PLASMA DYNAMICS

VII. PLASMAS AND CONTROLLED NUCLEAR FUSION*

E. Feedback Stabilization

Academic Research Staff

Prof. R. R. Parker
Prof. L. D. Smullin
Prof. K. I. Thomassen

Graduate Students

S. P. Hirshman
R. S. Lowder
A. R. Millner

1. INVESTIGATION OF A PLASMA GUN

We are undertaking a theoretical investigation of the Marshall coaxial plasma gun, which is capable of accelerating a dense plasma to velocities of 10^7 cm/s. It does not suffer from the space-charge limitations of electrostatic accelerators, and thus particle densities of order $10^{14}/\text{cm}^3$ are readily achievable.

We have chosen a magneto quasi-static slug model in an attempt to analyze the dynamics of the current sheet. In this formulation, the complicated breakdown process can be avoided by assigning an initial mass to the current sheet. The resulting "planar" sheet is then treated as a thin mass obeying the dynamics of a snowplow model. This model has the advantage that we may easily take account of complicated boundary effects. Physically, the sheet corresponds to an ionization front in which the current is fed continuously from the gas which is snowplowed and entrained from ahead of the sheet.[1] An ion drag term, which corresponds to randomization of the ion axial motion at the cathode (with a subsequent loss in axial momentum of the sheet), has been included in the momentum equation. It follows by mass and charge conservation in the plasma that the ions must carry one-half the total current to the cathode, thereby losing axial momentum at the rate of $\frac{1}{2}\frac{|I|}{e}m_iV$. The momentum equation for the sheet has been coupled with the circuit equations to yield a complete set of equations. These nonlinear equations can only be solved (exactly) numerically, but under certain reasonable assumptions, analytic expressions may be obtained. The snowplow momentum equation for the sheet is

$$\frac{d}{dt}(Mv) + \frac{|I|}{2e}m_iv = f^e, \tag{1}$$

*This work was supported by the U.S. Atomic Energy Commission (Contract AT(11-1)-3070).

where the second term is the ion drag term. For the coaxial system under consideration $f^e = \frac{1}{2} \mathscr{L} I^2$ and $\mathscr{L} = \frac{\mu_o \ln r_o/r_i}{2\pi}$. In terms of the magnetic pressure, this equation (without the drag term) is equivalent to the more familiar form

$$\frac{B^2}{2\mu_o} = \rho v^2 + \frac{dv}{dt} \int_0^z \rho d\xi. \tag{2}$$

Note that although the magnetic pressure varies over the surface of the sheet, experiment indicates that the sheet remains approximately planar. This fact might be accounted for by an uneven mass pickup.[2]

Consider Fig. VII-1. The circuit equation is

$$e = L_e \frac{dI}{dt} + \frac{d}{dt}(LI) = (L_e+L) \frac{dI}{dt} + (\mathscr{L} v)I, \tag{3}$$

where we have used $dL/dt = \mathscr{L} v$. To complete this set of equations, we need an initial gas distribution (mass loading) in the gun. If we assume that the initial puff of gas has diffused with a diffusion constant D for a time T_o before the gun is fired ($T_o > T_{exper}$), then

$$\frac{dM}{dt} = \frac{(1-\gamma)}{\sqrt{\pi D T_o}} M_o v \exp\left[\frac{-x^2}{4DT_o}\right], \tag{4}$$

where

$$\gamma = \frac{M(t=0)}{M_o}.$$

It is important to note what has been left out of these equations. The initial breakdown is taken into account only phenomenologically when we assume an initial current sheath mass and that the ionized gas is a (perfect) conductor, allowing the current to flow impeded only by the inductance of the circuit and gun. A look at the initial processes in the gun will indicate, at least qualitatively, that breakdown should be (and is experimentally observed to be) dependent upon the polarity of the applied voltage, as well as the gas pressure (E/p ratio). The polarity dependence arises because the breakdown occurs in a nonuniform ($1/r$) electric field. The ionization ratio as a function of p will go through a maximum. The current sheet is most likely to occur at these high values of breakdown current.[1] Experimentally, the initial breakdown is observed to be somewhat resistive and imperfect up to ~0.5 μs (typically). Resistive losses have not been included in the model.

Numerous other physical processes have been neglected. For instance, in the circuit equation we should take account of losses attributable to ionization, recombination radiation, and thermal-heating effects. The ionization effect is usually negligibly small. The power input resulting from ionization and recombination can be at most

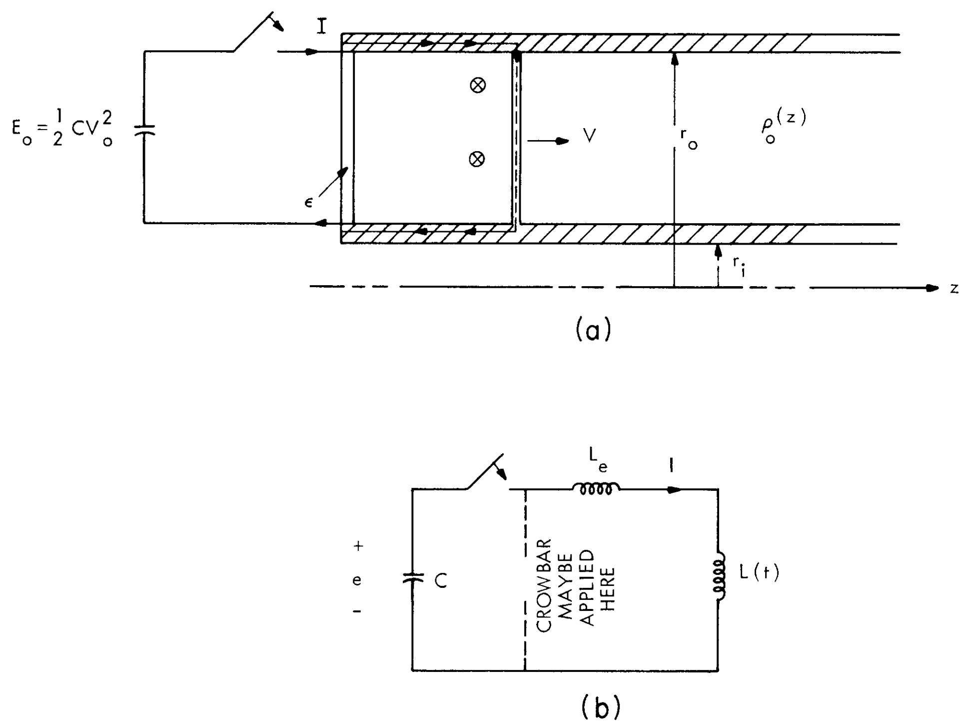

Fig. VII-1. (a) Coaxial plasma gun.
(b) Equivalent circuit for plasma gun.

$P_{max} = V_e I$, where $V_e = 13.6$ volts, and hence appears to be negligible in comparison with the powers involved with other processes. A more comprehensive model would consider these effects, as well as the self-consistent field problem associated with the electron current flow in the sheet.[1] The ions in fact are dragged along by an E-field because of a slight axial charge separation. In the present account we have neglected the drag force attributable to ablation of material from the gun walls. We have found that the shape of the calculated velocity graph matches closely the shape of the experimental graph, when we include the ablation drag. (These graphs will be submitted in a future quarterly report.)

To discuss the efficiency of the gun in converting the stored energy E_o in the capacitor bank into plasma kinetic energy, it is necessary to compute $\eta = W_K/E_o$. Mendel[3] computes $\eta' = W_p/E_o$, where W_p is the total energy imparted to the plasma (which, in the absence of ion drag, is equivalent to W_K). His result is

$$\eta' = \frac{1 - \left(\int_0^{L_f} EdL\right)/E_o L_f}{1 + L_e/L_f}. \tag{5}$$

From this he concludes that for a fixed L_f, it is desirable to have $L_e \ll L_f$, to get the energy out of the source as quickly as possible, and to keep it out (by applying a crowbar at the source, for instance), thereby minimizing $\tilde{\gamma} = \int_0^{L_f} EdL$. In our model, the desirability of the last criterion is debatable, since at high enough current and velocity, the ion drag force may overwhelm the $J \times B$ force and actually decelerate the current sheet. Nevertheless, the results of applying a crowbar may lead to an increased final velocity.

To modify Mendel's calculation, we define the kinetic energy of the plasma $\left(= \frac{1}{2} Mv^2\right)$

$$W_K = W_p - \int \frac{1}{2} m_i v^2 \frac{|I|}{e} dt = W_p - \frac{m_i}{M} \int W_K \frac{|I|}{e} dt, \tag{6}$$

where

$$W_p = \int \frac{1}{2} i^2 \frac{dL}{dt} dt = \int f^e \cdot v \, dt.$$

Using the energy equation for the circuit (Fig. VII-1b),

$$E_o = E(t) + W_p + \frac{1}{2} (L_e+L)I^2, \tag{7}$$

and integrating, we find that

$$I^2 = \frac{2}{(L_e+L)^2} \int_0^t (L_e+L)(-\dot{E}) \, dt. \tag{8}$$

Substitution of this value in Eq. 6 yields

$$\frac{dW_K}{dt} + \gamma_o(t) W_K = E_o \dot{\eta}', \tag{9}$$

where

$$\gamma_o(t) = \frac{m_i}{eM} \frac{1}{L_e + L} \left(\int_0^t (L_e+L)(-2\dot{E}) \, d\xi\right)^{1/2} \geq 0.$$

Note that the ion drag force is dissipative and leads to damping of W_K. At this point we would like to substitute the capacitor energy source for E and a value for L(t) to obtain $\gamma(t)$ explicitly. We are now integrating the equations of motion, both numerically and analytically, and our research will continue in this direction.

S. P. Hirshman, L. D. Smullin

References

1. O. K. Mawardi, "Bounded Current Sheets," Phys. Fluids, Vol. 7S, pp. 9-16, November 1964.
2. L. Burkhardt and R. Lovberg, "Current Sheet in a Coaxial Plasma Gun," Phys. Fluids 5, 341-347 (1962).
3. C. W. Mendel, Jr., "Conical Z-Pinch Gun," J. Appl. Phys. 42, 5483-5491 (1971).

VII. PLASMAS AND CONTROLLED NUCLEAR FUSION*

F. High-Temperature Plasma Physics

Academic and Research Staff

Prof. B. Coppi
Dr. D. B. Montgomery†
Prof. G. Bekefi
Prof. A. Bers
Prof. R. A. Blanken
Prof. R. J. Briggs
Prof. L. M. Lidsky
Prof. R. R. Parker
Prof. K. I. Thomassen
Dr. E. Minardi
Dr. L. Ornstein
Dr. P. A. Politzer
Dr. J. Rem
Dr. D. Schram
Dr. F. C. Schüller
Dr. D. J. Sigmar
A. Hugenholtz

Graduate Students

E. L. Bernstein
D. L. Cook
R. Dagazian
D. P. Hutchinson
Y. Y. Lau
M. A. Lecomte
A. R. Millner
T. Orzechowski
N. R. Sauthoff
M. Simonutti
B. V. Waddell
D. C. Watson
S. M. Wolfe

1. PLASMA INSTABILITIES DRIVEN BY A DC ELECTRIC FIELD

In a previous report[1] we pointed out that a dc electric field E_0 parallel to the magnetic field B_0 can produce unstable electron-plasma waves. These instabilities are attributable to the presence of the E_0-field rather than the relative drift velocity u_e between electrons and ions. Thus, unstable electron-plasma waves may be generated by a dc electric field even if the electron drift velocity u_e is less than the electron thermal velocity v_{Te}. We also showed that these instabilities set in for fields E_0 that are comparable to, or larger than, the runaway field E_R ($E_R \sim m\nu_{ei} v_{Te}/e$, where ν_{ei} is the classical electron-ion collision frequency). These unstable waves have characteristic phase velocities that are in the tail of the electron velocity distribution function, and hence can make an important contribution (by enhancing the scattering of these electrons) to the inhibition or reduction of runaway electrons in applied electric fields. They may thus help to explain the observed lack of runaways in Tokamak TM-3 experiments where the induced electric field ranged from E_R to $15E_R$.[2]

In this report we present further results of this study. In particular, we extend our analysis to arbitrary shapes of the electron distribution function $f_{0e}(w)$; next we include the effects of an effective collision frequency that depends upon velocity; then we allow for the slow evolution of the distribution function $f_{0e}(w, t)$; and finally include the ion dynamics to determine the ion-wave instabilities that may also be driven by E_0. In the following discussion we consider only a one-dimensional description of the interaction,

*This work was supported by the U.S. Atomic Energy Commission (Contract AT(11-1)-3070).

†Dr. D. Bruce Montgomery is at the Francis Bitter National Magnet Laboratory.

which would be pertinent in a large applied magnetic field ($\omega_{ce} > \omega_{pe}$).

Effects Caused by the Shape of $f_0(w)$

The linearized Boltzmann equation for the electron dynamics is

$$\frac{\partial f_1}{\partial t} + w \frac{\partial f_1}{\partial z} + \frac{-eE_0}{m} \frac{\partial f_1}{\partial w} + \frac{-eE_1}{m} \frac{\partial (n_0 f_0)}{\partial w} = -\nu_{eff}(f_1 - n_1 f_{0L}), \tag{1}$$

where the right-hand side is an assumed Bhatnagar-Gross-Krook (BGK) collision model[3] with an assumed constant collision frequency ν_{eff}. Equation 1 can be solved exactly[4] and, together with Poisson's equation, gives the following dispersion relation.

$$\frac{\dfrac{\omega_p^2}{k^2} \displaystyle\int \frac{dw}{w - \frac{\omega + i\nu_{eff}}{k}} \exp\left[\frac{-ieE_0}{2mk} \frac{\partial^2}{\partial w^2}\right] \frac{\partial f_0}{\partial w}}{1 + \dfrac{i\nu_{eff}}{k} \displaystyle\int \frac{dw}{w - \frac{\omega + i\nu_{eff}}{k}} \exp\left[\frac{-ieE_0}{2mk} \frac{\partial^2}{\partial w^2}\right] f_{0L}} = 1. \tag{2}$$

If f_0 and f_{0L} are Maxwellian and with and without drift, respectively, then we obtain our previous results.[5] For arbitrary distribution functions (well-behaved at $w \to \pm\infty$) it is convenient to introduce the function

$$Y_0(\zeta) = \int \frac{f_0(w)\, dw}{\frac{w}{v_T\sqrt{2}} - \zeta}, \tag{3}$$

and $Y_0'(\zeta)$ its derivative with respect to ζ, where the integration is carried out over the usual Landau-contour prescription. Extracting the operator from Eq. 2, we then obtain

$$\frac{1}{2K^2} \left\{ \exp\left[\frac{i\gamma}{4} \frac{\partial^2}{\partial \zeta^2}\right] Y_0'(\zeta) \right\}_{\zeta=\zeta_0} - \frac{iN}{K\sqrt{2}} \left\{ \exp\left[\frac{i\gamma}{4} \frac{\partial^2}{\partial \zeta^2}\right] Y_{0L}(\zeta) \right\}_{\zeta=\zeta_0} = 1, \tag{4}$$

where the following normalizations have been introduced: $K = kv_T/\omega_p = k\lambda_D$; $\gamma = -eE_0/kmv_T^2$; $N = \nu_{eff}/\omega_p$; $\zeta_0 = (\Omega + iN)/K\sqrt{2}$, with $\Omega = \omega/\omega_p$.

Equation 4 can be solved to first-order in N and γ, and in the long-wavelength limit we find

$$\frac{\omega_r}{\omega_p} \approx 1 + \frac{k\langle w\rangle}{\omega_p^2} + \frac{3}{2}\frac{k^2}{\omega_p^2}\left[\langle w^2\rangle - \langle w\rangle^2\right] \tag{5a}$$

$$\frac{\omega_i}{\omega_r} \approx -\frac{\nu_{eff}}{2\omega_p} + \frac{\pi}{2}\frac{\omega_p^2}{k^2}\left(\frac{df_0}{dw}\right)_{\omega_r/k} + \frac{3}{2}\left(\frac{-eE_0}{m}\right)\frac{k}{\omega_p^2}, \tag{5b}$$

where $\langle w^n\rangle \equiv \int w^n f_0\, dw$ are the velocity moments of f_0.

For f_0 Maxwellian, Eqs. 5a and 5b reduce to our previous results.[6] For this case we can carry out an analysis of marginal stability by using Eq. 4, without restricting ourselves to long wavelengths. The results are shown in Fig. VII-2. The growth term arising from the applied dc electric field is proportional to $\mathcal{E} = E_0/E_R$, and the damping term that is due to collisions is proportional to $a = \nu_{eff}/\nu_{ei}$. Thus, in the absence of Landau damping, the marginal stability line is a straight-line graph through the origin in the $\mathcal{E}$-a plane. The slope is the ratio of the coefficient of a in the collision damping term to the coefficient of $\mathcal{E}$ in the growth term, namely 1/3K. The introduction of Landau damping moves the whole marginal stability line upward. The intercept on the $\mathcal{E}$ axis now occurs not at the origin but at that positive value of $\mathcal{E}$ for which the growth term is equal and opposite to the Landau damping evaluated at the phase velocity of the wave. Since the coefficient of $\mathcal{E}$ in the growth term is very small, Landau damping must also be very small if the intercept is to occur at reasonable (1-50) values of $\mathcal{E}$. For a Maxwellian velocity distribution, this requires phase velocities around 7.2 times the thermal velocity, that is, $7.0 \lesssim \omega/kv_T \approx \omega_p/kv_T \equiv 1/K$, and hence $d\mathcal{E}/da = K/3 \gtrsim 7/3 \approx 2.3$. These conclusions are valid for a wide range of drift velocities and plasma parameters, as shown in Fig. VII-2.

For a non-Maxwellian velocity distribution, these conclusions must be modified. Figure VII-3 illustrates possible forms assumed by a velocity distribution evolving under the influence of the dc electric field. The distribution in Fig. VII-3a is a Maxwellian, drifted but otherwise undistorted. In Fig. VII-3b, a "hot tail" has developed and has caused the region of negligible slope to extend farther in toward thermal velocities. In Fig. VII-3c, the distortion is severe. The growth in density of the tail and the depletion of the main hump have combined to extend the flat part of the distribution down almost to thermal velocity. The previous discussion of the $\mathcal{E}$-a graphs still applies, with the exception that the required phase velocity is a function of the extent to which the distribution is distorted. Using Eqs. 4 and 5, we find

For distribution (a):	$7.0 \lesssim 1/K$	$d\mathcal{E}/da \gtrsim 2.3.$
For distribution (b):	$4.0 \lesssim 1/K$	$d\mathcal{E}/da \gtrsim 1.33.$
For distribution (c):	$2.5 \lesssim 1/K$	$d\mathcal{E}/da \gtrsim 0.80.$

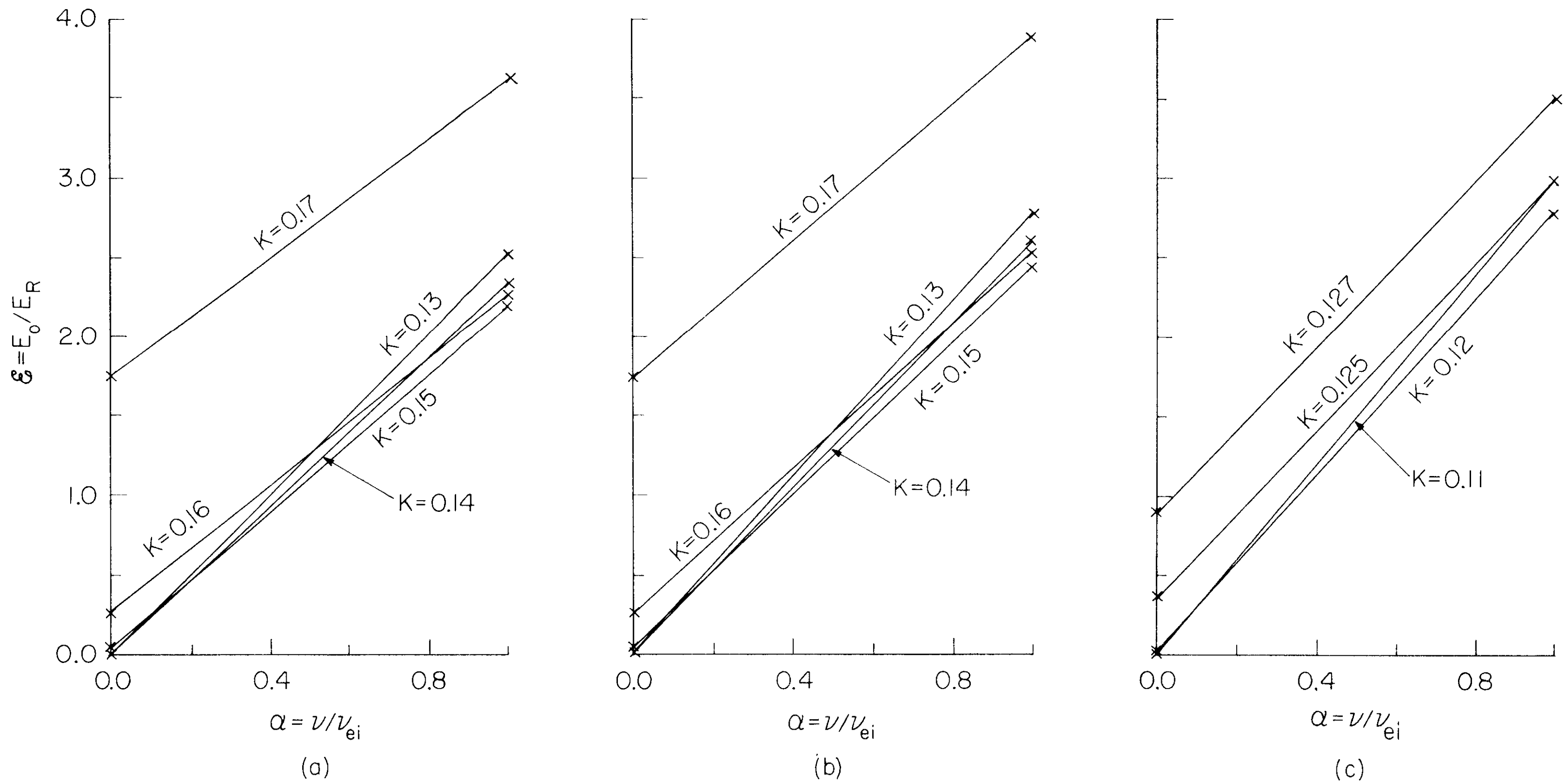

Fig. VII-2. Parametric representation of plasma-wave instability onset at various wavelengths and for a drifted Maxwellian distribution function.

(a) $u_e = 0$ $\quad n_0\lambda_{De}^3 = 10^5$.

(b) $u_e = v_{Te}$ $\quad n_0\lambda_{De}^3 = 10^5$.

(c) $u_e = 0$ $\quad n_0\lambda_{De}^3 = 10^{10}$.

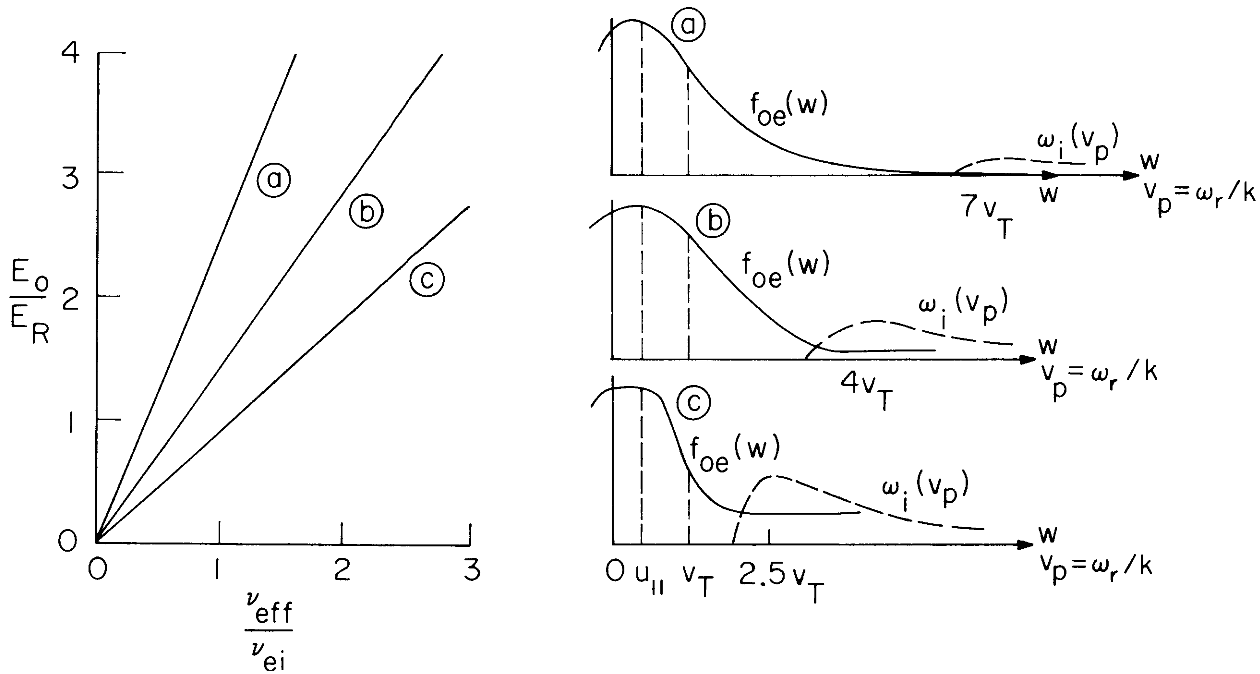

Fig. VII-3. Plasma-wave instability characteristics for Maxwellian and non-Maxwellian distribution functions.

Thus, as the distortion of the velocity distribution increases, the threshold value of the electric field required for instability decreases, as shown in Fig. VII-3. In the Tokamak device TM-3, $\mathscr{E}$ may assume values of order 5-50, and the electron distribution function is certainly distorted. Thus, if the high-frequency effective collision rate ν_{eff} does not exceed the classical collision rate ν_{ei} by more than a factor of 5 or so, electron plasma waves may be driven unstable.

Effects of a Velocity-Dependent Collision Frequency

We now turn to a more detailed analysis of the effective collision frequency. Under conditions in which $E_0 \gtrsim E_R$ the experiments in TM-3 exhibit a large anomalous resistivity.[2] It is also clear that under these conditions large-amplitude ion-acoustic waves are present, and they are in large part responsible for the anomalous resistivity. The effect of this strong turbulence on the electrons is difficult to model. We shall assume that we can represent this as an effective "hard collision" which is dependent upon the electron's velocity, and again use the BGK collision model[3] in the one-dimensional Boltzmann equation.

$$\frac{\partial f}{\partial t} + w \frac{\partial f}{\partial z} + \frac{-e}{m} E \frac{\partial f}{\partial w} = -\nu_e(w) \left[f - f_{0L} \frac{\int \nu_e(w)\, f\, dw}{\int \nu_e(w)\, f_{0L}\, dw} \right]. \tag{6}$$

Consider first the zero-order distribution function $f_0(w)$. From Eq. 6 we have

$$\frac{-eE_0}{m} \frac{\partial f}{\partial w} = -\nu_e(w) \left[f_0 - f_{0L} \frac{\int \nu_e(w)\, f_0\, dw}{\int \nu_e(w)\, f_{0L}\, dw} \right]. \tag{7}$$

Under the assumption that $f_0 = f_{0L} + f_{01}$, where f_{0L} is Maxwellian and f_{01} is due to E_0, the linearized solution of Eq. 7 gives us the dc current

$$\begin{aligned} J_0 &= -en_0 \int w f_{01}\, dw \\ &\equiv \frac{e^2 n_0}{m\nu_{eff}^{(0)}} E_0, \end{aligned} \tag{8}$$

from which we can define the dc effective collision frequency $\nu_{eff}^{(0)}$. We thus find

$$\frac{1}{\nu_{eff}^{(0)}} = \int \frac{-w \frac{\partial f_{0L}}{\partial w}}{\nu_e(w)}\, dw. \tag{9}$$

For high-frequency oscillations the linearized Boltzmann equation from Eq. 6 is

$$\frac{\partial f_1}{\partial t} + w\frac{\partial f_1}{\partial z} + \frac{-eE_0}{m}\frac{\partial f_1}{\partial w} + \frac{-eE_1}{m}\frac{\partial n_0 f_0}{\partial w} = -\nu_e(w)\left[f_1 - f_{0L}\frac{\int \nu_e(w)\, f_1\, dw}{\int \nu_e(w)\, f_{0L}\, dw}\right]. \tag{10}$$

Solving Eq. 10 to first order in the effects of the dc electric field and of the collisions, we again find in the long-wavelength limit, Eqs. 5a and 5b, where now

$$\nu_{eff} \equiv \int \nu_e(w)\left[2f_0 - f_{0L}\right] dw \tag{11}$$

is the appropriate high-frequency collision frequency. We may expect $\nu_e(w)$ to be sharply peaked below the electron drift velocity where the ion-acoustic growth rates are largest, and hence that $\nu_{eff}^{(0)} \gg \nu_{eff} \sim \nu_{ei}$. We thus infer that the electric field instability threshold imposed by collisions need not be directly related to the dc anomalous resistivity (namely, $\nu_{eff}^{(0)}$), but rather should be related to the high-frequency resistivity (namely, ν_{eff}), which may be considerably smaller. Our approximate representation of an effective collision frequency for the electrons in strongly turbulent fields remains to be justified in detail by further theoretical work.

Effects Caused By Evolution of the Distribution Function

For reasonable values of the applied electric field ($E_0 \sim E_R$) the growth rates predicted by Eq. 5b will be of the order of the effective electron collision frequency. Thus, the time scales for the instability may be comparable to the time scale of evolution of the distribution function $f_0(w,t)$. Since the time scale of evolution is small compared with the plasma period, we can examine its effect on the instability by WKB techniques. The linearized Boltzmann equation assumes the same form as in Eq. 1 except that $f_0 = f_0(w,t)$ is now a slowly varying function of time. Assuming

$$f_1 = f_{10} \exp i\left[kz - \int^t \omega(t')\, dt'\right], \tag{12}$$

and similarly for the other field quantities, we obtain

$$\left(w - \frac{\omega + i\nu}{k}\right) f_{10} = \left(\frac{\omega_p^2}{k^2}\frac{\partial f_0}{\partial w} - i\frac{\nu}{k} f_{0L}\right) n_{10} + \frac{i}{k}\left(\frac{-eE_0}{m}\frac{\partial}{\partial w} + \frac{\partial}{\partial t}\right) f_{10}. \tag{13}$$

Solving Eq. 13 to first order in the effects of collisions, dc electric field, and slow evolution of f_0, we find in the long-wavelength limit

$$\frac{\omega_r}{\omega_p} \cong 1 + \frac{k\langle w\rangle}{\omega_p} + \frac{3}{2}\frac{k^2}{\omega_p^2}\left[\langle w^2\rangle - \langle w\rangle^2\right] \tag{14a}$$

$$\frac{\omega_i}{\omega_p} \cong -\frac{\nu_{eff}}{2\omega_p} + \frac{\pi}{2}\frac{\omega_p^2}{k^2}\left(\frac{\partial f_0}{\partial w}\right)_{\omega_r/k} + \frac{3}{2}\frac{k}{\omega_p^2}\left(\frac{-eE_0}{m} - \frac{\partial\langle w\rangle}{\partial t}\right)$$

$$-\frac{15}{4}\frac{k^2}{\omega_p^2}\frac{\partial}{\partial t}\left[\langle w^2\rangle - \langle w\rangle^2\right]. \tag{14b}$$

Equation 14a is of the same form as Eq. 5a except that in it the average velocity $\langle w\rangle$ and the thermal velocity $[\langle w^2\rangle - \langle w\rangle^2]^{1/2}$ are slowly varying functions of time through their dependence on $f_0(w,t)$. The growth rate as given by Eq. 14b has three new features. The Landau damping, second term on the right-hand side, is changing in time as f_0 evolves. The growth term attributable to the dc electric field is counterbalanced by the evolution of the average velocity $\langle w\rangle$, as shown in the third term. In fact, for a freely evolving average velocity, this growth term is identically zero. Finally, the last term shows that an additional damping is introduced by increasing the thermal energy of the electrons. In order to determine the stability or instability of these long-wavelength plasma oscillations, we would have to evaluate these three new features from a knowledge of the evolution of f_0. Consider, first, ordinary Joule heating. As has been shown,[7] the classical results are only strictly valid for $E \ll 10^{-2} E_R$. For such low fields, introducing the usual formulas for the evolution of the average velocity and temperature,[8] we find that Eq. 14b predicts stability. At electric fields $E \sim 0.1E_R$ the validity of the classical calculations is in doubt, and saturation of the average velocity may set in.[7] Under such conditions Eq. 14b would predict instability, provided the distribution function were sufficiently distorted to reduce the Landau damping. It has been shown that for velocities $w^2 \gg (\kappa T/m)(E_R/E_0)^{1/2}$ the distribution function is indeed much flatter than a Maxwellian.[9] For $E_0 \gtrsim E_R$ there is still no proper theory for the evolution of f_0. The experiments in TM-3, however, do indicate a steady state in which the electron drift velocity is saturated, and hence the plasma oscillations of Eq. 14 may be expected to be unstable. Further confirmation of this could come from measurements of the RF emission spectrum of the plasma, and direct measurements of $f_0(\bar{w})$. Apparently, such measurements have not yet been made.

Ion-Acoustic Wave Instabilities Driven by E_0

Up to the present, we have considered the effect of the dc electric field on electron plasma waves, and ignored the ion dynamics. It is physically clear, however, that the dc electric field may also modify the ion-acoustic waves. We assume that the perturbed ion distribution function is described by an equation similar to Eqs. 1, 10, and 13, within an effective collision frequency model and with slow evolution of the distribution functions. We combine the solutions for the perturbed electron and ion distribution functions in Poisson's equation to obtain the dispersion relation, in the usual way. The ion sound waves are found at low frequencies ($\omega \ll \omega_{pe}$) in the limit $v_{Ti} \ll \omega/k \ll v_{Te}$, and have the following dispersion relation

$$\frac{\omega_r}{\omega_{pi}} \cong \frac{K}{(1+K^2)^{1/2}} \left[1 + \frac{3}{2} \frac{T_i}{T_e} (1+K^2)\right] \tag{15a}$$

$$\begin{aligned} \frac{\omega_i}{\omega_{pi}} \cong &- \frac{\nu_{eff}^i}{2\omega_{pi}} + \frac{K}{(1+K^2)^{1/2}} \frac{\pi}{2k^2} \left[\frac{\partial}{\partial w} \left(\omega_{pe}^2 f_{0e} + \omega_{pi}^2 f_{0i}\right)\right]_{\omega_r/k} \\ &+ \frac{(1+K^2)^{1/2}}{K} \frac{3}{2} \frac{k}{\omega_{pi}^2} \left\{\frac{eE_0}{m_i} \left[1 - \frac{1/3}{(1+K^2)^2}\right] - \frac{\partial u_i}{\partial t}\right\} - \frac{(1+K^2)}{K^2} \frac{15}{4} \frac{k^2}{\omega_{pi}^3} \frac{\partial v_{Ti}^2}{\partial t} \\ &+ \frac{1}{(1+K^2)} \frac{3}{4} \frac{1}{\omega_{pi} T_e} \frac{\partial T_e}{\partial t}, \end{aligned} \tag{15b}$$

where $K \equiv k\lambda_{De}$, u_i is the ion drift velocity, and T_e is the electron temperature. In Eq. 15, k is positive and the waves described by it are traveling in the positive z direction. For waves traveling in the negative z direction the dispersion relation is Eq. 15, with $(1+K^2)^{1/2}$ replaced by $-(1+K^2)^{1/2}$ and k remaining positive. Equation 15a is the well-known dispersion relation for ion-acoustic waves, including the correction that is due to finite ion-temperature. The first term in the growth-rate expression of Eq. 15b is the damping attributable to the effective ion collision frequency ν_{eff}^i. The second term contains the Landau damping and/or growth caused by the resonant electrons and/or ions; this term exhibits the usual growth of ion-acoustic waves from the relative drift velocity between ions and electrons. The third term has three parts: the electric field effect on the ions, on the electrons, and the evolution of the ion drift velocity; this term is destabilizing for waves propagating in the direction of E_0 (opposite to the electron drift), provided that the ion drift velocity is saturated or at least not evolving freely. The fourth term is similar to the last term of Eq. 14b, and represents a damping when the ion temperature is increasing. The last term is a growth caused by the heating of the electrons. It should be noted that we have exhibited in Eq. 15b only the lowest order terms [in $(m_e/m_i)^{1/2}$ and slow variation] caused by the evolution of f_{0e} and f_{0i}. Under steady-state conditions ion-acoustic waves in the direction of E_0 will be unstable if the electric field term can overcome both the collisional and Landau damping terms of Eq. 15b. Under the assumption that $T_e \gg T_i$, the most severe condition arises from the electron Landau-damping term. This damping can become sufficiently small, provided the electron distribution function flattens out for $w_{\parallel} < 0$. Calculations on the evolution of f_{0e} in strong electric fields ($E_0 > E_R$) indicate that such flattening does occur.[10] There are still no calculations on the evolution of the ion-distribution function which, for such large fields, may be mainly controlled by the strong ion-acoustic instabilities from the relative average drift velocity between electrons and ions.

If the E_0 growth term in Eq. 15b is to overcome the effective collisional damping, we require $(E_0/E_R) > \left(\nu_{\text{eff}}^{i}/\nu_{ii}\right)(T_e/T_i)^{3/2}/3(1+K^2)^{1/2}$, where ν_{ii} is the classical ion-ion collision frequency. In addition, the condition for neglecting ion-Landau damping requires $(1+K^2) \ll T_e/2T_i$. We thus conclude that the excitation of unstable ion-acoustic waves in the direction of E_0 requires applied fields $E_0 > E_R$. These requirements are satisfied in the high-field experiments on TM-3,[2] and in some turbulent heating experiments.[11]

Conclusions

We have shown that the presence of a dc electric field (or equivalently a slowly varying induced electric field) in a plasma can generate instabilities of electron plasma

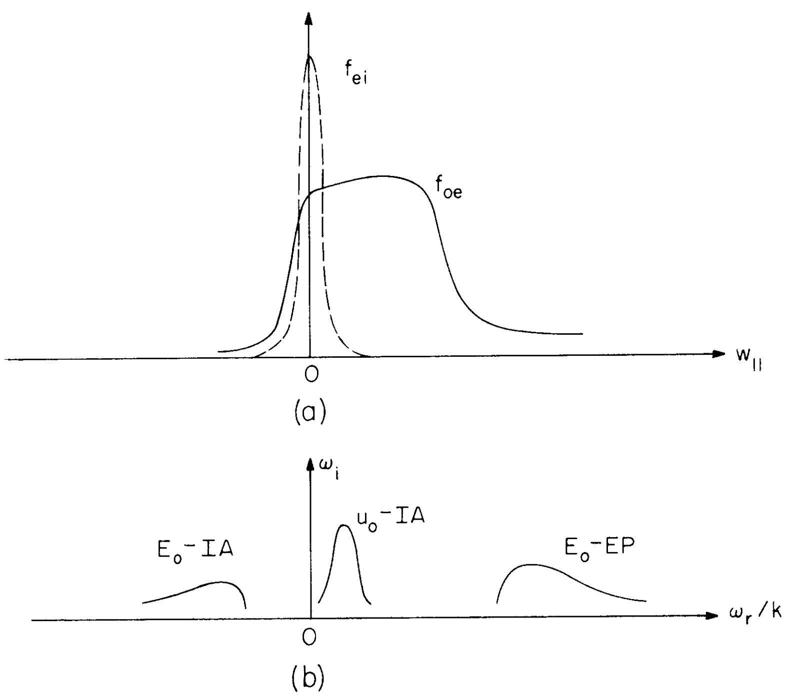

Fig. VII-4. Possible distribution functions and instabilities in an applied electric field. (a) Parallel distribution functions; the electron distribution function is assumed to be flattened at both positive and negative drift velocities. (b) Growth rates as a function of parallel phase velocity: E-IA are the electric field-driven ion-acoustic waves; E-EP are the electric field-driven electron-plasma waves; U_0-IA are the drift-driven ion-acoustic waves.

and ion-acoustic waves that are distinct from instabilities driven by the relative average drift velocity between electrons and ions. These new instabilities may be particularly prominent in high electric fields ($E_0 \gtrsim E_R$) where the evolving electron distribution function tends to flatten for velocities along E_0, and in high velocities opposite to E_0 (see Fig. VII-4a). Thus, in addition to ion-acoustic waves at low positive phase velocities that may be unstable because of the relative average drift, the electric field may generate unstable electron plasma waves at high positive phase velocities and ion-acoustic waves for negative phase velocities (see Fig. VII-4b). Both instabilities caused by the electric field are fluidlike (nonresonant) and may be expected to have important effects on the evolution of the distribution function.

A. Bers, D. C. Watson

References

1. A. Bers, Quarterly Progress Report No. 97, Research Laboratory of Electronics, M.I.T., April 15, 1970, pp. 65-68. (Eq. 8 should be corrected as follows: in the third term $\sqrt{\pi/2}$ should read $\sqrt{\pi/8}$; in the last term, replace e with -e.)
2. G. A. Bobrovskii, E. I. Kuzuetsov, and K. A. Razumova, Sov. Phys. – JETP 32, 599 (1971).
3. L. Bhatnagar, E. P. Gross, and M. Krook, Phys. Rev. 94, 511 (1954); E. P. Gross and M. Krook, Phys. Rev. 102, 593 (1956).
4. A. Bers and S. R. J. Brueck, Proc. 10th International Conference on the Physics of Semiconductors (U.S. Atomic Energy Commission Conf-700801, 1970), pp. 129-133; D. C. Watson, S.M. Thesis, Department of Electrical Engineering, M.I.T., 1972.
5. A. Bers, Quarterly Progress Report No. 97, op. cit., p. 66, Eq. 5 in the limit of $\omega_c \to \infty$.
6. Ibid., p. 67, Eqs. 7 and 8, with $\langle w \rangle = v_0$ and $\langle w^2 \rangle - \langle w \rangle^2 = v_T^2$.
7. A. Bers, B. Coppi, T. Dupree, R. Kulsrud, and F. Santini, Plasma Physics and Controlled Nuclear Fusion Research 1971, Vol. II (International Atomic Energy Agency, Vienna, 1971), pp. 247-263.
8. H. Dreicer, Phys. Rev. 115, 238 (1959).
9. A. V. Gurevich, Sov. Phys. – JETP 12, 904 (1961).
10. L. M. Kovrizhnykh, Sov. Phys. – JETP 10, 989 (1960).
11. Plasma Physics and Controlled Nuclear Fusion Research (op. cit.), Papers E-2,3, and 5-8.