PLASMA DYNAMICS

VIII. PLASMAS AND CONTROLLED NUCLEAR FUSION*

A. Waves and Radiation

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1. HARMONIC GENERATION BY AN ANTENNA IMMERSED IN A PLASMA

In a previous report,[1] we described measurements of harmonic generation produced when a slotted-sphere antenna is immersed in an essentially collisionless plasma. Assuming that the harmonics are generated by the nonlinear conductance of the plasma sheath surrounding the antenna, we found strong disagreement between the measured and theoretically predicted harmonic amplitudes. In this report we shall show that the model of an oscillating sheath (that is, a nonlinear capacitor) predicts harmonics that are in good agreement with observations.

The complex admittance Y of a slotted-sphere antenna oscillating in the dipole mode is given by[2]

$$Y = \frac{3}{4}\pi\omega\epsilon_o a\left[\frac{3}{Q} - 1\right]. \tag{1}$$

Here ω is the frequency of the applied voltage $V \cos \omega t$, a is the radius of the spherical antenna, and the factor Q is given by

$$Q = 1 + \frac{2(1-K)}{1 + 2K}\left(\frac{a}{R}\right)^3, \tag{2}$$

where R is the radius of the plasma sheath, and K the plasma dielectric coefficient.

Now suppose that the sheath oscillates as a result of the applied voltage V. Then the perturbed admittance Y' is of the form

$$Y'(R+\xi) = Y(R) + \left(\frac{\partial Y}{\partial R}\right)_{\xi=0} \cdot \xi, \tag{3}$$

*This work was supported by the U.S. Atomic Energy Commission (Contract AT(30-1)-3980).

where ξ is the radial sheath displacement (from its equilibrium value). The value of ξ can be estimated in terms of the known radial electric fields in the sheath and plasma regions, through application of Gauss' law, with the result that

$$E(\text{sheath}) - E(\text{plasma}) = \omega_p^2 \xi \left(\frac{m}{e}\right), \tag{4}$$

where ω_p is the plasma frequency, m the mass, and e the charge of the electron. Use of Eqs. 1, 2, and 4 now permits us to evaluate the perturbed susceptance. After somewhat lengthy manipulation, we obtain the following expression for the admittance Y'

$$\frac{Y'}{Y_o} = \frac{Y}{Y_o} + \frac{V \cos \omega t}{Ra} \left(\frac{e}{m}\right) \frac{1}{\omega_p^2} \left[1 - \frac{Y}{Y_o}\right]^2 \left[1 + 2 \frac{Y}{Y_o}\right]. \tag{5}$$

Here Y is the "linear" admittance given by Eq. 1, and $Y_o \equiv (3\pi/2)\omega\epsilon_o a$ is the admittance of the slotted-sphere antenna when radiating into free space.

The second term in Eq. 5 is the sought-for contribution to Y, caused by the oscillating sheath. We note that its magnitude is proportional to the amplitude of the applied voltage V, and thus gives rise to harmonic generation. The voltage $V = I \cos \omega t/Y'$ developed across the antenna has a harmonic component that drives a current whose magnitude is governed by the plasma-antenna impedance, evaluated at the harmonic frequencies. A detailed comparison between experiment and theory is under way.

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2. RADIOFREQUENCY ABSORPTION IN PLASMAS

Work has been completed on an investigation of the coupling of quasi-static RF magnetic fields with a plasma at frequencies near ion cyclotron resonance.[1,2] The plasma that was studied was a low-pressure reflex arc in hydrogen. Changes in the impedance of a Stix coil surrounding the plasma were measured as the DC magnetic field was swept through ion cyclotron resonance. Typical measurements of the normalized resistive component of the impedance are shown in Fig. VIII-1. The DC magnetic field increases from right to left. The ion cyclotron resonance line rises above a nonresonant background with increasing discharge current (ion density). The half-width at the higher

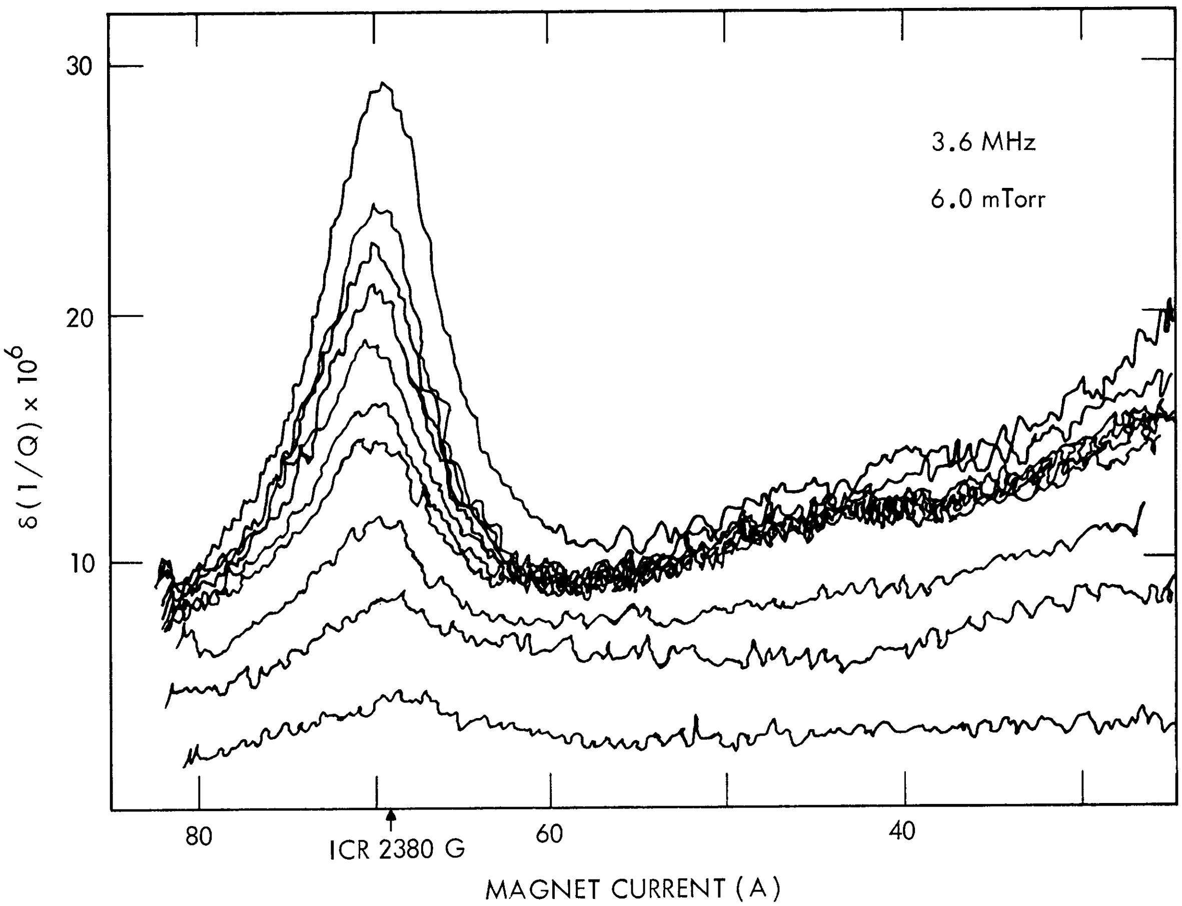

Fig. VIII-1. Conductance measurements in the arc mode. Each curve represents an increment of 20 mA in the discharge current.

currents is independent of ion density. The resonance line at low currents is weakly visible above the background, is distorted, and displays a shift to lower magnetic fields. Ion density at 100 mA discharge current is ~5×10^{10} cm.

The characteristics described here cannot be explained solely in terms of single-particle coupling of the RF magnetic field to the ions. Such a model predicts a symmetrical ion cyclotron line rising well above a small background. This model breaks down with increasing ion density above 10^{12} cm^{-3} because of the excitation of propagating ion cyclotron waves. The resonance line should shift to higher magnetic fields for this mechanism, in contrast to the observed shift to lower magnetic field.

The mechanism that allows the penetration of an RF magnetic field into an anisotropic plasma, despite the tendency to produce charge separation fields in the plasma by the Hall effect, is the neutralization of these fields by electrons moving along the DC magnetic field lines between adjacent half-periods of the driving field. The impedance component associated with this electron motion displays the characteristics of the measured impedance at low discharge current in Fig. VIII-1. While the electron response along the magnetic field lines does not vary strongly in the vicinity of ion cyclotron resonance, the Hall currents driving the electrons do display a strong variation. The background in Fig. VIII-1 is due to the electron Hall current, and the resonant contribution from the ions gives the distorted ion cyclotron line. The single-particle ion cyclotron resonance line appears at high currents in Fig. VIII-1, because of a saturation in the magnitude of the Hall component. The Hall component was found to be a strong function of the operating characteristics of the discharge. This is not unexpected, since the generation of radial charge separation should be strongly dependent on the detailed radial charge distribution in the plasma.

The half-width of the ion-cyclotron resonance line can be used as a measure of the ion dynamics in the discharge. The half-widths measured at the highest available current as a function of pressure for three different cathodes are shown in Fig. VIII-2. The

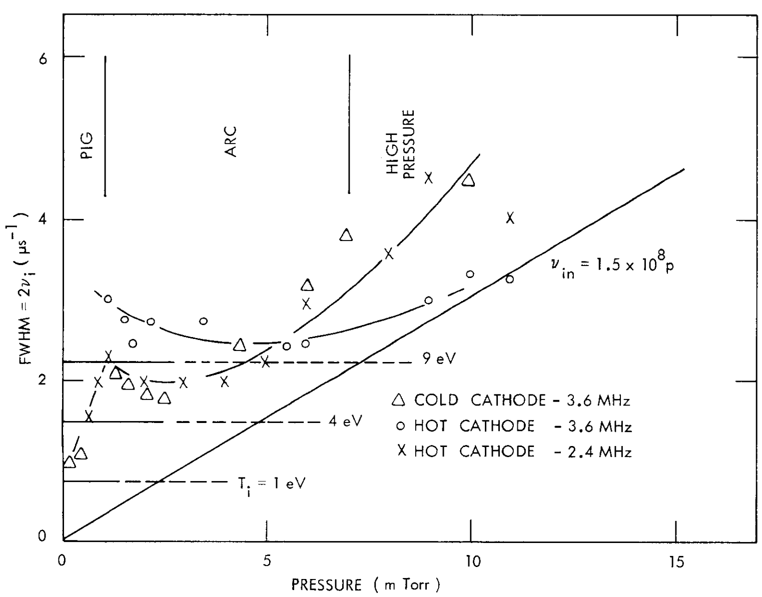

Fig. VIII-2. Half-width of the ion cyclotron resonance line as a function of pressure.

various pressure regions correspond to the modes of the reflex discharge.

The resonance lines in the arc mode rose well above the background and displayed the symmetry characteristic of single-particle coupling. The half-width was found to be relatively independent of pressure. The linewidth in this regime is attributed to Doppler broadening, arising from longitudinal motion of ions through the periodic fields of the coil. The ion temperatures were found to be a few electron volts in this mode and to be dependent on the type of cathode that was used.

At high pressures the resonance lines display more of the characteristics of the Hall component and the interpretation of the half-width becomes ambiguous. From the smallest measured half-width in this region it is possible to set an upper limit on the ion-neutral collision frequency of 1.5×10^8 $(\text{Torr-s})^{-1}$. This is in reasonable agreement with measurements of ion mobility in hydrogen measured at higher pressures.[3] At pressures below 1 m Torr the discharge changes to a PIG mode. The half-width of the ion cyclotron line drop corresponds to a lower ion temperature.

The conclusions reached in this study are that measurements of absorption near ion cyclotron resonance, interpreted on the basis of single-particle coupling to the RF magnetic field can provide significant diagnostic information on the ionic properties of a plasma. Its general use, however, is limited by the existence of coupling to electrons through the Hall effect, with a resulting ambiguity in the interpretation of half-widths of the ion cyclotron resonance line. Further research is needed to clarify the role of gradients in the development of charge-separation fields within the plasma as a result of internal or external driving fields.

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VIII. PLASMAS AND CONTROLLED NUCLEAR FUSION*

E. High-Temperature Toroidal Plasmas

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1. ALCATOR EXPERIMENT

Introduction

The idea for an advanced experiment in high-temperature plasmas started to take form at M.I.T. in the spring of 1969 with various motivations.

One motivation came from the realization that in high-temperature plasmas micro-instabilities can be excited in such a way that the plasma configuration is not destroyed but its transport property can be significantly modified. Thus one important consequence is the appearance of anomalous resistivity. This takes place when the interactions between particles and plasma collective effects arising from a relatively large current density add up to the known effects of interparticle collisions and produce an enhanced resistivity. Evidence of this is shown in Fig. VIII-3. Therefore it was natural to propose an experiment that could achieve different plasma regimes where the analysis of these transport processes would be possible and, at the same time, could utilize these effects to enhance the plasma ohmic heating process that occurs when a current flows into it.

A second motivation was that much of the research effort in high-temperature plasmas thus far has been devoted to testing new magnetic confinement configurations, rather than to achieving new plasma regimes, where magnetic confinement can be easier, and to further understanding plasma collective processes. So a departure from this trend was needed.

A third motivation came from realizing that at the Francis Bitter National Magnet Laboratory advanced technologies have been developed for the construction of large-volume high magnetic field magnets which are ideally suitable to the type of experiments indicated above. The object is in fact to obtain a plasma in which a high current density can flow so that anomalous resistivity is produced, but with a safe margin of stability against macroscopic (MHD) modes that are known to destroy confinement. For this, it is necessary to have a magnetic configuration (Fig. VIII-4) whose major

*This work was supported by the U.S. Atomic Energy Commission (Contract AT(30-1)-3980).

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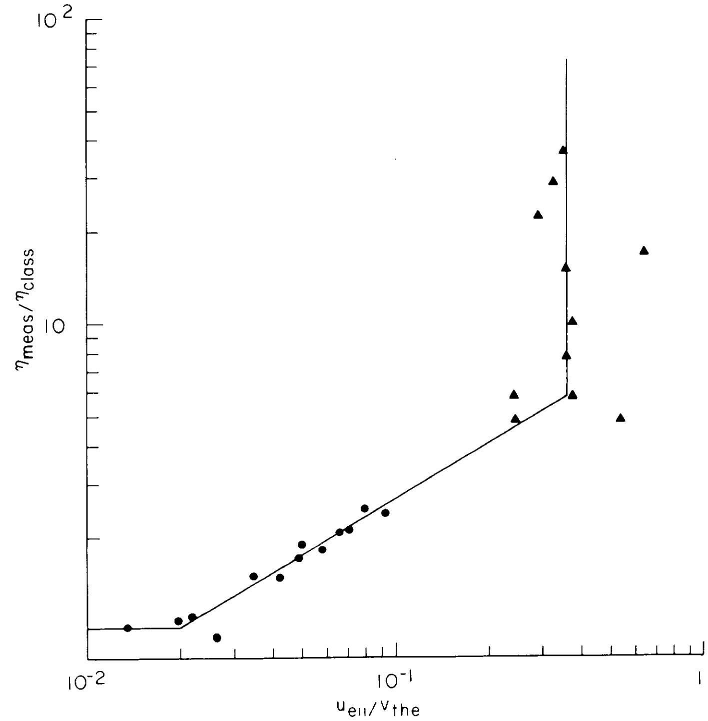

Fig. VIII-3. Evidence of anomalous resistivity as a function of electron current. Here η_{meas} is the ratio of the applied electric field to the average current; η_{class} is the resistivity evaluated from the classical Spitzer formula as a function of the measured electron temperature $T_{e\perp}$ and averaged over the plasma cross section; $u_{e\parallel}$ is the average electron flow velocity along the magnetic field, and v_{the} the electron thermal velocity. The dot points refer to an experiment carried out on the Princeton C-Stellarator, where the electron temperature was evaluated by light laser scattering. The triangular points refer to experiments carried out on the TM-3 Tokamak device, where the electron temperature was obtained by diamagnetic measurements.

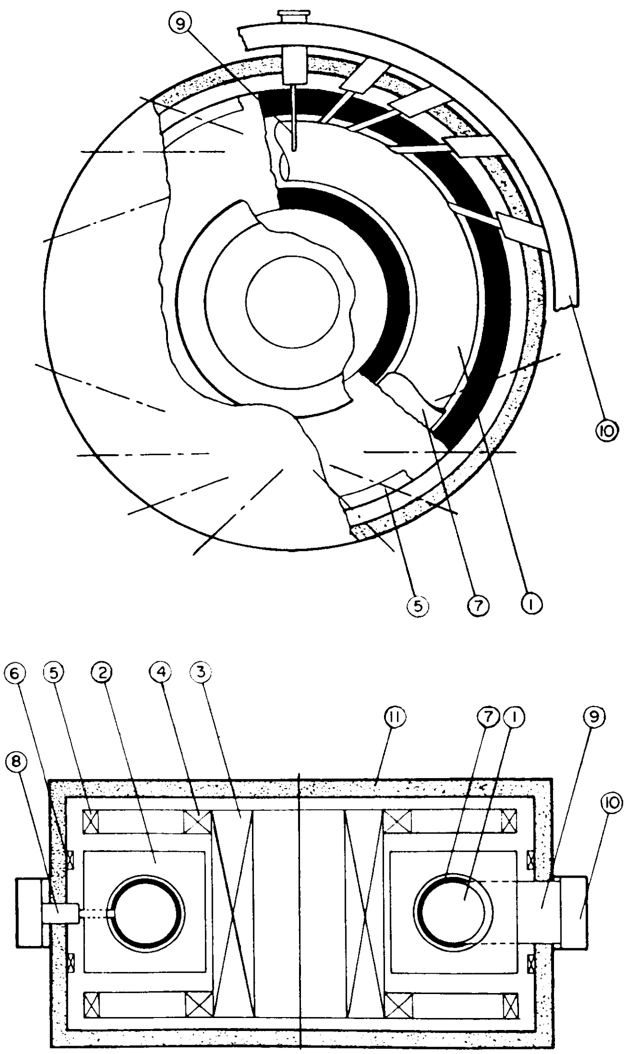

Fig. VIII-4. Proposed Alcator.
Key: (1). Toroidal cavity of stainless-steel vacuum vessel. (2). Bitter toroidal coil (clamping plates and retaining rings not shown). (3), (4), (5). Air-core transformer coils. (6). Transverse field coils. (7). Passive copper stabilizing torus. (8). Oblique diagnostic and pumping ports welded to central toroidal vacuum vessel. (9). Transverse access port to central vacuum to provide access for diagnostics or limiter section. (10). Vacuum manifold and diagnostic access. (11). Nitrogen cryostat, foam plastic and fiberglas construction.

radius, R, is as small as possible, the plasma cross section as large as possible, and the magnetic field as high as possible. So the design of Alcator [fr. L. alto campo torus, high field torus] started to take shape when we felt that all of these requirements could be met on a device of relatively modest size and compatible with our facilities.

The choice of operating Alcator at first in the Tokamak mode[1] was made for two reasons. The first was that our investigation of the phenomenon of anomalous resistivity had shown that, as evidenced by Fig. VIII-3, Tokamak experiments could cover two distinct types of regime. One of them, corresponding to electron flow velocities much less than the electron thermal velocity, could be identified with a regime studied experimentally on the Princeton C-Stellarator, and theoretically analyzed, in the sense that it was not dominated by the effects of run-away electrons.[2] In other words, the electron distribution in velocity space for this regime was considered to be quite close to a Maxwellian. Since the results available at that time[1] for the Russian T-3 Tokamak appeared to fall in this regime, it followed that the reported temperature had to be that of a nearly Maxwellian distribution and that Tokamak had superior confinement properties to any other type of device realized thus far. This conclusion was later (in the fall of 1969) confirmed[3] by the laser scattering experiments carried out by a British team at the Kurchatov Institute, in Moscow. The second reason was that only a diffuse pinch configuration, which Tokamak is, can tolerate magnetic fields of the order of the 10^5 G that could be provided by the advanced toroidal Bitter magnets proposed for Alcator. This magnetic field magnitude is of special technological significance, in that it represents the probable practical limit of future superconducting technology. Configurations proved out by inexpensive liquid-nitrogen coils such as the one considered for Alcator can be the forerunners of much more expensive superconducting devices.

Device Characteristics

Alcator is a toroidal plasma column with major radius R of 54 cm. The plasma (minor) radius can be as large as 13 cm. The magnetic field B_T parallel to the toroidal axis varies from 160 kG to 100 kG over the plasma column cross section.

The magnetic configuration has a high degree of azimuthal (toroidal) symmetry, so that it is very close to being two-dimensional. (Recall that a two-dimensional plasma configuration, besides being more accessible to a theoretical analysis than a three-dimensional configuration, has superior properties for particle confinement.) Diagnostic accessibility to the plasma is obtained, without excessive sacrifice of symmetry, by compensating coils in the neighborhood of perpendicular diagnostic ports and by the presence of a large number of oblique channels through the Bitter plates composing the magnet. Oblique channels for laser scattering diagnostics are important to gain information on the particle distribution as a function of the velocity components parallel and perpendicular to the magnetic field. (This optical configuration had been proposed earlier in the Frascati Laboratory's Tokamak design.)

In its initial mode of operation the Alcator plasma column will have a circular cross section, so chosen for reasons of both physics and simplicity, and the confining magnetic poloidal field B_p is provided by an electron current $j_{\parallel}$ inside the plasma, longitudinally to the main magnetic field B_T. This current also provides plasma heating by collisional resistive heating and weakly turbulent effects.

Additional methods of heating are under consideration. In fact, ohmic heating is slow in the keV temperature range that is under consideration, does not balance favorably at densities higher than 10^{14} particles/cm^3 with radiation losses, does not guarantee an efficient ion heating, and, from the point of view of investigating basic effects such as anomalous resistivity, has the disadvantage of not allowing a decoupling of the electron temperature T_e from the magnitude of the current $j_{\parallel}$ and of the resulting magnetic shear.

The initial Alcator device will be gradually transformed into more complex magnetic field configurations. In particular, a transition to a three-dimensional configuration will be provided by shorting groups of Bitter plates, thereby introducing predetermined periodicities of B_T along the toroidal axis. A variety of magnetic surface cross sections, with varying degrees of MHD stability are planned, to be realized by properly shaping the Bitter plates' inner bore, or by a proper choice of the vertical magnetic fields that can be supplied by the air-core transformer compensating coils, or by the additional transverse field coils provided for the proper positioning of the plasma column (see Fig. VIII-4), or by introducing new coils.

Finally, we notice that the Alcator magnetic structure can be adapted to enclose helical windings (at substantially lower values of the magnetic field) and obtain a very attractive Stellarator configuration with a high degree of symmetry, favorable aspect ratio, and good magnetic shear.

Plasma Parameters and Regimes of Operation

The accuracy of predictions of the best plasma parameters to be achieved in Alcator is limited by the lack of prior plasma experiments carried out with comparable optimal values of magnetic field and aspect ratio (R/a = torus radius/plasma radius) and a good degree of azimuthal symmetry. Theoretically, however, all of the most important parameters determining plasma confinement have strong and favorable dependences on the magnetic field strength: for example, $\beta \equiv nkT/B^2$, the collisional coefficients for particle and energy diffusion across the magnetic field, the growth rates and transport parameters associated with most microinstabilities and, in particular, with the trapped particle mode, and so forth.

The main considerations adopted in predicting the expected plasma parameters are the following.

1. On the basis of theoretical considerations and of a large number of experimental tests in different Russian Tokamak devices we expect the maximum kinetic energy density that can be contained to be

$$nkT_e \sim \frac{B_p^2}{8\pi}, \tag{1}$$

where B_p is the poloidal magnetic field that is limited by the Kruskal-Shafranov criterion against hydromagnetic kink modes

$$B_p \leqslant B_T \frac{a}{Rq_o}, \tag{2}$$

and q_o is the so-called stability margin that we shall take ≈2.5. Therefore the geometrical properties of Alcator have been chosen with the criterion of achieving the largest possible value of B_p, and compromising between confinement time (leading to relatively large values of a) and anomalous resistivity (leading to relatively large values of B_T/R). So if $\overline{B}_T = 130$ kG and $R/a = 4.5$, $B_p \approx 11.5$ kG. Notice that, by choosing a density of operation $n_o \approx 10^{14}$ cm^{-3}, the maximum electron temperature that can be contained is 20 keV. More generally, $T_{e_{max}}$ (keV) $\approx 20(n_o/n)(B_p/B_{po})^2$, where $B_{po} \approx 11.5$ kG.

In practice, the maximum value of T_e that can be achieved is determined by the plasma heating process, the radiation losses, and the stability against trapped particle modes. Then if 6 ÷ 7 keV is the upper temperature limit, as expected, the "poloidal beta" $\beta_p = 8\pi nkT_e/B_p^2$ will be approximately 1/3.

Notice that the expected ion-banana maximum width is

$$\overline{\Delta r} \sim 0.4\, T_i^{1/2}\,(\text{keV}) \left(\frac{B_{po}}{B_p}\right) \text{cm}$$

for hydrogen. This is one of the reasons why it is important to maintain a high value of B_p, even though the expected values of $\beta_p \equiv nkT_e/B_p^2$ will be less than unity.

2. With ohmic heating adopted for the contained plasma, the heating rate and the power input are determined by $j_{\parallel}^2$, $j_{\parallel}$ being the current density parallel to the magnetic field. On the basis of Eq. 2 the current density limitation is

$$j_{\parallel_o}^2 = \left(\frac{5}{\pi}\frac{B_p}{a}\right)^2 \leqslant \left(\frac{5}{\pi q_o}\frac{B_T}{R}\right)^2 \approx (0.64)^2 \frac{B_T^2\,(\text{G})}{R^2\,(\text{cm})}\ (\text{A/cm}^2)^2, \tag{3}$$

which in Alcator is much larger than in any existing or proposed experiment (see Table VIII-1).

A key factor in the heating process of the Alcator plasma is the enhanced (anomalous) resistivity that arises in the presence of collective modes that are excited when the electron flow velocity

$$u_{e\|} = \frac{j_{\|}}{ne} > u_{thr}, \tag{4}$$

where u_{thr} is the threshold velocity.[2] Notice that for $n = 10^{14}\ cm^{-3}$, the plasma frequency is

$$\omega_{pe} = 5.63 \times 10^{11}\ rad/s$$

and is less than the electron gyro frequency

$$\Omega_e = 2.3 \times 10^{12}\left(\frac{B_T}{\bar{\bar{B}}_T}\right)\ rad/s.$$

Under these conditions an evaluation of u_{thr} has been given,[2] but for an approximate estimate we can take

$$u_{thr} \approx \sqrt{\frac{\alpha kT_e}{m_i}},$$

where m_i is the ion mass, and α is a numerical factor larger than 1. So from the inequality (4) we have

$$\frac{n_a}{n_o} T_{ea}^{1/2}\ (keV) < 1.3 \times 10^3\ \frac{B_T}{R},$$

where n_a and T_{ea} are the upper limits under which the resistivity anomaly occurs. So for $n_a = n_o$ and $B_T = 130$ kG, $T_{ea} \approx \alpha^{-1}$ 10 keV.

Now an important condition to be satisfied is that the heating time be shorter than the expected confinement time. To evaluate the former time, we have used a series of numerical programs of increasing complexity, taking into account radiation and transport losses, as well as the functional dependence of anomalous resistivity on electric field, current, electron temperature, and density. The latter time has been deduced tentatively from the scaling laws given for the Russian Tokamaks such as

$$\tau_E \propto a^2 B_p^{\gamma},$$

where τ_E is the energy containment time, and γ is a positive coefficient larger than or of the order of one. There has been considerable discussion about the rationale behind these scaling laws and in fact one of the major purposes of Alcator will be to provide new information on the parametric dependence of τ_E.

A rough idea of the heating time can be obtained by considering times $t > t_c$ after which the current is maintained constant at the Kruskal-Shafranov limit. Thus, neglecting all losses, we have

$$t_n = t_c + \frac{3}{2} \frac{nkT_c}{\eta_c J_{\|c}^2} \int_1^{T_e/T_c} \frac{\eta_c}{\eta_\|} \left(\frac{T_e}{T_c}\right) \frac{dT_e}{T_c},$$

where T_c is the temperature reached at the time t_c, and η_c the corresponding classical resistivity that is due only to electron-ion collisions. This is given by

$$\eta_{\text{class}} = 1.3 \times \frac{10^{-6}}{T_e^{3/2}} \hat{\alpha}\ \Omega\text{-cm},$$

with T_e in keV, $\hat{\alpha} \equiv (\ln \Lambda)/15$, and $\ln \Lambda$ has a well-known definition. So we can evaluate the time

$$\tau_E^c = \frac{3}{2} \frac{nkT_c}{\eta_c J_{\|c}^2} = \frac{n}{n_o} T_c^{5/2}\ (\text{keV})\quad 7.7\ \text{ ms}.$$

In particular, if the resistivity remained classical for $T_e > T_c$, we would have

$$t = t_c + 2.4 \frac{n}{n_o} \left(T_e^{5/2} - T_e^{5/2}\right) \text{ ms}.$$

Therefore for $T_e = 6$ keV, t_n would be higher than 210 ms and would require too large a confinement time.

It is therefore clear that the quoted values for density and temperatures can be reached in Alcator only if anomalous resistivity sets in. To reach an acceptable value for the heating time, an anomaly factor not smaller than 3 is necessary. This is, however, well within the bounds of the observed values of the T-3 Tokamak experiment[3] and of theoretical expectations.

We notice at this point that the voltage drop, which is due to the longitudinal resistivity $\eta_\|$ around the toroid, is

$$V_R = 2\pi R \eta_\| \times \frac{5}{\pi} \frac{B_p}{a} \leqslant 10 \eta_\| \frac{B_T}{q_o} = V_o, \tag{5}$$

and if we use the classical resistivity for $\eta_\|$,

$$V_{o_{class}} = 0.52 \frac{B_T}{T_e^{3/2}} 10^{-5} \text{ V}. \tag{6}$$

So at 6 keV, $V_{o_{class}}$ is very small and it would be very surprising if electron-ion collisions were the only cause of voltage drop. On the other hand, the fluctuations that are present inevitably in a plasma and have an energy content that is a fraction of the particle thermal energy should therefore lead to a considerable amount of anomalous resistivity.

The evaluation of the plasma heating process and of the inductance associated with the Alcator configuration have lead to the choice of an air-core transformer to provide the necessary magnetic flux that has been estimated to be approximately 1.2 V-s.

Comparison with Other Experiments

We shall compare experimental facilities for the generation and the confinement of a toroidal plasma column, excluding open-ended devices such as mirrors which are typified by strongly non-Maxwellian particle distribution functions of the so-called loss-cone type. All of the devices to be considered have a constant toroidal magnetic B_T field generated by currents flowing in conductor coils around the plasma column, but may have the remaining confining field components generated by currents flowing inside conductors placed within or outside the plasma column, or flowing inside the plasma itself. Roughly we have the following devices.

Stellarators: Wherein the confining magnetic fields are generated by current conductors outside the plasma column, and are intrinsically three-dimensional, in the sense that there is no symmetry in the $\vec{e}_\theta$ direction.

Multipoles (quadrupole, octopole, levitron, spherator, etc.): Wherein the poloidal field is generated by current conductors inside the plasma that in principle are two-dimensional. In practice, however, the toroidal symmetry is often spoiled by the existence of conductor supports, which may also create an additional mechanism for particle losses.

Tokamaks: In which the poloidal field is generated by a current flowing inside the plasma. In the devices realized thus far, the same current also provides the plasma heating. A Tokamak-type configuration, in principle, is also two-dimensional.

Now we notice that no toroidal experimental facility has been extensively operated at magnetic fields higher than 35 kG. With regard to the realization of a two-dimensional configuration, we recall that the Russian Tokamaks have considerable magnetic field variations in the azimuthal direction, so that their degree of effective symmetry has been subject to discussion.

Azimuthally symmetric diffuse pinches, such as Tokamaks, have a special appeal to fusion research because of the simplicity with which $\beta = 8\pi nkT/B^2$ can be increased, and because of the possibility of extending their regimes of operation over a wide range of parameters. As we have indicated, it would not be practically possible to realize a Stellarator configuration with the high magnetic field values that are contemplated for Alcator. Also, ohmic heating is particularly effective for these types of device, since the plasma is heated preferentially in the center where the magnetic well is deepest. As we have mentioned, the appearance of anomalous resistivity increases the efficiency of electron heating and provides the possibility of achieving high electron temperatures in a relatively short experimental time.

The primary drawback of Tokamaks is that the current flowing inside the plasma provides heating and the confining magnetic field at the same time, so that they have a certain degree of inflexibility and interdependence of the various parameters.

According to the previous classification, Alcator belongs to the Tokamak category, but potentially it will outclass all devices of this kind that have been built because of its unusually high magnetic field, high degree of symmetry, favorable aspect ratio R/a,

Table VIII-1. Comparison of three devices.

	T-3	T-10	Alcator
B_T	30 kG	40 kG	130 kG
a	15 cm	50 cm	12 cm
R	100 cm	150 cm	54 cm
R/a	6.7	3	4.5
$B_{po}^2 = \left(\frac{a}{R}\frac{B_T}{q_o}\right)^2$	$3.30\ (\text{kG})^2$	$28\ (\text{kG})^2$	$134\ (\text{kG})^2$
$j_{\parallel o}^2 = \left(0.64\ \frac{B_T}{R}\right)^2$	$3.65 \times 10^4\ (\text{A/cm}^2)^2$	$2.9 \times 10^4\ (\text{A/cm}^2)^2$	$2.40 \times 10^6\ (\text{A/cm}^2)^2$
n	$2 \times 10^{13}\ \text{cm}^{-3}$	$10^{14}\ \text{cm}^{-3}$	$10^{14}\ \text{cm}^{-3}$
T_e	1 keV	3.5 keV	6 ÷ 7 keV
τ	20 ms	?	?

Note: The quoted value of B_T is the average of the toroidal magnetic field over the plasma cross section. The parameters and data for T-3 are from a set of experimental data. The parameters for T-10 are based on present uncertain information, and its expected temperature, T_e, is derived by assuming that the maximum energy density, nkT, equals $B_p^2/8\pi$, and by choosing the particle density.

high values of current density that can be achieved, and diagnostic accessibility. Table VIII-1 summarizes the main parameters, some figures of merit, and typical experimental parameters expected from the results obtained on the best existing Tokamak device (T-3), for the device presented here, and for the device (T-10) that has been proposed by the Kurchatov Institute of Moscow, according to the available information.

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2. DESTRUCTION OF FLUX SURFACES IN TOKAMAKS ARISING FROM PERTURBATIONS FROM AXISYMMETRY

In a low-beta plasma confinement device, a particle orbit follows the magnetic field line very closely. Hence before a device can contain a plasma it must contain its own magnetic field lines. This criterion is especially pertinent in toroidal devices in which a small perturbation in the magnetic field line may be greatly amplified as the line goes many times around the machine.[1] We shall report on some very preliminary studies of the effect of slight nonaxisymmetry on magnetic field lines in a machine like the Alcator.

To simplify the analysis, we assume that the machine has low aspect ratio, $1/R \ll 1$ (ratio of plasma radius to toroidal radius), and also that the magnetic field line to be considered is near the center of the plasma. The results of the analysis are so clear-cut that we can make reasonable speculations about the behavior of the field lines when the restrictions mentioned above do not hold.

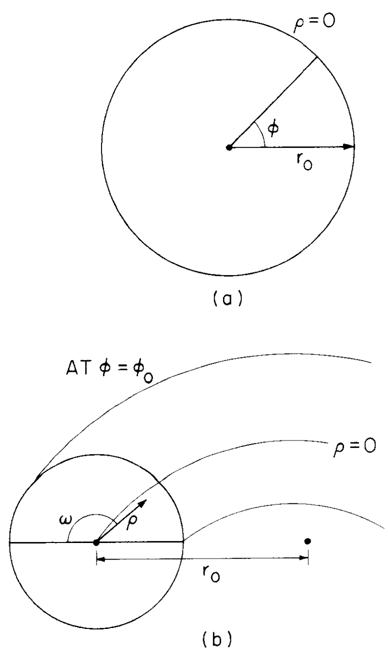

Fig. VIII-5.
Simplified toroidal coordinate system.

In case they do hold, it is convenient to determine magnetic surfaces by using the simplified coordinate system[2] specified by (ρ, ϕ, ω), and related to standard cylindrical coordinates by

$$\begin{aligned} \phi &= \phi \\ Z &= \rho \sin \omega \\ r &= r_o + \rho \cos \omega, \end{aligned} \qquad (1)$$

rather than the standard toroidal coordinates.[3] The coordinate system is illustrated in Fig. VIII-5. The quantity r_o is the toroidal radius, and $\rho = 0$ specifies the center of the plasma.

In any Tokamak device like the Alcator at M.I.T., there are two components to the magnetic field: the toroidal field generated by external conductors, and the poloidal field generated by the plasma current. To find the equation of the field line, it is necessary to have both. We shall only consider perturbation to axisymmetry from the toroidal field, however. This is certainly a reasonable assumption in the Russian machines in which the toroidal field is set up by a number of coils spaced around a circle. In the Alcator at M.I.T., which will use Bitter coils, this assumption is less obvious, but may still be valid.

On the inside the toroidal field can be expressed as the gradient of a scalar potential so that $\underline{B} = \underline{\nabla}V$. Maxwell's equation $\underline{\nabla} \cdot \underline{B} = 0$ then reduces to $\nabla^2 V = 0$. The eigenfunctions of V have been calculated in standard toroidal coordinates in terms of half-order Legendre functions, generally called toroidal harmonics.[3] The ϕ dependence naturally goes as $\cos(n\phi+B)$. Under the assumption that $n\rho/r_o \ll 1$, the solution for V can be written

$$V = V_n\left(1 - \frac{\rho \cos \omega}{2r_o} + \frac{n^2\rho^2}{4r_o^2} + \dots\right), \tag{2}$$

by making use of standard formulas for toroidal harmonics.[4]

The next problem is to find the poloidal field. Here it is sufficient to work to zero order in aspect ratio, so that

$$B_\omega = \frac{1}{\rho} \int_0^\rho \frac{4\pi\rho' J(\rho')}{C}\, d\rho', \tag{3}$$

where J is the toroidal current density, assumed to be a function of ρ only.

Now we want to solve the equation for a magnetic field line to see if small deviations from axisymmetry can cause secular deviations of a field line from its equilibrium position. The equation for the radial position of the field line is

$$\rho - \rho_o = \int_{\phi_o}^{\phi} d\phi' \frac{B_\rho}{B_\phi} (r_o + \rho \cos \omega), \tag{4}$$

where

$$\omega - \omega_o = \int_{\phi_o}^{\phi} \frac{r_o + \rho \cos \omega}{\rho} \frac{B_\omega(\rho)}{B_\phi}\, d\phi', \tag{5}$$

and ρ_o, ω_o, and ϕ_o are the initial coordinates of the field line. Then, correct to lowest order in aspect ratio,

$$\frac{d\omega}{d\phi} = \alpha(\rho),$$

where

$$\alpha = \frac{r_o B_\omega(\rho)}{\rho B_\phi}. \tag{6}$$

To lowest order in powers of $n\rho/r_o$, Eq. 4 becomes

$$\frac{d\rho}{d\phi} \approx \frac{V_n}{B_\phi} \cos\left[\alpha(\rho)(\phi-\phi_o)\right] \cos(n\phi+B). \tag{7}$$

In the limit of $V_n \to 0$, Eq. 7 can be integrated approximately to yield

$$\rho - \rho_o = \frac{V_n}{2B_o}\left[\frac{\cos\left[(\alpha(\rho)-n)\phi-\phi_o+B\right] - \cos(B-\phi_o)}{n-\alpha(\rho)} + n \to -n\right], \tag{8}$$

so that the maximum deviation of the field line is then $V_n/2B_\phi[\alpha(\rho_o)-n]$, and the approximation is valid as long as $(d\alpha/d\rho)(V_n/2B_\phi[\alpha(\rho_o)-n]) \ll 1$.

If $\alpha(\rho_o) = n$, however, the perturbation of the field caused by axiasymmetry is resonant with the natural rotation of the torus. In this case, there will be a secular perturbation of the field line from its initial position. By neglecting the dependence of α on ρ, Eq. 7 can be integrated once to yield

$$\rho - \rho_o = \frac{V_n}{2B}(\phi-\phi_o), \tag{9}$$

where various phase factors have been omitted for simplicity. But when

$$\frac{d\alpha}{d\rho}\frac{V_n}{2B_\phi}(\phi-\phi_o)^2 \approx \pi, \tag{10}$$

the phase of the cosine term in Eq. 7 will be 180° out of phase with that calculated from the unperturbed solution, Eq. 9. Thus the field line will start to curve back to its original position. The maximum deviation of the field line is given roughly by

$$(\Delta\rho)_{max} \approx \sqrt{\pi \frac{V_n}{2B_\phi} \Big/ \frac{d\alpha}{d\rho}}. \tag{11}$$

Notice the $d\alpha/d\rho = d/d\rho(r_o B_\omega(\rho)/\rho B_\phi) = 1/L_s$, the reciprocal of the shearing length of the magnetic field. Thus, the stronger the shear, the less a field line deviates from its unperturbed position because of small deviations from axisymmetry. In the case of a uniform toroidal current density, $L_s = \omega$, so that a small perturbation in field coils could cause wild fluctuations in the position of the magnetic field line.

If the small perturbation to axisymmetry does not have a single azimuthal mode, but many, separated in mode number by Δn, it is essential that the field line not wander from one resonant region to the next. Therefore the condition for stability becomes

$$\sqrt{\frac{\pi V_n}{2B_\phi} \frac{d\alpha}{d\rho}} < \Delta n. \tag{12}$$

One obvious result of Eq. 12 is that it is best to make perturbations as symmetrical as possible. For instance, rather than drill one large hole, say, for diagnostics, at one point in the torus giving $\Delta n = 1$, it would be better to drill 10 small holes uniformly spaced around, so that $\Delta n = 10$.

Finally, let us point out that if terms of higher order in aspect ratio are retained, the resonance condition would no longer be only $\alpha(\rho) = n$, but rather

$$m\alpha(\rho) = n, \tag{13}$$

for a resonance of m^{th} order in aspect ratio. The expression for $(\Delta\rho)_{max}$ would follow exactly as in Eq. 11, except that V_n should be replaced by $(\rho_o/r_o)^{m-1}\ V_n$.

In conclusion, we have shown how small deviations from axisymmetry can lead to secular deviations of the magnetic field lines. The final magnitude of this deviation is limited by magnetic shear as shown in Eq. 11. Thus shear is important not only for stabilizing hydrodynamic instabilities, but also in limiting the random wanderings of flux lines arising from slight nonaxisymmetry. Furthermore, as stated in Eq. 12, we have shown that it is best to keep these deviations as symmetrical as possible.

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3. PLASMA INSTABILITY IN AN ELECTRIC FIELD

In relation to Tokamak-type fusion plasmas it is of considerable interest to re-examine the stability properties of a plasma in an externally applied electric field. Recent analyses of experimental data on the resistivity of such plasmas as a function of electric field have shown three distinct regimes.[1] At very low electric fields the resistivity is normal, that is, essentially what one would expect from electron-ion collisions, and independent of the electric field. At electric fields for which the electron drift velocity equals and exceeds the ion-sound velocity the resistivity becomes anomalous; that is, it is found to be higher than electron-ion collisions would predict, and increases monotonically with the electric field.[2] Various theories of instabilities have been proposed to explain this regime.[1,3] As the electric field is further increased, a third resistivity regime sets in when the electron drift velocity equals approximately one-third of the electron thermal velocity. Here the resistivity starts to increase very rapidly, and a regime of strong anomalous resistance is encountered.[4] We shall seek to explain the onset of this strong anomalous resistance as being due to an instability of the plasma electrons in the presence of a moderately high applied electric field.

The stability of a plasma in an electric field has been analyzed by many authors and for various physical situations. If we assume that the only collisions in the plasma are between electrons and ions, an applied electric field leads to run-away electrons in the tail of the distribution function and a stationary state does not exist, even in low electric fields.[5] Under such conditions, the evolution of the electron distribution function is also accompanied by unstable ion-acoustic oscillations that have been studied in detail.[6] Experiments on Tokamak-type plasmas, however, present us with a quite different physical reality. To begin with, at very low electric fields the plasma is stable and the resistivity is normal. When the anomalous resistivity regime sets in low-frequency fluctuations appear, but the plasma remains over-all stable and with essentially no run-away electrons. On the basis of these experimental facts, we can assume that in this regime the low-frequency fluctuations, which are responsible for the anomalous resistance, permit the establishment of a quasi-stationary electron distribution function. The onset of the regime of strong anomalous resistance can hence be studied by investigating the stability of the quasi-stationary state as a function of the applied electric field.

The electron distribution function, $f_o(w)$, in the quasi-stationary state will be taken to satisfy the Boltzmann equation in the presence of the applied electric ($\overline{E}_o$) and magnetic ($\overline{B}_o$) fields,

$$\frac{-e}{m}(\overline{E}_o+\overline{w}\times\overline{B}_o)\cdot\frac{\partial f_o}{\partial \overline{w}} = -\nu_e(f_o-f_{oL}), \tag{1}$$

where the effective collision frequency ν_e and the local distribution function $f_{oL}(w)$, to which the electrons relax, are taken as part of a phenomenological model of the quasi-stationary state. Small-amplitude perturbations will satisfy the linearized Boltzmann equation, which for electrostatic perturbations $(\overline{E}_1 = -\nabla\Phi_1)$ becomes

$$\frac{\partial f_1}{\partial t} + \overline{w} \cdot \frac{\partial f_1}{\partial \overline{r}} + \frac{-e}{m} (\overline{E}_o + \overline{w} \times \overline{B}_o) \cdot \frac{\partial f_1}{\partial \overline{w}} + \nu_e f_1 = \frac{en_o}{m} \overline{E}_1 \cdot \frac{\partial f_o}{\partial \overline{w}} + \nu_e n_1 f_{oL}, \tag{2}$$

where $n_1 = \int f_1 \, d^3w$ is the electron density perturbation. For perturbations with time-space dependence $\exp i(-\omega t + \overline{k} \cdot \overline{r})$, the dispersion relation for the perturbed modes is given by the zeros of the longitudinal dielectric response function $K_\ell(\omega, \overline{k}, \overline{E}_o, \overline{B}_o) = 1 + en_1/\epsilon_o k^2 \Phi_1$, which can be found from the solution of Eq. 2 with Poisson's equation. In the following discussion we shall assume that $\overline{B}_o$ is parallel to $\overline{E}_o$ for the plasmas of interest.

The simplest solution of Eq. 2 is obtained by assuming that

$$f_o(w) = \frac{(2\pi)^{-3/2}}{v_{\|} v_{\perp}} \exp\left[-\frac{w_{\perp}^2}{2v_{\perp}^2} - \frac{(w_{\|} - v_o)^2}{2v_{\|}^2}\right] \tag{3}$$

and

$$f_{oL}(w) = \frac{(2\pi)^{-3/2}}{v_{\|} v_{\perp}} \exp\left[-\frac{w_{\perp}^2}{2v_{\perp}^2} - \frac{w_{\|}^2}{2v_{\|}^2}\right], \tag{4}$$

where $v_{\|} = (\kappa T_{\|}/m)^{1/2}$ and $v_{\perp} = (\kappa T_{\perp}/m)^{1/2}$ are the electron thermal velocities along and across $\overline{B}_o$, and v_o is the electron drift velocity along $\overline{B}_o$. The dispersion relation is found to be[7]

$$\frac{\omega_p^2}{k^2 u_{\|}^2} \sum_n e^{-\lambda} I_n(\lambda) \left[\frac{Z'(\zeta_n')}{2} - \frac{u_{\|}^2}{v_{\perp}^2} \frac{n\omega_c Z(\zeta_n')}{k_{\|} u_{\|} \sqrt{2}}\right] = 1 + \frac{i\nu_e}{k_{\|} u_{\|} \sqrt{2}} \sum_n e^{-\lambda} I_n(\lambda) Z(\zeta_n), \tag{5}$$

where $\lambda = (k_{\perp} v_{\perp}/\omega_c)^2$, $\zeta_n = (\omega + i\nu - n\omega_c)/k_{\|} u_{\|} \sqrt{2}$, $\zeta_n' = (\omega - k_{\|} v_o + i\nu - n\omega_c)/k_{\|} u_{\|} \sqrt{2}$, Z and Z' are the plasma dispersion function[8] and its derivative, and

$$u_{\|} = v_{\|} \left(1 + \frac{ieE_o}{k_{\|} \kappa T_{\|}}\right)^{1/2}. \tag{6}$$

We note that the effect of E_o enters explicitly into our dispersion relation through

$u_{\|}$ of Eq. 6, and results from the $\overline{E}_o \cdot \partial f_1/\partial \overline{w}$ term in the linearized Boltzmann equation. This term is clearly important for the resonant electrons of the distribution function when $eE_o/m\nu_e$ becomes comparable to $v_{\|}$. More specifically, as Eq. 6 shows, the plasma modes are significantly affected by the electric field E_o when the energy gained from E_o by an electron in traversing a distance of one wavelength along the field becomes comparable to the electron's thermal energy. Taken together, these two conditions also imply that an electron's mean-free path must be comparable to the wavelength. We now examine Eq. 5 for plasma oscillations in a strong magnetic field, so that $\lambda \ll 1$ and we need only retain the $n = 0$ terms in the sums. The long-wavelength plasma modes are then obtained in the limit of $|\zeta_o'| \gg 1$ and $|\zeta_o| \gg 1$. Solving the resultant dispersion relation for complex $\omega = \omega_r + i\omega_i$ with k real, and to order $(k_{\|}v_{\|}/\omega_r)^2$ and (ν_e/ω_r), we obtain

$$\omega_r \approx \omega_p'^2 \left(1 + \frac{k_{\|}v_o}{\omega_p'} + \frac{3}{2}\frac{k_{\|}^2 v_{\|}^2}{\omega_p'^2}\right) \tag{7}$$

$$\frac{\omega_i}{\omega_r} \approx -\frac{\nu_e}{2\omega_p'} - \sqrt{\frac{\pi}{2}}\, e^{-3/2}\, \frac{e^{-1/2(k_{\|}\lambda_D')^2}}{(k_{\|}\lambda_D')^3} + \frac{3}{2}\frac{k_{\|}\frac{e}{m}E_o}{\omega_p'^2}, \tag{8}$$

where $\omega_p'^2 = \omega_p^2\left(1+k_\perp^2/k_{\|}^2\right)$ and $\lambda_D' = v_{\|}/\omega_p'$. Equation 7 gives the frequency of electron plasma oscillations in a strong magnetic field, and is seen to be affected (Doppler-shifted) by the drift velocity but not by the electric field E_o. Equation 8 gives the growth ($\omega_i > 0$) or decay ($\omega_i < 0$) rate of these oscillations. The first term gives the damping that is due to collisions, and the second term gives the decay rate that is due to Landau damping. The third term which is due to the presence of the electric field may give rise to growth of the oscillations. For negligible Landau damping the condition for instability is

$$\frac{eE_o}{m\nu_e} > \frac{1}{3}\frac{\omega_p'}{k_{\|}}. \tag{9}$$

From Eq. 9 we can estimate the minimum electric field (or drift velocity) for the onset of instability as $eE_{o\,min}/m\nu_e \geqslant v_{\|}/3$ or $v_{o\,min} \geqslant v_{\|}/3$, which corresponds to the observed onset of the region of strong anomalous resistivity in Tokamak plasmas.

The major weakness in the analysis just presented lies in the assumption of a specific electron distribution function for the quasi-stationary state in Tokomak plasmas. It can be shown, however, that in the limits considered our conclusions are not particularly sensitive to the shape of the distribution function. Further details will be presented in a subsequent report.

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