III. PHYSICAL ACOUSTICS

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A. ATTENUATION OF SOUND IN TURBULENT PIPE FLOW

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In turbulent pipe flow there is a static pressure drop along the length of the pipe, which is caused by turbulent friction. This turbulent friction can also attenuate the fundamental acoustic mode propagating in the duct. A theoretical analysis of this effect is presented here. The variation of static density along the duct axis and the change of friction factor with Reynolds number are included in the theoretical model. Also, some experimental data are compared with results of this analysis.

It has been found experimentally that the static pressure gradient dp_o/dx is almost a constant along the pipe when the entrance and the exit sections are ignored. In the present analysis dp_o/dx is taken from experiment. Let

$$\frac{dp_o}{dx} = -a = \text{experimental constant.} \tag{1}$$

Here $a > 0$ if x increases in the direction of flow. We assume a one-dimensional model and also that the fluid is "barotropic"; that is, $p \sim \rho^{\ell}$, where $1 \leqslant \ell \leqslant \gamma$, with γ the ratio of specific heats. The continuity and momentum equations for the steady basic unperturbed flow in the duct are

$$\rho_o u_o = \text{constant} \equiv m_o \tag{2}$$

$$m_o \frac{du_o}{dx} + \frac{\alpha}{2} \rho_o u_o^2 = -\frac{dp_o}{dx}, \tag{3}$$

where α is related to the friction factor f and is defined as $\frac{dp}{dx} = -\alpha \frac{\rho u^2}{2}$. The term $\frac{1}{2} \alpha \rho_o u_o^2$ represents the turbulent shear stress. The equation of state is assumed to be

$$\frac{p_o}{\rho_o^{\ell}} = \frac{p_{ref}}{\rho_{ref}^{\ell}}, \tag{4}$$

where $\ell = 1$ for isothermal basic flow, and $\ell = \gamma$ for adiabatic basic flow. Equations 1-4 can be combined to give the basic flow.

$$p_o = p_{ref} - ax \tag{5}$$

$$\rho_o = \rho_{ref} - \frac{b}{\ell} x \tag{6}$$

$$u_o = \frac{m_o}{\rho_{ref} - \frac{b}{\ell} x} \approx \frac{m_o}{\rho_{ref}} \left(1 + \frac{a}{p_{ref} \ell} x\right), \tag{7}$$

where

$$b = \frac{a\rho_{ref}}{p_{ref}} = \frac{a\rho_o}{p_o} \tag{8}$$

and subscript ref denotes the conditions at $x = 0$.

We retain only first-order terms in the pressure gradient a or "density" gradient b, and ignore all second-order terms, such as ab, b^2, a^2. Thus, d^2u_o/dx^2 and $d^2\rho_o/dx^2$ are neglected. Also, α is assumed to be a function of u_o alone.

The continuity and momentum equations for the sound field are

$$\frac{\partial \rho_1}{\partial t} + \frac{\partial}{\partial x} (\rho_o u_1 + \rho_1 u_o) = 0 \tag{9}$$

$$\begin{aligned} \rho_o \frac{\partial u_1}{\partial t} + m_o \frac{\partial u_1}{\partial x} + (\rho_o u_1 + \rho_1 u_o) \frac{\partial u_o}{\partial x} + \alpha m_o u_1 + \frac{1}{2} \alpha \rho_1 u_o^2 \\ + \frac{1}{2} m_o \frac{\partial \alpha}{\partial u_o} u_o u_1 + 2\beta_v \rho_o c_o u_1 = - \frac{\partial p_1}{\partial x}. \end{aligned} \tag{10}$$

Here β_v is the attenuation of sound caused by viscosity and heat conduction.

To separate the effects of variations in static density, we redefine the acoustic variables.

$$R = \frac{\rho_1(x,t)}{\rho_o(x)} \quad \text{and} \quad V = \frac{u_1(x,t)}{u_o(x)}. \tag{11}$$

The first-order acoustic flow is taken as adiabatic so that

$$p_1 = c_o^2 \rho_1. \tag{12}$$

Equations 9 and 10 when transformed to new variables become

$$\left(\frac{\partial}{\partial t} + u_o \frac{\partial}{\partial x}\right) R + u_o \frac{\partial V}{\partial x} = 0 \tag{13}$$

$$\left[\frac{\partial}{\partial t} + u_o \frac{\partial}{\partial x} - \frac{2}{m_o} \frac{\partial p_o}{\partial x} + 2\beta_v c_o + \frac{1}{2} \frac{\partial \alpha}{\partial u_o} u_o^2\right] V$$

$$+ \left[\frac{\partial u_o}{\partial x} + \frac{\alpha}{2} u_o + \frac{c_o^2 \rho_o}{m_o} \frac{\partial}{\partial x} + \frac{\gamma}{m_o} \frac{\partial p_o}{\partial x}\right] R = 0. \tag{14}$$

Differentiating Eq. 14 with respect to x, eliminating $\partial V/\partial x$, using Eq. 13 and the basic flow solution leads to the wave equation

$$\frac{\partial^2 R}{\partial t^2} + 2u_o \frac{\partial^2 R}{\partial x \partial t} + u_o^2 \frac{\partial^2 R}{\partial x^2} + \frac{\partial R}{\partial t} \left(\frac{2a}{m_o} + 2\beta_v c_o + \frac{1}{2} \frac{\partial \alpha}{\partial u_o} u_o^2\right)$$

$$+ u_o \frac{\partial R}{\partial x} \left[2\beta_v c_o + \frac{1}{2} \frac{\partial \alpha}{\partial u_o} u_o^2 - \frac{\alpha u_o}{2} + \frac{2a(\gamma+1)}{m_o}\right] = c_o^2 \frac{\partial^2 R}{\partial x^2}. \tag{15}$$

This is a linear second-order partial differential equation with variable coefficients for the density perturbation R. The coefficients β_v, $\frac{2a}{m_o}$, α, $\frac{\partial \alpha}{\partial u_o}$ and $\frac{2a}{m_o}(\gamma+1)$ are constants, but c_o and u_o are functions of x.

For a wave solution of the form $\exp[-i\omega t + i \int k(x)\, dx]$ Eq. 15 gives a dispersion relation for the wave number k.

$$\left(1-M_o^2\right)k^2 - i\left(1-M_o^2\right) \frac{\partial k}{\partial x}$$

$$+ kM_o \left[2 \frac{\omega}{c_o} + i\left\{\frac{2a(\gamma+1)}{m_o c_o} + 2\beta_v + \frac{1}{2} \frac{\partial \alpha}{\partial M_o} M_o^2 - \frac{\alpha M_o}{2}\right\}\right]$$

$$- \frac{\omega^2}{c_o^2} \left[1 + i \frac{c_o}{\omega} \left\{\frac{2a}{m_o c_o} + 2\beta_v + \frac{1}{2} \frac{\partial \alpha}{\partial M_o} M_o^2\right\}\right] = 0, \tag{16}$$

where $M_o = u_o/c_o$ is the Mach number of the basic flow.

The real part of the wave number k is likely to be $\frac{\omega/c_o}{1 + M_o}$ for the right traveling wave, and $\frac{-\omega/c_o}{1 - M_o}$ for the left traveling wave. We shall use this to find $\partial k/\partial x$ to be used in Eq. 16. After obtaining the solution we can check to see whether this is a consistent approximation. The solution of Eq. 16 yields

$$k_+ = \frac{\omega/c_o}{1+M_o} + i\,\frac{1}{1+M_o}\left\{\begin{array}{l}\beta_v + \dfrac{\alpha M_o^2}{4} + \dfrac{a}{m_o c_o}\left(1 - \dfrac{3}{4}\gamma M_o - \dfrac{\gamma M_o^2}{4}\right) \\ - \dfrac{b}{4\ell\rho_o}(1+2M_o)(1-M_o) + \dfrac{1}{4}\dfrac{\partial\alpha}{\partial M_o} M_o^2\end{array}\right\} \tag{17}$$

$$k_- = \frac{-\omega/c_o}{1-M_o} - i\,\frac{1}{1-M_o}\left\{\begin{array}{l}\beta_v - \dfrac{\alpha M_o^2}{4} + \dfrac{a}{m_o c_o}\left(1 + \dfrac{3}{4}\gamma M_o - \dfrac{\gamma M_o^2}{4}\right) \\ + \dfrac{b}{4\ell\rho_o}(1-2M_o)(1+M_o) + \dfrac{1}{4}\dfrac{\partial\alpha}{\partial M_o} M_o^2\end{array}\right\} \tag{18}$$

provided

$$\frac{c_o}{\omega}\left\{\begin{array}{l}2\beta_v + \dfrac{2a}{m_o c_o}\left(1+\gamma M_o^2\right) + \dfrac{1}{2}\dfrac{\partial\alpha}{\partial M_o} M_o^2 - \dfrac{\alpha M_o^3}{2} \\ + \dfrac{\gamma a}{2\rho_o c_o^2}(1-M_o)^2 - \dfrac{b(1+2M_o)(1-M_o)^2}{2\ell\rho_o}\end{array}\right\} \ll 1 \quad \text{for} \quad k_+$$

and

$$\frac{c_o}{\omega}\left\{\begin{array}{l}2\beta_v + \dfrac{2a}{m_o c_o}\left(1+\gamma M_o^2\right) + \dfrac{1}{2}\dfrac{\partial\alpha}{\partial M_o} M_o^2 - \dfrac{\alpha M_o^3}{2} \\ - \dfrac{\gamma a}{2\rho_o c_o^2}(1+M_o)^2 + \dfrac{b(1-2M_o)(1+M_o)^2}{2\ell\rho_o}\end{array}\right\} \ll 1 \quad \text{for} \quad k_-\,.$$

We note that $\partial k_i/\partial x$ is a second-order quantity in a, b, α, β_v, so we were right in approximating $\partial k/\partial x$. We note that

$$\frac{a}{m_o c_o} = \frac{\alpha M_o}{2} \tag{19}$$

$$\frac{b}{4\ell\rho_o} = \frac{\alpha}{8}\,\frac{\gamma}{\ell}\,M_o^2. \tag{20}$$

Therefore

$$k_{i_+} = \frac{1}{(1+M_o)} \left\{ \begin{array}{l} \beta_v + \frac{1}{4} \frac{\partial \alpha}{\partial M_o} M_o^2 + \frac{\alpha M_o}{4} (2+M_o) \\ - \frac{\alpha}{8} \frac{\gamma}{\ell} M_o^2(1+3\ell) + M_o(1+\ell) - 2M_o^2 \end{array} \right\} \tag{21}$$

$$k_{i_-} = \frac{1}{1 - M_o} \left\{ \begin{array}{l} \beta_v + \frac{M_o^2}{4} \frac{\partial \alpha}{\partial M_o} + \frac{\alpha M_o}{4} (2-M_o) \\ + \frac{\alpha}{8} \frac{\gamma}{\ell} M_o^2\left[(1+3\ell) - M_o(1+\ell) - 2M_o^2\right] \end{array} \right\} \tag{22}$$

Obviously the attenuations are a function of the Mach number. At zero Mach number the attenuation is due to viscosity and heat conduction. It is interesting to note that the attenuation at very low Mach numbers can be made smaller than that resulting from viscosity and heat conduction if the following relations between α, β_v, and M_o are satisfied. Only the first three terms in Eqs. 21 and 22 are kept for this purpose.

$$\frac{\partial k_{i_+}}{\partial M_o} < 0 \quad \text{if}$$

$$- \beta_v + \frac{\left(2-M_o^2-M_o^3\right)}{4(1-M_o)} \alpha + \frac{M_o\left(4-3M_o^2-M_o^3\right)}{4(1-M_o)} \frac{\partial \alpha}{\partial M_o} + \frac{M_o^2(1+M_o)}{4} \frac{\partial^2 \alpha}{\partial M_o^2} < 0$$

and

$$\frac{\partial k_{i_-}}{\partial M_o} < 0 \quad \text{if}$$

$$\beta_v + \frac{\left(2-M_o^2+M_o^3\right)}{4(1+M_o)} \alpha + \frac{M_o\left(4-3M_o^2+M_o^3\right)}{4(1+M_o)} \frac{\partial \alpha}{\partial M_o} + \frac{M_o^2(1-M_o)}{4} \frac{\partial^2 \alpha}{\partial M_o^2} < 0.$$

At high Mach numbers (at high Reynolds numbers) $\partial\alpha/\partial M_o = 0$ and $\alpha/2 \gg \beta_v$. Hence attenuation increases with Mach number in that region.

For the sake of simplicity, we shall use the power law for the friction factor for smooth pipes. Then $\alpha = (\text{const})M_o^{-1/4}$. In this approximation $M_o \frac{\partial\alpha}{\partial M_o}$ and $M_o^2 \frac{\partial^2\alpha}{\partial M_o^2}$ are of the order of α. Then, to lowest order, $\partial k_{i_-}/\partial M_o < 0$ if $\beta_v + 21\alpha/64 < 0$, which is impossible. Therefore, upstream attenuation always increases with Mach number. Similarly, for the downstream attenuation, if $\beta_v > \frac{21\alpha}{64}$, then $\partial k_{i_+}/\partial M_o < 0$. This means that if $\alpha < (\text{const})\sqrt{f}$, then the downstream attenuation will decrease initially with Mach number. If the pipe roughness is so controlled that $\alpha < (\text{const})\sqrt{f}$, then by turning on a very low speed flow the sound attenuation can be reduced to a level less than that caused by viscosity and heat conduction. This result is very interesting and may have useful application in the transmission of sound signals through long pipes. Note that β_v is proportional to the square root of the acoustic frequency f. The value of the constant depends on the viscosity and thermal conductivity of the gas and the hydraulic diameter of the pipe.

Now we correct Eqs. 21 and 22 for the change in static density. We use the notation

$$R \sim \exp[ikx-i\omega t] \tag{23a}$$

$$\rho_1 \sim \exp[ik^*x-i\omega t] \tag{23b}$$

to obtain

$$k_{i_+}^* = k_{i_+} + \frac{\alpha}{2}\frac{\gamma}{\ell} M_o^2 \tag{24a}$$

$$k_{i_-}^* = k_{i_-} - \frac{\alpha}{2}\frac{\gamma}{\ell} M_o^2 \tag{24b}$$

$$k_{i_+}^* = \frac{1}{1+M_o}\left[\beta_v + \frac{M_o^2}{4}\frac{\partial\alpha}{\partial M_o} + \frac{\alpha M_o}{4}(2+M_o) + \frac{\alpha}{8}\frac{\gamma}{\ell} M_o^2\left\{3(1-\ell) + M_o(3-\ell) + 2M_o^2\right\}\right] \tag{25}$$

$$k_{i_-}^* = \frac{1}{1-M_o}\left[\beta_v + \frac{M_o^2}{4}\frac{\partial\alpha}{\partial M_o} + \frac{\alpha M_o}{4}(2-M_o) + \frac{\alpha}{8}\frac{\gamma}{\ell} M_o^2\left\{3(\ell-1) + M_o(3-\ell) - 2M_o^2\right\}\right] \tag{26}$$

Note that for small Mach numbers k_{i_+}/k_{i_-} is approximately equal to $(1+M_o)/(1-M_o)$. But for large Mach numbers

$$\frac{k_{i_+}}{k_{i_-}} \neq \frac{1-M_o}{1+M_o}$$

$$\frac{k^*_{i_+}}{k^*_{i_-}} \neq \frac{1-M_o}{1+M_o}.$$

For isothermal basic flow these relations give

$$k^*_{i_+} = \frac{1}{1+M_o}\left[\beta_v + \frac{M_o^2}{4}\frac{\partial\alpha}{\partial M_o} + \frac{\alpha M_o}{2} + \frac{\alpha M_o^2}{4}\left(1+\gamma M_o+\gamma M_o^2\right)\right] \tag{27}$$

$$k^*_{i_-} = \frac{1}{1-M_o}\left[\beta_v + \frac{M_o^2}{4}\frac{\partial\alpha}{\partial M_o} + \frac{\alpha M_o}{2} - \frac{\alpha M_o^2}{4}\left(1-\gamma M_o+\gamma M_o^2\right)\right], \tag{28}$$

whereas for adiabatic basic flow they yield

$$k^*_{i_+} = \frac{1}{1+M_o}\left[\beta_v + \frac{M_o^2}{4}\frac{\partial\alpha}{\partial M_o} + \frac{\alpha M_o}{2} + \frac{\alpha M_o^2}{8}\left\{(5-3\gamma) + M_o(3-\gamma) + 2M_o^2\right\}\right] \tag{29}$$

$$k^*_{i_-} = \frac{1}{1-M_o}\left[\beta_v + \frac{M_o^2}{4}\frac{\partial\alpha}{\partial M_o} + \frac{\alpha M_o}{2} + \frac{\alpha M_o^2}{8}\left\{(3\gamma-5) + M_o(3-\gamma) - 2M_o^2\right\}\right]. \tag{30}$$

We note that the "attenuation" caused by change in static density is the same in both directions, whereas the attenuation caused by turbulence is different. Also note that the increase in static density in the upstream direction tends to amplify the sound signal, hence to decrease its attenuation resulting from turbulence.

For isothermal basic flow we note from Eqs. 21, 22, and 24 that the ratio of attenuation resulting from turbulence and static density variation goes as follows. When $\partial\alpha/\partial M_o = 0$, we get for the upstream direction

$$\frac{2+(2\gamma-1)M_o - \gamma M_o^2 - \gamma M_o^3}{2\gamma M_o(1-M_o)},$$

and for the downstream direction,

$$\frac{2-(2\gamma-1)M_o-\gamma M_o^2+\gamma M_o^3}{2\gamma M_o(1+M_o)}.$$

At low Mach numbers this ratio is fairly large, so that the effect of static density variation may be ignored. But at large Mach numbers the effect of static density variation cannot be ignored. As the Mach number increases, the contribution of static density completely overshadows the turbulent attenuation on the downstream side.

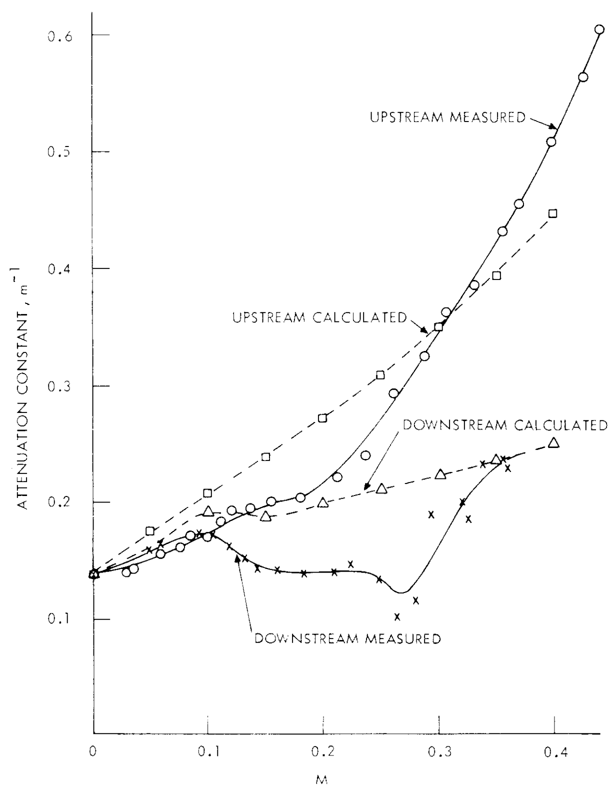

Fig. III-1. Measured and calculated upstream and downstream attenuation of sound in turbulent pipe flow.

The attenuation of an acoustic pulse was measured with and without flow in a duct of square cross section with a 3/4 in. side. Two microphones were placed 6 ft apart. The speaker was pulsed at 1240 Hz. Interference from the pipe termination was avoided by proper design. Figure III-1 shows the experimental results, as well as the values calculated from Eqs. 27 and 28 with

$$\alpha=\frac{4}{d}\left(0.0014+0.0125\,\mathrm{Rey}^{-0.32}\right), \tag{31}$$

where d is the side of the square cross section of the duct, and Rey is the Reynolds number of the mean flow.

The attenuation increases slowly with Mach number at low Mach numbers but for Mach numbers greater than 0.25 the effect of turbulence is very large indeed. The downstream attenuation increases at a slower rate than does the upstream attenuation. That the downstream attenuation can decrease with increasing Mach number is evident at $M = 0.1$ on the calculated curve.

The downstream measured curve deviates significantly from the downstream calculated curve in the Mach number range 0.1-0.32. The reason for this is that in this region the basic flow is in the transition range from laminar to turbulent flow. This effect also appears on the upstream measured curve.

Turbulence scatters sound waves and high-frequency scattering cannot be detected quantitatively with the simple pulse technique that we used. This scattering of the incident harmonic sound pulse train is neglected in the present analysis and may account for some of the observed discrepancies.

The friction-factor law, Eq. 31, used for the calculations is the commonly accepted expression. Our measurements show higher friction factors. In the transition region it is very hard to get a good estimate of a. The errors in the constant a are obviously reflected as errors on the calculated curves in Fig. III-1.

B. ORIFICE FLOW NOISE

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U. Ingard

In one of our recent experiments in which air was sucked through an orifice in a plate forming one of the walls of a suction chamber, we focused attention mainly on the noise on the upstream side of the orifice plate. Of particular interest was the dependence of the noise emission on the static pressure ratio in the vicinity of the critical value of this ratio.

In these experiments we also studied the sound emission into the suction chamber. Figure III-2 shows the measured sound-pressure level (SPL) inside the chamber as a function of the static pressure ratio across the orifice plate.

It is interesting to try to interpret the observed dependence of the SPL on the pressure ratio in terms of the expression

$$W_2 = C_n\left(\frac{V_o}{c_2}\right)^n \frac{\rho_2 V_o^3}{2} A_o \tag{1}$$

for the acoustic power W_2 emitted from the flow on the downstream side of the orifice plate. In this expression C_n is a numerical constant, V_o is the velocity in the orifice, ρ_2 is the fluid density in the suction chamber, and A_o is the orifice area. The factor $C_n(V_o/c_2)^n$ can be regarded as the efficiency of noise emission by the jet stream discharging kinetic energy at the rate $\rho_2 V_o^3 A_o/2$ (the difference between the density ρ_o in the jet at the orifice and the density ρ_c in the chamber is ignored). The exponent n depends on the nature of the flow fluctuations in the stream. For flow pulsations n = 1 (monopole), lateral-flow fluctuations and corresponding pressure fluctuations on the orifice plate correspond to n = 3 (dipole), and fully developed turbulence in the stream (quadrupole) corresponds to n = 5. In general, the emitted noise is contributed by all three effects and we may set

$$W_2 = \left[C_m \frac{V_o}{c_2} + C_d\left(\frac{V_o}{c_2}\right)^3 + C_q\left(\frac{V_o}{c_2}\right)^5\right] \frac{\rho_2 V_o^3}{2} A_o. \tag{2}$$

We shall show in this report that our experimental data can be fitted well to an expression of this form.

1. Pressure Drop

In our experiments we did not measure directly the Mach number in the orifice but rather the static pressures P_1 and P_2 outside and inside the orifice plate. We shall relate these pressures by making the assumption that the flow is isentropic from the outside of the chamber to the orifice and that there is very little "pressure recovery" in the suction chamber on the downstream side of the orifice plate. The pressure in the jet, P_o, as it leaves the orifice is then assumed to be the same as the pressure P_2 in the chamber. The local temperature T_o and the density ρ_o in the jet at the orifice will be somewhat different from the corresponding quantities in the almost quiescent air in the suction chamber. Under these conditions, we obtain

$$V_o^2 = \frac{2}{\gamma - 1} c_1 \left[1 - \left(\frac{P_o}{P_1}\right)^{(\gamma-1)/\gamma}\right] = \frac{2}{\gamma - 1} c_1^2 \left[1 - \left(1 - \frac{\Delta P}{P_1}\right)^{(\gamma-1)/\gamma}\right], \tag{3}$$

where c_1 is the sound speed outside the chamber, P_1 the static pressure outside the chamber, and ΔP the pressure drop $P_1 - P_o$.

If we neglect pressure recovery, we have $\Delta P \approx P_1 - P_2$, where P_2 is the static pressure inside the chamber. Since the temperatures inside and outside the chamber are the same, we may set $c_1 = c_2$. The expression for the acoustic power in Eq. 2 can then be written

$$W_2 = \left[C_m F(x) + C_d F^3(x) + C_q F^5(x)\right] F^3(x) \frac{\rho_2 c_1^3}{2} A_o. \tag{4}$$

Finally, if we set $\rho_2 = \rho_1(1-x)$, we obtain

$$\frac{W_2}{\left(\rho_1 c_1^3/2\right)} = \left[C_m F^4(x) + C_d F^6(x) + C_q F^8(x)\right](1-x), \tag{5}$$

where $x = (P_1-P_2)/P_1 \equiv \Delta P/P_1$.

In Fig. III-2 the functions represented by the three terms in Eq. 5 (including the factor (1−x)) are shown, with the constants C_q, C_d, and C_m adjusted to produce a best fit with the experimental data.

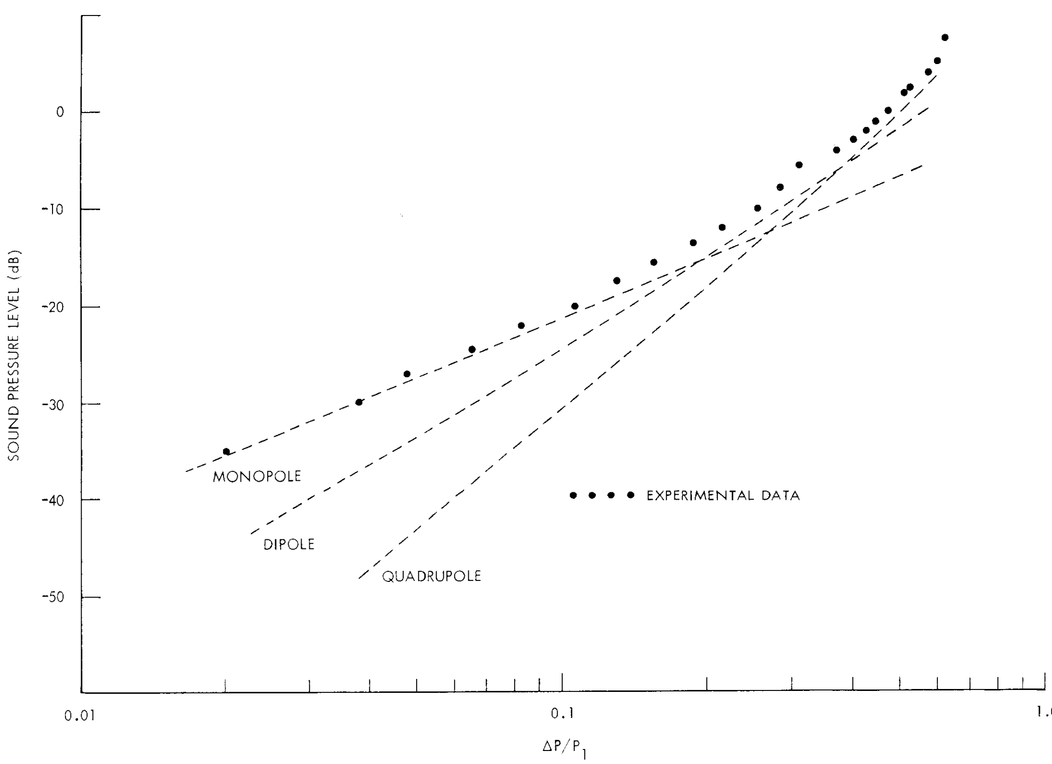

Fig. III-2. Orifice flow noise.

We find that the best fit is obtained if $C_m \approx 140\ C_q$, $C_d \approx 19\ C_q$ so that our empirical expression for W_2 becomes

$$\frac{W_2}{\left(\rho_1 c_1^3/2\right)} \approx C_q\left[140\, F^4 + 19\, F^6 + F^8\right](1-x). \tag{6}$$

Once our "reverberation" chamber (suction chamber) has been calibrated, the value of C_q can be determined from our measurements.