittance growth and an effective heating of the beam, as it was observed e.g. in the CELSIUS cooler storage ring [1].

Electron cooling at medium energies will play an essential role in the proposed FAIR storage rings [2]. Electron cooling is already available to improve the beam quality of the intense ion beams at low energy in the existing SIS synchrotron. On the other hand, the transverse tune shift and spread due to the direct space charge force plays an important role at low or medium energies. The resonances excited by the non-linear space charge field of the cooler electron can potentially limit the reachable beam intensity and quality.

In this work, we calculated the resonances driven by the electron space charge field in the cooler installed in the SIS–18 heavy ion synchrotron at GSI Darmstadt. This theoretical study provides the necessary information for dedicated measurements of cooler induced resonances and effects in SIS.

MODEL

In our calculations we used a simple model consisting of a rotation matrix providing the phase advance of the lattice of SIS–18 and a non-linear transverse kick introducing the force of the electron cooler in thin lens approximation. The coordinates of a particle after the $(n+1)$-st revolution are calculated from those of the n-th revolution by

$$\begin{pmatrix} z_{n+1} \\ z'_{n+1} \end{pmatrix} = \begin{pmatrix} \cos 2\pi\nu_z & \hat{\beta}_z \sin 2\pi\nu_z \\ -\frac{1}{\hat{\beta}_z}\sin 2\pi\nu_z & \cos 2\pi\nu_z \end{pmatrix} \times \begin{pmatrix} z_n \\ z'_n + \Delta z'(x_n, y_n) \end{pmatrix} \quad (1)$$

with $z = x, y$. Here, ν_z is the bare tune of the lattice, $\hat{\beta}_z$ is the unperturbed beta function in z direction at the location of the electron cooler, and

$$\Delta z'(x,y) = \frac{qq'N'}{2\pi\varepsilon_0 m_0 c^2 \beta_0^2 \gamma_0^3} \frac{z}{R^2} \int_0^R \mathrm{d}r\ r\ n_0(r) \quad (2)$$

with $R = \sqrt{x^2+y^2}$ is the transverse momentum kick depending on both spatial directions x, y. Here, we applied a radial shape of the electron beam $n_0(r)$ having a constant density in the centre and a thin edge region being characteristic for an electron cooler as shown in figure 1. The parameters used in the calculations are given in table 1.

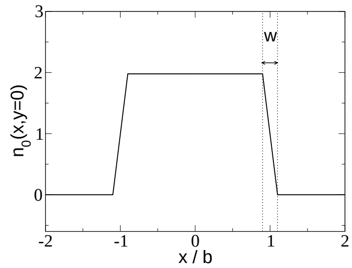

Figure 1: Normalised charge density profile used for the electron beam as provided by the beambeam element of MAD-X as a function of x for $y = 0$. An edge layer with a width $w = 0.01\ b$ was used in the calculations.

Particle	U^{73+}
Injection energy E	11.4 MeV/u
Relativistic factors $\beta_0,\ \gamma_0$	$0.15,\ 1.01$
Cooling length $L_{\rm cool}$	3 m
Electron current I_e	0.3 A
Cathode radius $r_{\rm cath}$	12.7 mm
Adiab. expansion factor f_E: used, (range)	$3,\ (1\ ...\ 8)$
Electron beam radius ($b = r_{\rm cath}\sqrt{f_E}$)	22 mm
Beta function in the cooler ($\hat{\beta}_x,\ \hat{\beta}_y$)	8 m, 15 m

Table 1: Parameter of SIS–18 used in the calculations and taken from [3] and [4].

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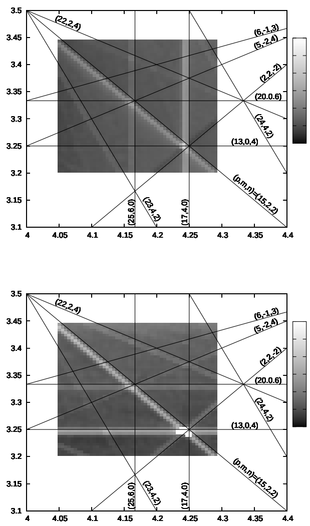

Figure 2: Relative rms beam width in x and y direction, upper and lower picture, respectively, depending on ν_x and ν_y (x and y axis in both figures). The colour scale is within $w_{\rm rel} \leq 1$ (dark grey) and $w_{\rm rel} \geq 1.25$ (white).

RESULTS

To make the positions of the resonances visible, the relative rms beam width was calculated as a function of the tune values ν_x, ν_y of the rotation matrix.

We explored for resonances the range given by $\nu_x \in [4.05, 4.3]$ and $\nu_y \in [3.2, 3.45]$, which is near the working point $(\nu_x, \nu_y) = (4.2, 3.4)$ [3], and which does not contain a half integer resonance. On the other hand, it was shown in [1] within an analytic model, that an electron cooler with a round electron beam excites only resonances of even order, where, additionally, the resonances strength decreases with increasing order. Hence, we searched only for resonances of order 4 and 6. Figure 2 shows the positions of the resonances found in the $\nu_x - \nu_y -$ plane. The black lines denote the positions of the resonances given by the condition

$$p = m\nu_x + n\nu_y. \tag{3}$$

So, all resonances found in our scan could be identified, and they show a quite reasonable behaviour. We found, that only sum resonances and uncoupled resonances lead to an increase of the beam width.

Figure 3 shows the relative beam width depending on the vertical tune. So, one can see, that the beam width is enhanced up to a factor 1.5 under the conditions considered, what is not visible in figure 2 due to the resolution.

In both figures, one can see, that the regions of enhanced beam width are always slightly shifted to smaller values compared to the lines defined by the resonance condition (3). This is because, in contrast to the lattice non-linearities, the electron cooler provides a finite linear tune shift in addition to the non-linear part.

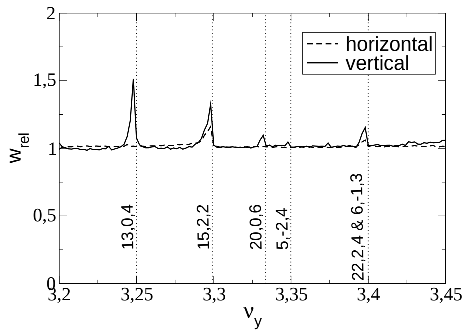

Figure 3: Relative beam width as a function of the vertical tune within the range $\nu_y \in [3.1, 3.45]$ for a fix horizontal tune $\nu_x = 4.2$. So, this figure is an extract of figure 2.

SUMMARY

We studied the resonances generated by the space charge force of the electron beam in the SIS–18 cooler. The initial rms radius of the ion beam was adjusted to the radius of the electron beam. Resonances up to the 6th order could be identified. Furthermore, we could qualitatively reproduce the dependency of the resonance width on the resonance order as given by an analytic model in reference [1]. Within that model, the widths of the resonances driven by a transverse momentum kick representing an electron cooler are given as integrals over the angle variable of the betatron motion. It predicts that the resonance width decreased, when the order of a the resonances is enhanced. A quantitative reproduction of the beam width using an analytic model was possible only for the half integer resonance, see [6].

The motivation for this work was, that the resonances are an additional possible constraint for the choice of the tune, because they could limit the extension of the space charge tune spread due to the self fields of the beam and therefore leading to the reduction of the space charge limit.

Additionally, there are many resonances driven by the non-linearities of the lattice in the real SIS, see e.g. [5]. In contrast to them, the resonances driven by the electron cooler do not lead to a beam loss, but to an increase of the beam with. Furthermore, the non-linearities and the cooler drive partially the same resonances. So, a major task of forthcoming studies will be to distinguish between the effects of both sources of resonances. Hence, further theoretical studies are necessary to investigate, the interplay of both sources of resonances and, that subject will also be an important task within measurements of the resonances driven by the electron cooler of SIS–18.

This work is presented in more detail in [6].

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