## IX. COGNITIVE INFORMATION PROCESSING*


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### A. ESTIMATES OF THE HUMAN VISUAL LINE-SPREAD AND POINT-SPREAD FUNCTIONS

There has been much interest in the human visual spatial modulation transfer function, line-spread function, and point-spread function. These three functions are meaningful only under conditions of vision when the human visual system is linear and space-invariant. The modulation transfer function is the system function relating sensation (the output) to stimulus (the input). The line-spread function is the output attributable to a knife impulse input of unit area per unit length, and the point-spread function is the output resulting from a point impulse input of unit volume. This report shows the relationship among the three functions when circular symmetry is present and uses experimental values of the easily measured modulation transfer function[1] to calculate values for the two spread functions.

Cutrona[2] approximates the human visual spatial modulation transfer function, H(k), as

$$H(k) = 470\left(e^{-.153|k|} - e^{-.54|k|}\right),$$

where k is spatial frequency in cpd (cycles per degree). The line-spread function,


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$h_\ell(x)$, is the one-dimensional impulse response of the visual system and is related to H(k).

$$h_\ell(x) = \int_{-\infty}^{\infty} H(k) \cos(2\pi kx)\, dk$$

$$= 940 \left[\frac{.153}{(.153)^2 + (2\pi x)^2} - \frac{.54}{(.54)^2 + (2\pi x)^2}\right],$$

where x is distance in degrees. The point-spread function is the two-dimensional impulse response. If the point-spread function, $h_p(r)$, has circular symmetry, then $h_\ell(x)$ and $h_p(r)$ are related by the Abel-like[3] integral equation

$$h_\ell(x) = \int_{-\infty}^{\infty} h_p\left((x^2+z^2)^{1/2}\right) dz,$$

which is solved by substituting $\alpha = x^2 + z^2$, and using Laplace transform techniques.[4]

$$h_p(r) = -\frac{1}{\pi}\int_r^{\infty} \frac{dh_\ell(y)}{dy} \frac{1}{(y^2-r^2)^{1/2}}\, dy$$

$$= 470 \cdot 2\pi \left[\frac{.153}{\left((.153)^2+(2\pi r)^2\right)^{3/2}} - \frac{.54}{\left((.54)^2+(2\pi r)^2\right)^{3/2}}\right],$$

where r is radial distance in degrees. $h_\ell$ and $h_p$ are shown in Fig. IX-1.

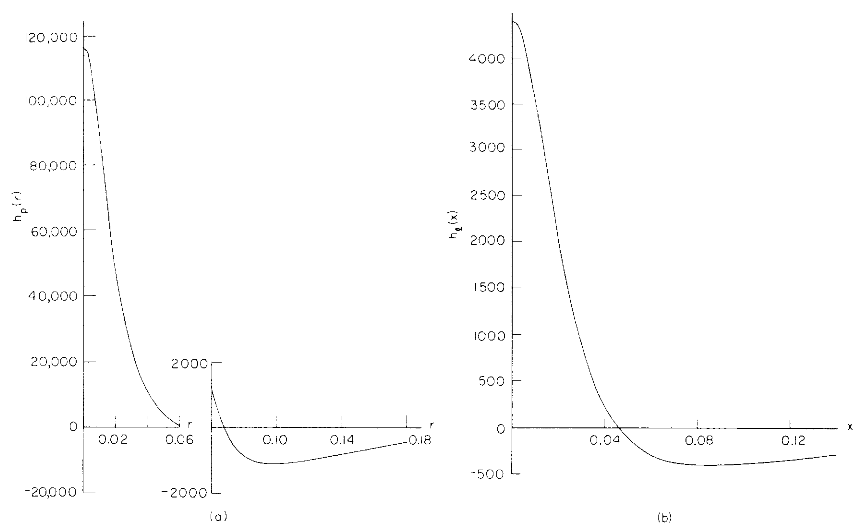


Fig. IX-1. (a) Point-spread function vs angular distance in degrees.
(b) Line-spread function vs angular distance in degrees.

Although an experiment by Mitchell, Freeman, and Westheimer[4] indicates circular asymmetries in the visual system, the formula given for $h_p$ is probably a fair approximation.

It is interesting to note that the formula for $h_p$ can be determined directly. $h_p(r)$ is the Fourier-Bessel transform of H(k) and is given by

$$h_p(r) = \int_0^\infty H(k)\, 2\pi k\, J_o(2\pi kr)\, dk,$$

where $J_o$ denotes the zero-order Bessel function of the first kind. The two computations agree.[4]

R. E. Greenwood

References

1. The numerical analysis is based on Robson's work at 1 Hz. See J. G. Robson, J. Opt. Soc. Am. 56, 1141 (1966).
2. L. Cutrona, Ph.D. Thesis, Department of Psychology, M.I.T., 1970.
3. G. F. Carrier, M. Krook, and C. E. Pearson, Functions of a Complex Variable (McGraw-Hill Book Company, Inc., New York, 1966). I would like to thank David Benney for suggesting that the equation was similar to Abel's equation.
4. I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products (Academic Press, Inc., New York, 1965). Formulas 2.562.2, 2.563.2, and 6.621.1 were useful.
5. D. E. Mitchell, R. D. Freeman, and G. Westheimer, J. Opt. Soc. Am. 57, 246 (1967).

## B. TRANSFORMATIONS ON DIGITIZED PICTURES USING LOCAL PARALLEL OPERATORS

### 1. Introduction

A vital area of concern in automatic pattern recognition and image-processing applications is that of preprocessing. Usually, a picture presented for analysis to a computer is in digitized form. This report is concerned with the (common) case in which a picture is represented as a rectangular array of 1's and 0's with 1 corresponding to black and 0 corresponding to white. Preprocessing can then be defined to be any transformation on the array such that some points are changed from 1 to 0 (erased), and some are changed from 0 to 1 (filled in).

A family of such transformations can be defined with the useful property that the connectivity of the picture is not altered. The following properties can also be imposed.

Local – Each point in the new pattern is a function of the corresponding point in the old pattern and its eight neighbors (see Fig. IX-2).

| | | |
|---|---|---|
| P4 | P3 | P2 |
| P5 | P | P1 |
| P6 | P7 | P8 |

Fig. IX-2. A point P and its eight neighbors.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | | 1 | 1 | 1 | | 0 | 1 | 0 |
| 1 | P | 0 | | 0 | P | 1 | | 1 | P | 1 |
| 1 | 0 | 0 | | 0 | 0 | 1 | | 0 | 1 | 1 |

Fig. IX-3. Changing P will not alter connectivity.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | | 1 | 0 | 1 | | 1 | 0 | 1 |
| 0 | P | 0 | | 1 | P | 0 | | 1 | P | 0 |
| 1 | 1 | 1 | | 0 | 1 | 1 | | 1 | 0 | 1 |

Fig. IX-4. Changing P will alter connectivity.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | | 0 | 0 | 0 | 0 |
| 0 | P | Q | 0 | | 1 | 1 | P | 1 |
| 0 | 1 | 1 | 0 | | 1 | 1 | Q | 1 |
| 0 | 1 | 1 | 0 | | 0 | 0 | 0 | 0 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | | 0 | 0 | 1 | 1 |
| 0 | P | Q | 0 | | 0 | P | 1 | 0 |
| 0 | 0 | 1 | 1 | | 1 | Q | 0 | 0 |
| 0 | 0 | 0 | 1 | | 1 | 0 | 0 | 0 |

Fig. IX-5. Removal of only P or Q will not disconnect the set of black points in these pictures. Removal of both will.

Parallel – A new copy of the picture is generated separately from the old one, so that each newly generated point is a function only of points on the original pattern.

A transformation, then, can be viewed as an array of local operators, one for each point, operating in parallel. In practice this is implemented sequentially, with one operator generating a new picture point-by-point from the original.

Any transformation can therefore be completely specified by specifying the conditions under which a single point may be changed, either from 1 to 0 or 0 to 1. One of these conditions is that changing the point does not alter connectivity. This condition is discussed next, followed by a description of two particular transformations.

2. Connectivity

A subset of a pattern is connected if for any two points p and q of the subset there exists a sequence of points in the subset:

$$p = p_o, p_1, \ldots, p_{n-1}, p_n = q$$

such that $p_i$ is a neighbor of $p_{i-1}$, $1 \leq i \leq n$. As in Fig. IX-2, each point is considered to have 8 neighbors.

Consider Fig. IX-3. In each of the three pictures, changing P either from black to white or from white to black will not alter the connectivity of the black points. This is because the set of 1's (blacks) around P is already connected. In each of the pictures in Fig. IX-4, the set of 1's will be connected only if P is black. Changing P either way alters connectivity.

From this, we can state a rule that must be satisfied to change a point: A point may not be changed, either from black to white or white to black unless the set comprising its 8 neighbors contains at most one connected subset of black points.

A simple means of calculating the number of connected black components in the neighborhood of P (excluding P) has been reported by Hilditch.[1] Define the crossing number of P with respect to the subset containing its black neighbors as the number of times a "bug" taking a walk around P by way of its neighbors would have to cross from outside to inside the subset. The bug is permitted to take diagonal steps to avoid leaving the black area. If the crossing number is 0, the number of connected black components is 0 or 1. Otherwise, the number of components is equal to the crossing number. In Fig. IX-3, for example, the first two pictures have crossing number 1 and the last picture has crossing number 0. In Fig. IX-4, the first two pictures have crossing number 2 and the last picture has crossing number 3.

The crossing number X(P) is calculated as follows:

$$X(P) = \sum_{i=1}^{4} b_i,$$

where $b_i = 1$ if $P_{2i-1} = 0$, and either $P_{2i}$ or $P_{2i+1} = 1$, and $b_i = 0$ otherwise.

3. Hole Filling

Consider the transformation specified by the following set of rules. A point P will be filled in, that is, changed from white to black, if and only if all of the following are true:

1. $P = 0$. The point must be white initially.

2. $\sum_{i=1}^{8} P_i > 4$. Most of its neighbors are black.

3. $X(P) = 0$ or 1.

All other points remained unchanged.

This transformation changes some points from white to black. A point is changed if most of its 8 neighbors are black and if connectivity is preserved. Such a transformation can be used to eliminate isolated small sets of 0's, that is, to fill in small holes in black regions. To achieve this effect, the transformation is applied repeatedly, always using the most recent version of the picture, until no more points are changed.

4. Thinning

Another algorithm that can be specified is thinning. Thinning is the process of reducing a picture to a stick figure or skeleton. A linelike black region is reduced to a black line which is only one point thick. Again, this effect is achieved by repeated application of a transformation. The transformation to be applied removes boundary points in the picture. A boundary point is a black point, at least one of whose axially adjacent neighbors (P1, P3, P5, P7) is white. Each application peels away the outer layer of the black regions. This is continued until all black regions have been shrunk to minimum thickness without erasing them.

This algorithm is somewhat complicated to explain, because of an effect produced by the parallel nature of the computation. The effect can be observed by noting the pictures of Fig. IX-5. In each case, the removal of either P or Q (for example, changing P or Q from black to white) does not alter connectivity. The removal of both these boundary points does, however.

One way out of this difficulty is to perform the thinning operation sequentially, changing points directly on the original picture. Unfortunately, as points are removed from the boundary of an object, subsequent nearby points become eligible for removal. The result is that the skeleton will tend to be biased toward one side or the other of the original set of black points, depending on the order in which the points are taken.

To maintain an essentially parallel type of computation, the following procedure is adopted: each point P is tested with its 8 original neighbors. If the point is eligible for removal, it must then satisfy the further condition that its removal in conjunction with any of its neighbors that has been previously removed does not alter connectivity.

Since the sequence in which the points are being tested is known, it is only necessary to test for this condition for those neighbors that precede P. If the picture is examined starting at the top and going left to right, then it is only necessary to consider P2, P3, P4, and P5.

Now notice that P2 may be ignored. The reasoning is as follows:

The only case of interest is that in which P is black, and P2 has been changed from black to white, and the crossing number X(P) with P2 black is one. This is the case in which we wish to change P and must check that this change plus the change in P2 does not alter connectivity.

Such a change in connectivity would take place only if X(P) with P2 white were greater than one.

Suppose that P1 and P3 are white. If P is P2's only black neighbor, then, as we shall show, P2 would not have been erased. On the other hand, if P2 has other black neighbors, then $X(P2) > 1$ and P2 still would not have been erased.

It follows that at least one of P1 and P3 is black. Therefore, removal of P2 cannot change the value of X(P), and so P can be erased without altering connectivity.

The same line of reasoning applies to P4. To guarantee that the parallel removal of two adjacent points does not alter connectivity, it is sufficient to say that P may not be erased if either P3 or P5 has been.

The thinning transformation can now be specified. A point P will be erased, that is, changed from black to white, if and only if all of the following are true:

1. $P = 1$. The point must be black initially.
2. $P1 + P3 + P5 + P7 < 4$. The point must be a boundary point.
3. $\sum_{i=1}^{8} P_i \neq 1$. If the point is on the tip of a thin line, it should not be erased, thereby shortening the line.
4. $X(P) = 1$.
5. Neither P3 nor P5 has been erased.

All other points remained unchanged.

## 5. Conclusion

Two examples of connectivity-preserving transformations on digitized pictures have been given. A family of such transformations can easily be specified. Examples of other functions that could be performed are thickening black regions, erasing isolated small sets of black points, and reducing each connected black region to a single point.

W. W. Stallings

References

1. C. J. Hilditch, "Linear Skeletons from Square Cupboards," Proc. Machine Intelligence Workshop 4 (1969).