XIV. SIGNAL PROCESSING

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A. RESULTS OF THE APPLICATION OF WINDOW FUNCTIONS TO THE PROBLEM OF ECHO REMOVAL

As reported in Quarterly Progress Report No. 100 (page 241), solutions to the problem of echo removal have been sought by using the theory of window functions developed by M. V. Cerrillo. Performance data have been obtained from computer simulations of the filters using the A. P. L. language; programs have been written, first, to calculate the impulse weights for filters designed to reject (single) echoes of known amplitude and delay, and second, to evaluate the time- and frequency-domain performance of these filters. First, the coefficients of a McLaurin series expansion of the required transfer function

$$H(s) = \frac{1}{1 + be^{-sT_D}} \tag{1}$$

were calculated, from which, by using Cerrillo's methods,[1] we then found the set of impulse weights $\{a_i\}$ for the filter. Of the four parameters required for these calculations only three, the echo delay T_D, the echo transmission coefficient b, and the order of the filter m, could be given a priori values. The window spacing δ had to be specified somewhat arbitrarily, since no methods have been devised for optimizing it for a given problem.

The time response of the system, as shown in Fig. XIV-1, was obtained by using a convolution program that evaluated

$$\hat{f}(t) = [f(t) + bf(t-T_D)\, U_{-1}(t-T_D)] * h_w(t), \tag{2}$$

where $h_w(t)$ is the impulse response of the filter. The delayed unit step was included to simulate the real situation in which the echo is not present until after the delay time

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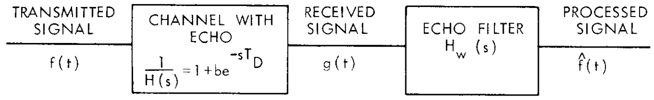

BLOCK DIAGRAM OF SIMULATED SYSTEM.

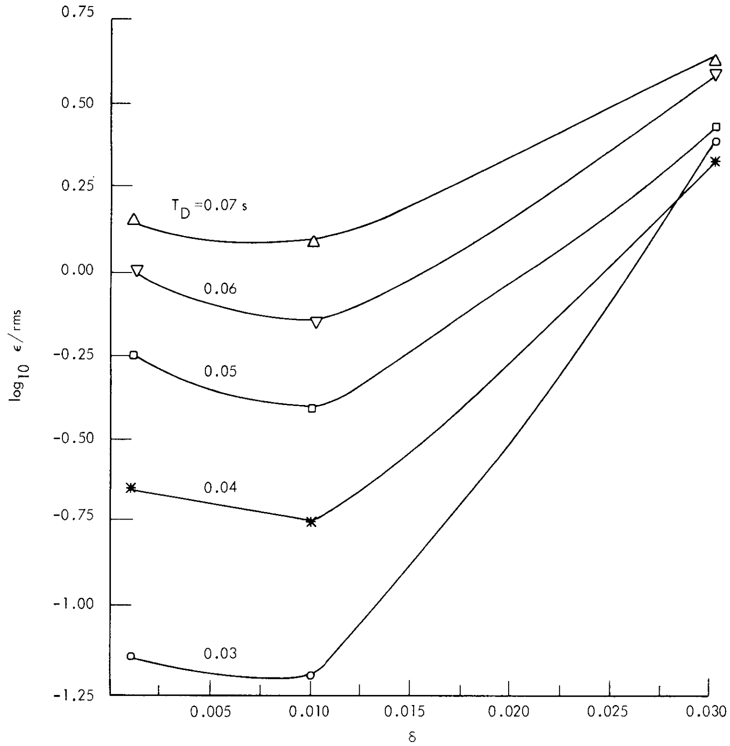

Fig. XIV-1.
Log ϵ_{rms} vs δ with T_D parametrized.

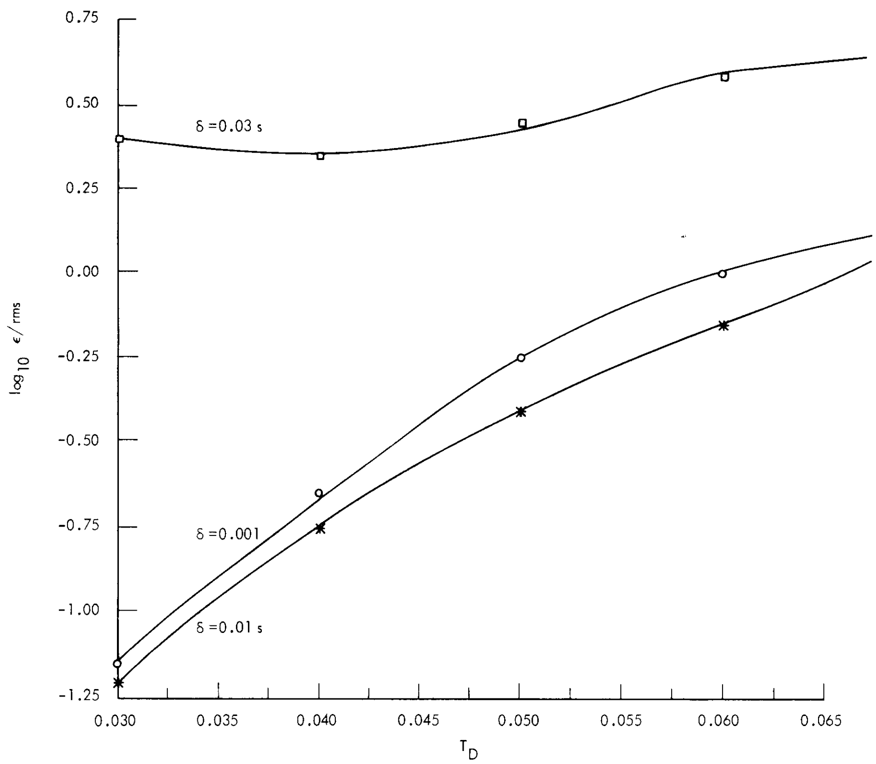

Fig. XIV-2.
Log ϵ_{rms} vs T_D with δ parametrized.

has elapsed. Furthermore, times $t < (m+1)\delta + T_D$ were not considered in the response calculations. The results were evaluated, first, by calculating the rms error,

$$\epsilon_{rms} = \left[\frac{\sum_i (\hat{f}(t_i) - f(t_i))^2}{\sum_i (f(t_i))^2} \right]^{1/2}, \tag{3}$$

committed by each of a large number of filters, and then by examining in detail the actual waveforms produced by the best of these filters. Finally, for comparison, the waveforms of one of the worst filters were examined in order to determine more specifically the types of errors.

The results obtained for 5 simulated systems, each with an echo transmission coefficient of b = 0.5 and with delays of 0.03-0.07 sec are shown in Figs. XIV-1 and XIV-2. The signal that was applied was

$$f(t) = 9 \sin 10t + 4 \sin 20t + 9 \sin 30t + 8 \sin 40t + 4 \sin 50t + \sin 60t + 3 \sin 70t + 2 \sin 80t + 2 \sin 90t + \sin 100t, \tag{4}$$

and the filters used were of order m = 3 (4 windows) with δ equal to 0.001, 0.01, and 0.03, respectively. In Table XIV-1 the rms error of the processed signals $\hat{f}(t)$ for both the best and the worst cases, $T_D = 0.03$ and $T_D = 0.06$, respectively, are compared with the errors in the received signals corrected for the amplitude differences. Signals corrupted by echoes resulting from a delay of 0.03 sec were improved approximately 12 dB over the amplitude-corrected received signals. The same processing techniques applied to signals corrupted by a 0.06 sec echo produced more distortion, however, than existed in the unprocessed received signals.

Frequency response curves (Fig. XIV-3) for the ideal and realized filters for the $T_D = 0.03$ case are in substantial agreement over the range of the test signal, hence the low values of rms error found previously. For a delay of 0.06 sec the filter exhibited a rising characteristic at high frequencies, which explains both the large rms errors shown in Table XIV-1 and the distortion found in detailed time plots of the waveforms.

The data indicate that satisfactory results, for filters of order m = 3 and channels with an echo transmission coefficient b = 0.5, can be obtained only if the signal bandwidth and the channel delay are such that

$$\omega_{max} T_D < 3.6$$

or

$$f_{max} T_D < 0.57.$$

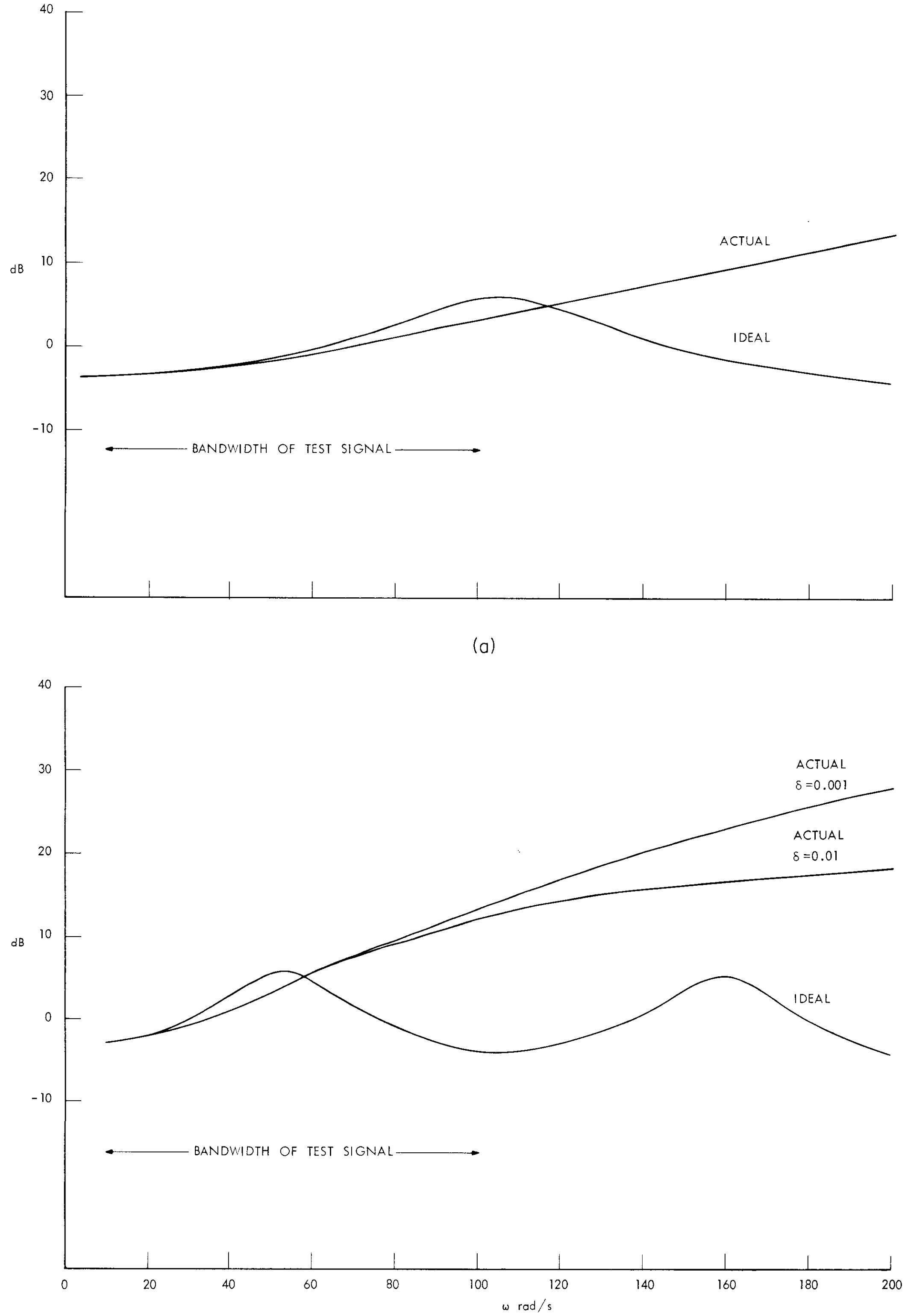

Fig. XIV-3. (a) Ideal and actual filter response for T_D = 0.03 sec, δ = 0.001 sec, b = 0.5, m = 3.
(b) Ideal and actual filter response for T_D = 0.06 sec, δ = 0.001 and 0.01 sec, b = 0.5, m = 3.

Table XIV-1. Summary of rms errors.
($m = 3$, $b = 0.5$)

	Error in Processed Signal $[\hat{f} - f]_{rms}$	Error in Amplitude-Corrected Received Signal $\left[\frac{g}{1+b} - f\right]_{rms}$
$T_D = 0.03$, $\delta = 0.001$	0.07	0.31
$T_D = 0.06$, $\delta = 0.001$	0.98	0.50
$T_D = 0.06$, $\delta = 0.01$	0.74	0.50

The window-function filters appear to be best suited for processing signals containing large amounts of echo produced by channels with short delays. Although such characteristics preclude the use of these filters in telephone and broadcast applications, they could be used in the processing of the type of seismic data encountered in oil explorations.

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References

1. M. V. Cerrillo, Technical Report 269, Research Laboratory of Electronics, M.I.T., Cambridge, Mass., May 3, 1950, pp. 122-139.