X. PHYSICAL ACOUSTICS*

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RESEARCH OBJECTIVES

Our general objective involves the study of the emission, propagation, and absorption of acoustic waves in matter. Specific areas of present research include (i) the interaction of waves with coherent light beams in fluids and solids, (ii) nonlinear acoustics in fluids, and (iii) the generation and propagation of sound waves in moving fluids, with particular emphasis on waves propagated in ducts and generation of sound by turbulent flow.

K. U. Ingard

A. UPSTREAM AND DOWNSTREAM SOUND RADIATION INTO A MOVING FLUID

ONR (Contract N00014-67-A-0204-0019)

K. U. Ingard, V. K. Singhal

The problem of sound radiation from a source in relative motion with respect to the surrounding fluid has become of considerable interest, particularly in connection with noise generation in aircraft and in various types of fluid machinery. Although the basic effects, as well as many details of the influence of fluid motion on sound radiation, have been identified by several investigators,[1-4] these studies have been limited to mathematical analysis of the idealized case of moving point sources. The problem of sound emission from real sources of finite dimensions has received much less attention. We present an explicit demonstration of the influence of relative fluid motion on sound radiation from a stationary source, with particular attention to the relation between the sound pressure fields radiated upstream and downstream.

1. Experimental Arrangement

The experimental arrangement is shown schematically in Fig. X-1. A sound source is mounted in one of the side walls of a rectangular duct with inner dimensions

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$3/4 \times 7/8$ in. The duct is connected to a steam ejector through a plenum chamber, and the flow speed in the tube can be varied from 0 to ~100 m/s, which corresponds to Mach number ~0.3.

The loudspeaker is driven by means of a pulse generator and produces harmonic sound-pressure wave trains in the duct. The carrier frequency of these waves is chosen to be considerably lower than the cutoff frequency for the first higher order acoustic mode in the duct, so that at the carrier frequency only the plane-wave mode will be able to propagate.

The pressure pulses are detected by two identical pressure transducers mounted in the side walls of the duct on the upstream and downstream sides of the sound source at equal distances from it. In the absence of flow, M = 0, the recorded pulses from these transducers are simultaneous and identical in shape, as demonstrated by the results shown in Fig. X-1. Any difference in the sensitivity of the transducers is compensated for by gain adjustment of the transducer amplifiers, so that in the absence of flow the amplitudes of the recorded pulses from the transducers are equal.

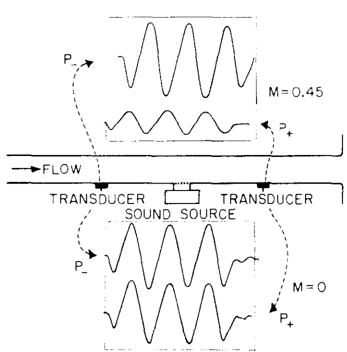

Fig. X-1.
Recorded pressure pulses in the upstream (p_-) and the downstream (p_+) directions at flow Mach numbers M = 0 and M = 0.3.

When the air in the duct is moving, however, the pressure amplitudes are no longer equal; the amplitude in the upstream direction is larger than in the downstream direction, and the difference increases monotonically with the flow speed in the duct. An example of recorded upstream and downstream pressure waves at a flow Mach number 0.3 is shown in Fig. X-1. In addition to the obvious difference in the pressure amplitudes, a difference in the time of arrival of the two pulses is also apparent in this figure. From this time difference and the known distance between the sound source and the receivers, the flow Mach number in the duct can be determined.

The Mach number dependence of the ratio between the upstream and downstream pressure amplitudes obtained in this manner is illustrated in Fig. X-2. Measurements were carried out at two frequencies, 800 Hz and 2000 Hz. As can be seen, there is no marked difference in the amplitude ratio at these frequencies.

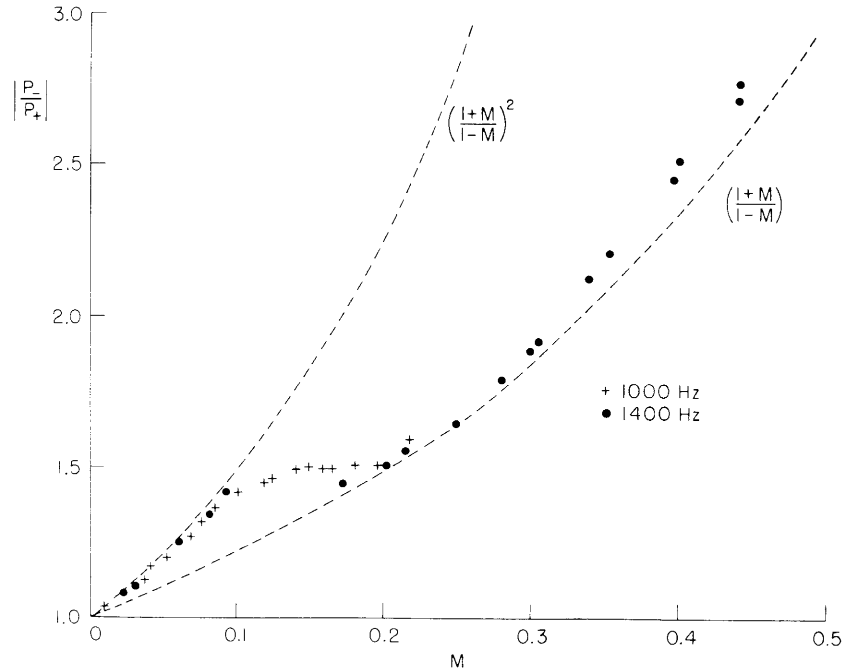

Fig. X-2. Mach number dependence of the measured ratio $|p_-/p_+|$ between the pressure amplitudes radiated in the upstream and downstream directions.

2. Mathematical Analysis

In the mathematical analysis of this problem we start from the wave equation for the sound pressure field p(x, y, z, t)

$$(1-M^2)\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}-\frac{2M}{c}\frac{\partial^2 p}{\partial x\partial t}-\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}=0, \tag{1}$$

where c is the sound speed, and M the Mach number. The coordinates x, y, z refer to a stationary laboratory frame of reference with respect to which the unperturbed fluid is assumed to move with uniform speed Mc in the positive x direction. We wish to solve this equation, subject to the boundary conditions peculiar to our experimental arrangement.

The duct walls, placed in the planes y = 0, y = a and z = 0, z = b, are assumed to be rigid everywhere except in the source region, as indicated in Fig. X-3. Consequently, the normal components of the fluid velocity and the corresponding pressure gradients are zero at the boundaries except at the source. If the source, located in the wall in the plane y = 0, produces a velocity perturbation u_y in the fluid flow in the plane y = 0, the effect of the source can be expressed as the boundary condition

$$u_y = u_o f(x, z, t) \qquad (y=0). \tag{2}$$

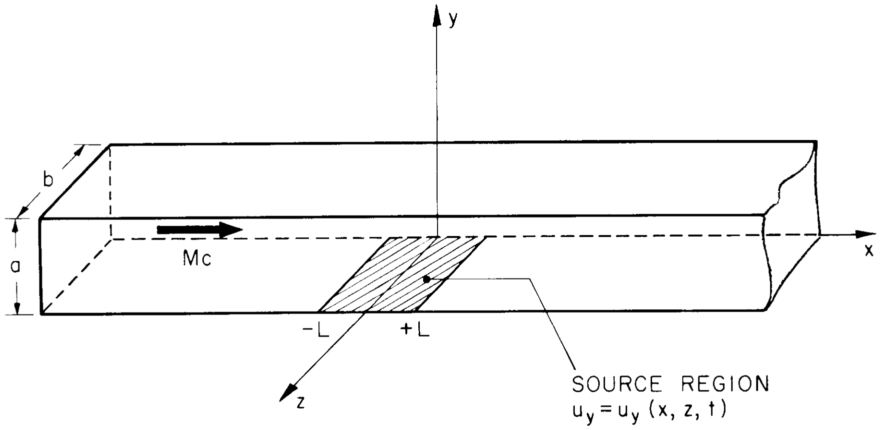

Fig. X-3. The source region is in the plane y = 0 of the duct and is defined by the perturbation of the fluid velocity u_y in the duct.

It is important to realize, however, that this velocity perturbation of the fluid in the duct is not necessarily the same as the velocity of the oscillating air column in the throat of the loudspeaker source

$$u_s = \frac{\partial\eta_s(x, z, t)}{\partial t}, \tag{3}$$

where $\eta_s(x, z, t)$ is the displacement of the air column. Although $u_y = u_s$ is valid when there is no mean flow in the duct, the situation is more complex when mean flow is present. For example, if the flow over the source region is streamlined, the transverse oscillatory motion of the air out of the source will result in a displacement of the streamlines, and this displacement gives rise to a velocity perturbation in the fluid flow given by

$$u_y(x, z, t) = \left(\frac{\partial}{\partial t} + Mc\,\frac{\partial}{\partial x}\right)\eta_s(x, z, t). \tag{4}$$

This model of the flow perturbation produced by the source may not be quite realistic in a highly turbulent duct flow, in which case the contribution from the space derivative in Eq. 4 is expected to be reduced by the irregularities in the flow. In the absence of this contribution, the boundary condition in (4) reduces to $u_y = u_s$.

In our experiment, since only the plane-wave mode is transmitted along the duct, it is expedient to introduce the average sound pressure $\overline{p}$ over the duct cross section,

$$\overline{p} = \frac{1}{A}\iint p\,dydz. \tag{5}$$

We now obtain a wave equation for $\overline{p}$ by integrating Eq. 1 over the transverse coordinates y, z. We make use of the fact that the duct walls are rigid, except in the source region, and note that the average of $\partial^2 p/\partial z^2$ is zero. Similarly, the average of $\partial^2 p/\partial y^2$ is $(1/A) \int (-\partial p/\partial y)_0 \, dz$, where $(\partial p/\partial y)_0$ is evaluated in the source region of the wall at $y = 0$. We can express $(\partial p/\partial y)_0$ in terms of the velocity perturbation u_y from the momentum equation

$$\rho\left(\frac{\partial}{\partial t} + Mc \frac{\partial}{\partial x}\right) u_y = -\frac{\partial p}{\partial y}. \tag{6}$$

The wave equation for the average pressure $\overline{p}$ can then be expressed as

$$(1-M^2) \frac{\partial^2 \overline{p}}{\partial x^2} - 2 \frac{M}{c} \frac{\partial^2 \overline{p}}{\partial x \partial t} - \frac{1}{c^2} \frac{\partial^2 \overline{p}}{\partial t^2} = s(x, t), \tag{7}$$

where

$$s(x, t) = -\frac{\rho}{A} \left(\frac{\partial}{\partial t} + Mc \frac{\partial}{\partial x}\right) \int_0^b u_y(x, z, t) \, dz. \tag{8}$$

In this inhomogeneous wave equation the right-hand side is considered to be a known source function $s(x, t)$ defined in the source plane $y = 0$, where u_y is given by Eq. 4. To solve this equation, we introduce the Fourier transforms

$$\overline{p}(x, t) = \iint P(k, \omega) \, e^{ikx} \, e^{-i\omega t} \, dk d\omega \tag{9}$$

$$s(x, t) = \iint S(k, \omega) \, e^{ikx} \, e^{-i\omega t} \, dk d\omega, \tag{10}$$

and from Eq. 7 obtain

$$P(k, \omega) = \frac{-S(k, \omega)}{(1-M^2)(k-k_+)(k-k_-)}, \tag{11}$$

where

$$k_+ = \frac{\omega}{c(1+M)} \qquad k_- = \frac{\omega}{c(1-M)}. \tag{12}$$

If we let the source region be limited to $-L < x < L$, so that $s(x, t)$ is zero outside this region, we can express $S(k, \omega)$ as

$$S(k, \omega) = \frac{1}{2\pi} \int_{-L}^{+L} s(x_o, \omega) \exp(-ikx_o) \, dx_o, \tag{13}$$

where $s(x, \omega)$ is the temporal Fourier transform of $s(x, t)$. Then, from Eqs. 4 and 7, we have

$$S(k, \omega) = (i\omega)\left(\frac{A_s}{A}\right) \rho u_o\left(1 - Mc \frac{k}{\omega}\right)^2 \eta_s(k, \omega), \tag{14}$$

where

$$\eta_s(k, \omega) = \frac{1}{2\pi\eta_o} \frac{1}{b2L} \int_0^b dz_o \int_{-L}^{+L} \eta_s(x_o, z_o, \omega) \exp(-ikx_o)\, dx_o \tag{15}$$

and $A_s = b2L$ is the source area, $u_o = (-i\omega\eta_o)$ is the velocity amplitude of the source, and η_o = the displacement amplitude. In the special case of a pistonlike displacement such that $\eta_s(x_o, z_o, \omega) = \eta_o$ in the source region, we have

$$\eta_s(k, \omega) = \frac{\sin kL}{kL}. \tag{16}$$

Having obtained $S(k, \omega)$, we obtain the pressure amplitude $p(x, \omega)$ from

$$p(x, \omega) = \int_{-\infty}^{+\infty} P(k, \omega)\, e^{ikx}\, dk = \int_{-\infty}^{+\infty} \frac{-S(k, \omega)\, e^{ikx}}{(1-M^2)(k-k_+)(k-k_-)}\, dk, \tag{17}$$

where $S(k, \omega)$ is given by Eqs. 14 and 15.

The poles $k = k_+$ and $k = k_-$, although located on the k axis in the present analysis, would contain a small positive and negative part, respectively, if some damping mechanism, such as viscosity or heat conduction, had been included in the analysis. Evaluating the integral by contour integration in the complex k plane, we can complete the contour in the upper k plane for $x > L$, and thus include the pole at $k = k_+$. The corresponding solution is then

$$p_+ = \frac{A_s}{A} \frac{1}{2} (\rho c u_o) \frac{1}{(1+M)^2} \eta_s(k_+, \omega) \exp\left(i \frac{\omega}{c(1+M)} x\right). \tag{18}$$

Similarly, by closing the path in the lower half plane, we find, for $x < -L$,

$$p_- = \frac{A_s}{A} \frac{1}{2} (\rho c u_o) \frac{1}{(1-M)^2} \eta_s(k_-, \omega) \exp\left(-i \frac{\omega}{c(1-M)} x\right). \tag{19}$$

These solutions represent the waves transmitted in the downstream and upstream directions traveling with speeds $c(1+M)$ and $c(1-M)$, respectively, as expected. It is interesting to note that the amplitudes of these waves are different in the presence of flow in the duct. If the source region is acoustically compact, that is, if L is much smaller than the acoustic wavelength λ, the value of the source function η_s is close to unity, as can be seen in the special example given in Eq. 16, and the ratio of the upstream and downstream pressure amplitudes becomes

$$\frac{|p_-|}{|p_+|} = \frac{(1+M)^2}{(1-M)^2}. \tag{20}$$

3. Discussion

It should be emphasized that the Mach number dependence of the wave amplitudes, expressed by the factors $(1+M)^{-2}$ and $(1-M)^{-2}$ in Eqs. 18 and 19 and leading to the amplitude ratio in Eq. 20, depends intimately on the nature of the source and the velocity perturbation that it produces in the fluid. In the analysis carried out here the relationship between the (known) displacement of the air column in the loudspeaker throat and the corresponding velocity perturbation produced in the duct flow has been assumed to be described by Eq. 4, a relation based on the model of an oscillatory displacement of streamlined flow over the source region. In highly turbulent flow it may be more realistic to use as a boundary condition $u_y = u_s = \partial\eta_s/\partial t$ (obtained by neglecting $\partial/\partial x$ in Eq. 4), which means that the velocity perturbation in the duct flow equals the velocity in the loudspeaker throat. The amplitude ratio $|p_-/p_+|$ obtained in this case is $(1+M)/(1-M)$.

In Fig. X-2, which shows the measured amplitude ratio $|p_-/p_+|$ as a function of the flow Mach number, we have also plotted the functions $F_1(M) = (1+M)^2/(1-M)^2$ and $F_2(M) = (1+M)/(1-M)$, which represent the theoretical results obtained on the basis of the two different boundary conditions that were considered. It is interesting to find that the experimental results fall between these theoretical curves. At Mach numbers less than ~0.1, the data are in good agreement with the function F_2, which indicates that the streamline model of the flow is meaningful. As the Mach number is increased, however, the data show a trend toward the function F_1, which favors the boundary condition $u_y = u_s$.

The experiments were carried out at the M. I. T. Gas Turbine Laboratory, and we wish to thank Angelo Moretta for assistance in setting up the flow facility for the experiments.

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B. SOUND ABSORPTION BY A SINGLE RESONATOR IN A DUCT

ONR (Contract N00014-67-A-0204-0019)

A. G. Galaitsis

1. Introduction

The absorption characteristics of an acoustic resonator as a duct termination have been studied in detail.[1,2] Here we consider the influence of a nonlinearly responding resonator attached to a side wall of a duct on a sound wave propagating along the duct.

The basic features of the problem are depicted in Fig. X-4. A plane sound wave p_i propagates to the left along the duct D and excites the resonator C which communicates with the duct through the orifice O. Consequently, a fraction of the incident wave is dissipated in the resonator, another part, p_r, is reflected back toward the sound source and the rest, p_t, propagates freely beyond the resonator.

We shall first derive expressions for the transmitted and reflected fractions of the incident energy and compare them with measured values.

2. Theory

Consider the system in Fig. X-4. The radius and cross section area of the orifice are r_o and A_o, the width and cross section of the duct are d and A, and the wavelength of the sound wave is λ. We shall assume that d, $r_o \ll \lambda$, so that we deal only with plane

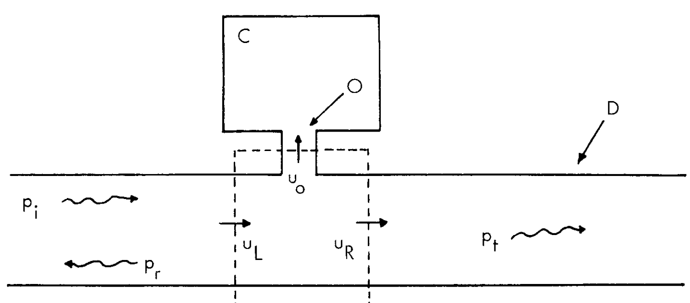

Fig. X-4. Acoustic resonator connected in parallel to a duct.

waves. In the absence of reflection at the far end of the duct the total fields p_L and p_R to the left and right of the resonator are

$$p_L = p_o(e^{ikx} + R\,e^{-ikx})\,e^{-i\omega t}$$
$$p_R = p_o T\,e^{ikx-i\omega t}, \tag{1}$$

and the corresponding acoustic velocities are

$$u_L = \frac{p_o}{\rho c}(e^{ikx} - R\,e^{-ikx})\,e^{-i\omega t}$$
$$u_R = \frac{p_o}{\rho c} T\,e^{ikx-i\omega t}. \tag{2}$$

The coefficients R and T are obtained by combining the boundary conditions

$$u_o = \frac{1}{\sigma}[u_L - u_R]$$
$$p_L = p_R \qquad (x = 0) \tag{3}$$

with the equation giving the response of the resonator

$$p_R = \rho c \zeta_o u_o \qquad (x = 0), \tag{4}$$

where u_o is the acoustic velocity at the orifice, $\sigma = A_o/A$, and ζ_o (the impedance of the orifice and resonator combination) is given by

$$\zeta_o = \theta - i\chi = 1.1\,\frac{u_1}{c} + \theta_o - i(\chi_o + \chi_r). \tag{5}$$

Expression (3) contains the linear contributions θ_o, χ_o, and χ_r^3, as well as the nonlinear contribution proportional to

$$u_1 \equiv |u_o|.$$

We substitute (1) and (2) in (3)-(5), and get

$$R = -\frac{\sigma}{2\zeta_o + \sigma}, \qquad T = \frac{2\zeta_o}{2\zeta_o + \sigma}. \tag{6}$$

Here R and T depend on u_1 through ζ_o. Therefore u_1 must be calculated before (6) can be evaluated. We calculate u_1 by observing that

$$u_1 = |u_o| = \frac{|p_R(x=0)|}{\rho c|\zeta_o|}, \tag{7}$$

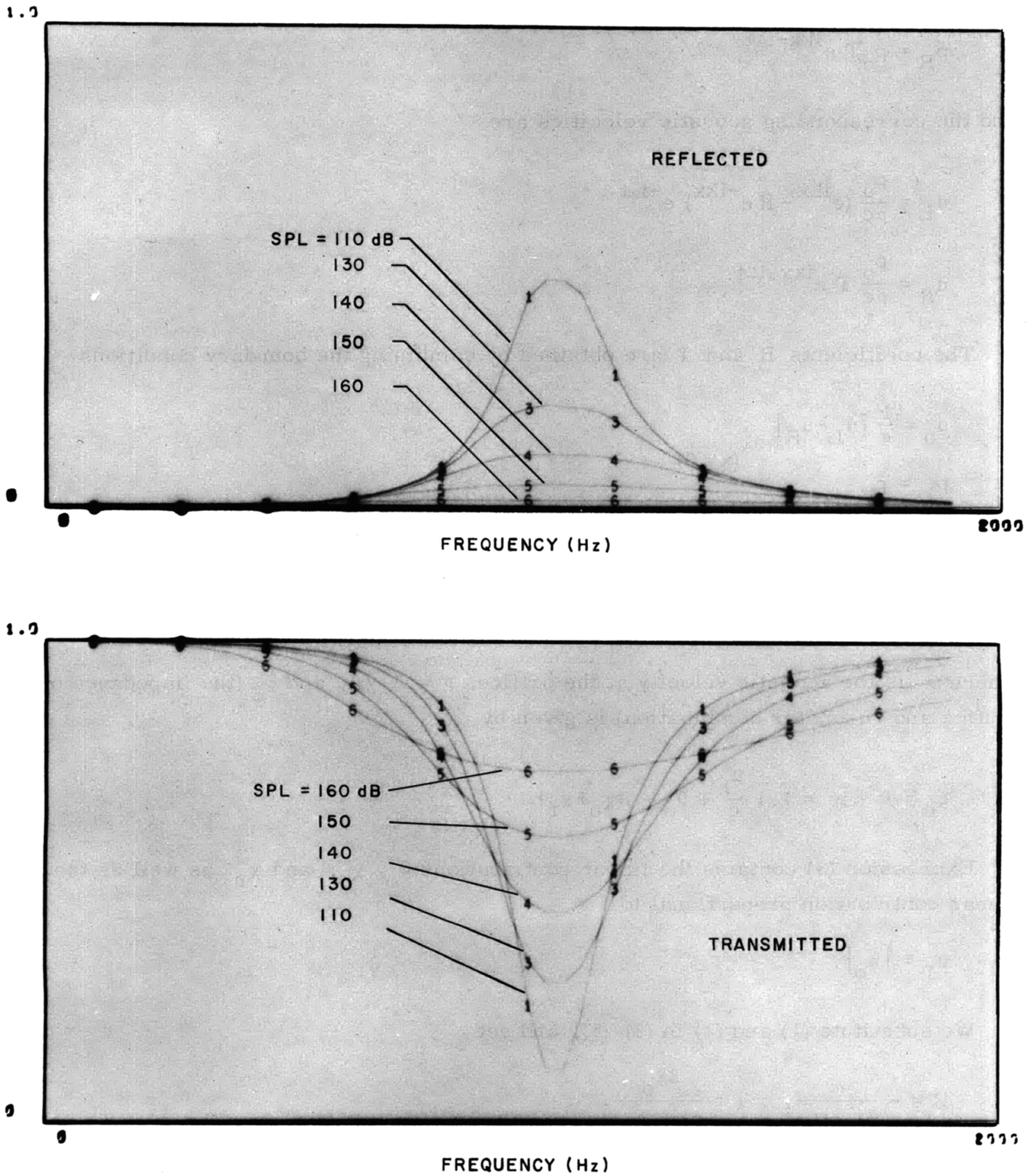

Fig. X-5. Predicted reflected (RE) and transmitted (TR) fractions of the incident power.

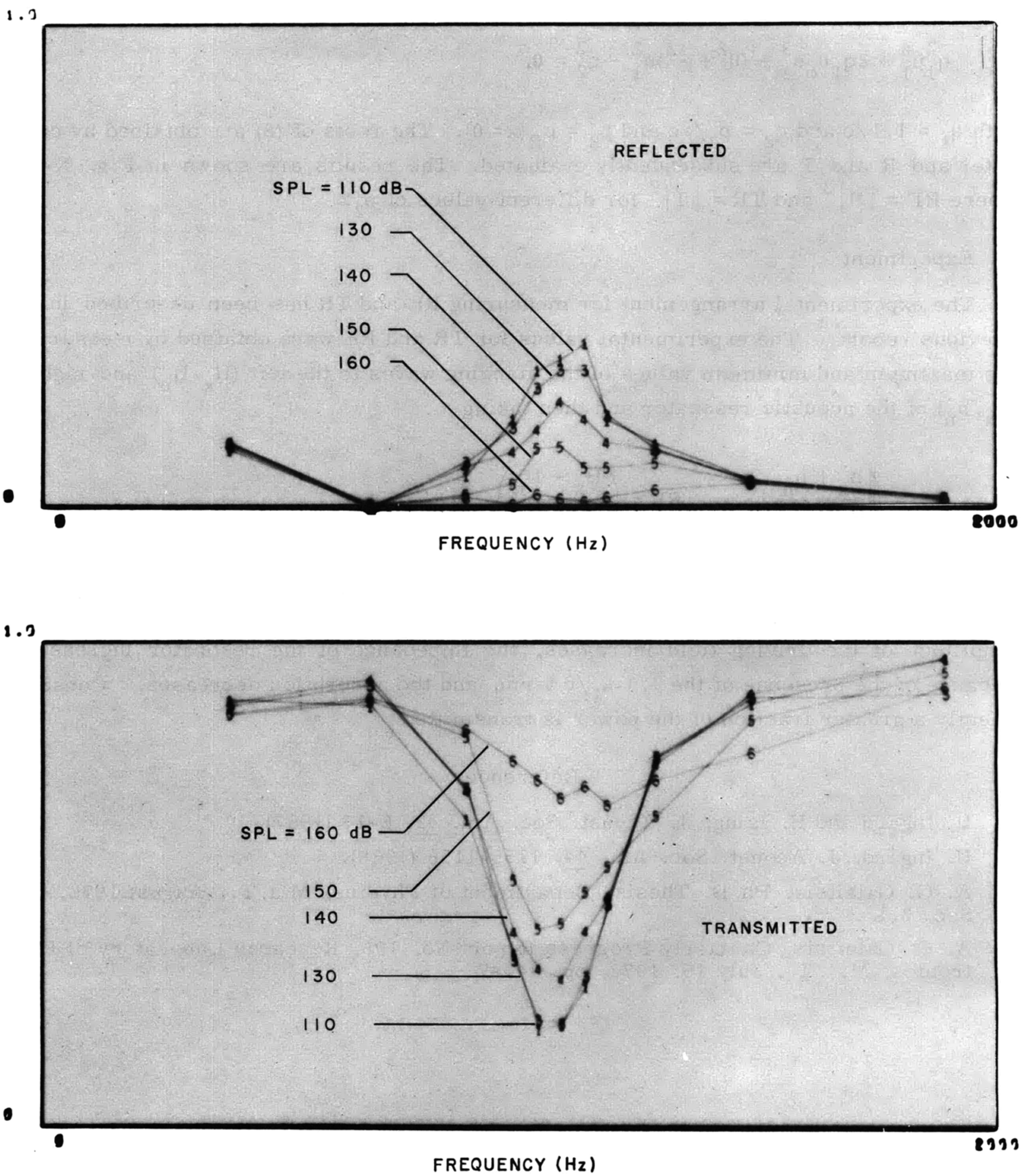

Fig. X-6. Measured reflected (RE) and transmitted (TR) fractions of the incident power.

which leads to

$$q_1^2 u_1^4 + 2q_1 \theta_o u_1^3 + (\theta^2 + \chi^2) u_1^2 - q_2^2 = 0, \tag{8}$$

with $q_1 = 1.1/c$ and $q_2 = p_2/\rho c$ and $p_2 = p_R(x=0)$. The roots of (8) are obtained by computer and R and T are subsequently evaluated. The results are shown in Fig. X-5, where $RE = |R|^2$ and $TR = |T|^2$ for different values of p_2.

3. Experiment

The experimental arrangement for measuring RE and TR has been described in a previous report.[4] The experimental values for TR and RE were obtained by measuring the maximum and minimum values of the standing waves to the left (H_x, H_n) and right (h_x, h_n) of the acoustic resonator and then taking

$$TR = \left(\frac{h_x + h_n}{H_x + H_n}\right)^2 \qquad RE = \left(\frac{H_x - H_n}{H_x + H_n}\right)^2 \tag{9}$$

for different values of p_2. The results are shown in Fig. X-6. The agreement is satisfactory and the nonlinear nature of the interaction is obvious in both cases. As the amplitude of the driving field increases, the impedance of the resonator increases because of the presence of the $1.1\ u_1/c$ term, and the absorption decreases. Consequently a greater fraction of the power is transmitted.

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