GENERAL PHYSICS

I. MICROWAVE SPECTROSCOPY

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A. PROPAGATING WAVES IN HIGH DIELECTRIC WAVEGUIDE OF SQUARE CROSS SECTION

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Work reported previously[1] has continued, and we have obtained field solutions for propagating waves in a slab of high dielectric material. This waveguide is of square cross section, there are no metal walls, and it is surrounded entirely by free space. The permittivity is sufficiently great that displacement current outside the waveguide may be neglected.

The transverse electric field inside the waveguide is described by a homogeneous differential integral equation.

$$k_z^2\vec{E}_T - \nabla_T(\nabla_T \cdot \vec{E}_T) = \frac{\omega^2\mu_o\epsilon}{2\pi} \int_A d^2r' \left[k_z^2\vec{E}'_T - \nabla'_T(\nabla'_T \cdot \vec{E}'_T)\right] K_o(k_zR)$$

$$+ \frac{\omega^2\mu_o\epsilon}{2\pi} \int_A d^2r' \, \nabla'_T[K_o(k_zR)\nabla'_T \cdot \vec{E}'_T], \tag{1}$$

where

$\vec{E}_T = \vec{E}_T(\vec{r}_T)$

$\vec{E}'_T = \vec{E}_T(\vec{r}'_T)$

$R = |\vec{r}_T - \vec{r}'_T|$

K_o = modified Bessel function.

Solutions to this equation must also satisfy the boundary condition

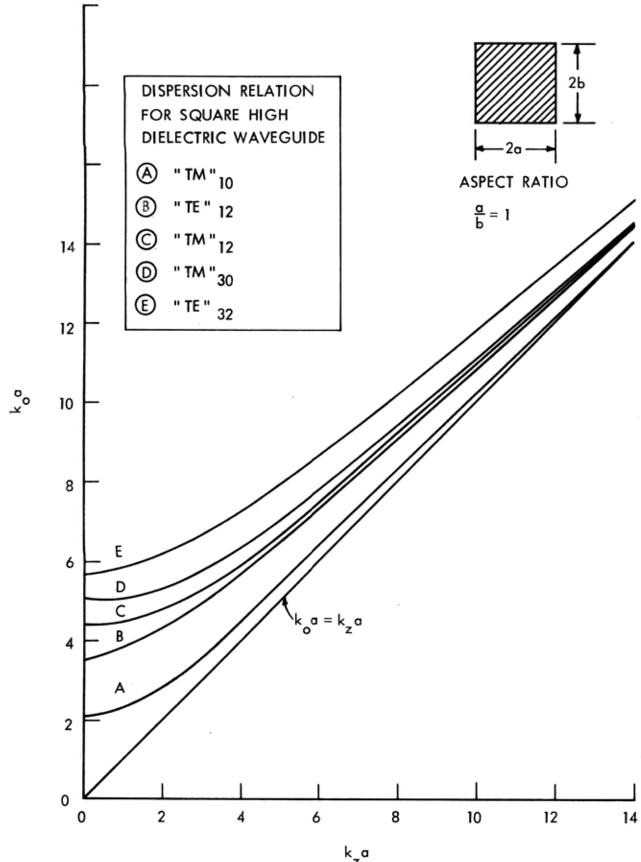


Fig. I-1.

Dispersion relation for square waveguide and for E_z of even symmetry in both transverse dimensions.

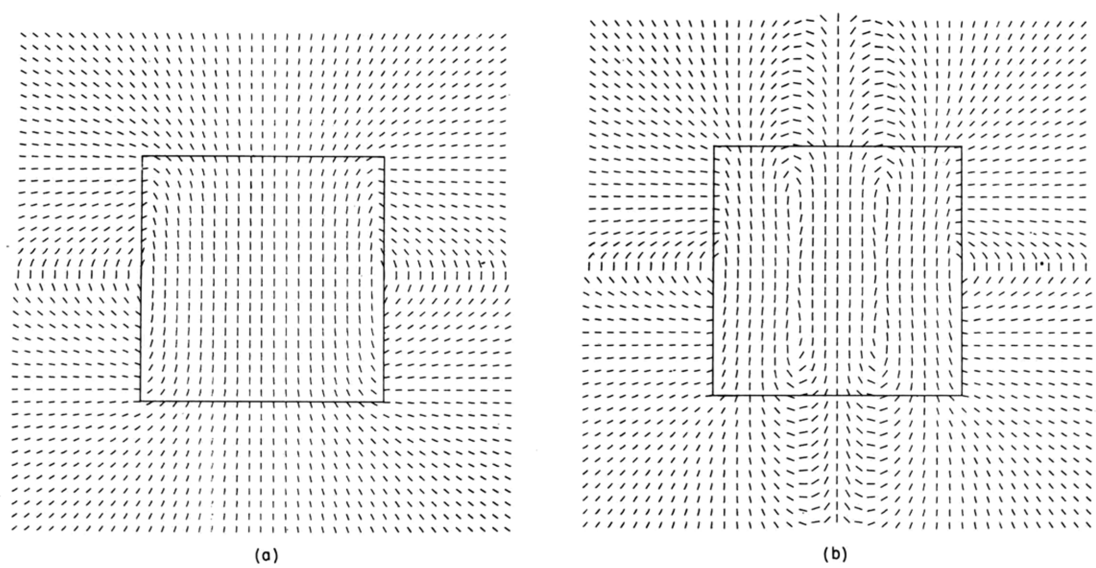
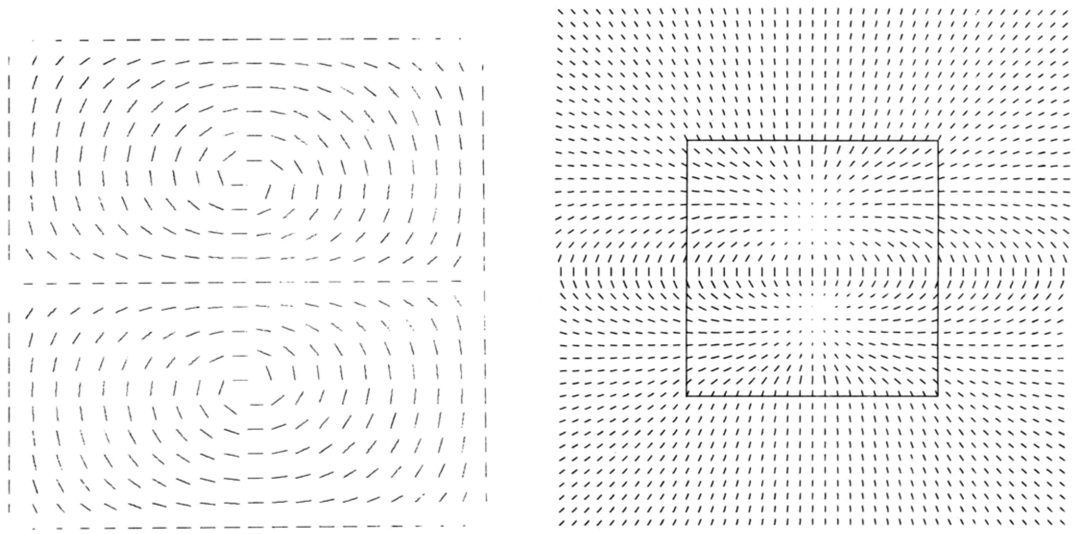


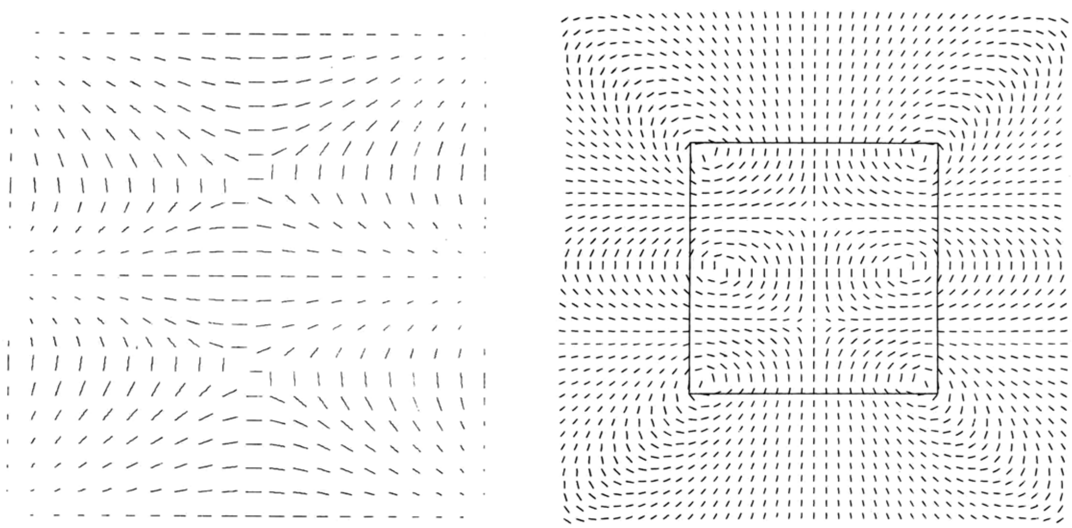
Fig. I-2. (a) Magnetic field inside and outside the guide for "TM"$_{10}$ mode and $\lambda_z/2a = 1$.

(b) Magnetic field for "TM"$_{30}$ mode and $\lambda_z/2a = 1$.



(a) (b)

Fig. I-3. (a) Electric field for "TE"$_{12}$ mode and $\lambda_z/2a = 1$.
(b) Magnetic field for "TE"$_{12}$ mode and $\lambda_z/2a = 1$.



(a) (b)

Fig. I-4. (a) Electric field for "TM"$_{12}$ mode and $\lambda_z/2a = 1$.
(b) Magnetic field for "TM"$_{12}$ mode and $\lambda_z/2a = 1$.

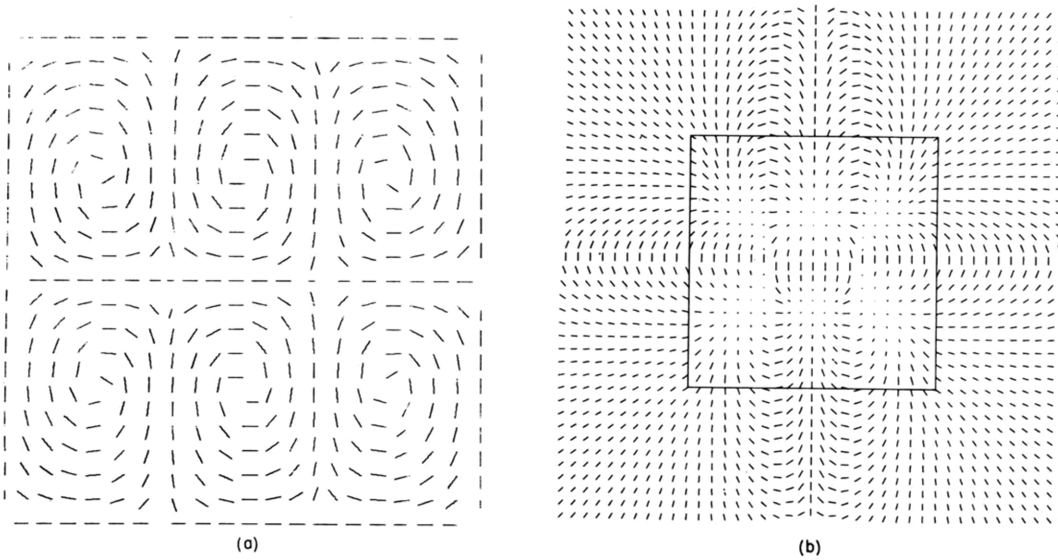


Fig. I-5. (a) Electric field for "TE"$_{32}$ mode and $\lambda_z/2a = 1$.
(b) Magnetic field for "TE"$_{32}$ mode and $\lambda_z/2a = 1$.

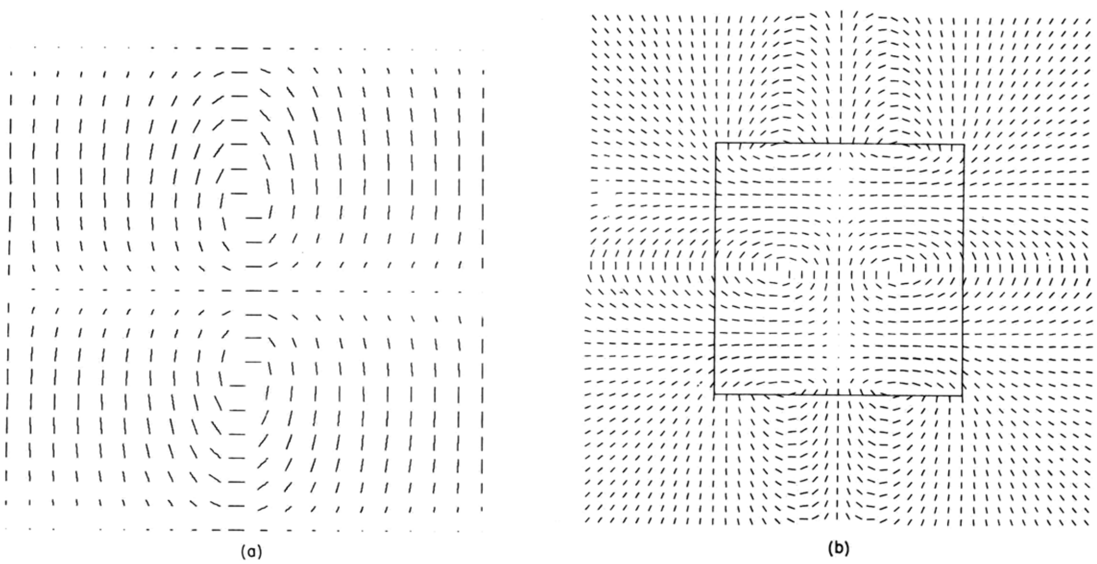


Fig. I-6. (a) Electric field for "TE"$_{12}$ mode and $\lambda_z/2a = 0.3$.
(b) Magnetic field for "TE"$_{12}$ mode and $\lambda_z/2a = 0.3$.

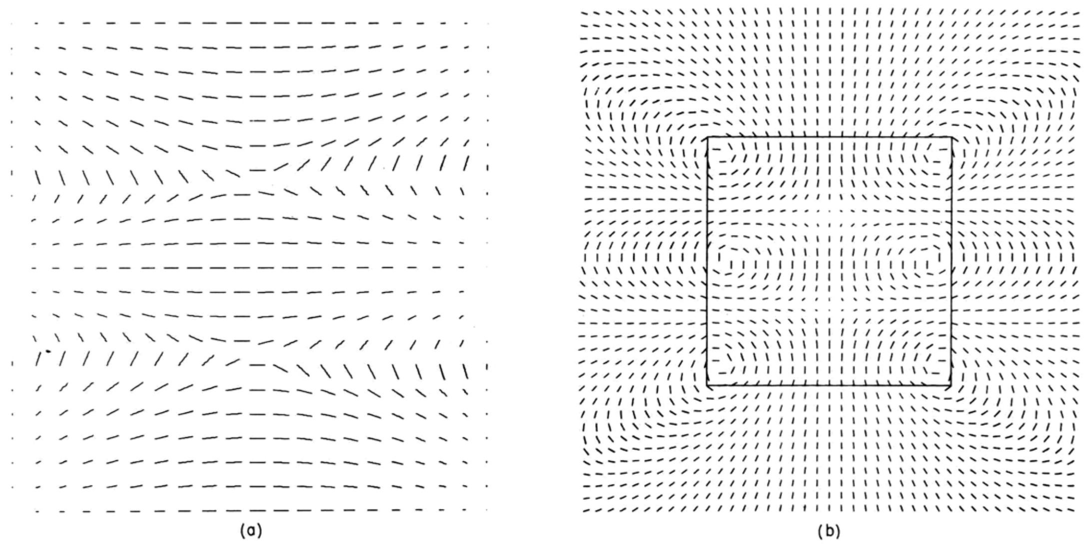


Fig. I-7. (a) Electric field for "TM"$_{12}$ mode and $\lambda_z/2a = 0.3$.
(b) Magnetic field for "TM"$_{12}$ mode and $\lambda_z/2a = 0.3$.

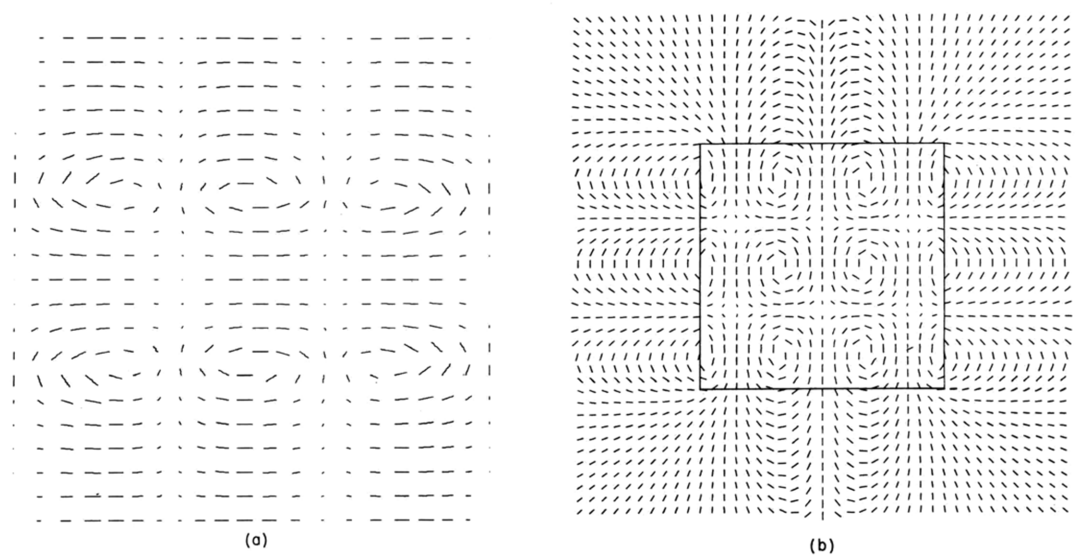


Fig. I-8. (a) Electric field for "TE"$_{32}$ mode and $\lambda_z/2a = 0.3$.
(b) Magnetic field for "TE"$_{32}$ mode and $\lambda_z/2a = 0.3$.

$$\hat{n} \cdot \vec{E}_T = 0. \tag{2}$$

Using a collocation technique, we transform the equation into a straightforward matrix eigenvalue problem of the form

$$\overline{\overline{m}}_1 \begin{pmatrix} \vec{A} \\ \vec{B} \end{pmatrix} = \Omega^2 \overline{\overline{m}}_2 \begin{pmatrix} \vec{A} \\ \vec{B} \end{pmatrix}. \tag{3}$$

The eigenvalues are normalized frequencies

$$\Omega^2 = \frac{\omega^2 \mu_o \epsilon ab}{2\pi}. \tag{4}$$

The eigenvectors $\vec{A}$ and $\vec{B}$ are coefficients in the expansions for the transverse field components E_x and E_y, respectively. The expansions used in this study were in terms of Tchebyshev polynomials, weighted by factors such as $(1-x^2)$ or $(1-y^2)$, so that Eq. 2 is satisfied identically.

Equation 1 and the procedure for solving it are valid not only for a square waveguide but also for a rectangular waveguide of arbitrary aspect ratio. They are also valid for a dielectric waveguide of arbitrary cross section, provided that the problem is done in Cartesian coordinates.

Equations 1 and 2 determine the transverse electric field, and the remaining field components follow directly from Maxwell's equations

$$\nabla \cdot \epsilon \vec{E} = 0$$

$$\vec{H} = \frac{j}{\omega \mu_o} \nabla \times \vec{E}.$$

Figure I-1 is a dispersion relation obtained for the square waveguide and for E_z of even symmetry in both transverse dimensions. The first five modes were obtained for field expansions of 4 or 5 terms in each dimension.

Figures I-2 through I-8 are vector plots of selected electric and magnetic fields. What is shown is the projection on the transverse plane of an array of unit vectors for displaying the <u>direction only</u> of the field indicated. Where the marks are very short, the field vector is mostly in the z direction. <u>No</u> indication of field magnitude is shown.

The fields on some of the figures seem to spiral. This is only an optical illusion because the array points are at one end of each of the marks.

This work may be extended to dielectric resonators. A formulation analogous to Eq. 1 has been obtained.[2]

References

1. J. K. Raines, "Propagating Waves on a High Dielectric Slab," Quarterly Progress Report No. 108, Research Laboratory of Electronics, M. I. T., January 15, 1973, pp. 23-27.

2. J. K. Raines, "Application of Integral Equations to Dielectric Waveguides of Rectangular Cross Section," Ph.D. Thesis, submitted to the Department of Electrical Engineering, M. I. T., September 1973.