Study of beam-beam interaction with a large Piwinski angle at LHC

K. Ohmi *
KEK, 1-1 Oho, Tsukuba, 305-0801, Japan

Abstract

Collision with a large Piwinski angle is one of the update scenarios of LHC toward the luminosity 10^{35} cm^{-2}s^{-1}. The large Piwinski angle is realized by a small beta function at the collision point and longer bunch length. The Piwinski angle is increased from 0.6 to 2 in the scenario. The bunch population is increased so as to keep the beam-beam parameter.

The beam-beam performance is degraded by crossing angle which induces additional nonlinear terms due to a symmetry breaking of the collision especially for the high beam-beam parameter. Effect of crossing angle for the nominal LHC design and the large Piwinski angle scheme are studied.

INTRODUCTION

We discuss effects of crossing angle in LHC and its upgrade plans. Piwinski angle for horizontal crossing is defined by

$$\phi = \frac{\theta\sigma_z}{\sigma_x}. \quad (1)$$

where θ, σ_z and σ_x are a half crossing angle, bunch length and horizontal beam size, respectively. The nominal LHC is $\theta = 140$ μrad, $\sigma_z = 7$ cm and $\sigma_x = 17$ μm, thus the Piwinski angle is $\phi = 0.6$.

The crossing angle induces [7] various nonlinear terms, which degrade the luminosity performance. The large Piwinski angle scheme $\phi = 2$ expects linear luminosity increase for the bunch population without increasing the beam-beam parameter. We study the beam-beam performance for the crossing collision in the nominal design and upgrade options using computer simulations.

EFFECT OF CROSSING ANGLE

Lorentz transformation is used so that the two beams move completely opposite direction. Electro-magnetic field is formed in the perpendicular to the moving direction, thus colliding beam experiences the electro-magnetic field in the perpendicular to the moving direction [1, 2, 3]. This feature simplifies treatment of the beam-beam force. The schematic view is seen in Figure 1.

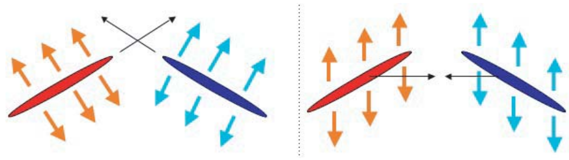

Figure 1: Collision in the laboratory and head-on frame. Light blue and orange arrows display the electric field line of the colliding bunches. Black arrow displays the traveling direction of the bunches.

The Lorentz transformation from the laboratory frame to the head-on frame ($\mathcal{M}_L$) is given for a half crossing angle θ by [3]

$$\begin{aligned}
x^* &= \tan\theta z + \left(1 + \frac{p_x^*}{p_s^*}\sin\theta\right)x \\
y^* &= y + \sin\theta\frac{p_y^*}{p_s^*}x \\
z^* &= \frac{z}{\cos\theta} - \frac{H^*}{p_s^*}\sin\theta x \\
p_x^* &= \frac{p_x - \tan\theta H}{\cos\theta} \\
p_y^* &= \frac{p_y}{\cos\theta} \\
p_z^* &= p_z - \tan\theta p_x + \tan^2\theta H,
\end{aligned} \quad (2)$$

where

$$H = (1+p_z) - \sqrt{(1+p_z)^2 - p_x^2 - p_y^2}$$

$$p_s = \sqrt{(1+p_z)^2 - p_x^2 - p_y^2}.$$

A star designates a dynamical variable in the head-on frame. H^* and p_s^* are $H(\boldsymbol{p}^*)$ and $p_s(\boldsymbol{p}^*)$, respectively. Note that the x^* and y^* axes are defined in the same direction for both beams, while the s^* axis is defined in opposite directions, since the two beams travel in opposite directions.

The linear part of the transformation is expressed by a matrix

$$M_L = \begin{pmatrix} 1 & 0 & 0 & 0 & \tan\theta & 0 \\ 0 & 1/\cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/\cos\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/\cos\theta & 0 \\ 0 & -\tan\theta & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (3)$$

These transformations, Eqs.(2) and (3), are not symplectic. In fact, the determinant of the transfer matrix M_L is not 1, but $\cos^{-3}\theta$. This is not a problem because the inverse factor of $\cos^3\theta$ is applied by the inverse transformation. This is due to the fact that the Lorenz transformation is not symplectic for the accelerator coordinate, because the Hamiltonian is divided by a reference momentum. Needless to say, the Lorentz transformation is symplectic for the physical coordinate, thus the transformations, Eqs.(2) and (3), are symplectic in the physical coordinate. The adiabatic damping is the concept in the accelerator coordinate. This discussion can be applied to the nonlinear transformation of Eq.(2) [4].

SIMULATION FOR NOMINAL LHC

We first evaluate luminosity for the nominal LHC using weak-strong and strong-strong simulations. Crossing angle induces linear $x - z$ coupling, with the result that the beam distribution diffuses and the luminosity degrades [4]. The diffusion rate strongly depends on the beam-beam parameter. For electron-positron colliders, the diffusion rate is faster than radiation damping rate $> 10^{-4}$/turn for $\xi > 0.05$. Here damping rate of LHC is the order of one day 10^9 turns and the luminosity life time is expected 10^9 turns. Tolerable diffusion rate or luminosity decrement is 10^{-9}/turn. The simulations was carried out during $\sim 10^6$ turns in this paper. The decrement of 10^{-3} should be cared to predict the luminosity life time of 10^9 turns.

Figure 2 shows evolution of the beam-size and luminosity given by the weak-strong and strong-strong simulation for the nominal bunch population. Plot (a) depicts beam size evolution given by the weak-strong and strong-strong simulations. A bunch is sliced in 10 pieces along its length in the weak-strong simulation. Macro-particles of 10^4 was used in the weak-strong simulation. The beam size of the weak beam is averaged in each 100 turns. No emittance growth nor luminosity degradation were seen in the weak-strong simulation.

Two dimensional model is used for the strong-strong simulation to save the calculation time. This approximation may give optimistic results. However an emittance growth is seen in the strong-strong simulation. The emittance growth is considered by numerical noise of macro-particle statistics. Macro-particles of 10^6 are used the simulation. The statistical noise of collision offset (0.1%) can be introduced collision by collision in the simulation [5]. Needless to say, the weak-strong simulation is noise free. Plot (b) depicts luminosity evolution for the nominal, twice and 4 times bunch populations. Luminosity degradations are 10^{-9}, 5×10^{-9} and 3×10^{-8} in one turn, respectively. If we believe this result, the bunch population is limited to the nominal value by the beam-beam effect. Here we consider this degradation is due to the numerical noise again. More discussions for noises in macro-particle simulations are seen in Ref.[6]. We use only the weak-strong simulation hereafter.

Figure 3 shows the luminosity degradation for $2\times$, $4\times$, $6\times$ and $8\times$ more bunch populations than the nominal value. The red and green lines depict the evolution of the luminosity zero or finite crossing angle. In the nominal bunch population, there was no difference between zero and finite crossing angle. The difference was visible for more than 6 times population. Anyway, the nominal LHC is no problem for finite crossing angle.

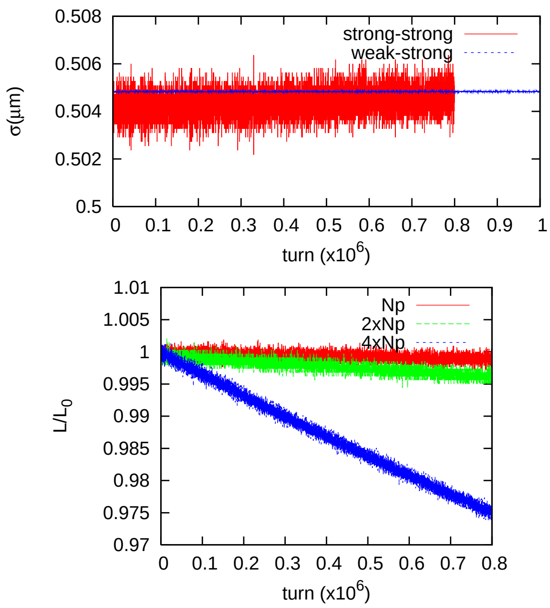

Figure 2: Beam size increment and luminosity decrement given by the strong-strong simulation.

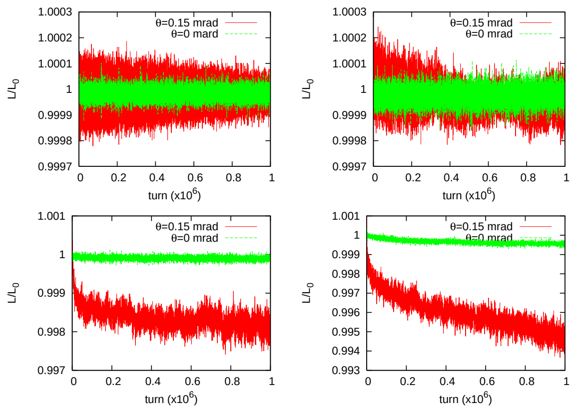

Figure 3: Luminosity degradation due to the crossing angle given by the weak-strong simulation. Plots (a)-(d) depicts for $2\times$, $4\times$, $6\times$ and $8\times$ more bunch population than the nominal value, respectively.

LARGE PIWINSKI ANGLE OPTION

We study a large Piwinski angle option for LHC. Table 1 [8] shows parameter list of the large Piwinski angle options. The Piwinski angle $\phi = 2$ is realized in the first option with long flat bunch, a half beta and 5 times bunch population. The angle $\phi = 3$ is realized in the second option with a quarter low beta and 2 times bunch population. In this paper we study the first scheme (LPA1).

LHC has two collision points. Both of the two collision points are designed so as horizontal-horizontal crossing in the nominal design. Hybrid crossing, in which horizontal and vertical crossing[9] are adopted for the two interaction points, can be considered for the upgrade plan. The tune spread due to the nonlinear beam-beam interaction is narrower for the hybrid crossing than the nominal crossing. The horizontal crossing induces the nonlinear terms xy^2 while vertical crossing induces skew terms x^2y. This means the hybrid crossing induces more resonances than the nominal crossing. It is very difficult which is better the two cases, less resonance with wider tune spread, or more resonances with narrower resonances. The answer depends on the case by case, operating point, beam-beam parameter and so on. Simulation only gives the answer.

The nominal crossing induces the same nonlinear interactions at the two interaction point. This means some nonlinear terms can be cancelled depending on the betatron phase difference. In the hybrid crossing, some terms can be cancelled but terms with different symmetry (parity) can not be cancelled.

Table 1: Basic parameters of LHC nominal and large Piwinski angle option. * The bunch length is total length with a flat longitudinal distribution.

variable	nominal	LPA-1	LPA-2
circumference (m)		26,658	
beam energy (TeV)		7	
bunch population (10^{11})	1.15	4.9	2.5
half crossing angle (mrad)	0.14	0.19	-
beta function at IP (m)	0.55	0.25	0.14
emittance (m)		5.07×10^{-10}	
beam-beam tune shift	0.0033		
bunch length (cm)	7	41*	7.5
synchrotron tune, ν_s		0.0019	
betatron tune, $\nu_{x(y)}$		63.31/59.32	
revolution frequency		10^9/day	
Piwinski angle ϕ	0.4	2	3
luminosity (cm^{-2}s^{-1})	1	10	

Simulation for the nominal and hybrid crossings

The weak-strong simulation was executed to study the large Piwinski angle scheme. The number of the longitudinal slices are increased for proportional to the Piwinski angle. We show examples of the nominal and hybrid crossings. It should be emphasized that the results depend on the betatron phase difference between two IP. Here the phase difference is chosen to be $\Delta\psi = 0.2 \times 2\pi$ for both of x-y plane. The parasitic interactions are included in the simulation.

Figure 4 shows the simulation results for $N_p = 4.9 \times 10^{11}$ with including 7 parasitic collisions both side of upstream and down stream of the collision point. Plots (a), (b) and (c) depict the evolution of luminosity and beam size, and beam particle distribution in x-y plane after 10^6 revolutions, respectively. Red and green lines are turn by turn beam size and its average during 100 turns. Luminosity does not change, while the beam size fluctuates for the revolutions. It is seen that some particles have a large amplitude in the final distribution (c).

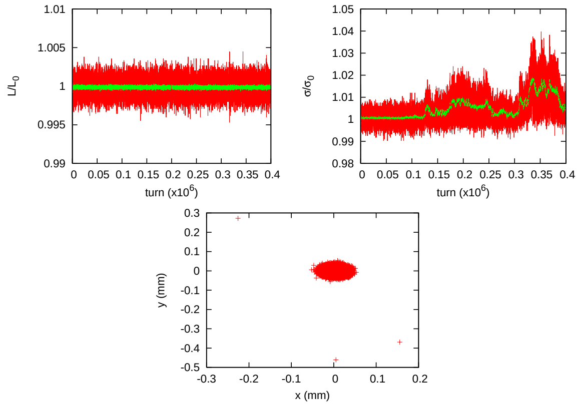

Figure 4: $N_p = 4.9 \times 10^{11}$ Evolution of (a) luminosity and (b)beam size. (c) Beam particle distribution in x-y plane after 10^6 revolutions.

A higher bunch population, $N_p = 6 \times 10^{11}$ was tried to make clear the luminosity degradation and emittance growth. Figure 5 shows the simulation results for $N_p = 6 \times 10^{11}$ with including 7 parasitic collisions each side. Again the luminosity does not degrade, but beam size increases faster than that of the nominal population, $N_p = 4.9 \times 10^{11}$. More particles have large amplitudes in the final distributions.

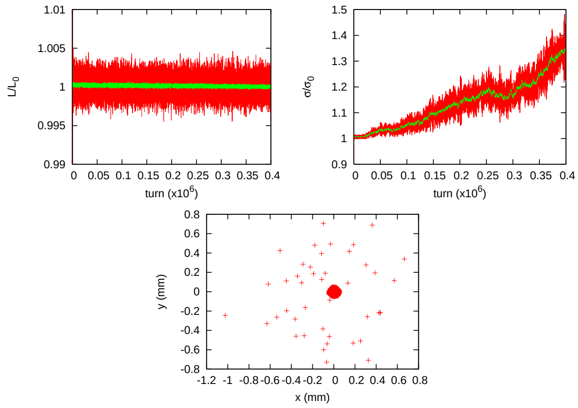

Figure 5: $N_p = 6 \times 10^{11}$ Evolution of (a) luminosity and (b)beam size. (c) Beam particle distribution in x-y plane after 10^6 revolutions.

We next cut off the parasitic interactions to understand why particles have large amplitudes. Figure 6 shows the simulation results for $N_p = 6 \times 10^{11}$ without parasitic collisions.

The same simulation was carried out for the nominal collision scheme, horizontal-horizontal. Emittance growth and luminosity degradation were not seen in the nominal collision. We would like to say the tune spread is not universal parameter to characterize the emittance growth and/or beam-beam performance, and we do not conclude

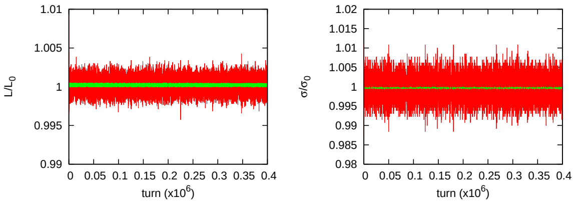

Figure 6: $N_p = 6 \times 10^{11}$ Evolution of (a) luminosity and (b)beam size. (c) Beam particle distribution in x-y plane after 10^6 revolutions.

that the nominal collision scheme is better than the hybrid scheme in this example.

Taylor map analysis for the nominal and hybrid crossings

Nonlinear terms in one turn map depend on the collision scheme; the nominal or hybrid crossing, or betatron phase difference between the two interaction points. The beam-beam interaction can be expanded by Taylor polynomial for the dynamic variables. The one turn map including two interaction points and two linear arcs is represented by Taylor polynomial. The one turn map characterizes resonance behaviors of the beam particles. For example, $x^n y^m$ term in the map, $\exp(-a : x^n y^m :)$, drives resonances of $n\nu_x \pm m\nu_y$. Details of the analysis is seen in Ref. [7] We discuss nonlinear terms up to 4-th order in this paper. Higher order terms may be important for proton rings without radiation damping. Further studies will be done elsewhere.

Figure 7 shows the coefficient of x^4 term of the beam-beam interaction as a function of the betatron phase difference between the two interaction points. The coefficient for the nominal crossing is small than that for hybrid crossing. This means the x^4 term is weakened by long bunch collision in the horizontal plane, perhaps. The coefficient has peaks for the phase difference of 0.3 and 0.8. The total tune is $(\nu_x, \nu_y) = (0.31, 0.32)$. The phase difference is another arc is 0.5 or 1 at the peaks, respectively.

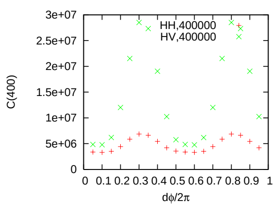

Figure 7: Coefficient of x^4 term for the betatron phase difference between the two interaction points. Red (HH) and green (HV) lines are for the horizontal-horizontal crossing adn horizontal-vertical crossing, respectively.

Figure 8 shows the coefficients of x^3z, y^3z and related terms. y^3z terms are very small for the nominal crossing, because the terms are suppressed by symmetry of the horizontal-horizontal crossing.

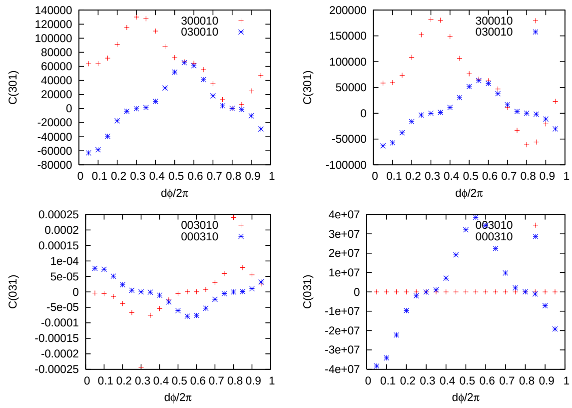

Figure 8: Coefficients of x^3z and y^3z terms of the beam-beam interaction. (a) x^3z (300010) and p_x^3z (030010) for the horizontal-horizontal crossing. (b) x^3z (300010) and p_x^3z (030010) for the horizontal-vertical crossing. (c) y^3z (003010) and p_y^3z (000310) for the horizontal-horizontal crossing. (d) y^3z (003010) and p_y^3z (000310) for the horizontal-vertical crossing.

Figure 9 shows the coefficients of x^2yz, xy^2z and related terms. xy^2z terms are very small for the nominal crossing, because the terms are suppressed by symmetry of the horizontal-horizontal crossing.

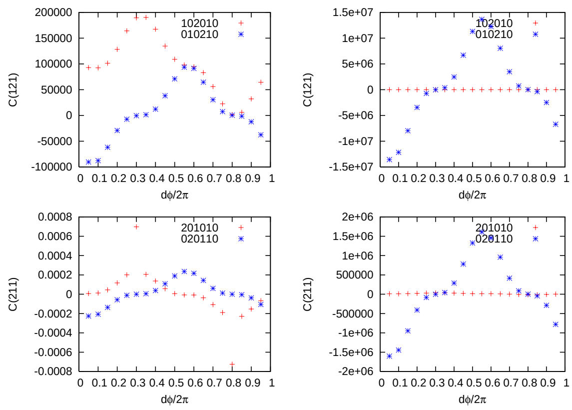

Figure 9: Coefficients of xy^2z and x^2yz terms of the beam-beam interaction. (a) xy^2z (102010) and $p_xp_y^2z$ (010210) for the horizontal-horizontal crossing. (b) xy^2z (102010) and $p_xp_y^2z$ (010210) for the horizontal-vertical crossing. (c) x^2yz (201010) and $p_x^2p_yz$ (020110) for the horizontal-horizontal crossing. (d) x^2yz (201010) and $p_x^2p_yz$ (020110) for the horizontal-vertical crossing.

SUMMARY

Effect of crossing angle are evaluated for the nominal LHC. The weak-strong simulation showed a visible luminosity degradation in a day for 6 times higher bunch population: that is, there is no problem for the nominal design. The strong-strong simulation gave luminosity degradation stronger than that of the weak-strong simulation. This degradation is considered due to numerical noise of macro-particle statistics at present.

High Piwinski angle scheme with a half beta and twice longer bunch length was investigated. The simulation included 7 parasitic interactions both of upstream and downstream of the collision point. Two type of collision scheme for two collision points, the nominal horizontal-horizontal crossing and the hybrid horizontal-vertical crossing, was studied. An example for each scheme was investigated with the weak-strong simulation. The hybrid crossing gave a halo formation due to the parasitic interactions in this example. We should not conclude that the nominal collision scheme is better than the hybrid scheme in this example.

Preliminary results for Taylor map analysis of the beam-beam interactions were presented. Nonlinear terms depending on the symmetry (parity) of the colliding system appear in the map. The nominal crossing gives a wide tune spread but less resonance term, while the hybrid crossing gives a narrow tune spread but more resonance terms. It is difficult to say simply which is better; depending on the operating point, betatron phase difference between the two interaction points.

ACKNOWLEDGEMENT

The author thanks to K. Takayama and F. Zimmermann for fruitful discussions.

REFERENCES

[1] J. Augustin, Orsay Report, No 36-69 (1969).

[2] K. Oide and K. Yokoya, Phys. Rev. A40, 315 (1989).

[3] K. Hirata, Phys. Rev. Lett. 74, 2228 (1995).

[4] K. Ohmi et al., Phys. Rev. ST-AB 7,104401 (2004).

[5] K. Ohmi et al., Proceedings of PAC07 (2007).

[6] K. Ohmi, Proceedings of ECLOUD07, KEK proceedings 2007-10, 189 (2007)

[7] K. Ohmi et al., Proceedings of PAC07 (2007).

[8] F. Zimmermann, Proceedings of PAC07 (2007).

[9] K. Takayama et al., Phys. Rev. Lett. 88, 144801 (2002).

[10] F. Ruggiero and F. Zimmermann, Phys. Rev. ST-AB 5, 061001 (2002).